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The phase diagram and the pathway of phase transitions for traffic flow in a circular one-lane roadway

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ABSTRACT

This paper demonstrates that patient driving habits lead to homogenous congested flow while impatient driving habits lead to wide-moving jam flow in the high density region based on the numerical simulation of the intelligent driver model proposed by M.Treiber [M. Treiber, A.Hennecke, D. Helbing, Phys. Rev. E 62 (2) (2000), 1805–1824]. In a circular one lane traffic system which includes homogeneous drivers, we obtain the stable condition of homogenous flow and the phase diagram of traffic flow based on the linearization analysis. The phase diagram shows three possible pathways of phase transition along with the increase of global density: from the homogenous free flow to the homogenous congested flow directly, from the homogenous free flow to the synchronized flow then to the homogenous congested flow, or from the homogenous free flow to synchronized flow then to the wide-moving jam flow. The paper also analyzes the traffic flow including heterogenous drivers, and the results indicate that homogenous congested flow will lose its stability when the proportion of impatient drivers reaches a critical value and some new kinds of traffic flow emerge: wide-moving jam flow or a mixture of synchronized flow and wide-moving jam flow.

1. Introduction

Looked as a typical complex system containing many self-driven interacting bodies, the traffic flow has been investigated by statistical and nonlinear methods for decades. A community of physicists have taken great efforts to understand the different phases of traffic flow, such as free flow at low density regions, synchronized flow at intermediate density regions, and jammed flow at higher density regions, and the mechanism of transition between different phases [1–8]. As shown by many empirical data, many macroscopic traffic flow models and some microscopic car-following traffic models, the traffic flow is a kind of free flow with low global density and evolves into homogenous congested flow with high global density. However, Kerner and his collaborators discovered "synchronized flow" and wide-moving jam flow based on empirical data [1,8–10]. Therefore, there are several possible paths from homogenous free flow to jammed flow along with the increase of global density. In order to solve this problem, Kerner had proposed a "three phase theory" implemented by a piecewise function of acceleration and incorporated the response of the driver into a time delay acceleration model [9]. In this paper, we will tackle the same problem using an intelligent driver model. This model is an easy to calibrate, accident free car following model. It describes what drivers actually do when they drive and already reproduces some empirical data [2].

The basic microscopic mechanism of traffic flow is the driver's behavior which has more and more influence on the traffic flow state with the increases of global density. It is easy to imagine that different driving habits will lead to different

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types of traffic flow even if the global density is identical. Therefore, we consider two kinds of driving habits: patient and impatient driving. Based on the stability analysis of the intelligent driver model and numerical simulation, we show that patient driving habit leads to homogenous congested flow and impatient driving habit results in wide-moving jam flow if the global density is high. At the same time, we work out the phase diagram of traffic flow and propose the possible routes of transition between different traffic phases. Then we discuss the effects of a heterogenous driver on the behavior of traffic flow. Finally, we come to the conclusion and discussion.

2. The model

We follow the intelligent driver model and consider N vehicles running on a closed circular one-lane roadway of length L. The nth vehicle's driver adjusts the speed according to the velocity v_n, the relative velocity with respect to its leading car $\Delta v_n = v_{n+1} - v_n$, and the headway $s_n = x_{n+1} - x_n$ (where x_n is the coordinate of the nth vehicle, and $x_{N+1} = x_1$, $v_{N+1} = v_1$) as follows [5]:

$$\dot{v}_n = a_n \left\{ 1 - \left(\frac{v_n}{v_{0,n}} \right)^{\delta} - \left(\frac{s_n^*}{s_n} \right)^2 \right\} \tag{1}$$

where a_n is the maximum acceleration, $v_{0,n}$ is the desired velocity of the nth vehicle, δ is the acceleration exponent, and s_n^* is the minimum desired headway which varies dynamically with the velocity and the relative velocity:

$$s_n^* = s_{0,n} + T_n v_n - \frac{v_n \Delta v_n}{2\sqrt{a_n b_n}} \tag{2}$$

where $s_{0,n}$ is the jam distance that must be kept at least even if the traffic flow comes to a complete standstill, T_n is the safe time headway, and b_n is the desired deceleration. We set the parameters as follows: $N = 150$, $v_{0,n} = v_0 = 20$ m/s, $a_n = a = 0.8$ m/s^2, $b_n = b = 1.8$ m/s^2, $l_n = l = 5$ m, and $\delta = 4$.

3. Simulation of patient and impatient driving habits

The intelligent driver model has a trivial homogenous flow, whose velocity v_h is determined by $s_h = \frac{s_0 + T v_h}{\sqrt{1 - (\frac{v_h}{v_0})^{\delta}}}$, and whose flux is $Q_h = \rho v_h$, where $\rho = \frac{N}{N(l+s_h)} = \frac{1}{l+s_h}$ is global density. If the homogenous flow loses its stability, the velocity of a vehicle will depend on the time. We denote standard deviation of velocity by $\sigma = \sqrt{\frac{1}{N}\sum(v_n - \bar{v})^2}$, where $\bar{v} = \frac{1}{N}\sum_n v_n$, and the relative deviation of velocity by $r = \frac{\sigma}{\bar{v}}$. Let Q denote flux of traffic flow, and it may be different from that of homogenous flow, and the ratio $q = \frac{Q}{Q_h} = \frac{\bar{v}}{v_h}$ measures the difference.

