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# Finite-time stabilization and stabilizability of a class of controllable systems

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**Abstract**

In this paper, finite-time control problem of a class of controllable systems is considered. Explicit formulae are proposed for the finite-time stabilization of a chain of power-integrators, and then discussions about a generalized class of nonlinear systems are given. 

*Keywords:* Finite-time stabilization; Nonlinear systems; Homogeneity

## 1. Introduction

Finite-time stabilization problems have been studied mostly in the contexts of optimality, controllability, and deadbeat control for several decades. These control laws are usually time-varying, discontinuous, or even depending directly on the initial conditions of considered systems [1]. Recently, finite-time stability and finite-time stabilization via continuous time-invariant feedback have been studied and finite-time controllers involving terms containing fractional powers were constructed for second-order systems ([3–5,9] and references therein). Furthermore, output feedback finite-time control design was also studied [12,13].

In addition, many results were obtained for the stabilization of analytic small-time local controllable (STLC) nonlinear systems in a special triangular form, or the systems that can be approximated by the STLC systems in this triangular form [6,7].

The work in this paper extends the previous results on asymptotic stabilization for the nonlinear systems discussed in some references such as [6,7,14] to continuous finite-time stabilization in some sense. The main result is given on the construction of continuous time-invariant finite-time controllers for a class of STLC systems.

## 2. Preliminaries

First of all, the concepts related to finite-time control are given (see [9]).

**Definition 2.1.** Consider a time-invariant system in the form of

$$\dot{x} = f(x), \quad f(0) = 0, \ x \in R^n, \tag{1}$$

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where $f:\hat{U}_0 \to R^n$ is continuous on an open neighborhood $\hat{U}_0$ of the origin. The equilibrium $x=0$ of the system is (locally) finite-time stable if (i) it is asymptotically stable, in $\hat{U}$, an open neighborhood of the origin, with $\hat{U} \subseteq \hat{U}_0$; (ii) it is finite-time convergent in $\hat{U}$, that is, for any initial condition $x_0 \in \hat{U}\backslash\{0\}$, there is a settling time $T > 0$ such that every solution $x(t,x_0)$ of system (1) is defined with $x(t,x_0)\in\hat{U}\backslash\{0\}$ for $t\in[0,T)$ and satisfies

$$\lim_{t\to T} x(t,x_0) = 0, \tag{2}$$

and $x(t,x_0)=0$, if $t \geqslant T$. Moreover, if $\hat{U} = R^n$, the origin $x=0$ is globally finite-time stable.

**Definition 2.2.** Consider a controlled system

$$\dot{x} = f(x) + g(x)u \quad x\in R^n,\ u\in R^m \tag{3}$$

with $f(0)=0$ and $g(0)\neq 0$. It is finite-time stabilizable via continuous time-invariant state feedback if there is a continuous feedback law $u=\mu(x)$ such that the origin $x=0$ of the closed-loop system $\dot{x}=f(x)+g(x)\mu(x)$ is a (locally) finite-time stable equilibrium.

Next, let us introduce the concept of homogeneity (see [6,11,17]) for the following analysis.

**Definition 2.3.** A family of dilations $\delta^r_\varepsilon$ is a mapping that assigns to every real $\varepsilon > 0$ a diffeomorphism

$$\delta^r_\varepsilon(x_1,\dots,x_n) = (\varepsilon^{r_1}x_1,\dots,\varepsilon^{r_n}x_n),$$

where $x_1,\dots,x_n$ are suitable coordinates on $R^n$ and $r=(r_1,\dots,r_n)$ with the dilation coefficients $r_1,\dots,r_n$ positive real numbers. A function $V(x)$ is homogeneous of degree $\alpha > 0$ with respect to the family of dilations $\delta^r_\varepsilon$ if

$$V(\varepsilon^{r_1}x_1,\dots,\varepsilon^{r_n}x_n) = \varepsilon^\alpha V(x). \tag{4}$$

A vector field $f(x) = (f_1(x),\dots,f_n(x))^{\mathrm{T}}$ is homogeneous of degree $k\in R$ with respect to the family of dilations $\delta^r_\varepsilon$ if

$$f_i(\varepsilon^{r_1}x_1,\dots,\varepsilon^{r_n}x_n) = \varepsilon^{k+r_i} f_i(x),\quad i=1,\dots,n,\ \varepsilon>0. \tag{5}$$

System (1) is called homogeneous if its vector field $f$ is homogeneous.

