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The Active Disturbance Rejection Control for a Class of MIMO Block Lower-triangular system

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Abstract: This paper studies the control problem for a class of multiple-input multiple-output (MIMO) lower-triangular systems which have uncertain dynamics and disturbance. The active disturbance rejection control (ADRC) is designed to achieve two objects: 1). the convergence of the closed-loop system; 2). the performance of the closed-loop system can be controlled to be close to the reference system by tuning the bandwidth of Extended state observer (ESO).

Key Words: active disturbance rejection control (ADRC), Extended state observer (ESO), Lower-triangular system

1 Introduction

Dealing with uncertainties is always a dominate issue in various control problems. Basically, the uncertainties stem from two sources: internal (parameter or unmodeled dynamics) uncertainties and external (disturbance) uncertainties. The idea of the invariance principle [1] gives some suggestions for controlling uncertain systems: the uncertainties causing changes in the controlled variable can be used to generate an activating signal to cancel the effect of the same uncertainties, no matter they are internal or external. Obviously, the activating signal to attenuate uncertainties can be easily constructed if the uncertainties are measurable. However, most uncertainties are not measurable. Hence how to estimate the uncertainties by the known information, for example, the control input and the output of system, become a significant problem. Lots of uncertainty observer based control methods are proposed which include the disturbance accommodation control (DAC) [2]-[3], the unknown input disturbance observer (UIDO) [4]-[5], the disturbance observer (DOB) [6]-[10], some special kinds of PID (SPID) [11]-[12], the extended high-gain observer (EHGO) [13]-[14] and the active disturbance rejection control (ADRC) [15]-[16]. These methods divide the process of controller design into two stages: one is to compensate for the uncertainties, which is reconstructed by certain kind of observer, and the other is to design the desired (tracking or regulating) performance. Some comparisons for these uncertainty observers based controller are discussed in [8], [17]-[18]. DAC and UIDO are shown to be certain special kinds of DOB [8]. The EHGO based control, which can estimate the uncertain dynamics of nonlinear system, is illustrated as a special kind of ADRC, which can deal with mixed internal and external uncertainties [17]-[18]. And by comparing DOB, SPID and ADRC, [18] declared that although external disturbance is the main concern in the DOB design and internal uncertainty is the main concern in the SPID design, the frame of ADRC provides a theoretical base for generalizing DOB and SPID for nonlinear systems with mixed internal and external uncertainties under some conditions.

The key in ADRC is to on-line estimate the total uncertainty, which lumps the internal uncertain dynamics and the external disturbances, by the extended state observer (ESO) [19]-[20]. ADRC's framework does not set strict mathematical constraints on the uncertainties, and the error feedback structure in ADRC are originally proposed to be in general nonlinear forms. Therefore, there have been lots of application researches [21]-[25]. On the other hand, ADRC's framework leaves much room for its theoretical analysis. In [26]-[27], it was proved that the observing errors of the linear ESO (LESO) would be bounded if either the uncertainty or its derivative is bounded. In [18], the stability analysis for a class of nonlinear non-parameterized system with the external disturbance was given, and a bounded error between the trajectory of the ADRC based closed-loop system and that of the reference system was analyzed.

The above mentioned researches [17]-[18], [26]-[27] for ADRC stability were carried out for the systems satisfying the matching condition, that is the uncertainties and control input are in the same channel. However, many uncertain systems are in the lower-triangular form [28]-[29], which have uncertainties not only in the control input channel. By using the backstepping tool, both the robust control and the adaptive control can stabilize the lower-triangular nonlinear systems [30]-[31]. In the robust control design, the uncertain dynamics are usually assumed to be bounded. In the adaptive control design, the lower-triangular system concerned should be parameterized. In [20], the ADRC design has been given for some lower-triangular systems, where the uncertainties are not assumed to be bounded or only be parameterized, but rigorous theoretical analysis lacks. In this paper, the performances of the ADRC based control for a class of multiple-input multiple-output (MIMO) block lower-triangular nonlinear system are discussed.

The paper is organized as follows. In section 2, the problem is presented and the ADRC is introduced. The performance of the closed-loop system is analyzed in section 3. The conclusions are given in the last section.

2 Problem Formulation and ADRC Design

Consider the following MIMO block lower-triangular system:

$$\dot{x}_1 = b_1(x_1,t)x_2 + f_1(x_1,t) \tag{1}$$

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$$\dot{x}_2 = b_2(x_1,x_2,t)u + f_2(x_1,x_2,t) \tag{2}$$

where $x_1 = [x_{11},...,x_{1n}]^n \in R^n$ and $x_2 = [x_{21},...,x_{2n}]^n \in R^n$ are the states that can be measured, $u \in R^n$ is the control input. $b_1(x_1,t)_{n\times n}$ is a nonlinear function matrix which is known and nonsingular, $f_1(x_1,t)_{n\times 1}, f_2(x_1,x_2,t)_{n\times 1}$ and $b_2(x_1,x_2,t)_{n\times n}$ are nonlinear and contain uncertainties, $b_2(x_1,x_2,t)$ is nonsingular and have a known nonsingular estimate $\hat{b}_2(t)$.

The control object is that $x_1(t)$ approaches 0 at an exponentially convergence rate. Thus if $f_1(x_1,t)$, $f_2(x_1,x_2,t)$, $b_2(x_1,x_2,t)$ are all known, the controller can be designed by the back-stepping and feedback linearization method

$$\begin{cases} \bar{x}_2 = b_1^{-1}(x_1,t)(-k_1x_1 - f_1(x_1,t)) \\ u = b_2^{-1}(x_1,x_2,t)\left(-f_2(x_1,x_2,t) - k_2(x_2-\bar{x}_2) + \dot{\bar{x}}_2\right) \end{cases} \tag{3}$$

where $\bar{x}_2(t)$ is the the virtual control for the subsystem (1) and

$$k_1 = \begin{bmatrix} k_{11} & 0 & ... & 0 \\ 0 & k_{12} & ... & 0 \\ ... & & & \\ 0 & \vdots & & k_{1n} \end{bmatrix}, k_2 = \begin{bmatrix} k_{21} & 0 & ... & 0 \\ 0 & k_{22} & ... & 0 \\ ... & & & \\ 0 & \vdots & & k_{2n} \end{bmatrix},$$

$k_{1i} > 0, k_{2i} > 0, i = 1,...,n.$

From (1), (2) and (3), we get the reference error system:

$$\begin{cases} \dot{\bar{e}}_1 = b_1(\bar{e}_1,t)\bar{e}_2 - k_1\bar{e}_1 \\ \dot{\bar{e}}_2 = -k_2\bar{e}_2 \end{cases} \quad t \geqslant t_0 \tag{4}$$

where $\bar{e}_1 = x_1, \bar{e}_2 = x_2 - \bar{x}_2$. The following Lemma shows that the feed back matrices k_1, k_2 can be designed such that $\bar{e}_1(t)$ converges with an exponentially rate and satisfying certain transition performance.