In the intelligent driver model, the jam distance $s_{0,n}$ and safe time T_n reflect the driving habit of the nth driver. Different driving habits will cause different dynamic behavior of traffic flow. Here we investigate two different driving habits: patient and impatient driving. Imagine that one driver plunges into a traffic jam; the driver must brake the car with deceleration a_{brake} to avoid a collision with its leading vehicle:

$$a_{brake} \sim \frac{s_{0,n}}{T^2}. \tag{3}$$

The patient driver needs a relative smaller deceleration to brake the vehicle while the impatient driver must have a relative larger deceleration to brake the vehicle.

To get an intuitional impression about the effect of driving habits on the traffic flow, we carry out the numerical simulation and show the results in Fig. 1. In order to simulate the traffic flow, we use the discrete form of Eqs. (1) and (2): $x_n(t + \Delta t) = x_n(t) + v_n(t + \Delta t)\Delta t$, $v_n(t + \Delta t) = max(0, v_n(t) + \dot{v}_n(t)\Delta t)$, where $\Delta t = 0.1s$. We assume that a patient driver has a headway $s_0 = 1.5$ m, and safe time $T = 2$ s while the impatient driver has a headway $s_0 = 1.5$ m, and safe time $T = 1.2$ s. As shown in Fig. 1, traffic flow of patient drivers and impatient drivers exhibit different behaviors along with the increase of global density. At the low density region, $r = 0$ and $q = 1$ hold for traffic flows composed by patient or impatient drivers, which means the traffic flow is homogenous free flow. When the global density is greater than a critical value $\rho_{1,\gamma}(\gamma = patient, impatient)$, homogenous flow loses its stability, the flux of traffic flow becomes smaller than that of homogenous flow ($q < 1$), and the relative deviation of a vehicle quickly increases to a certain value ($r > 0$). Before the density increases to another critical value $\rho_{2,\gamma}$, the driving habit has no remarkable influence on the behavior of traffic flow. The vehicles slow down but still move on. It can be verified that traffic flow is a kind of synchronized flow or mixture of synchronized flow with wide-moving jam flow. The driving habit leads to quite different dynamical behaviors after the density reaches $\rho_{2,\gamma}$. The homogenous flow reobtains (the flow became homogenous but congested) the stability habit since $r = 0$, $q = 1$ hold for high density if the drivers are patient. But r and q increase sharply in the case of impatient drivers, which implies that speed of vehicle changes quickly and the flux is much bigger than that of homogenous flow. Fig. 1c shows the velocity of traffic flow produced by patient and impatient drivers with high global density ($\rho > \rho_{2,\gamma}$). It is easy to see that a patient driving habit leads to a congested homogenous flow, and the impatient driving habit leads to a wide-moving

