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Periodic solutions in non autonomous predator prey system with delays

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Abstract

In this paper, we consider a biological model for two predators and one prey with periodic delays. By assuming that one predator consumes prey according to Holling II functional response while the other predator consumes prey according to the Beddington–DeAngelis functional response, based on the coincidence degree theory, the existence of positive periodic solutions for this model is obtained under suitable conditions.

Keywords: Periodic solution; Periodic delay; Predator–prey system; Coincidence degree theory

1. Introduction

In the pioneering work on predator–prey systems by Lotka [11] and Volterra [14] in the 1920s, they concluded that the coexistence of two or more predators competing for fewer prey resources is impossible, which was later known as the principle of competitive exclusion. For instance, in the case of two predators competing for a single prey species, one considers the following Lotka–Volterra predator–prey model

$$\begin{aligned}\frac{dx}{dt} &= rx\left[1-\frac{x}{k}\right] - axy - Axz,\\ \frac{dy}{dt} &= y[-d+ex],\\ \frac{dz}{dt} &= z[-D+Ex],\end{aligned} \tag{1.1}$$

where y and z are the densities of the two predators and x is the density of the prey. The principle of competitive exclusion then implies that one of the two predators of system (1.1) usually becomes extinct.

Afterwards, it was recognized that the principle of competitive exclusion was somehow at odds with the reality of natural systems. For example, Ayala [1] demonstrated experimentally that two species of *Drosophila* could coexist

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upon a single limiting resource. By assuming that both predators consume prey according to Holling II functional response, then system (1.1) was modified to the following predator–prey system

$$\begin{aligned} \frac{dx}{dt} &= rx\left[1-\frac{x}{k}\right]-\frac{axy}{1+bx}-\frac{Axz}{1+Bx},\\ \frac{dy}{dt} &= y\left[-d+\frac{ex}{1+bx}\right],\\ \frac{dz}{dt} &= z\left[-D+\frac{Ex}{1+Bx}\right]. \end{aligned} \tag{1.2}$$

Based on dynamical system techniques and the geometrical singular perturbation theory, Liu, Xiao and Yi [10] showed the coexistence of predators and prey of system (1.2) which happened along a stable periodic orbit in the positive octant of $\mathbb{R}^3$.

By assuming that one predator consumes prey according to the Holling II functional response and the other predator consumes prey according to the Beddington–DeAngelis functional response, then system (1.1) was modified to the following predator–prey system

$$\begin{aligned} \frac{dx}{dt} &= rx\left[1-\frac{x}{k}\right]-\frac{axy}{1+bx}-\frac{Axz}{1+Bx+Cz},\\ \frac{dy}{dt} &= y\left[-d+\frac{ex}{1+bx}\right],\\ \frac{dz}{dt} &= z\left[-D+\frac{Ex}{1+Bx+Cz}\right]. \end{aligned} \tag{1.3}$$

Based on dynamical system theory, particularly persistence theory, Cantrell, Cosner and Ruan [2] showed the coexistence of predators and prey of system (1.3) which occurred along a stable positive equilibrium.

In this paper, we consider the following delay version of system (1.3)

$$\begin{aligned} \frac{dx(t)}{dt} &= r(t)x(t)\left[1-\frac{x(t)}{k}\right]-\frac{a(t)x(t)y(t)}{1+bx(t)}-\frac{A(t)x(t)z(t)}{1+Bx(t)+Cz(t)},\\ \frac{dy(t)}{dt} &= y(t)\left[-d(t)+\frac{e(t)x(t-\tau(t))}{1+bx(t-\tau(t))}\right],\\ \frac{dz(t)}{dt} &= z(t)\left[-D(t)+\frac{E(t)x(t-\sigma(t))}{1+Bx(t-\sigma(t))+Cz(t-\sigma(t))}\right], \end{aligned} \tag{1.4}$$

where $x(t)$ stands for the prey's density, $y(t)$ and $z(t)$ stand for the predators' densities respectively. Functions $r(t), a(t), A(t), d(t), e(t), D(t), E(t)$ are positive ω-periodic continuous functions, $\tau(t)$ and $\sigma(t)$ are nonnegative ω-periodic continuous functions, k, b, B and C are positive constants.

One of the reasons for introducing a delay into a predator–prey system is that the rate of reproduction of predators depends on the rate at which they have consumed the prey in the past, and this idea is well justified [6,7,15]. Usually Hopf bifurcation theory is used to investigate the existence of periodic solutions for predator–prey system with a single constant delay [5,12,13]. Recently, the geometrical singular perturbation theory [4] has been employed by Lin and Yuan [8,9] to study the existence of periodic solutions for predator–prey systems with a single delay or multiple delays.

In this paper, based on the coincidence degree theory developed by Gains and Mawhin [3], we will justify the existence of periodic solutions for predator–prey system with periodic delays (1.4) under some natural and easily verifiable conditions. This method has been adopted by Li and Kuang [7] to prove the existence of positive periodic solutions for Lotka–Volterra equations or systems with periodic delays.

2. Existence of positive periodic solution

In order to obtain the existence of a positive periodic solution for system (1.4), we present the following result from Gains and Mawhin [3] about the coincidence degree theory.

Let X and Z be two Banach spaces. Let $L : \text{Dom}\, L \subset X \to Z$ be a linear operator and $N : X \to Z$ be a continuous operator. The operator L is call a Fredholm operator of index zero if $\dim\text{Ker}\, L = \text{CodimIm}\, L = Z/\text{Im}\, L < \infty$ and $\text{Im}\, L$ is closed in Z.