Lemma 1 *Assume $\|\bar{e}_1(t_0)\| \leq \rho_1$ and k_1 satisfy that*

$$\|k_1\| \geq \frac{\|\bar{e}_2(t_0)\|}{\rho_1} \max_{\|\bar{e}_1\|\leq\rho_1} \|b_1(\bar{e}_1,t)\| \tag{5}$$

then the system (4) is exponentially stable.

Remark 1 *In this paper, $\|\cdot\|$ stands for $\|\cdot\|_2$ or the induced matrix 2-norm.*

Proof. Using the second equation of (4), we can get

$$\|\bar{e}_2\| \leq e^{-\|k_2\|(t-t_0)}\|e_2(t_0)\|. \tag{6}$$

Using the first equation of (4) and (6), there is

$$\begin{aligned} \frac{d}{dt}\|\bar{e}_1\|^2 \leq & -2\|k_1\|\|\bar{e}_1\|^2 \\ & + 2\|e_1\|\|b_1(\bar{e}_1,t)\|e^{-\|k_2\|(t-t_0)}\|e_2(t_0)\|. \end{aligned} \tag{7}$$

By (5) and (7), we can get that

$$\|\bar{e}_1(t)\| = \rho_1 \Rightarrow \frac{d}{dt}\|\bar{e}_1\|^2 \leq 0,$$

which means $\{\|\bar{e}_1\| \leq \rho_1\}$ is an invariant set for $\bar{e}_1$. Since $\|\bar{e}_1(t_0)\| \leq \rho_1$,

$$\|\bar{e}_1(t)\| \leq \rho_1, t \in [t_0,\infty). \tag{8}$$

From (7) and (8), there is

$$\begin{aligned} \frac{d}{dt}\|\bar{e}_1\| \leq & -\|k_1\|\|\bar{e}_1\| + \\ & \max_{\|\bar{e}_1\|\leq\rho_1} \|b_1(\bar{e}_1,t)\|\|e^{-\|k_2\|(t-t_0)}\|e_2(t_0)\|. \end{aligned} \tag{9}$$

Using the Gronwall-Bellman inequality for (9), we can get

$$\begin{aligned} \|\bar{e}_1(t)\| \leq & e^{-\|k_1\|(t-t_0)}\|\bar{e}_1(t_0)\| \\ & + \frac{e^{-\|k_2\|(t-t_0)} - e^{-\|k_1\|(t-t_0)}}{\|k_1\| - \|k_2\|}\|e_2(t_0)\| \max_{\|\bar{e}_1\|<\rho_1} \|b_1(\bar{e}_1,t)\|. \end{aligned}$$

Q.E.D.

To deal with the uncertainties in $f_1(\cdot)$, $f_2(\cdot)$, $b_2(\cdot)$, the ADRC provides a novel frame to achieve the function of the control (3). Rewrite (1)-(2) as

$$\begin{cases} \dot{x}_1 = b_1(x_1,t)x_2 + f_1(x_1,t) \\ \dot{x}_2 = \hat{b}_2(t)u + f_2(x_1,x_2,t) + (b_2(x_1,x_2,t) - \hat{b}_2(t))u \end{cases} \tag{10}$$

Then to estimate the unknown uncertainties $f_1(x_1,t)$ and $f_2(x_1,x_2,t) + (b_2(x_1,x_2,t) - \hat{b}_2(t))u$, the following linear extended state observer (LESO) can be designed [32],

$$\begin{cases} \dot{\hat{x}}_1 = b_1(x_1,t)x_2 - \beta_{1,1}(\hat{x}_1 - x_1) + \Delta_1 \\ \dot{\Delta}_1 = -\beta_{1,2}(\hat{x}_1 - x_1) \end{cases} \tag{11}$$

$$\begin{cases} \dot{\hat{x}}_2 = \hat{b}_2(t)u - \beta_{2,1}(\hat{x}_2 - x_2) + \Delta_2 \\ \dot{\Delta}_2 = -\beta_{2,2}(\hat{x}_2 - x_2) \end{cases} \tag{12}$$

where the parameters

$$\begin{aligned} \beta_{i,1} &= \begin{bmatrix} \beta_{i,1,1} & 0 & ... & 0 \\ 0 & \beta_{i,1,2} & ... & 0 \\ \vdots & & & \\ 0 & ... & & \beta_{i,1,n} \end{bmatrix} = \frac{1}{\varepsilon_i}\begin{bmatrix} \bar{\beta}_{i,1,1} & 0 & ... & 0 \\ 0 & \bar{\beta}_{i,1,2} & ... & 0 \\ \vdots & & & \\ 0 & ... & & \bar{\beta}_{i,1,n} \end{bmatrix} \\ \beta_{i,2} &= \begin{bmatrix} \beta_{i,2,1} & 0 & ... & 0 \\ 0 & \beta_{i,2,2} & ... & 0 \\ \vdots & & & \\ 0 & ... & & \beta_{i,2,n} \end{bmatrix} = \frac{1}{\varepsilon_i^2}\begin{bmatrix} \bar{\beta}_{i,2,1} & 0 & ... & 0 \\ 0 & \bar{\beta}_{i,2,2} & ... & 0 \\ \vdots & & & \\ 0 & ... & & \bar{\beta}_{i,2,n} \end{bmatrix} \\ & i = 1,2. \end{aligned} \tag{13}$$