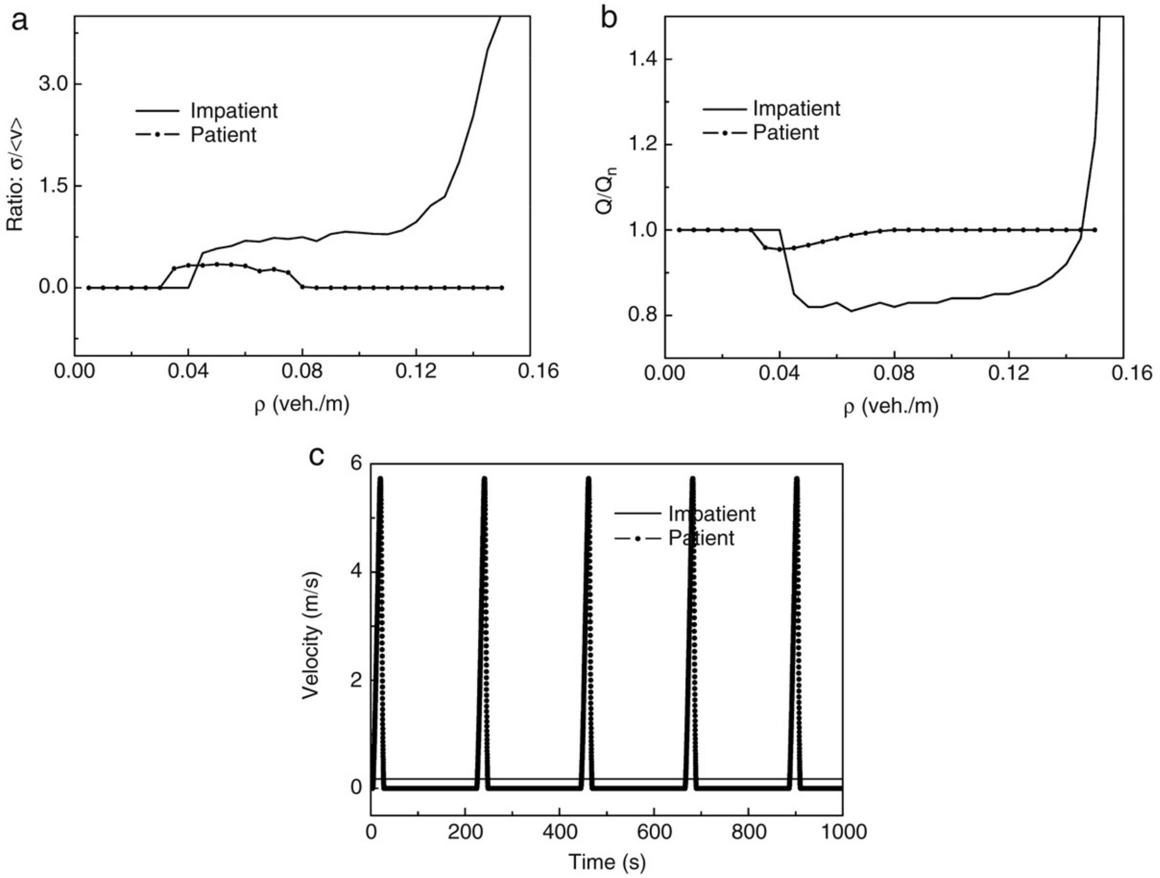

Fig. 1. (a): The relative deviation of velocity r. $r = 0$ means that each vehicle has identical velocity and the traffic flow is homogenous. (b): The ratio q. Along with the increase of density, r and q experience three stages. For an impatient driver, $r = 0$, $q = 1$ for $\rho < \rho_{1,\gamma}$, $r > 0$, $q < 1$ for $\rho_{1,\gamma} < \rho < \rho_{2,\gamma}$, and r, q increases dramatically for $\rho_{2,\gamma} < \rho$; and for a patient driver, $r = 0$, $q = 1$ for $\rho < \rho_1$, $r > 0$, $q < 1$ for $\rho_1 < \rho < \rho_2$, and $r = 0$, $q = 1$ for $\rho_2 < \rho$; (c): The velocity of vehicle driven by impatient and patient drivers, where $\rho = 0.146$ veh/m, and $s_0 = 1.5$ m. The impatient driver has a long period of rest and a short period of high speed motion, which is the typical characteristic of wide moving flow. However, the patient driver has a nonzero constant velocity. In the simulation, the initial states are set as follows: all vehicles are positioned in the lane with the same headway $s = \frac{L}{N} - l$ and their velocities range from 0 to 1 m/s randomly.

jam flow. These results of the intelligent driver model indicate that a patient driving habit leads to homogenous congested flow while an impatient driving habit results in wide-moving jam flow in the high density traffic flow.

4. Linear stability analysis

In order to quantitatively illustrate the effect of the patient and impatient driving habits on the traffic flow, we use the linearization method to analyze the stability of homogenous traffic flow using a generalized car-following model:

$$\dot{v}_n = V(s_n, \Delta v_n, v_n) \tag{4}$$

where Δv_n is the relative velocity with respect to its leading car, s_n is the headway. In reality, the partial derivation of acceleration $V(s_n, \Delta v_n, v_n)$ satisfies $\frac{\partial V}{\partial s_n} \geqslant 0$, $\frac{\partial V}{\partial \Delta v_n} \geqslant 0$, and $\frac{\partial V}{\partial v_n} \leqslant 0$ for $n = 1, 2, \ldots, N$. The homogenous flow solution of (4) follows: $\Delta v_n = 0$ and $x_n^*(t) = (n-1)s_h + v_h t$, where v_h is the velocity of vehicles in homogenous flow and $x_n^*(t)$ is the coordinate of vehicle n at time t. The small perturbation of homogenous flow is chosen as: $\delta x_n(k, t) = x_n - x_n^* = c\exp\{i\alpha_k n + zt\}$, where $\alpha_k = \frac{2\pi}{N}k$ $(n = 1, 2, \ldots, N)$, $z = u + vi$, u and v are real numbers. Then, the evolution of perturbation is:

$$\delta\dot{v}_n = \{f_1\delta s_n + f_2\delta\Delta v_n + f_3\delta v_n\} \tag{5}$$

where $f_1 = \frac{\partial V}{\partial s_n}|_{(s_h,0,v_h)}$, $f_2 = \frac{\partial V}{\partial \Delta v_n}|_{(s_h,0,v_h)}$, $f_3 = \frac{\partial V}{\partial v_n}|_{(s_h,0,v_h)}$, and $\delta x_{N+1} = \delta x_1$. Putting $\delta x_n(k, t)$ into the linearizing formula, the algebraic equation for z can be obtained:

$$z^2 - \{f_2(e^{i\alpha_k} - 1) + f_3\}z - f_1(e^{i\alpha_k} - 1) = 0. \tag{6}$$

The stable condition of the homogenous flow is that perturbation eliminates with time elapsing, therefore, any mode of perturbation (including $k = N$ and $k = 1$) must approach zero, which requires $u < 0$ for any mode of perturbation.