Define a function

$$\Gamma_n(x) = (|x_1|^{c/r_1} + \cdots + |x_n|^{c/r_n})^{1/c},\quad c > \max\{r_1,\dots,r_n\}, \tag{6}$$

which satisfies: $\Gamma_n(\varepsilon^{r_1}x_1,\dots,\varepsilon^{r_n}x_n) = \varepsilon\Gamma_n(x)$, $\varepsilon>0$ with $\Gamma_n(0)=0$ and $\Gamma_n(x)>0$ if $x\neq 0$. Where no confusion arises, we denote $\Gamma_n(x)$ by $\Gamma(x)$ for simplicity.

The following lemma was presented in some references like [2,3,12].

**Lemma 2.1.** *Suppose that system* (1) *is homogeneous of degree $k<0$ with respect to the family of dilations $\delta^r_\varepsilon$, $f(x)$ is continuous and $x=0$ is its asymptotically stable equilibrium. Then the equilibrium of system* (1) *is globally finite-time stable. Moreover, if $\tilde{f}(x)$ is a continuous vector field defined on $R^n$ such that*

$$\lim_{\varepsilon\to 0}\frac{\tilde{f}_i(\varepsilon^{r_1}x_1,\dots,\varepsilon^{r_n}x_n)}{\varepsilon^{k+r_i}} = 0,\quad i=1,\dots,n,\ \tilde{f}=(\tilde{f}_1,\dots,\tilde{f}_n)^{\mathrm{T}} \tag{7}$$

*uniformly for $\|x\|=1$, then the zero solution of*

$$\dot{x} = f(x) + \tilde{f}(x),\quad \tilde{f}(0)=0 \tag{8}$$

*is* (*locally*) *finite-time stable.*

## 3. Finite-time stabilization

At first we focus on the construction of continuous finite-time stabilizing feedback laws for a class of STLC system (see [7]):

$$\begin{aligned}
\dot{x}_1 &= x_2^{m_1},\\
&\ \vdots\\
\dot{x}_{n-1} &= x_n^{m_{n-1}},\\
\dot{x}_n &= u,
\end{aligned} \tag{9}$$

where $m_i>0$, $i=1,\dots,n-1$ are odd integers.

For convenience, as in [9], set $\mathrm{sig}(y)^\alpha = |y|^\alpha\,\mathrm{sgn}(y)$ for $\alpha>0$, where $|y|$ denotes the absolute value of real number $y$ and $\mathrm{sgn}(\cdot)$ the sign function. Clearly, $\mathrm{sig}(y)^\alpha = y^\alpha$ if $\alpha = q_1/q_2$ where $q_i>0$, $i=1,2$ are odd integers. Note that

$$\frac{\mathrm{d}}{\mathrm{d}y}|y|^{\alpha+1} = (\alpha+1)\,\mathrm{sig}(y)^{\alpha},$$

and

$$\frac{\mathrm{d}}{\mathrm{d}y}\mathrm{sig}(y)^{\alpha+1} = (\alpha+1)|y|^{\alpha} \quad \alpha > 0.$$

**Theorem 3.1.** *Let $r_i$, $\beta_{i-1}$, $i = 1,\dots,n$ and $k$ be constants satisfying the following given inequalities*:

$$r_1 = 1,\dots, \quad r_i = \frac{r_{i-1}+k}{m_{i-1}}, \;\; r_i > -k > 0, \quad i = 1,\dots,n, \tag{10}$$

$$\beta_0 = r_2, (\beta_i m_i + 1)r_{i+1} \geqslant (\beta_{i-1}m_{i-1}+1)r_i > 0, \quad i = 1,\dots,n-2, \;\; \beta_{n-1} > 0, \tag{11}$$

*where we denote $m_0 = 1$. Then there exist constants $l_i > 0$ for $i = 1,2,\dots,n$ such that the control law*

$$u(x) = u_n(x) \tag{12}$$

*renders the system* (9) *finite-time stable, where $u_i$, $i = 1,\dots,n$ are defined as follows*:

$$u_0 = 0,$$

$$\begin{aligned} u_{i+1}(x_1,\dots,x_{i+1}) &= -\,l_{i+1}\,\mathrm{sig}[\mathrm{sig}(x_{i+1})^{m_i\beta_i} \\ &\quad -\mathrm{sig}(u_i(x_1,\dots,x_i))^{\beta_i}]^{(r_{i+1}+k)/(r_{i+1}m_i\beta_i)}, \\ &\quad i = 1,\dots,n-1. \end{aligned}$$