If L is a Fredholm operator of index zero, then continuous projections $P : X \to X$ and $Q : Z \to Z$ exist such that $\text{Im}\, P = \text{Ker}\, L$ and $\text{Im}\, L = \text{Ker}\, Q = \text{Im}(I - Q)$. It follows that $L|\text{Dom}\, L \cap \text{Ker}\, P : (I - P)X \to \text{Im}\, L$ is invertible and its inverse is denoted by K_P.

If Ω is a bounded open subset of X, the operator N is called L-compact on $\overline{\Omega}$ if $(QN)(\overline{\Omega})$ is bounded and $K_P(I - Q)N : \overline{\Omega} \to X$ is compact. Because $\text{Im}\, Q$ is isomorphic to $\text{Ker}\, L$, there exists an isomorphism $J : \text{Im}\, Q \to \text{Ker}\, L$.

Lemma 2.1. *Let L be a Fredholm operator of index zero and let N be L-compact on $\overline{\Omega}$. Suppose:*

(a) *for each* $\lambda \in (0, 1)$, $x \in \partial\Omega \cap \text{Dom}\, L$, *then* $Lx \neq \lambda Nx$;

(b) *for each* $x \in \partial\Omega \cap \text{Ker}\, L$, *then* $QNx \neq 0$;

(c) $\deg\{JQN, \Omega \cap \text{Ker}\, L, 0\} \neq 0$;

Then the operator $Lx = Nx$ has at least one solution lying in $\text{Dom}\, L \cap \overline{\Omega}$.

For convenience, we shall introduce the following notation

$$\overline{u} = \frac{1}{\omega}\int_0^\omega u(t)\mathrm{d}t,$$

where u is a periodic continuous function with period ω.

Let $\alpha = \frac{\overline{d}}{\overline{e} - b\overline{d}}$. Now we can state the main result in this paper as follows

Theorem 2.2. *Assume*

$$\overline{e} > b\overline{d}, \qquad \overline{E} > B\overline{D}, \qquad \alpha > \mathrm{e}^{2\overline{r}\omega}\frac{\overline{D}}{(\overline{E} - B\overline{D})}, \qquad k > \mathrm{e}^{2\overline{r}\omega}\alpha$$

and

$$\overline{r}\mathrm{e}^{-2\overline{r}\omega}\alpha\left(1 - \frac{\mathrm{e}^{2\overline{r}\omega}\alpha}{k}\right) - \frac{\overline{A}}{C\overline{E}}[(\overline{E} - B\overline{D})\mathrm{e}^{2\overline{r}\omega}\alpha - \overline{D}]\mathrm{e}^{2\overline{D}\omega} > 0.$$

Then system (1.4) has at least one positive ω-periodic solution.

In the following, we first give three important lemmas which are crucial to proving Theorem 2.2. For this purpose, we should make some preparation.

Let

$$x(t) = \exp\{x_1(t)\}, \qquad y(t) = \exp\{x_2(t)\}, \qquad z(t) = \exp\{x_3(t)\}. \tag{2.1}$$

Then system (1.4) is changed to the following system

$$\begin{aligned}
\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} &= r(t)\left[1 - \frac{\exp\{x_1(t)\}}{k}\right] - \frac{a(t)\exp\{x_2(t)\}}{1 + b\exp\{x_1(t)\}} - \frac{A(t)\exp\{x_3(t)\}}{1 + B\exp\{x_1(t)\} + C\exp\{x_3(t)\}},\\
\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} &= \left[-d(t) + \frac{e(t)\exp\{x_1(t - \tau(t))\}}{1 + b\exp\{x_1(t - \tau(t))\}}\right],\\
\frac{\mathrm{d}x_3(t)}{\mathrm{d}t} &= \left[-D(t) + \frac{E(t)\exp\{x_1(t - \sigma(t))\}}{1 + B\exp\{x_1(t - \sigma(t))\} + C\exp\{x_3(t - \sigma(t))\}}\right].
\end{aligned} \tag{2.2}$$

In order to apply Lemma 2.1 to system (2.2), we take

$$X = Z = \{x^*(t) = [x_1(t), x_2(t), x_3(t)]^{\mathrm{T}} \in C(\mathbb{R}, \mathbb{R}^3) : x^*(t+\omega) = x^*(t)\}$$

and denote

$$\|x^*\| = \|[x_1(t), x_2(t), x_3(t)]^{\mathrm{T}}\| = \max_{t\in[0,\omega]} |x_1(t)| + \max_{t\in[0,\omega]} |x_2(t)| + \max_{t\in[0,\omega]} |x_3(t)|.$$

Then it can be shown that X and Z are Banach spaces.

Let

$$Nx^* = \begin{pmatrix} r(t)\left[1 - \dfrac{\exp\{x_1(t)\}}{k}\right] - \dfrac{a(t)\exp\{x_2(t)\}}{1 + b\exp\{x_1(t)\}} - \dfrac{A(t)\exp\{x_3(t)\}}{1 + B\exp\{x_1(t)\} + C\exp\{x_3(t)\}} \\ \left[-d(t) + \dfrac{e(t)\exp\{x_1(t-\tau(t))\}}{1 + b\exp\{x_1(t-\tau(t))\}}\right] \\ \left[-D(t) + \dfrac{E(t)\exp\{x_1(t-\sigma(t))\}}{1 + B\exp\{x_1(t-\sigma(t))\} + C\exp\{x_3(t-\sigma(t))\}}\right] \end{pmatrix} \tag{2.3}$$

and let

$$Lx^* = x^{*\prime}, \qquad Px^* = \frac{1}{\omega}\int_0^\omega x^*(t)\mathrm{d}t, \quad x^* \in X, \qquad Qz^* = \frac{1}{\omega}\int_0^\omega z^*(t)\mathrm{d}t, \quad z^* \in Z.$$