Now we let $\beta_1 = f_2(\cos\alpha_k - 1) + f_3$, $\beta_2 = f_2 \sin\alpha_k$, $\gamma_1 = f_1(\cos\alpha_k - 1)$, $\gamma_2 = f_1 \sin\alpha_k$. Then we modify (6) as $u^2 - v^2 - \beta_1 u - \beta_2 v - \gamma_1 = 0$ and $2uv + \beta_2 u - \beta_1 v + \gamma_2 = 0$. By the latter equation, we get $v = -\frac{\beta_2 u+\gamma_2}{2u-\beta_1}$ and the equation of u:

$$u^2 - \left(\frac{\beta_2 u + \gamma_2}{2u - \beta_1}\right)^2 - \beta_1 u + \beta_2 \frac{\beta_2 u + \gamma_2}{2u - \beta_1} - \gamma_1 = 0. \tag{7}$$

Let $G(u) = u^2 - (\frac{\beta_2 u+\gamma_2}{2u-\beta_1})^2 - \beta_1 u + \beta_2 \frac{\beta_2 u+\gamma_2}{2u-\beta_1} - \gamma_1$, we have: $\frac{dG(u)}{du} = \frac{1}{(2u-\beta_1)^3}[(2u - \beta_1)^4 + (\beta_1\beta_2 + 2\gamma_2)^2]$, which means that $G(u)$ monotonically increases from a negative value $u = 0.5\beta_1$ (because of $f_2 \geq 0$ and $f_3 < 0$). Therefore, all roots of $G(u) = 0$ are negative if and only if $G(0) \geq 0$: $(\frac{\gamma_2}{\beta_1})^2 + \beta_2\frac{\gamma_2}{\beta_1} + \gamma_1 \leq 0$ for any perturbation mode, which implies:$f_1 < \min_k \frac{1}{\sin^2\alpha_k}\{1 - \cos\alpha_k - \frac{f_2\sin^2\alpha_k}{f_2(\cos\alpha_k - 1)+f_3}\}[f_2(\cos\alpha_k - 1) + f_3]^2$, and we have $f_1 < (\frac{1}{2} - \frac{f_2}{f_3})f_3^2$ along with $\cos\alpha_k$ approaching 1. Then, the stable condition of homogenous traffic flow is described as follows:

$$f_1 + f_2 f_3 - \frac{1}{2}f_3^2 < 0. \tag{8}$$

Actually, we can directly find out the roots of Eq. (7) and derive the same formula (see the Appendix). According to the stable condition, we define a stable function according to (8):

$$F(\rho)|_{(s_h(\rho),0,v_h(\rho))} = f_1 + f_2 f_3 - \frac{1}{2}f_3^2 \tag{9}$$

which depicts the boundary of the different behavior of traffic flow and provides us a criterion to verify the stability of homogenous flow, or detect the homogenous congested flow at high density region. Actually, this criterion could be applied to all car-following models. Take the intelligent driver model for example, $f_1 = 2a\frac{(s_0+Tv_h)^2}{s_h^3}$, $f_2 = \sqrt{\frac{a}{b}}\frac{(s_0+Tv_h)v_h}{s_h^2}$, $f_3 = -2a\{\frac{(s_0+Tv_h)T}{s_h^2} + \frac{\delta}{2v_h}(\frac{v_h}{v_0})^\delta\}$, we have the stable condition for the homogenous solution:

$$s_h - \sqrt{\frac{a}{b}}Tv_h - aT^2 - \frac{\delta s_h^2(\frac{v_h}{v_0})^\delta}{2v_h(s_0 + Tv_h)}\left(aT + \sqrt{\frac{a}{b}}v_h + \frac{a\delta(\frac{v_h}{v_0})^\delta s_h^2}{2v_h(s_0 + Tv_h)}\right) < 0. \tag{10}$$

Given the high density, both $\frac{v_h}{v_0} \to 0$ and $s_h - \sqrt{\frac{a}{b}}Tv_h - aT^2 < 0$ hold. Using $s_h = \frac{s_0+Tv_h}{\sqrt{1-(\frac{v_h}{v_0})^\delta}} = \frac{1}{\rho} - l$, we have $s_0 + (\sqrt{\frac{b}{a}} - 1)(\frac{1}{\rho} - l) - \sqrt{ab}T^2 < 0$ and high density $\frac{1}{\rho} \to l + s_0$, we have the stable condition of homogenous congested flow of the intelligent driver model:

$$s_0 - aT^2 < 0. \tag{11}$$

Recalling the deceleration, $a_{brake} \approx \frac{s_0}{T^2}$, the above formula implies that an impatient driver has a larger deceleration $a_{brake} > a$, while a patient driver has a smaller deceleration $a_{brake} < a$. The homogenous congested flow is stable for the patient driver and unstable for the impatient driver. Here we plot stable function (9) of the patient and impatient drivers in Fig. 2a. As shown in Fig. 2a, the homogenous flow of the intelligent driver model is stable at low density, and loses its stability at intermediate density. The homogenous flow of a patient driver reobtains the stability at high density region, however, the homogenous flow of impatient flow is still instable.