satisfy the conditions that

$$L_{i,j}(s) \triangleq s^2 + \bar{\beta}_{i,1,j}s + \bar{\beta}_{i,2,j}, \quad i = 1,2, j = 1,...,n$$

are Hurwitz polynomials. The output of ESO (11)-(12), $\hat{x}_i (i = 1,2)$, $\Delta_1(\cdot)$ and $\Delta_2(\cdot)$ should be designed to estimate $x_i (i = 1,2)$, $f_1(\cdot)$ and $f_2(\cdot) + \left(b_2(\cdot) - \hat{b}_2(\cdot)\right)u$. Next we will prove that the estimate errors $\|f_1(\cdot) - \Delta_1(\cdot)\|$ and $\|f_2(\cdot) - \left(b_2(\cdot) - \hat{b}_2(\cdot)\right)u - \Delta_2(\cdot)\|$ can be as small as possible. Then, based on LESO(11)-(12), the control can be designed as:

$$\begin{cases} x_2^* = b_1(\cdot)^{-1}(-k_1x_1 - \Delta_1(\cdot)) \\ u = \hat{b}_2(\cdot)^{-1}\left(-k_2(x_2 - x_2^*) - \Delta_2(\cdot) + \dot{x}_2^*\right). \end{cases} \tag{14}$$

Remark 2 *The parameters in (13) are usually tuned be*

$$\beta_{11} = \frac{2}{\varepsilon_1}I_{n\times n}, \beta_{12} = \frac{1}{\varepsilon_1^2}I_{n\times n}, \beta_{21} = \frac{2}{\varepsilon_2}I_{n\times n}, \beta_{22} = \frac{1}{\varepsilon_2^2}I_{n\times n}.$$

3 Analysis of the Closed-Loop System

Firstly, the following mild assumptions for the system (1)-(2) are given:

A1. If $x_i \in \{x_i | \|x_i\| \le \rho_i\}, i = 1,2$, then $b_1(\cdot)$, $f_1(\cdot)$, $b_2(\cdot)$, $f_2(\cdot)$, $b_1(\cdot)^{-1}$, $b_2(\cdot)^{-1}$ and their first two order partial derivative have known bounds which are only dependent on ρ_i.

The Assumption A1 is used for guaranteeing the existence and uniqueness of the solutions for the closed-loop system.

To study the closed-loop system which is based on the ADRC (11)-(14), we denote the tracking errors as

$$e_1 = x_1, \quad e_2 = x_2 - x_2^*$$

and denote the estimate errors of ESO as

$$\begin{cases} \hat{e}_{11} = x_1 - \hat{x}_1 \\ \hat{e}_{12} = f_1(x_1,t) - \Delta_1 \end{cases} \tag{15}$$

$$\begin{cases} \hat{e}_{21} = x_2 - \hat{x}_2 \\ \hat{e}_{22} = f_2(x_1,x_2,t) + \left(b_2(x_1,x_2,t) - \hat{b}_2(t)\right)u - \Delta_2. \end{cases} \tag{16}$$

Then the closed-loop system is

$$\begin{cases} \dot{e}_1 = b_1(e_1,t)e_2 - k_1 e_1 + \hat{e}_{12} \\ \dot{e}_2 = -k_2 e_2 + \hat{e}_{22} \end{cases} \tag{17}$$

$$\begin{cases} \dot{\hat{e}}_{11} = -\beta_{11}\hat{e}_{11} + \hat{e}_{12} \\ \dot{\hat{e}}_{12} = -\beta_{12}\hat{e}_{11} + \dfrac{d}{dt}f_1(\cdot) \end{cases} \tag{18}$$

$$\begin{cases} \dot{\hat{e}}_{21} = -\beta_{21}\hat{e}_{21} + \hat{e}_{22} \\ \dot{\hat{e}}_{22} = -\beta_{22}\hat{e}_{21} + \dfrac{d}{dt}\left(f_2(\cdot) + \Lambda(\cdot)\left(-k_2 e_2 - \Delta_2 + \dot{x}_2^*\right)\right). \end{cases} \tag{19}$$

(17)-(19) show that the tracking errors e_1, e_2 and the estimate errors $\hat{e}_{11}, \hat{e}_{12}, \hat{e}_{21}, \hat{e}_{22}$ are coupled. In order to estimate the dynamical uncertainties, the convergence rate of ESO's estimate error usually is designed to be larger than the tracking errors. Introducing the following transformation:

$$\begin{cases} \hat{e}_{11} = \varepsilon_1 \xi_{11}, \hat{e}_{12} = \xi_{12} \\ \hat{e}_{21} = \varepsilon_2 \xi_{21}, \hat{e}_{22} = \xi_{22} \end{cases} \tag{20}$$

(17)-(19) can be transformed to:

$$\begin{cases} \dot{e}_1 = b_1(e_1,t)e_2 - k_1 e_1 + \xi_{12} \\ \dot{e}_2 = -k_2 e_2 + \xi_{22} \end{cases} \tag{21}$$

$$\begin{bmatrix} \dot{\xi}_{11} \\ \dot{\xi}_{12} \end{bmatrix} = \frac{1}{\varepsilon_1}\begin{bmatrix} -\bar{\beta}_{11} & I \\ -\bar{\beta}_{12} & 0 \end{bmatrix}\begin{bmatrix} \xi_{11} \\ \xi_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{d}{dt}f_1(x_1,t) \end{bmatrix} \tag{22}$$

$$\begin{bmatrix} \dot{\xi}_{21} \\ \dot{\xi}_{22} \end{bmatrix} = \frac{1}{\varepsilon_2}\begin{bmatrix} -\bar{\beta}_{21} & I \\ -\bar{\beta}_{22} & 0 \end{bmatrix}\begin{bmatrix} \xi_{21} \\ \xi_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{d}{dt}\left(f_2(x_1,x_2,t) + \Lambda(x_1,x_2,t)\left(-k_2 e_2 - \Delta_2 + \dot{x}_2^*\right)\right) \end{bmatrix}. \tag{23}$$

From (22) and (23), the parameters $\varepsilon_i, i = 1,2$ can be used for tuning the convergence rate of ESO.