**Remark 3.1.** From Proposition 3 and Corollary 1 in [8], the existence of (finite-time) stabilizing feedback (together with Lemma 2.1) of a rather general system was obtained. The idea of Theorem 3.1 is based on work in [8]. However, here we propose a constructive method to give explicit controllers. Moreover, with such a constructive Lyapunov function, we can get an upper bound or an estimate of the settling time for any initial condition by Theorem 1 in [4] and discussions in [5].

**Remark 3.2.** With homogeneity, Praly proposed a feedback law

$$u(x) = \sum_{i=1}^{n} \rho(x)^{-(n+1-i)\tau(x)} x_i,$$

which is finite-time stabilizing if $\tau > 0$, for linear systems [16]. In fact, like the discussion in [16], the result of this theorem can also be extended: if $k \geqslant 0$, (12) renders system (9) asymptotically stable, with assumption (instead of (11)): $\beta_i m_i r_{i+1} \geqslant \beta_{i-1}m_{i-1}r_i$, $i = 1,\dots,n-2$ for the $C^1$ smoothness of $\mathrm{sig}(u_i)^{\beta_i}$, $i = 1,\dots,n-2$.

**Proof of Theorem 3.1.** With the feedback, the closed-loop system is homogeneous of degree $k < 0$ with respect to the family of dilations $\delta_\varepsilon^r$ for $r = (r_1,\dots,r_n)$. According to Lemma 2.1, system (9) under the feedback (12) is globally finite-time stable once it is asymptotically stable. Thus, we only need to prove the asymptotic stability of the closed-loop system. The asymptotic stability can be proved together using induction method.

*Step* 1: Consider system $\dot{x}_1 = u$. Take a continuous feedback law, for any given $l_1 > 0$,

$$\begin{aligned} u = u_1(x) &= -l_1\,\mathrm{sig}[\mathrm{sig}(x_1)^{\beta_0} - 0]^{(r_1+k)/(r_1\beta_0)} \\ &= -l_1\,\mathrm{sig}(x_1)^{1+k}, \end{aligned}$$

(of homogeneity degree $r_1 + k$). Then take a homogeneous Lyapunov function $V_1(x) = x_1^{1+r_2}$ of degree $(m_0\beta_0+1)r_1$ and therefore, its derivative, $\dot{V}_1 = -(1+r_2)l_1V_1(x)^{(1+r_2+k)/(1+r_2)}$, is negative definite with respect to $x_1$. Therefore, $u_1$ stabilizes the homogeneous closed-loop system $\dot{x}_1 = u_1$. Obviously, $\mathrm{sig}(u_1(x))^{\beta_1}$ is $C^1$ with $u_1(0) = 0$ since $\beta_1 \geqslant 1/m_1r_2 = 1/(1+k)$ by (11).

*After Step* $j-1$: We already have that $\mathrm{sig}(u_{j-1}(x_1,\dots,x_{j-1}))^{\beta_{j-1}}$ is $C^1$ of homogeneity degree $r_{j-1}+k > 0$ with $u_j(0,\dots,0) = 0$ and $u_{j-1}(x_1,\dots,x_{j-1})$ is a continuous stabilizing feedback for system

$$\begin{aligned} \dot{x}_1 &= x_2^{m_1}, \\ &\;\;\vdots \\ \dot{x}_{j-1} &= u_{j-1} \end{aligned} \tag{13}$$

which is homogeneous of degree $k$ with respect to the family of dilations $\delta_\varepsilon^r$ for $r = (1, r_2,\dots,r_{j-1})$. Its Lyapunov function is $V_{j-1}$, which is positive definite with respect to $x_1,\dots,x_{j-1}$ and homogeneous of degree $(m_{j-2}\beta_{j-2}+1)r_{j-1}$, and moreover, its derivative

$\dot V_{j-1}|_{(13)}$ is negative definite. The construction of $V_{j-1}$, based on $V_{j-2}$, will be introduced in detail in Step $j$.