The optimal velocity model is one kind of car following model [12–15], but its behavior in the high global density is different from that of the intelligent driver model. Fig. 2b illustrates a typical case of optimal velocity model, $\dot{v} = \frac{1}{\tau}\{f(s) - v\} + \Delta v g(s, v)$, which indicates that homogenous free traffic flow loses its stability and then the homogenous congested flow obtains the stability.

5. The phase diagram of traffic flow

Let $F(\rho) = f_1 + f_2 f_3 - \frac{1}{2}f_3^2 = 0$, we obtain the stable domain of homogenous flow in the parameter space. Considering the intelligent driver model, we have a hyperplane separating the space (v_h, s_0, T) into two domains, the stable one and the unstable one to the homogenous flow solution. We can also easily solve the same hyperplane of the space (ρ, s_0, T) by letting the global density

$$\rho = \frac{1}{l + s_h} = \frac{\sqrt{1 - (\frac{v_h}{v_0})^\delta}}{s_0 + Tv_h + l\sqrt{1 - (\frac{v_h}{v_0})^\delta}}.$$

As shown in Fig. 3a, outside of the hyperplane, homogeneous flow is stable in domain I, where $F(\rho, s_0, T) < 0$, while inside of the hyperplane, homogeneous flow is unstable in domain II where $F(\rho, s_0, T) > 0$.

The homogenous flow is stable and sustainable in domain I, and the homogenous flow loses its stability and evolves into the synchronized flow or the wide moving flow in domain II. Fig. 3a gives us the stable region of the homogenous flow, but

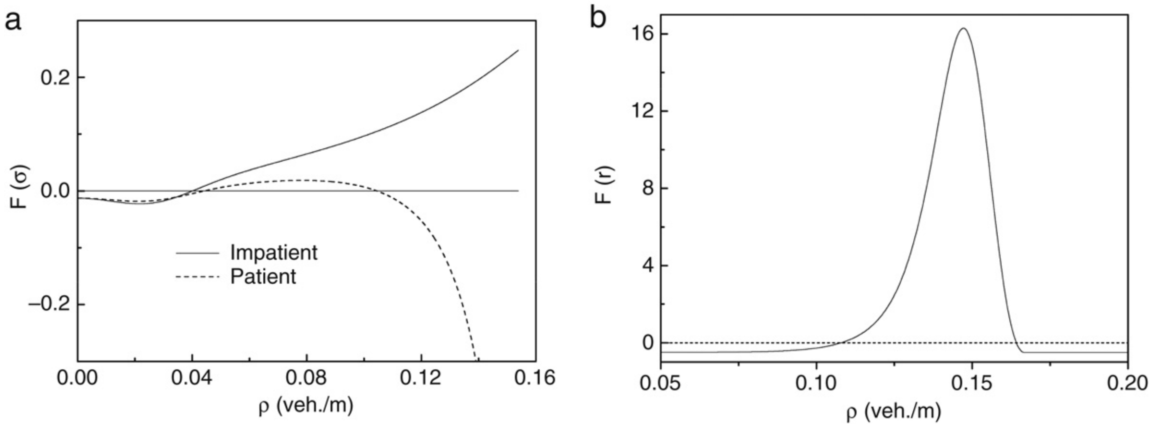

Fig. 2. Stable function. The density is from 0 to the maximum $\frac{1}{l}$. (a): Aforementioned three different cases of intelligent driver model. The stable function of patient driver traverses the zero line, $F(\rho) = 0$, twice, which means that the homogenous flow loses the stability at intermediate region and then regains the stability at high density region. The stable function of impatient driver traverses the zero line only once, which indicates that the homogenous flow loses its stability at the intermediate and high density region. (b): A typical case of optimal velocity models defined by Ref. [11]. And the stable function traverses the zero line twice.