Define

$$\begin{cases} \xi_i = \begin{bmatrix} \xi_{i1} & \xi_{i2} \end{bmatrix}^T, i = 1,2, \\ \chi = (e_1, e_2, \xi_1, \xi_2), \\ \mu = \left(k_1, k_2, \bar{\beta}_{i,j,l}(i,j = 1,2, l = 1,...n)\right). \end{cases}$$

Since

$$\begin{aligned} \dot{x}_2^* &= \frac{d}{dt}\left(b_1^{-1}(\cdot)(-k_1 x_1 - \Delta_1(\cdot))\right) \\ &= \frac{d}{dt}\left(b_1^{-1}(\cdot)\right)\left(-k_1 x_1 - \Delta_1(\cdot)\right) + b_1(\cdot)^{-1}\left(-k_1 \dot{x}_1 - \dot{\Delta}_1(\cdot)\right) \\ &= \frac{d}{dt}\left(b_1^{-1}(\cdot)\right)\left(-k_1 x_1 - f_1(\cdot) + \xi_{12}\right) \\ &\quad + b_1^{-1}(\cdot)\left(-k_1\left(b_1(x_1,t)e_2 - k_1 x_1 + \xi_{12}\right) - \frac{1}{\varepsilon_1}\bar{\beta}_{12}\xi_{11}\right), \end{aligned}$$

$$\dot{\Lambda}(\cdot) = \frac{d}{dt}\left(b_2(\cdot) - \hat{b}_2(\cdot)\right)\hat{b}_2^{-1}(\cdot) + \left(b_2(\cdot) - \hat{b}_2(\cdot)\right)\frac{d}{dt}(\hat{b}_2^{-1}(\cdot)),$$

$$\begin{aligned} \ddot{x}_2^* &= \frac{d^2}{dt^2}\left(b_1^{-1}(\cdot)\right)\left(-k_1 x_1 - f_1(\cdot) + \xi_{12}\right) + \frac{d}{dt}\left(b_1^{-1}(\cdot)\right)\left(-k_1 \dot{x}_1 - \dot{f}_1(\cdot) + \dot{\xi}_{12}\right) \\ &\quad + \frac{d}{dt}(b_1^{-1}(\cdot))\left(-k_1\left(b_1(x_1,t)e_2 - k_1 x_1 + \xi_{12}\right) - \frac{1}{\varepsilon_1}\bar{\beta}_{12}\xi_{11}\right) \\ &\quad + b_1^{-1}(\cdot)\left(-k_1\left(\dot{b}_1(x_1,t)e_2 + b_1(x_1,t)\dot{e}_2 - k_1\dot{x}_1 + \dot{\xi}_{12}\right) - \frac{1}{\varepsilon_1}\bar{\beta}_{12}\dot{\xi}_{11}\right) \\ &= \frac{d^2}{dt^2}\left(b_1^{-1}(\cdot)\right)\left(-k_1 x_1 - f_1(\cdot) + \xi_{12}\right) + \frac{d}{dt}\left(b_1^{-1}(\cdot)\right)\left(-k_1\dot{x}_1 - \frac{1}{\varepsilon_1}\bar{\beta}_{12}\xi_{11}\right) \\ &\quad + \frac{d}{dt}(b_1^{-1}(\cdot))\left(-k_1\left(b_1(x_1,t)e_2 - k_1 x_1 + \xi_{12}\right) - \frac{1}{\varepsilon_1}\bar{\beta}_{12}\xi_{11}\right) \\ &\quad - b_1^{-1}(\cdot)k_1\left(\dot{b}_1(x_1,t)e_2 + b_1(x_1,t)\dot{e}_2 - k_1\dot{x}_1 - \frac{1}{\varepsilon_1}\bar{\beta}_{12}\xi_{11} + \dot{f}_1(x_1,t)\right) \\ &\quad + b_1^{-1}(\cdot)\left(-\frac{1}{\varepsilon_1^2}\bar{\beta}_{12}(-\bar{\beta}_{11}\xi_{11} + \xi_{12})\right), \end{aligned}$$

we can get

$$\begin{aligned} \dot{f}_1(x_1,t) &= \frac{\partial f_1}{\partial x_1}\left(b_1(x_1,t)e_2 - k_1 x_1 + \xi_{12}\right) + \frac{\partial f_1}{\partial t} = \phi_1(\chi,\mu,t) \\ \frac{d}{dt}\left(f_2(x_1,x_2,t) + \Lambda(\cdot)\left(-k_2 e_2 - \Delta_2 + \dot{x}_2^*\right)\right) &= \frac{\partial f_2}{\partial x_1}\dot{x}_1 + \frac{\partial f_2}{\partial x_2}\dot{x}_2 + \frac{\partial f_2}{\partial t} \\ &\quad + \dot{\Lambda}(\cdot)\left(-k_2 e_2 - f_2 - (b(\cdot) - \hat{b}(\cdot))u + \xi_{22} + \dot{x}_2^*\right) + \Lambda(\cdot)\left(-k_2\dot{e}_2 - \dot{\Delta}_2 + \ddot{x}_2^*\right) \\ &= \phi_2(\chi,\mu,t) + \phi_3(\chi,\mu,t)\frac{1}{\varepsilon_1}\xi_1 + \phi_4(\chi,\mu,t)\frac{1}{\varepsilon_1^2}\xi_1 - \Lambda(\cdot)\frac{1}{\varepsilon_2}\bar{\beta}_{22}\xi_{21}. \end{aligned}$$