*Step j*: Consider system

$$\dot x_1 = x_2^{m_1},$$

$$\vdots$$

$$\dot x_{j-1} = x_j^{m_{j-1}},$$

$$\dot x_j = u_j. \tag{14}$$

Take a continuous feedback law as follows:

$$u_j(x_1,\dots,x_j) = -\,l_j\,\mathrm{sig}[\mathrm{sig}(x_j)^{m_{j-1}\beta_{j-1}} - \mathrm{sig}(u_{j-1}(x_1,\dots,x_{j-1}))^{\beta_{j-1}}]^{(r_j+k)/(r_j m_{j-1}\beta_{j-1})}, \tag{15}$$

where $l_j$ will be determined later. Note that the closed-loop system is homogeneous of degree $k$ and $u_j$ is homogeneous of degree $r_j+k>0$ with respect to the family of dilations $\delta_\varepsilon^r$ for $r=(1,r_2,\dots,r_j)$.

Set

$$W_j(x) \triangleq \int_{\mathrm{sig}(u_{j-1}(x_1,\dots,x_{j-1}))^{1/(m_{j-1})}}^{x_j} [\mathrm{sig}(s)^{m_{j-1}\beta_{j-1}} - \mathrm{sig}(u_{j-1}(x_1,\dots,x_{j-1}))^{\beta_{j-1}}]\,\mathrm{d}s$$

which is nonnegative and even positive when $x_j^{m_{j-1}} \neq u_{j-1}(x_1,\dots,x_{j-1})$ (can also be proved using Young's inequality). It is easy to see that

$$W_j(x) = \frac{1}{m_{j-1}\beta_{j-1}+1}[|x_j|^{m_{j-1}\beta_{j-1}+1} + m_{j-1}\beta_{j-1}|u_{j-1}|^{\beta_{j-1}+1/m_{j-1}}] - x_j\,\mathrm{sig}(u_{j-1})^{\beta_{j-1}}.$$

which implies that $W_j$ is $C^1$ considering that $\mathrm{sig}(u_{j-1})^{\beta_{j-1}}$ is $C^1$. Then construct a Lyapunov function candidate as follows:

$$V_j(x) = W_j(x) + V_{j-1,j}(x_1,\dots,x_{j-1}), \tag{16}$$

where

$$V_{j-1,j} \triangleq V_{j-1}^{(m_{j-1}\beta_{j-1}+1)r_j/(m_{j-2}\beta_{j-2}+1)r_{j-1}}$$

is $C^1$ because $(m_{j-1}\beta_{j-1}+1)r_j \geqslant (m_{j-2}\beta_{j-2}+1)\,r_{j-1}$. Thus, $V_j$ is $C^1$ and homogeneous of degree $(m_{j-1}\beta_{j-1}+1)r_j$ with respect to the family of dilations $\delta_\varepsilon^r$ for $r=(1,r_2,\dots,r_j)$ because $W_j$ and $V_{j-1,j}$ are so. Quite clearly, $V_j$ is positive definite with respect to $x_1,\dots,x_j$.

Its derivative along system (14) under the feedback law (15) is

$$\dot V_j(x)|_{(14)} = \sum_{i=1}^{j-1}\frac{\partial W_j}{\partial x_i}x_{i+1}^{m_i} + w_j(x)u_j + \dot V_{j-1,j}(x_1,\dots,x_{j-1})\big|_{(13)} + \frac{\partial V_{j-1,j}}{\partial x_{j-1}}(x_j^{m_{j-1}} - u_{j-1}(x_1,\dots,x_{j-1})), \tag{17}$$

where the homogeneity degree of the system is $(m_{j-1}\beta_{j-1}+1)r_j+k$ and

$$w_j(x_1,\dots,x_j) \triangleq \frac{\partial W_j}{\partial x_j} = \mathrm{sig}(x_j)^{m_{j-1}\beta_{j-1}} - \mathrm{sig}(u_{j-1}(x_1,\dots,x_{j-1}))^{\beta_{j-1}}. \tag{18}$$

(17) can be rewritten in a compact form: $\dot V_j(x) = V_j^0(x) + w_j(x)u_j$ with

$$V_j^0(x) \triangleq \sum_{i=1}^{j-1}\frac{\partial W_j}{\partial x_i}x_{i+1}^{m_i} + \dot V_{j-1,j}(x_1,\dots,x_{j-1})|_{(13)} + \frac{\partial V_{j-1,j}}{\partial x_{j-1}}(x_j^{m_{j-1}} - u_{j-1}(x_1,\dots,x_{j-1})), \tag{19}$$

where $\dot V_{j-1,j}|_{(13)}$ is negative definite because $\dot V_j|_{(13)}$ is so.