Then it follows that

$$\operatorname{Im} L = \left\{z^* | z^* \in Z, \int_0^\omega z^*(t)\mathrm{d}t = 0\right\} \quad \text{is closed in } Z,$$
$$\operatorname{Ker} L = \{x^* | x^* \in X, x^* = \mathbb{R}^3\}, \quad \operatorname{dimker}\ L = \operatorname{CodimIm} L = 3.$$

Therefore, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_p : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ has the following form

$$K_p(z^*) = \int_0^t z^*(s)\mathrm{d}s - \frac{1}{\omega}\int_0^\omega \int_0^t z^*(s)\mathrm{d}s\mathrm{d}t.$$

Thus

$$QNx^* = \begin{pmatrix} \dfrac{1}{\omega}\displaystyle\int_0^\omega \left[r(t)\left(1 - \dfrac{\exp\{x_1(t)\}}{k}\right) - \dfrac{a(t)\exp\{x_2(t)\}}{1 + b\exp\{x_1(t)\}} \right. \\ \left. - \dfrac{A(t)\exp\{x_3(t)\}}{1 + B\exp\{x_1(t)\} + C\exp\{x_3(t)\}}\right]\mathrm{d}t \\ \dfrac{1}{\omega}\displaystyle\int_0^\omega \left[-d(t) + \dfrac{e(t)\exp\{x_1(t-\tau(t))\}}{1 + b\exp\{x_1(t-\tau(t))\}}\right]\mathrm{d}t \\ \dfrac{1}{\omega}\displaystyle\int_0^\omega \left[-D(t) + \dfrac{E(t)\exp\{x_1(t-\sigma(t))\}}{1 + B\exp\{x_1(t-\sigma(t))\} + C\exp\{x_3(t-\sigma(t))\}}\right]\mathrm{d}t \end{pmatrix} \tag{2.4}$$

and

$$
K_p(I-Q)Nx^* = \begin{pmatrix}
\int_0^t \left[r(s)\left(1-\frac{\exp\{x_1(s)\}}{k}\right) - \frac{a(s)\exp\{x_2(s)\}}{1+b\exp\{x_1(s)\}} - \frac{A(s)\exp\{x_3(s)\}}{1+B\exp\{x_1(s)\}+C\exp\{x_3(s)\}}\right]\mathrm{d}s \\
\quad -\frac{1}{\omega}\int_0^\omega\int_0^t \left[r(s)\left(1-\frac{\exp\{x_1(s)\}}{k}\right) - \frac{a(s)\exp\{x_2(s)\}}{1+b\exp\{x_1(s)\}} \right. \\
\quad \left. - \frac{A(s)\exp\{x_3(s)\}}{1+B\exp\{x_1(s)\}+C\exp\{x_3(s)\}}\right]\mathrm{d}s\mathrm{d}t \\
\quad -\left[\frac{t}{\omega}-\frac{1}{2}\right]\int_0^\omega \left[r(s)\left(1-\frac{\exp\{x_1(s)\}}{k}\right) - \frac{a(s)\exp\{x_2(s)\}}{1+b\exp\{x_1(s)\}} \right. \\
\quad \left. - \frac{A(s)\exp\{x_3(s)\}}{1+B\exp\{x_1(s)\}+C\exp\{x_3(s)\}}\right]\mathrm{d}s \\
\int_0^t \left[-d(s)+\frac{e(s)\exp\{x_1(s-\tau(s))\}}{1+b\exp\{x_1(s-\tau(s))\}}\right]\mathrm{d}s \\
\quad -\frac{1}{\omega}\int_0^\omega\int_0^t \left[-d(s)+\frac{e(s)\exp\{x_1(s-\tau(s))\}}{1+b\exp\{x_1(s-\tau(s))\}}\right]\mathrm{d}s\mathrm{d}t \\
\quad -\left[\frac{t}{\omega}-\frac{1}{2}\right]\int_0^\omega \left[-d(s)+\frac{e(s)\exp\{x_1(s-\tau(s))\}}{1+b\exp\{x_1(s-\tau(s))\}}\right]\mathrm{d}s \\
\int_0^t \left[-D(s)+\frac{E(s)\exp\{x_1(s-\sigma(s))\}}{1+B\exp\{x_1(s-\sigma(s))\}+C\exp\{x_3(s-\sigma(s))\}}\right]\mathrm{d}s \\
\quad -\frac{1}{\omega}\int_0^\omega\int_0^t \left[-D(s)+\frac{E(s)\exp\{x_1(s-\sigma(s))\}}{1+B\exp\{x_1(s-\sigma(s))\}+C\exp\{x_3(s-\sigma(s))\}}\right]\mathrm{d}s\mathrm{d}t \\
\quad -\left[\frac{t}{\omega}-\frac{1}{2}\right]\int_0^\omega \left[-D(s)+\frac{E(s)\exp\{x_1(s-\sigma(s))\}}{1+B\exp\{x_1(s-\sigma(s))\}+C\exp\{x_3(s-\sigma(s))\}}\right]\mathrm{d}s.
\end{pmatrix} \tag{2.5}
$$

It can be seen that QN and $K_p(I-Q)N$ are continuous. Moreover, it can be verified that $\overline{K_p(I-Q)N(\overline{\Omega})}$ is compact for any open bounded set $\Omega \subset X$ by using Arzela–Ascoli theorem and $QN(\overline{\Omega})$ is bounded. Therefore, N is L-compact on $\overline{\Omega}$ for any open bounded subset Ω in X.

Now we are in a position to construct an appropriate open bounded subset Ω in X for the application of Lemma 2.1 to system (2.2).