According to A1, $\|\phi_i(\cdot)\|, i = 1,..,4$ are bounded as long as $\|\chi\|$, $\|\hat{b}_2(t)\|$, $\|\hat{b}_2^{-1}(t)\|$ and $\|\dot{\hat{b}}_2(t)\|$ are bounded. Moreover, the closed-loop system (21)-(23) can be rewritten as

$$\begin{cases} \dot{e}_1 = b_1(e_1,t)e_2 - k_1 e_1 + \xi_{12} \\ \dot{e}_2 = -k_2 e_2 + \xi_{22} \end{cases} \tag{24}$$

$$\dot{\xi}_1 = \frac{1}{\varepsilon_1}A_1\xi_1 + B\phi_1(\chi,\mu,t) \tag{25}$$

$$\begin{aligned} \dot{\xi}_2 &= \frac{1}{\varepsilon_2}A_2\xi_2 + B\left(\phi_2(\chi,\mu,t) + \phi_3(\chi,\mu,t)\frac{1}{\varepsilon_1}\xi_1\right) \\ &\quad + B\phi_4(\chi,\mu,t)\frac{1}{\varepsilon_1^2}\xi_1 \end{aligned} \tag{26}$$

where

$$A_1 = \begin{bmatrix} -\bar{\beta}_{11} & I \\ -\bar{\beta}_{12} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ I \end{bmatrix}, A_2 = \begin{bmatrix} -\bar{\beta}_{21} & I \\ -\bar{\beta}_{22}(I + \Lambda(\cdot)) & 0 \end{bmatrix}$$

$$\Lambda(\cdot) = \left(b_2(\cdot) - \hat{b}_2(\cdot)\right)\hat{b}_2(\cdot)^{-1}.$$

Lemma 2 *If A_1 are Hurwitz, there exists a positive matrix P_1 and positives c_{11}, c_{12} such that*

$$\begin{aligned} &A_1^T P_1 + P_1 A_1 = -I_{n\times n}, \\ &c_{11} I \le P_1 \le c_{12} I. \end{aligned}$$

Lemma 3 *[14] If there exists α_Λ such that*

$$\prod_{i=1}^{n} \left\| \frac{\bar{\beta}_{2,2,i}}{L_{2,i}(s)} \right\|_\infty \cdot \|\Lambda(\cdot)\| \le \alpha_\Lambda < 1 \tag{27}$$

then there exist positives c_0, c_{21}, c_{22} and a positive matrix P_2 such that

$$A_2^T P_2 + P_2 A_2 \le -c_0 I_{n\times n},$$
$$c_{21} I \le P_2 \le c_{22} I.$$

Remark 3 *It can be easily verified that if the eigenvalues of $L_{2i}, i = 1,...,n$ are real, then*

$$\prod_{i=1}^{n} \| \frac{\bar{\beta}_{2,2,i}}{L_{2,i}(s)} \|_\infty = 1$$

which means the condition (27) equals to $\|\Lambda(\cdot)\| \le \alpha_\Lambda < 1$.

Assume the initial condition of system (24)-(26) satisfy

$$\begin{cases} \|e_1(t_0)\| \le \zeta_1, \|e_2(t_0)\| \le \zeta_2 \\ \|\hat{e}_{12}(t_0)\| \le \hat{\zeta}_1, \|\hat{e}_{22}(t_0)\| \le \hat{\zeta}_2. \end{cases} \tag{28}$$

Theorem 1 *Consider the system (1)-(2) and ADRC (11)-(14). Design k_1, k_2 as*

$$\|k_1\| \ge \max\{ \frac{\sqrt{c_{12}}}{\sqrt{c_{11}}} \frac{\hat{\zeta}_1}{\zeta_1} + \max_{\|e_1\| \le \zeta_1} \|b_1(\cdot)\| \frac{\zeta_2}{\zeta_1}, L_1 \frac{\sqrt{c_{22}}}{\sqrt{c_{21}}} \hat{\zeta}_2 \} \tag{29}$$

$$\|k_2\| \ge \frac{\sqrt{c_{22}}}{\sqrt{c_{21}}} \hat{\zeta}_2 / \zeta_2 \tag{30}$$

where L_1 satisfies

$$\|b_1(a,t) - b_1(b,t)\| \le L_1 \|a-b\|, \forall \|a\| \le \zeta_1, \forall \|b\| \le \zeta_1,$$

and design $\hat{b}_2(t)$ such that $\|\hat{b}_2(t)\|$, $\|\hat{b}_2^{-1}(t)\|$ and $\|\dot{\hat{b}}_2(t)\|$ are bounded and (27) is satisfied. Then there exist positives $\gamma_i^*(\zeta_1, \zeta_2, \hat{\zeta}_1, \hat{\zeta}_2, \mu), i = 1,2,3,4$, and $\varepsilon_1^*, \varepsilon_2^*(\varepsilon_1)$ such that $\forall \varepsilon_1 \in (0, \varepsilon_1^*]$ and $\forall \varepsilon_2 \in (0, \varepsilon_2^*]$, the closed-loop system has the following properties:

1).
$$\begin{cases} \|e_1(t) - \bar{e}_1(t)\| \\ \quad \le \gamma_1^* \left(\max\{|\varepsilon_1 ln \varepsilon_1|, |\varepsilon_2 ln \varepsilon_2|\} + \frac{\varepsilon_2}{\varepsilon_1} \right), t \in [t_0, \infty). \\ \|e_2(t) - \bar{e}_2(t)\| \\ \quad \le \gamma_2^* \left(\max\{|\varepsilon_1 ln \varepsilon_1|, |\varepsilon_2 ln \varepsilon_2|\} + \frac{\varepsilon_2}{\varepsilon_1} \right), t \in [t_0, \infty). \end{cases} \tag{31}$$

2).
$$\begin{cases} \lim_{t\to\infty} \|e_1(t) - \bar{e}_1(t)\| = \gamma_3^* \left(\frac{\varepsilon_2}{\varepsilon_1} + \varepsilon_1 \right), \\ \lim_{t\to\infty} \|e_2(t) - \bar{e}_2(t)\| = \gamma_4^* \left(\frac{\varepsilon_2}{\varepsilon_1} \right). \end{cases} \tag{32}$$

3).
$$\begin{cases} \|\xi_1(t)\| \le O(\varepsilon) \\ \|\xi_2(t)\| \le O(\frac{\varepsilon_2}{\varepsilon_1}) \end{cases} \quad t \ge t_p. \tag{33}$$

where $\bar{e}_i$ satisfies the dynamics (4) with the initial condition

$$\bar{e}_1(t_0) = e_1(t_0), \quad \bar{e}_2(t_0) = e_2(t_0)$$

and t_p satisfies

$$t_p - t_0 \le \max\{O(|\varepsilon_1 ln \varepsilon_1|), O(|\varepsilon_2 ln \varepsilon_2|)\}.$$

Theorem 1 shows that the system (1)-(2) can be semi-globally stabilized by ADRC (11)-(14).