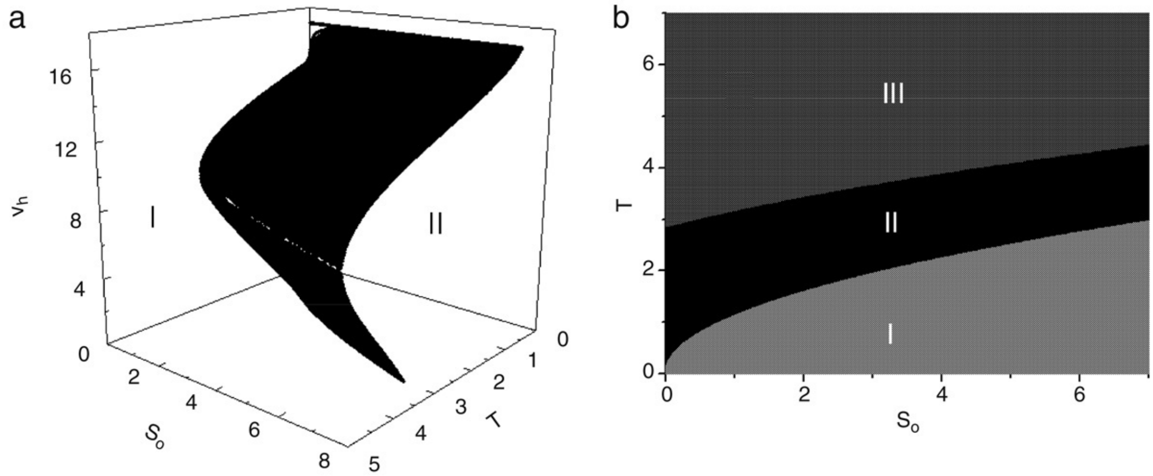

Fig. 3. Stable domain of homogenous flow solution and the pathways of phase transitions of traffic flow. (a) The hyperplane separates the space of (v_h, s_0, T) into two domains; One is stable and the other is unstable to homogenous traffic flow solution. (b) The projection of the hyperplane into plane (s_0, T), which illustrates the three different pathways of the phase transition of traffic flow along with the increase of density: from the homogenous flow to the synchronized flow, then to the wide jammed flow (I); from the homogenous flow to the synchronized flow, and then to the homogenous congested flow (II); and from the homogenous flow at the low density region directly to the homogenous flow at the high density region (III).

it does not clearly tell us the pathway of phase transition. We project the hyperplane into the parameter space of (s_0, T) and get three different regions according to the number of points of the hyperplane projected to the plane (s_0, T). Region I has only one point of hyperplane projected. The traffic flow with parameters in this region will experience the phase transition from homogenous flow at low density, to synchronized flow at intermediate density, and to the wide moving jam flow at high density. The second region, II, has two points of hyperplane projected, which indicates that the homogenous flow can reobtain the lost stability and the traffic flow undergoes the phase transition from homogenous flow at low density, to synchronized flow at intermediate density and then to the homogenous congested flow at high density. Region III has no point of hyperplane projected, which implies that the homogenous flow maintains stability for all possible densities. The homogeneous free traffic flow will directly change into homogeneous congested flow. The velocity of a vehicle in the traffic flow with the parameters in this region will continuously decrease along with the increase of the density.

The boundary between region III and region II can be looked as a critical line, which lays out a critical hyperplane in the space of (v_h, s_0, T). The critical line is determined by $\frac{\partial}{\partial T}F(\rho) = 0$. The traffic flow at this hyperplane will directly transfer from free flow to homogenous congested flow. The boundary between region I and region II follows the function $s_0 = aT^2$ and lays out another hyperplane in the space of (v_h, s_0, T), we call it a wide jam hyperplane. The stable hyperplane, critical hyperplane and the wide jam hyperplane separate the space (v_h, s_0, T) into 6 domains. The domain, inside the stable hyperplane and satisfying $s_0 - aT^2 > 0$, includes two phases: the synchronized flow and the wide moving flow. The boundary

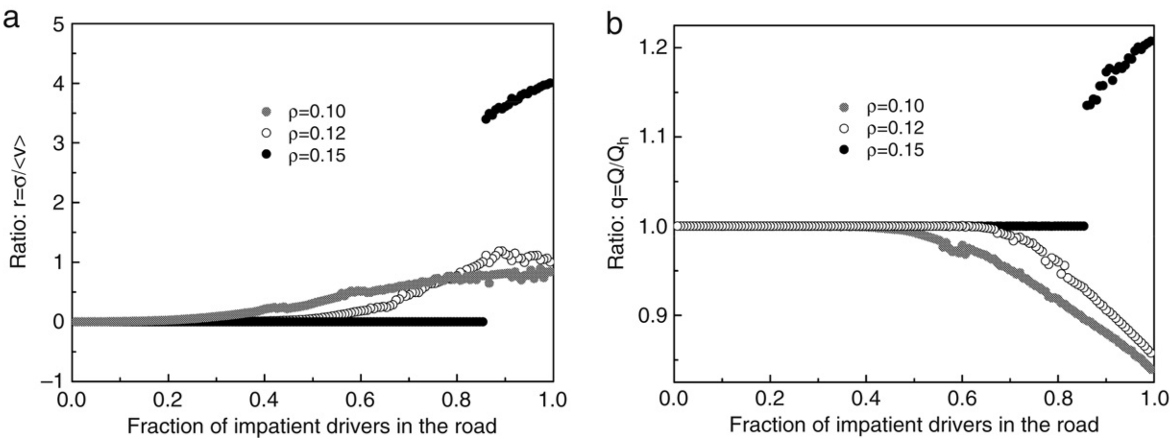

Fig. 4. The effects of impatient driver on the traffic flow. (a), The relative deviation of velocity in traffic flow becoming larger than zero indicates that vehicles in the traffic flow are not a constant and the homogenous flow loses its stability. (b), the ratio of flux in real traffic flow to that of homogenous traffic flow. All parameters are the same as those in Fig. 1. The figure shows that homogenous congested flow loses its stability along with the increase of impatient drivers in the traffic flow. But, the critical value of impatient driver and its consequent final state is related to the global density. The critical fraction of impatient driver p is: 0.21, 0.51, 0.85 for ρ equal to 0.1, 0.12, and 0.15, respectively.