Corresponding to the operator equation $Lx^* = \lambda Nx^*$, we have

$$
\begin{aligned}
\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} &= \lambda\left[r(t)\left(1-\frac{\exp\{x_1(t)\}}{k}\right) - \frac{a(t)\exp\{x_2(t)\}}{1+b\exp\{x_1(t)\}} - \frac{A(t)\exp\{x_3(t)\}}{1+B\exp\{x_1(t)\}+C\exp\{x_3(t)\}}\right], \\
\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} &= \lambda\left[-d(t)+\frac{e(t)\exp\{x_1(t-\tau(t))\}}{1+b\exp\{x_1(t-\tau(t))\}}\right], \\
\frac{\mathrm{d}x_3(t)}{\mathrm{d}t} &= \lambda\left[-D(t)+\frac{E(t)\exp\{x_1(t-\sigma(t))\}}{1+B\exp\{x_1(t-\sigma(t))\}+C\exp\{x_3(t-\sigma(t))\}}\right].
\end{aligned} \tag{2.6}
$$

Suppose that $x^*(t) = [x_1(t), x_2(t), x_3(t)]^{\mathrm{T}} \in X$ is a solution of system (2.6) for a certain $\lambda \in (0, 1)$. By integrating system (2.6) over the interval $[0, \omega]$, we obtain

$$
\begin{aligned}
&\int_0^\omega \left[r(t)\left(1-\frac{\exp\{x_1(t)\}}{k}\right) - \frac{a(t)\exp\{x_2(t)\}}{1+b\exp\{x_1(t)\}} - \frac{A(t)\exp\{x_3(t)\}}{1+B\exp\{x_1(t)\}+C\exp\{x_3(t)\}}\right]\mathrm{d}t = 0, \\
&\int_0^\omega \left[-d(t)+\frac{e(t)\exp\{x_1(t-\tau(t))\}}{1+b\exp\{x_1(t-\tau(t))\}}\right]\mathrm{d}t = 0, \\
&\int_0^\omega \left[-D(t)+\frac{E(t)\exp\{x_1(t-\sigma(t))\}}{1+B\exp\{x_1(t-\sigma(t))\}+C\exp\{x_3(t-\sigma(t))\}}\right]\mathrm{d}t = 0.
\end{aligned} \tag{2.7}
$$

Therefore,

$$\int_0^\omega \left[\frac{r(t)\exp\{x_1(t)\}}{k} + \frac{a(t)\exp\{x_2(t)\}}{1+b\exp\{x_1(t)\}} + \frac{A(t)\exp\{x_3(t)\}}{1+B\exp\{x_1(t)\}+C\exp\{x_3(t)\}}\right]\mathrm{d}t = \overline{r}\omega, \tag{2.8}$$

$$\int_0^\omega \frac{e(t)\exp\{x_1(t-\tau(t))\}}{1+b\exp\{x_1(t-\tau(t))\}}\mathrm{d}t = \overline{d}\omega \tag{2.9}$$

and

$$\int_0^\omega \frac{E(t)\exp\{x_1(t-\sigma(t))\}}{1+B\exp\{x_1(t-\sigma(t))\}+C\exp\{x_3(t-\sigma(t))\}}\mathrm{d}t = \overline{D}\omega. \tag{2.10}$$

By (2.6) and (2.8)–(2.10), we obtain

$$\begin{aligned}\int_0^\omega |x_1'(t)|\mathrm{d}t &\le \int_0^\omega |r(t)|\mathrm{d}t + \int_0^\omega \left[\frac{r(t)\exp\{x_1(t)\}}{k} + \frac{a(t)\exp\{x_2(t)\}}{1+b\exp\{x_1(t)\}}\right. \\ &\quad \left. + \frac{A(t)\exp\{x_3(t)\}}{1+B\exp\{x_1(t)\}+C\exp\{x_3(t)\}}\right]\mathrm{d}t = 2\overline{r}\omega, \end{aligned} \tag{2.11}$$

$$\int_0^\omega |x_2'(t)|\mathrm{d}t \le \int_0^\omega |d(t)|\mathrm{d}t + \int_0^\omega \frac{e(t)\exp\{x_1(t-\tau(t))\}}{1+b\exp\{x_1(t-\tau(t))\}}\mathrm{d}t = 2\overline{d}\omega \tag{2.12}$$

and

$$\int_0^\omega |x_3'(t)|\mathrm{d}t \le \int_0^\omega |D(t)|\mathrm{d}t + \int_0^\omega \frac{E(t)\exp\{x_1(t-\sigma(t))\}}{1+B\exp\{x_1(t-\sigma(t))\}+C\exp\{x_3(t-\sigma(t))\}}\mathrm{d}t = 2\overline{D}\omega. \tag{2.13}$$

Note that $[x_1(t), x_2(t), x_3(t)]^{\mathrm{T}} \in X$, then there exists $\xi_i, \eta_i \in [0, \omega]$, $i = 1, 2, 3$ such that

$$x_i(\xi_i) = \min_{t\in[0,\omega]} x_i(t), \qquad x_i(\eta_i) = \max_{t\in[0,\omega]} x_i(t), \quad i = 1, 2, 3. \tag{2.14}$$

The following three lemmas give priori estimates for the three components of the solution $x^*(t) = [x_1(t), x_2(t), x_3(t)]^{\mathrm{T}}$ for system (2.6) and their proofs will be given in the next section.