Proof. Define

$$V_1(\xi_1) = \xi_1^T P_1 \xi_1, \quad V_2(\xi_2) = \xi_2^T P_2 \xi_2,$$

$$\Omega_1 = \{\chi \,|\, |e_1| \le \zeta_1, |e_2| \le \zeta_2, \sqrt{V_1(\xi_1)} \le \sqrt{c_{12}}\hat{\zeta}_1, \sqrt{V_2(\xi_2)} \le \sqrt{c_{22}}\hat{\zeta}_2\}.$$

Firstly, we will prove $\exists \varepsilon_1^*, \varepsilon_2^*(\varepsilon_1)$ such that $\forall \varepsilon_1 \in (0, \varepsilon_1^*]$ and $\forall \varepsilon_2 \in (0, \varepsilon_2^*]$, Ω_1 is an invariant set for (24)-(26). The process is divided into the following 4 cases.

Case 1. Assume

$$\begin{cases} \|e_1\| = \zeta_1 \\ \|e_2\| \le \zeta_2, \sqrt{V_1(\xi_1)} \le \sqrt{c_{12}}\hat{\zeta}_1, \sqrt{V_2(\xi_2)} \le \sqrt{c_{22}}\hat{\zeta}_2 \end{cases}$$

then

$$\begin{aligned} \frac{d}{dt}\left(e_1^T e_1\right) &= -2\|k_1\|\|e_1\|^2 + 2e_1^T(b_1(e_1,t)e_2 + \xi_{12}) \\ &\le -2\|k_1\|\|e_1\| \left(\zeta_1 - \zeta_2/\|k_1\| \max_{\|e_1\|\le\zeta_1} \|b_1(\cdot)\| \right) \\ &\quad - 2\|k_1\|\|e_1\| \left(-\frac{\sqrt{c_{12}}}{\sqrt{c_{11}}} \hat{\zeta}_1/\|k_1\| \right) \\ &\le 0. \end{aligned} \tag{34}$$

Case 2. Assume

$$\begin{cases} \|e_2\| = \zeta_2 \\ \|e_1\| \le \zeta_1, \sqrt{V_1(\xi_1)} \le \sqrt{c_{12}}\hat{\zeta}_1, \sqrt{V_2(\xi_2)} \le \sqrt{c_{22}}\hat{\zeta}_2 \end{cases}$$

then

$$\begin{aligned} \frac{d}{dt}\left(e_2^T e_2\right) &= -2\|k_2\|e_2^2 + 2e_2^T \xi_{22} \\ &\le -2\|k_2\|\|e_2\| \left(\zeta_2 - \frac{\sqrt{c_{22}}}{\sqrt{c_{21}}} \hat{\zeta}_2/\|k_2\| \right) \le 0. \end{aligned} \tag{35}$$

Case 3. Assume

$$\begin{cases} \sqrt{V_1(\xi_1)} = \sqrt{c_{12}}\hat{\zeta}_1 \\ \|e_1\| \le \zeta_1, \|e_2\| \le \zeta_2, \sqrt{V_2(\xi_2)} \le \sqrt{c_{22}}\hat{\zeta}_2. \end{cases}$$

then there exists positive $\gamma_1(\zeta_1, \zeta_2, \hat{\zeta}_1, \hat{\zeta}_2, \mu)$, such that such that $\|\phi_1(\chi, \mu, t)\| \le \gamma_1$. Then it follows

$$\begin{aligned} \frac{d}{dt}V_1(\xi_1) &= -\frac{\|\xi_1\|^2}{\varepsilon_1} + 2\xi_1^T P_1 B \phi_1(\chi, \mu, t) \\ &\le -\frac{\|\xi_1\|}{\varepsilon_1} \left(\frac{\sqrt{c_{12}}}{\sqrt{c_{11}}} \hat{\zeta}_1 - 2\varepsilon_1 c_{12} \gamma_1 \right) \end{aligned} \tag{36}$$

Obviously, there exists ε_1^* such that $\forall \varepsilon_1 \in (0, \varepsilon_1^*]$

$$\frac{d}{dt}V_1(\xi_1) \le 0. \tag{37}$$

Case 4. Assume

$$\begin{cases} \sqrt{V_2(\xi_2)} = \sqrt{c_{22}}\hat{\zeta}_2 \\ \|e_1\| \le \zeta_1, \|e_2\| \le \zeta_2, \sqrt{V_1(\xi_1)} \le \sqrt{c_{12}}\hat{\zeta}_1 \end{cases}$$

then there exist positive $\gamma_i(\zeta_1, \zeta_2, \hat{\zeta}_1, \hat{\zeta}_2, \mu), i = 2,3,4$ such that $\|\phi_i(\chi, \mu, t)\| \le \gamma_i, i = 2,3,4$. Then it follows

$$\begin{aligned} &\frac{d}{dt}V_2(\xi_2) = -\frac{c_0}{\varepsilon_2}\|\xi_2\|^2 \\ &\quad + 2\xi_2^T P_2 B \left(\phi_2(\chi, \mu, t) + \frac{1}{\varepsilon_1}\phi_3(\chi, \mu, t)\xi_1 + \frac{1}{\varepsilon_1^2}\phi_4(\chi, \mu, t)\xi_1 \right) \\ &\le -\frac{c_0}{\varepsilon_2}\|\xi_2\| \left(\frac{\sqrt{c_{22}}}{\sqrt{c_{21}}}\hat{\zeta}_2 - 2\frac{\varepsilon_2}{c_0}c_{22}(\gamma_2 + \frac{1}{\varepsilon_1}\gamma_3\frac{\sqrt{c_{12}}}{\sqrt{c_{11}}}\hat{\zeta}_1 + \frac{1}{\varepsilon_1^2}\gamma_4\frac{\sqrt{c_{12}}}{\sqrt{c_{11}}}\hat{\zeta}_1) \right) \end{aligned} \tag{38}$$