When $w_j=0$, that is, $x_j^{m_{j-1}} = u_{j-1}(x_1,\dots,x_{j-1})$, we have that $W_j=0$ and $\partial W_j/\partial x_i = 0$, $i=1,\dots,j-1$, and therefore, if also $x\neq 0$,

$$\dot V_j(x) = V_j^0(x) = \dot W_j(x) + \dot V_{j-1,j}(x_1,\dots,x_{j-1})|_{(13)} = \dot V_{j-1,j}(x_1,\dots,x_{j-1})|_{(13)} < 0. \tag{20}$$

Define the 'unit sphere' $S_0^j = \{x\in R^j\colon \Gamma_j(x)=1\}$, $S_+^j = \{x\in R^j\colon V_j^0(x)\geqslant 0\}$, and $S_-^j = \{x\in R^j\colon V_j^0(x) < 0\}$. If $S_+^j$ is empty, then $\dot V_j$ is negative positive. Hence suppose $S_+^j$ is nonempty. Then, by homogeneity, $S_0^j\cap S_+^j\neq\emptyset$. By continuity, $S_+^j$ is closed and hence

$S_0^j \cap S_+^j$ is compact. Define

$$M_1 \triangleq \max_{x \in S_0^j} V_j^0(x)$$

and

$$M_2 \triangleq \min_{x \in S_0^j \cap S_+^j} -\frac{w_j(x) u_j(x_1,\ldots,x_j)}{l_j}.$$

Note that $u_j/l_j$ does not depend on $l_j$ and $M_2 \geqslant 0$ observing that $w_j(x)u_j(x_1,\ldots,x_j) \leqslant 0$. In fact, $M_2 \neq 0$ because $M_2 = 0$ implies $w_j = 0$ and $\{x \in R^j:\ w_j = 0\} \subset S_-^j$ from (20). In other words, we have $M_2 > 0$. Take $l_j > M_1/M_2 \geqslant 0$, then $\dot{V}_j(x) < 0$ for all $x \in S_0^j$. Consider $0 \neq x \in R^j$ and let $\varepsilon = \Gamma_j(x) > 0$ and $e = (x_1/\Gamma_j(x)^{r_1},\ldots,x_j/\Gamma_j(x)^{r_j})^{\mathrm{T}} \in S_0^j$. Then, $\dot{V}_j(x) = \varepsilon^{(\beta_{j-1}m_{j-1}+1)r_j+k}\dot{V}_j(e) < 0$ for any $x \neq 0$ due to homogeneity.

In addition, $\mathrm{sig}(u_j(x_1,\ldots,x_j))^{\beta_j}$ (when $j < n-1$) is $C^1$ because

$$\mathrm{sig}(u_j)^{\beta_j} = -l_j^{\beta_j}\,\mathrm{sig}[\mathrm{sig}(x_j)^{\beta_{j-1}m_{j-1}} - \mathrm{sig}(u_{j-1})^{\beta_{j-1}}]^{\beta_j m_j r_{j+1}/m_{j-1}\beta_{j-1}r_j}$$

with $\mathrm{sig}(u_{j-1}(x_1,\ldots,x_{j-1}))^{\beta_{j-1}}$ being $C^1$ and

$$\beta_j(r_j + k) = \beta_j m_j r_{j+1} > \beta_{j-1} m_{j-1} r_j$$

from (11) and $r_i > r_{i+1} > 0$.

*Up to Step n*: The result of the theorem follows by induction. Note that the homogeneity of $u = u_n(x)$ with degree of $r_n + k > 0$ implies its continuity. □

**Remark 3.3.** Note that at step $j$ ($j = 2,\ldots,n$) when $l_j$ is given, any sufficient large $l_j^{\mathrm{new}} > l_j$ also works for the finite-time control law $u_j$.