Lemma 2.3.

$$\max_{t\in[0,\omega]} |x_1(t)| \le \max\left\{\left|\ln\left(\frac{\overline{d}}{\overline{e}-b\overline{d}}\right) + 2\overline{r}\omega\right|, \left|\ln\left(\frac{\overline{d}}{\overline{e}-b\overline{d}}\right) - 2\overline{r}\omega\right|\right\} := B_1.$$

Lemma 2.4.

$$\begin{aligned}\max_{t\in[0,\omega]} |x_3(t)| \le \max&\left\{\left|\ln\left(\frac{\left(\overline{E}-B\overline{D}\right)\mathrm{e}^{-2\overline{r}\omega}\alpha - \overline{D}}{C\overline{D}}\right) - 2\overline{D}\omega\right|,\right. \\ &\left.\left|\ln\left(\frac{\left(\overline{E}-B\overline{D}\right)\mathrm{e}^{2\overline{r}\omega}\alpha - \overline{D}}{C\overline{D}}\right) + 2\overline{D}\omega\right|\right\} := B_3.\end{aligned}$$

Lemma 2.5.

$$\begin{aligned}\max_{t\in[0,\omega]} |x_2(t)| \le \max&\left\{\left|\ln\left[\frac{\overline{e}}{\overline{a}\,\overline{d}}\left[\overline{r}\mathrm{e}^{-2\overline{r}\omega}\alpha\left(1-\frac{\mathrm{e}^{2\overline{r}\omega}\alpha}{k}\right) - \frac{\overline{A}}{C\overline{E}}\left[\left(\overline{E}-B\overline{D}\right)\mathrm{e}^{2\overline{r}\omega}\alpha - \overline{D}\right]\mathrm{e}^{2\overline{D}\omega}\right]\right] - 2\overline{d}\omega\right|,\right. \\ &\left.\left|\ln\left[\frac{\overline{e}}{\overline{a}\,\overline{d}}\left[\overline{r}\mathrm{e}^{2\overline{r}\omega}\alpha\left(1-\frac{\mathrm{e}^{-2\overline{r}\omega}\alpha}{k}\right) - \frac{\overline{A}}{C\overline{E}}\left[\left(\overline{E}-B\overline{D}\right)\mathrm{e}^{-2\overline{r}\omega}\alpha - \overline{D}\right]\mathrm{e}^{-2\overline{D}\omega}\right]\right] + 2\overline{d}\omega\right|\right\} := B_2.\end{aligned}$$

Proof of Theorem 2.2. Based on the Lemmas 2.3–2.5, it can be seen that the constants B_i, $i = 1, 2, 3$ are independent of λ. Moreover, under the assumption in Theorem 2.2, it can be verified that the following system of algebraic equations

$$\begin{aligned}
&\bar{r}\left[1 - \frac{u_1}{k}\right] - \frac{\bar{a}u_2}{1 + bu_1} - \frac{\overline{A}u_3}{1 + Bu_1 + Cu_3} = 0,\\
&-\bar{d} + \frac{\bar{e}u_1}{1 + bu_1} = 0,\\
&-\overline{D} + \frac{\overline{E}u_1}{1 + Bu_1 + Cu_3} = 0
\end{aligned} \tag{2.15}$$

has a unique positive solution $(u_1, u_2, u_3)^{\mathrm{T}}$.

Denote

$$B = B_1 + B_2 + B_3 + B_4,$$

where $B_4 > 0$ is taken to be sufficiently large such that

$$\|(\ln u_1, \ln u_2, \ln u_3)^{\mathrm{T}}\| = |\ln u_1| + |\ln u_2| + |\ln u_3| < B_4.$$

Define

$$\Omega = \{x^*(t) \in X : \|x^*\| < B\}.$$

It is clear that Ω satisfies the condition (a) of Lemma 2.1.

Let $x^* = [x_1(t), x_2(t), x_3(t)]^{\mathrm{T}} \in \partial\Omega \cap \mathbb{R}^3$, then x^* is a constant vector in $\mathbb{R}^3$ with $\|x^*\| = B$. Therefore

$$QNx^* = \begin{pmatrix} \bar{r}\left[1 - \dfrac{\exp\{x_1\}}{k}\right] - \dfrac{\bar{a}\exp\{x_2\}}{1 + b\exp\{x_1\}} - \dfrac{\overline{A}\exp\{x_3\}}{1 + B\exp\{x_1\} + C\exp\{x_3\}} \\ -\bar{d} + \dfrac{\bar{e}\exp\{x_1\}}{1 + b\exp\{x_1\}} \\ -\overline{D} + \dfrac{\overline{E}\exp\{x_1\}}{1 + B\exp\{x_1\} + C\exp\{x_3\}} \end{pmatrix} \neq 0. \tag{2.16}$$

On the other hand, by the assumption in Theorem 2.2 and the definition of topology degree, direct calculation yields

$$\deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} = \operatorname{sgn}\begin{vmatrix} f_{x_1} & f_{x_2} & f_{x_3} \\ g_{x_1} & 0 & 0 \\ h_{x_1} & 0 & h_{x_3} \end{vmatrix}, \tag{2.17}$$

where

$$\begin{aligned}
f_{x_1} &= -\frac{\bar{r}\exp\{u_1\}}{k} + \frac{\bar{a}b\exp\{u_1 + u_2\}}{[1 + b\exp\{u_1\}]^2} + \frac{\overline{A}B\exp\{u_1 + u_3\}}{[1 + B\exp\{u_1\} + C\exp\{u_3\}]^2},\\
f_{x_2} &= -\frac{\bar{a}\exp\{u_2\}}{1 + b\exp\{u_1\}} < 0, \qquad f_{x_3} = -\frac{\overline{A}\exp\{u_3\}[1 + B\exp\{u_1\}]}{[1 + B\exp\{u_1\} + C\exp\{u_3\}]^2}\\
g_{x_1} &= \frac{\bar{e}\exp\{u_1\}}{[1 + b\exp\{u_1\}]^2} > 0, \qquad h_{x_1} = \frac{\overline{E}\exp\{u_1\}[1 + C\exp\{u_3\}]}{[1 + B\exp\{u_1\} + C\exp\{u_3\}]^2}\\
h_{x_3} &= -\frac{C\overline{E}\exp\{u_1 + u_3\}}{[1 + B\exp\{u_1\} + C\exp\{u_3\}]^2} < 0.
\end{aligned}$$