Obviously, there exists $\varepsilon_2^*(\varepsilon_1)$ such that $\forall \varepsilon_2 \in (0, \varepsilon_2^*]$

$$\frac{d}{dt}V_2(\xi_2) \leq 0. \tag{39}$$

From (34)-(39), we can conclude that Ω_1 is a positive invariant set for the closed-loop system (24)-(26). Since $\chi(t_0) \in \Omega_1$, there is

$$\chi(t) \in \Omega_1, \quad t \geq t_0. \tag{40}$$

Define $t_{p1}(\varepsilon_1) = \max\{t_0 - 2c_{12}\varepsilon_1 ln\varepsilon_1, t_0\}$ and $t_{p2}(\varepsilon_2) = \max\{t_0 - \frac{2c_{22}}{c_0}\varepsilon_2 ln\varepsilon_2, t_0\}$. From (36), there is

$$\frac{d}{dt}\sqrt{V_1(\xi_1)} \leq -\frac{1}{\varepsilon_1}\frac{\sqrt{V_1(\xi_1)}}{2c_{12}} + \frac{c_{12}\gamma_1}{\sqrt{c_{11}}}. \tag{41}$$

Using the Gronwall-Bellman inequality for (41), we can get

$$\sqrt{V_1(\xi_1(t))} \leq \frac{2c_{12}^2\gamma_1}{\sqrt{c_{11}}}\varepsilon_1 + \sqrt{V_1(\xi_1(t_0))}e^{-\frac{1}{2\varepsilon c_{12}}(t-t_0)}. \tag{42}$$

From the definition of t_{p1}, we can have

$$\|\xi_1(t)\| \leq \varepsilon_1\tilde{\gamma}_1, \quad t \geq t_{p1} \tag{43}$$

where $\tilde{\gamma}_1 = \frac{2c_{12}^2}{c_{11}}\gamma_1 + \frac{\sqrt{c_{12}}}{\sqrt{c_{11}}}\hat{\zeta}_1$.

Similarly, from (38), when $t \geq t_p \triangleq \max\{t_{p1}, t_{p2}\}$, there is

$$\frac{d}{dt}\sqrt{V_2(\xi_2)} \leq -\frac{c_0}{2\varepsilon_2 c_{22}}\sqrt{V_2(\xi_2)} + \frac{c_{22}}{\sqrt{c_{21}}}(\gamma_2 + \gamma_3\tilde{\gamma}_1 + \frac{\gamma_4\tilde{\gamma}_1}{\varepsilon_1}).$$

By the Gronwall-Bellman inequality, there is

$$\sqrt{V_2(\xi_2(t))} \leq \frac{2c_{22}^2}{c_0\sqrt{c_{21}}}(\gamma_2 + \gamma_2\tilde{\gamma}_1 + \frac{\gamma_2\tilde{\gamma}_1}{\varepsilon_1}) + \sqrt{V_2(\xi_2(t_0))}e^{-\frac{c_0(t-t_0)}{2\varepsilon c_{22}}}.$$

Hence, we have $\tilde{\gamma}_2$

$$\|\xi_2(t)\| \leq \frac{\varepsilon_2}{\varepsilon_1}\tilde{\gamma}_2 \quad t \geq t_p. \tag{44}$$

where $\tilde{\gamma}_2 = \frac{2c_{22}^2}{c_0c_{21}}(\varepsilon_1(\gamma_2 + \gamma_3\tilde{\gamma}_1) + \gamma_4\tilde{\gamma}_1) + \varepsilon_1\hat{\zeta}_2\frac{\sqrt{c_{22}}}{\sqrt{c_{21}}}$.

It can be concluded from (43)-(44) that LESO (25)-(26) satisfy the properties (33).

Consider the errors between the system (4) and (24), where the initial conditions of (4) are

$$\bar{e}_1(t_0) = e_1(t_0), \quad \bar{e}_2(t_0) = e_2(t_0).$$

Define $\tilde{e}_1 = e_1 - \bar{e}_1, \tilde{e}_2 = e_2 - \bar{e}_2$, we can get

$$\dot{\tilde{e}}_1 = -k_1\tilde{e}_1 + b_1(e_1,t)e_2 - b_1(\bar{e}_1,t)\bar{e}_2 + \xi_{12} \tag{45}$$
$$\dot{\tilde{e}}_2 = -k_2\tilde{e}_2 + \xi_{22} \tag{46}$$
$$\tilde{e}_1(t_0) = 0, \tilde{e}_2(t_0) = 0 \tag{47}$$

Since Ω_1 is an invariant set and $\xi_2(t)$ satisfies (44), we can get for $t \in [t_0, \infty)$,

$$\begin{aligned}
\|\tilde{e}_2(t)\| &\leq \int_{t_0}^{t} e^{-\|k_2\|(t-\tau)}\|\xi_2(\tau)\|d\tau \\
&\leq \int_{t_0}^{t} e^{-\|k_2\|(t-\tau)}\|\xi_2(\tau)\|d\tau + \int_{t_p}^{t} e^{-\|k_2\|(t-\tau)}\|\xi_2(\tau)\|d\tau \\
&\leq \max\{2c_{12}|\varepsilon_1 ln\varepsilon_1|, \frac{2c_{22}}{c_0}|\varepsilon_2 ln\varepsilon_2|\}\frac{\sqrt{c_{22}}}{\sqrt{c_{21}}}\hat{\zeta}_2 + \frac{\varepsilon_2}{\varepsilon_1}\frac{\tilde{\gamma}_2}{\|k_2\|} \\
&\leq \left(\max\{|\varepsilon_1 ln\varepsilon_1|, |\varepsilon_2 ln\varepsilon_2|\} + \frac{\varepsilon_2}{\varepsilon_1}\right)\tilde{\gamma}_4,
\end{aligned} \tag{48}$$