**Example 1.** Consider a third-order system

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = x_3,$$

$$\dot{x}_3 = u.$$

For instance, its finite-time stabilizing controllers can be taken in either of the following forms:

$$u = -l_3[l_2(x_1^5 + x_2^{45/7})^{1/9} + x_3]^{3/5}, \tag{21}$$

or

$$u = -l_2^{3/5} l_3 (x_1 + x_2^{9/7})^{1/3} - l_3 x_3^{3/5},$$

where $l_i > 0$, $i = 2, 3$, can be estimated according to the proof of Theorem 3.1. Note that both proposed controllers in (21) render the corresponding closed-loop system homogeneous of degree $k = -\frac{2}{9}$ with respect to the family of dilations $\delta_\varepsilon^r$ for $r = (1, \frac{7}{9}, \frac{5}{9})$. However, $(\beta_1, \beta_2)$ is $(\frac{45}{7}, 1)$ and $(\frac{9}{7}, \frac{3}{5})$, respectively, in the two cases (with $\beta_0 = \frac{7}{9}$).

Then we consider a class of single-input STLC systems that can be expressed in the following triangular form by diffeomorphism transformation (discussed in [6–8]):

$$\dot{x}_1 = f_1(x_1, x_2) = \lambda_1 x_2^{m_1} + \hat{h}_1(x_2) + x_1 h_1^1(x_1, x_2),$$

$$\hat{h}_1(x_2) = \mathrm{o}(x_2^{m_1}),$$

$$\vdots$$

$$\dot{x}_i = f_i(x_1,\ldots,x_{i+1}) = \lambda_i x_{i+1}^{m_i} + \hat{h}_i(x_{i+1}) + \sum_{j=1}^{i} x_j h_i^j(x_1,\ldots,x_{i+1}), \quad \hat{h}_i(x_{i+1}) = \mathrm{o}(x_{i+1}^{m_i}),$$

$$\vdots$$

$$\dot{x}_n = f_n(x) + g_n(x)u, \tag{22}$$

where $f_i(0) = 0$, $i = 1,\ldots,n$ are analytic, $m_i > 0$, $i = 1,\ldots,n-1$ are odd integers, and $\lambda_i \neq 0$, $i = 1,\ldots,n-1$, and $g_n(0) \neq 0$. The systems of the form (22) include controllable linear systems, feedback linearizable nonlinear systems [15], and some nonlinear systems having unstable uncontrollable eigenvalue of approximate linearization, which cannot be stabilized using smooth feedback [6,14].

Without changing local controllability and local stabilizability, we assume that $\lambda_i = 1$, $i = 1,\ldots,n-1$ and $g_n(x) = 1$ (otherwise, we take

$$x_1^{\mathrm{new}} = x_1, \quad x_2^{\mathrm{new}} = \frac{x_2}{\lambda_1^{1/m_1}}, \ldots,$$

$$x_n^{\mathrm{new}} = \frac{x_n}{\lambda_1^{1/m_1 m_2 \ldots m_{n-1}} \ldots \lambda_{n-1}^{1/m_{n-1}}}$$

and $u^{\mathrm{new}} = u/g_n(x)$ in a neighborhood of the origin). Then it is easy to see that system (22) can be locally finite-time stabilized under the control law in the form of (12), by Theorem 3.1 and (7) in Lemma 2.1 with homogeneous dilation coefficients given in (10). In other words,

**Corollary 3.1.** *System* (22) *is finite-time stabilizable via continuous time-invariant feedback.*

In fact, a multi-input controllable linear system can be transformed into Brunovsky canonical form, which is, in fact, a collection of mutually independent controllable single-input linear systems (finite-time stabilizable using continuous feedback by Theorem 3.1). On the other hand, uncontrollable modes of linear stabilizable system at best converge exponentially. Therefore,

**Corollary 3.2.** *A multi-input linear system is finite-time stabilizable* (*via continuous time-invariant state feedback*) *if and only if it is controllable.*

## 4. Concluding remarks

The paper focuses on continuous finite-time control (with fractional powers) for some classes of controllable nonlinear systems. The motivation of the research is to understand finite-time stabilizability via continuous time-invariant feedback. Moreover, we also wish to get ideas about how to employ nonsmoothness actively or skillfully for effective systems synthesis. In fact, from many numerical and experimental results, rather than only theoretic studies, it was observed that this class of nonsmooth controllers (we should modify them in practice as people did to sliding mode control) could improve the transient behaviors and robustness properties of considered systems (already discussed in [4,5,10]). In addition, feedback design with fractional powers was also found to be viable for realizing desired system dynamic behavior in bifurcation control [18]. However, further detailed formulation and rigorous derivation might be needed for studying all the relevant problems.

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