Therefore, we have

$$\begin{vmatrix} f_{x_1} & f_{x_2} & f_{x_3} \\ g_{x_1} & 0 & 0 \\ h_{x_1} & 0 & h_{x_3} \end{vmatrix} = -f_{x_2}g_{x_1}h_{x_3} < 0.$$

It follows that

$$\deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} = \operatorname{sgn} \begin{vmatrix} f_{x_1} & f_{x_2} & f_{x_3} \\ g_{x_1} & 0 & 0 \\ h_{x_1} & 0 & h_{x_3} \end{vmatrix} = -1 \neq 0. \tag{2.18}$$

By now we have proved that Ω satisfies all the requirements in Lemma 2.1. Therefore, system (2.2) has at least one ω-periodic solution. Hence, by (2.1), system (1.4) has at least one positive ω-periodic solution, this completes the proof of Theorem 2.2. □

3. Proof of lemmas

In this section, we give the proof of Lemmas 2.3–2.5.

Proof of Lemma 2.3. By (2.9) and (2.14), we have

$$\overline{d}\omega \geq \frac{\overline{e}\omega \exp\{x_1(\xi_1)\}}{1 + b\exp\{x_1(\xi_1)\}}, \tag{3.1}$$

that is

$$\frac{\exp\{x_1(\xi_1)\}}{1 + b\exp\{x_1(\xi_1)\}} \leq \frac{\overline{d}}{\overline{e}}. \tag{3.2}$$

By the assumption in Theorem 2.2, it follows that

$$x_1(\xi_1) \leq \ln\left(\frac{\overline{d}}{\overline{e} - b\overline{d}}\right). \tag{3.3}$$

By (2.11) and (3.3), we have

$$x_1(t) \leq x_1(\xi_1) + \int_0^\omega |x_1'(t)|\mathrm{d}t \leq \ln\left(\frac{\overline{d}}{\overline{e} - b\overline{d}}\right) + 2\overline{r}\omega. \tag{3.4}$$

On the other hand, by (2.9) and (2.14), we have

$$\overline{d}\omega \leq \frac{\overline{e}\omega \exp\{x_1(\eta_1)\}}{1 + b\exp\{x_1(\eta_1)\}}, \tag{3.5}$$

that is

$$\frac{\exp\{x_1(\eta_1)\}}{1 + b\exp\{x_1(\eta_1)\}} \geq \frac{\overline{d}}{\overline{e}}. \tag{3.6}$$

By the assumption in Theorem 2.2, it follows that

$$x_1(\eta_1) \geq \ln\left(\frac{\overline{d}}{\overline{e} - b\overline{d}}\right). \tag{3.7}$$

By (2.11) and (3.7), we have

$$x_1(t) \geq x_1(\eta_1) - \int_0^\omega |x_1'(t)|\mathrm{d}t \geq \ln\left(\frac{\overline{d}}{\overline{e} - b\overline{d}}\right) - 2\overline{r}\omega. \tag{3.8}$$

Therefore, By (3.4) and (3.8), we have

$$\max_{t\in[0,\omega]} |x_1(t)| \leq \max\left\{\left|\ln\left(\frac{\overline{d}}{\overline{e} - b\overline{d}}\right) + 2\overline{r}\omega\right|, \left|\ln\left(\frac{\overline{d}}{\overline{e} - b\overline{d}}\right) - 2\overline{r}\omega\right|\right\}. \tag{3.9}$$

This completes the proof of Lemma 2.3. □

Proof of Lemma 2.4. By (2.10) and (2.14), we have

$$\overline{D}\omega \geq \int_0^{\omega} \frac{E(t)\exp\{x_1(t-\sigma(t))\}}{1+B\exp\{x_1(t-\sigma(t))\}+C\exp\{x_3(\eta_3)\}}\mathrm{d}t \geq \frac{\overline{E}\omega\exp\{x_1(\xi_1)\}}{1+B\exp\{x_1(\xi_1)\}+C\exp\{x_3(\eta_3)\}}, \tag{3.10}$$

that is

$$\frac{\exp\{x_1(\xi_1)\}}{1+B\exp\{x_1(\xi_1)\}+C\exp\{x_3(\eta_3)\}} \leq \frac{\overline{D}}{\overline{E}}. \tag{3.11}$$

It follows that

$$\exp\{x_3(\eta_3)\} \geq \frac{(\overline{E}-\overline{D}B)\exp\{x_1(\xi_1)\}-\overline{D}}{\overline{D}C}. \tag{3.12}$$

By (3.8), we have

$$\exp\{x_3(\eta_3)\} \geq \frac{(\overline{E}-\overline{D}B)\exp\left\{\ln\left(\frac{\overline{d}}{\overline{e}-\overline{b}\overline{d}}\right)-2\overline{r}\omega\right\}-\overline{D}}{\overline{D}C} = \frac{(\overline{E}-\overline{D}B)\mathrm{e}^{-2\overline{r}\omega}\frac{\overline{d}}{\overline{e}-\overline{b}\overline{d}}-\overline{D}}{\overline{D}C}. \tag{3.13}$$

By the assumption in Theorem 2.2, we have

$$x_3(\eta_3) \geq \ln\left[\frac{(\overline{E}-\overline{D}B)\mathrm{e}^{-2\overline{r}\omega}\alpha-\overline{D}}{\overline{D}C}\right]. \tag{3.14}$$

By (2.13) and (3.14), we have

$$x_3(t) \geq x_3(\eta_3) - \int_0^{\omega}|x_3'(t)|\mathrm{d}t \geq \ln\left[\frac{(\overline{E}-\overline{D}B)\mathrm{e}^{-2\overline{r}\omega}\alpha-\overline{D}}{\overline{D}C}\right] - 2\overline{D}\omega. \tag{3.15}$$