of these two phases can not be provided by the stable function, due to the existence of the mixed phase mentioned before. However, each of the other five domains describes only one phase of traffic flow.

6. The effect of heterogenous drivers on the phase transition of traffic flow

In the above sections, drivers in the traffic flow are homogenous, which obviously is an ideal condition and not the case in the real world. Actually, we can investigate more realistic traffic flows based on the above discussion by introducing heterogeneity of drivers into the traffic system. Given a homogenous congested flow in which all drivers are patient, a fraction of these patient drivers, p for example, become impatient drivers for some reasons at one moment. So, we have a traffic flow with heterogenous drivers at high density region. Because the minimum desired headway of impatient drivers is smaller than that of patient drivers, not only impatient drivers acquire acceleration but also the following drivers do. The consequence is that traffic flow evolves into a new state according to the dynamics of Eq. (1). If the traffic flow with heterogenous drivers can evolve into a new homogenous congested traffic flow in which impatient and patient drivers have the same velocity v_h and zero acceleration, we have the desired headway of impatient and patient drivers: $s_{hi} = \frac{s_0+v_h T_i}{\sqrt{1-(\frac{v_h}{v_0})^\delta}}$, $s_{hp} = \frac{s_0+v_h T_p}{\sqrt{1-(\frac{v_h}{v_0})^\delta}}$, and satisfies: $ps_{hi} + (1-p)s_{hp} = \frac{1}{\rho} - l$. These two conditions give us the velocity of homogeneous traffic flow: $v_h = \frac{(\frac{1}{\rho}-l)\sqrt{1-(\frac{v_h}{v_0})^\delta}-s_0}{pT_i+(1-p)T_p} \sim \frac{\frac{1}{\rho}-l-s_0}{pT_i+(1-p)T_p}$ which is the same as that of homogeneous traffic flow produced by an "effective" identical driver with parameters s and T as weighted averages of respective drivers [16]. If the new homogenous congested flow is stable, the necessary condition must satisfy: $p(s_{hi}-\sqrt{\frac{a}{b}}T_i v_h - aT_i^2)+(1-p)(s_{pi}-\sqrt{\frac{a}{b}}T_p v_h - aT_p^2) < 0$, which is not the same as that of "effective" identical drivers. Now, we can approximately work out the critical fraction of impatient drivers that homogenous congested flow loses its stability: $p_{cr} < \frac{aT_p^2-[s_0+(1-\sqrt{\frac{a}{b}})(\frac{1}{\rho}-l-s_0)]}{a(T_p^2-T_i^2)}$. Given different densities $\rho = 0.15, 0.12$, and 0.10, the corresponding critical fraction of impatient drivers is $p_{cr} = 0.80$, 0.53, and 0.26, which is close to the simulation results 0.85, 0.51 and 0.21 (see Fig. 4). The homogenous traffic flow will lose its stability and evolve into a different type of traffic flow if the fraction of impatient drivers is larger than p_{cr}. As shown in Fig. 4, the traffic flow with density $\rho = 0.15$ finally evolves into a wide-moving jam flow when the fraction of impatient drivers is larger than $p_{cr} = 0.85$. The traffic flow with density $\rho = 0.12$ and 0.10 loses stability and evolves into an inhomogenous traffic flow. Results in Fig. 5 show that traffic flow with density $\rho = 0.12$ and 93% impatient drivers evolves into a mixed state: several pieces of synchronized flow and a wide-moving jam flow simultaneously coexists in the traffic flow. Our results indicate that homogenous congested flow will lose its stability along with the increase of impatient drivers and the heterogeneous drivers enrich the traffic behavior of traffic flow, for example, the mixture of synchronized flow and wide-moving jam flow.