where $\tilde{\gamma}_4 = \left(2c_{12} + \frac{2c_{22}}{c_0}\right)\frac{\sqrt{c_{22}}\hat{\zeta}_2}{\sqrt{c_{21}}} + \frac{\tilde{\gamma}_2}{\|k_2\|}$. Furthermore, there is

$$\begin{aligned}
\lim_{t\to\infty}\|\tilde{e}_2(t)\| &\leq \lim_{t\to\infty}\int_{t_0}^{t_p} e^{-\|k_2\|(t-\tau)}\|\xi_2(\tau)\|d\tau \\
&\quad + \lim_{t\to\infty}\int_{t_p}^{t} e^{-\|k_2\|(t-\tau)}\|\xi_2(\tau)\|d\tau \\
&= \lim_{t\to\infty}\int_{t_p}^{t} e^{-\|k_2\|(t-\tau)}\|\xi_2(\tau)\|d\tau \leq \frac{\varepsilon_2}{\varepsilon_1}\frac{1}{\|k_2\|}\tilde{\gamma}_2.
\end{aligned} \tag{49}$$

Since $\dot{\bar{e}}_2 = -k_2\bar{e}_2, \|\bar{e}_2(t_0)\| = \|e_2(t_0)\| \leq \zeta_2$, there is

$$\|\bar{e}_2(t)\| \leq \zeta_2.$$

Thus

$$\|\left(b_1(e_1,t) - b_1(\bar{e}_1,t)\right)\bar{e}_2\| \leq L_1\zeta_2\|\tilde{e}_1\|.$$

Transform (45) to be as

$$\dot{\tilde{e}}_1 = -k_1\tilde{e}_1 + (b_1(e_1,t) - b_1(\bar{e}_1,t))\bar{e}_2 - b_1(e_1,t)\tilde{e}_2 + \xi_{12}$$

then we have

$$\frac{d}{dt}\|\tilde{e}_1\| \leq -\tilde{k}_1\|\tilde{e}_1\| + (\|b_1(e_1,t)\|\|\tilde{e}_2\| + \|\xi_1\|) \tag{50}$$

where $\tilde{k}_1 = \|k_1\| - L_1\zeta_2 > 0$.

Since Ω_1 is an invariant set, ξ_1 satisfies (43) and $\tilde{e}_2$ satisfies (48), we can get for $t \in [t_0, \infty)$,

$$\begin{aligned}
\|\tilde{e}_1(t)\| &\leq \int_{t_0}^{t} e^{-\tilde{k}_1(t-\tau)}(\|\xi_1(\tau)\| + \|b_1(e_1,t)\tilde{e}_2\|)d\tau \\
&\leq \int_{t_0}^{t_p} e^{-\tilde{k}_1(t-\tau)}\|b_1(e_1,t)\tilde{e}_2\|d\tau \\
&\quad + \int_{t_0}^{t_p} e^{-\tilde{k}_1(t-\tau)}\|\xi_1(\tau)\|d\tau + \int_{t_p}^{t} e^{-\tilde{k}_1(t-\tau)}\|\xi_1(\tau)\|d\tau \\
&\leq \tilde{\gamma}_5\tilde{\gamma}_4\left(\max\{|\varepsilon_1 ln\varepsilon_1|, |\varepsilon_2 ln\varepsilon_2|\} + \frac{\varepsilon_2}{\varepsilon_1}\right)\frac{1}{\tilde{k}_1} \\
&\quad + \tilde{\gamma}_6\left(\max\{|\varepsilon_1 ln\varepsilon_1|, |\varepsilon_2 ln\varepsilon_2|\} + \varepsilon_1\right)
\end{aligned} \tag{51}$$

where $\tilde{\gamma}_5 = \max_{\|e_1\|\leq\zeta_1}\|b_1(\cdot)\|$ and $\tilde{\gamma}_6 = \left(2c_{12} + \frac{2c_{22}}{c_0}\right)\frac{\sqrt{c_{12}}}{\sqrt{c_{11}}}\hat{\zeta}_1 + \frac{\tilde{\gamma}_1}{\tilde{k}_1}$.

Since ξ_1 satisfies (43) and $\tilde{e}_2$ satisfies (49), there is

$$\begin{aligned}
\lim_{t\to\infty}\|\tilde{e}_1(t)\| &\leq \lim_{t\to\infty}\left(\int_{t_0}^{t/2} e^{-\tilde{k}_1(t-\tau)}\|b_1(e_1,\tau)\tilde{e}_2\|d\tau + \int_{t_0}^{t_p} e^{-\tilde{k}_1(t-\tau)}\|\xi_1(\tau)\|d\tau\right) \\
&\quad + \lim_{t\to\infty}\left(\int_{t/2}^{t} e^{-\tilde{k}_1(t-\tau)}\|b_1(e_1,\tau)\tilde{e}_2\|d\tau + \int_{t_p}^{t} e^{-\tilde{k}_1(t-\tau)}\|\xi_1(\tau)\|d\tau\right) \\
&= \lim_{t\to\infty}\int_{t/2}^{t} e^{-\tilde{k}_1(t-\tau)}\|b_1(e_1,\tau)\tilde{e}_2\|d\tau + \lim_{t\to\infty}\int_{t_p}^{t} e^{-\tilde{k}_1(t-\tau)}\|\xi_1(\tau)\|d\tau \\
&\leq \tilde{\gamma}_5\frac{\varepsilon_2}{\varepsilon_1}\frac{1}{\|k_2\|}\tilde{\gamma}_2\frac{1}{\tilde{k}_1} + \varepsilon_1\tilde{\gamma}_1\frac{1}{\tilde{k}_1}.
\end{aligned} \tag{52}$$

Using (51)-(52), (48)-(49), we can get (31)-(32). Q.E.D.

4 Conclusion

We have analyzed the convergence of linear ADRC for a class of block MIMO lower-triangular system. The nonlinear system concerned in this paper can be non-parameterized and the unknown dynamics and disturbance are assumed to satisfy mild assumptions. Theorem 1 shows that the error between the trajectory of the ADRC based closed-loop system and that of the reference closed-loop system is limited by a bound which can be adjusted by tuning the bandwidth of ESO.

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