On the other hand, by (2.10) and (2.14), we have

$$\overline{D}\omega \leq \int_0^{\omega} \frac{E(t)\exp\{x_1(t-\sigma(t))\}}{1+B\exp\{x_1(t-\sigma(t))\}+C\exp\{x_3(\xi_3)\}}\mathrm{d}t \leq \frac{\overline{E}\omega\exp\{x_1(\eta_1)\}}{1+B\exp\{x_1(\eta_1)\}+C\exp\{x_3(\xi_3)\}}, \tag{3.16}$$

that is

$$\frac{\exp\{x_1(\eta_1)\}}{1+B\exp\{x_1(\eta_1)\}+C\exp\{x_3(\xi_3)\}} \geq \frac{\overline{D}}{\overline{E}}. \tag{3.17}$$

It follows that

$$\exp\{x_3(\xi_3)\} \leq \frac{(\overline{E}-\overline{D}B)\exp\{x_1(\eta_1)\}-\overline{D}}{\overline{D}C}. \tag{3.18}$$

By (3.4), we have

$$\exp\{x_3(\xi_3)\} \leq \frac{(\overline{E}-\overline{D}B)\exp\left\{\ln\left(\frac{\overline{d}}{\overline{e}-\overline{b}\overline{d}}\right)+2\overline{r}\omega\right\}-\overline{D}}{\overline{D}C} = \frac{(\overline{E}-\overline{D}B)\mathrm{e}^{2\overline{r}\omega}\frac{\overline{d}}{\overline{e}-\overline{b}\overline{d}}-\overline{D}}{\overline{D}C} = \frac{(\overline{E}-\overline{D}B)\mathrm{e}^{2\overline{r}\omega}\alpha-\overline{D}}{\overline{D}C}. \tag{3.19}$$

By the assumption in Theorem 2.2, we have

$$x_3(\xi_3) \le \ln\left[\frac{(\overline{E}-\overline{D}B)e^{2\overline{r}\omega}\alpha-\overline{D}}{\overline{D}C}\right]. \tag{3.20}$$

By (2.13) and (3.20), we have

$$x_3(t) \le x_3(\xi_3) + \int_0^\omega |x_3'(t)|dt \le \ln\left[\frac{(\overline{E}-\overline{D}B)e^{2\overline{r}\omega}\alpha-\overline{D}}{\overline{D}C}\right] + 2\overline{D}\omega. \tag{3.21}$$

Therefore, by (3.15) and (3.21), we have

$$\max_{t\in[0,\omega]} |x_3(t)| \le \max\left\{\left|\ln\left(\frac{\left(\overline{E}-B\overline{D}\right)e^{-2\overline{r}\omega}\alpha-\overline{D}}{C\overline{D}}\right)-2\overline{D}\omega\right|,\right.$$
$$\left.\left|\ln\left(\frac{\left(\overline{E}-B\overline{D}\right)e^{2\overline{r}\omega}\alpha-\overline{D}}{C\overline{D}}\right)+2\overline{D}\omega\right|\right\}. \tag{3.22}$$

This completes the proof of Lemma 2.4. □

Proof of Lemma 2.5. By (2.8) and (2.14), we have

$$\overline{r}\omega \le \frac{\overline{r}\omega\exp\{x_1(\eta_1)\}}{k} + \frac{\overline{a}\omega\exp\{x_2(\eta_2)\}}{1+b\exp\{x_1(\xi_1)\}} + \frac{\overline{A}\omega\exp\{x_3(\eta_3)\}}{1+B\exp\{x_1(\xi_1)\}+C\exp\{x_3(\eta_3)\}}. \tag{3.23}$$

By multiplying both sides of (3.23) by $\exp\{x_1(\xi_1)\}$, it follows that

$$\overline{r}\exp\{x_1(\xi_1)\} \le \frac{\overline{r}\exp\{x_1(\xi_1)\}\exp\{x_1(\eta_1)\}}{k}$$
$$+\frac{\overline{a}\exp\{x_1(\xi_1)\}\exp\{x_2(\eta_2)\}}{1+b\exp\{x_1(\xi_1)\}} + \frac{\overline{A}\exp\{x_1(\xi_1)\}\exp\{x_3(\eta_3)\}}{1+B\exp\{x_1(\xi_1)\}+C\exp\{x_3(\eta_3)\}}. \tag{3.24}$$

By (3.2) and (3.11), we have

$$\overline{r}\exp\{x_1(\xi_1)\} \le \frac{\overline{r}\exp\{x_1(\xi_1)\}\exp\{x_1(\eta_1)\}}{k} + \frac{\overline{a}\,\overline{d}\exp\{x_2(\eta_2)\}}{\overline{e}} + \frac{\overline{A}\,\overline{D}\exp\{x_3(\eta_3)\}}{\overline{E}}. \tag{3.25}$$

It follows that

$$\frac{\overline{a}\,\overline{d}}{\overline{e}}\exp\{x_2(\eta_2)\} \ge \overline{r}\ \exp\{x_1(\xi_1)\}\left[1-\frac{\exp\{x_1(\eta_1)\}}{k}\right] - \frac{\overline{A}\,\overline{D}\exp\{x_3(\eta_3)\}}{\overline{E}}. \tag{3.26}$$