7. Conclusion and discussion

As conclusions and discussion of this paper, we want to make several points. Firstly, the homogenous congested flow and the wide-moving jam flow are demonstrated by the numerical simulation of the intelligent driver model. According to this conclusion, we argue that it is the driver's habit, such as the jam distance s_0, safe time headway T, and even the

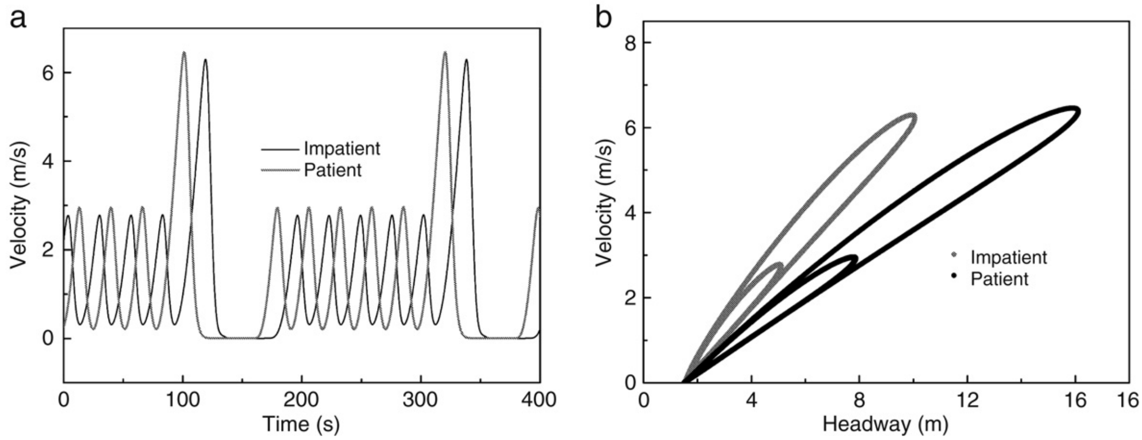

Fig. 5. The mixed traffic flow of synchronized flow and wide-moving jam flow. There are 93% impatient drivers in the traffic flow at density $\rho = 0.12$. (a), The typical velocity of impatient and patient drivers over time shows that there are several synchronized flows (velocity is not zero at the minimum) and a wide-moving jam flow (velocity equal to zero for a long time). (b), The typical velocity of impatient and patient drivers to their headway. This figure indicates two facts: patient driver has a larger headway than that of impatient driver at the same speed; patient and impatient drivers also experience approximately the same velocity evolution course but different headway evolving course in the free traffic flow [12], which implies that there are different jams in the traffic flow combining with results in (a).

acceleration form, that accounts for different phases of traffic flow at high density region. At the same time, the flux of wide-moving jam flow is larger than that of homogenous congested flow in the high density region, which is an interesting phenomenon.

Secondly, the phase diagram of traffic flow with homogenous drivers is presented using the linearization analysis method. The phase diagram helps us understand the behavior of traffic flow from a unified view. At the same time, our investigation is helpful for understanding the different phases and phase transition of the closed homogenous interacting many-particles system, since all of those phase transition pathways in our paper are presented in other physical, social, biological, and chemical systems [10,17,18]. In addition, even though the phase diagram is based on the intelligent driver model, other continuous car-following models' phase diagram and possible pathways of phase transition also could be worked out following the same lines in a closed circular one-lane roadway system.

Thirdly, we investigate the traffic flow including patient and impatient drivers. Homogenous congested flow will lose its stability along with the increase of impatient drivers, and the traffic flow becomes a wide-moving jam flow, mixture of synchronized and wide-moving jam flow, or a new homogenous congested flow. It can be concluded as the more impatient drivers, the easier the homogenous flow loses its stability and forms a more complex traffic flow.

At last, it should be noted that the traffic system we investigate is a closed system which is not common in the real world. At the same time, the closed circular system has some limitations to explore all possible phases of traffic flow because the closed homogeneous traffic system can not exhibit all possible phases of traffic flow, for example, the "pinned localized cluster" or "localized synchronized pattern" [6]. Therefore, we should explore the open traffic system and investigate all possible phases of dynamical congested states of traffic flow in the future.

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Appendix

Let $v = 2u - \beta_1$, rewrite (7) as follows:

$$v^2 - \frac{(2\gamma + \beta_1\beta_2)^2}{v^2} - \beta_1^2 + \beta_2^2 - 4\gamma_1 = 0 \tag{A.1}$$

so, we have:

$$u = \frac{\beta_1}{2} \pm \frac{1}{4}\sqrt{\theta} \tag{A.2}$$

where $\theta = 2(\beta_1^2 - \beta_2^2 + 4\gamma_1 \pm \sqrt{\delta})$ and $\delta = (\beta_1^2 - \beta_2^2 + 4\gamma_1)^2 + 4(\beta_1\beta_2 + 2\gamma_2)^2$. The stability requires that all root of $u < 0$, so we have $u = \frac{\beta_1}{2} - \frac{1}{4}\sqrt{\theta} < \frac{\beta_1}{2} < 0$ and $u = \frac{\beta_1}{2} + \frac{1}{4}\sqrt{\theta} < 0$ if and only if $\theta < 4\beta_1^2$, if and only if $\delta < (\beta_1^2 + \beta_2^2 - 4\gamma_1)^2$, if and only if $\gamma_2^2 + \beta_1\beta_2\gamma_2 + \beta_1^2\gamma_1 < 0$. This formula can be rewritten as the form of (8).

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