By (3.4), (3.8) and (3.21), we have

$$\frac{\overline{a}\,\overline{d}}{\overline{e}}\exp\{x_2(\eta_2)\} \ge \overline{r}\ \exp\left\{\ln\left(\frac{\overline{d}}{\overline{e}-b\overline{\overline{d}}}\right)-2\overline{r}\omega\right\}\left[1-\frac{\exp\left\{\ln\left(\frac{\overline{d}}{\overline{e}-b\overline{\overline{d}}}\right)+2\overline{r}\omega\right\}}{k}\right]$$
$$-\frac{\overline{A}\,\overline{D}\exp\left\{\ln\left[\frac{(\overline{E}-\overline{D}B)e^{2\overline{r}\omega}\alpha-\overline{D}}{\overline{D}C}\right]+2\overline{D}\omega\right\}}{\overline{E}}$$
$$=\overline{r}e^{-2\overline{r}\omega}\alpha\left(1-\frac{e^{2\overline{r}\omega}\alpha}{k}\right)-\frac{\overline{A}}{C\overline{E}}\left[\left(\overline{E}-B\overline{D}\right)e^{2\overline{r}\omega}\alpha-\overline{D}\right]e^{2\overline{D}\omega}. \tag{3.27}$$

By the assumption in Theorem 2.2, it follows that

$$x_2(\eta_2) \ge \ln\left[\frac{\overline{e}}{\overline{a}\,\overline{d}}\left[\overline{r}e^{-2\overline{r}\omega}\alpha\left(1-\frac{e^{2\overline{r}\omega}\alpha}{k}\right)-\frac{\overline{A}}{C\overline{E}}\left[\left(\overline{E}-B\overline{D}\right)e^{2\overline{r}\omega}\alpha-\overline{D}\right]e^{2\overline{D}\omega}\right]\right]. \tag{3.28}$$

Therefore, by (2.12) and (3.28), we have

$$\begin{aligned} x_2(t) &\geq x_2(\eta_2) - \int_0^{\omega} |x_2'(t)|\mathrm{d}t \\ &\geq \ln\left[\frac{\overline{e}}{\overline{a}\,\overline{d}}\left[\overline{r}\mathrm{e}^{-2\overline{r}\omega}\alpha\left(1-\frac{\mathrm{e}^{2\overline{r}\omega}\alpha}{k}\right)-\frac{\overline{A}}{C\overline{E}}\left[\left(\overline{E}-B\overline{D}\right)\mathrm{e}^{2\overline{r}\omega}\alpha-\overline{D}\right]\mathrm{e}^{2\overline{D}\omega}\right]\right]-2\overline{d}\omega. \end{aligned} \tag{3.29}$$

On the other hand, by (2.8) and (2.14), we have

$$\overline{r}\omega \geq \frac{\overline{r}\omega\exp\{x_1(\xi_1)\}}{k}+\frac{\overline{a}\omega\exp\{x_2(\xi_2)\}}{1+b\exp\{x_1(\eta_1)\}}+\frac{\overline{A}\omega\exp\{x_3(\xi_3)\}}{1+B\exp\{x_1(\eta_1)\}+C\exp\{x_3(\xi_3)\}}. \tag{3.30}$$

By multiplying both sides of (3.30) by $\exp\{x_1(\eta_1)\}$, it follows that

$$\begin{aligned} \overline{r}\exp\{x_1(\eta_1)\} \geq{}& \frac{\overline{r}\exp\{x_1(\eta_1)\}\exp\{x_1(\xi_1)\}}{k} \\ &+\frac{\overline{a}\exp\{x_1(\eta_1)\}\exp\{x_2(\xi_2)\}}{1+b\exp\{x_1(\eta_1)\}}+\frac{\overline{A}\exp\{x_1(\eta_1)\}\exp\{x_3(\xi_3)\}}{1+B\exp\{x_1(\eta_1)\}+C\exp\{x_3(\xi_3)\}}. \end{aligned} \tag{3.31}$$

By (3.6) and (3.17), we have

$$\overline{r}\exp\{x_1(\eta_1)\} \geq \frac{\overline{r}\exp\{x_1(\eta_1)\}\exp\{x_1(\xi_1)\}}{k}+\frac{\overline{a}\,\overline{d}\exp\{x_2(\xi_2)\}}{\overline{e}}+\frac{\overline{A}\,\overline{D}\exp\{x_3(\xi_3)\}}{\overline{E}}. \tag{3.32}$$

It follows that

$$\frac{\overline{a}\,\overline{d}}{\overline{e}}\exp\{x_2(\xi_2)\} \leq \overline{r}\;\exp\{x_1(\eta_1)\}\left[1-\frac{\exp\{x_1(\xi_1)\}}{k}\right]-\frac{\overline{A}\,\overline{D}\exp\{x_3(\xi_3)\}}{\overline{E}}. \tag{3.33}$$

By (3.4), (3.8) and (3.15), we have

$$\begin{aligned} \frac{\overline{a}\,\overline{d}}{\overline{e}}\exp\{x_2(\xi_2)\} &\leq \overline{r}\;\exp\left\{\ln\left(\frac{\overline{d}}{\overline{e}-b\overline{d}}\right)+2\overline{r}\omega\right\}\left[1-\frac{\exp\left\{\ln\left(\frac{\overline{d}}{\overline{e}-b\overline{d}}\right)-2\overline{r}\omega\right\}}{k}\right] \\ &\quad-\frac{\overline{A}\,\overline{D}\exp\left\{\ln\left[\frac{(\overline{E}-\overline{D}B)\mathrm{e}^{-2\overline{r}\omega}\alpha-\overline{D}}{\overline{D}C}\right]-2\overline{D}\omega\right\}}{\overline{E}} \\ &=\overline{r}\mathrm{e}^{2\overline{r}\omega}\alpha\left(1-\frac{\mathrm{e}^{-2\overline{r}\omega}\alpha}{k}\right)-\frac{\overline{A}}{C\overline{E}}\left[\left(\overline{E}-B\overline{D}\right)\mathrm{e}^{-2\overline{r}\omega}\alpha-\overline{D}\right]\mathrm{e}^{-2\overline{D}\omega}. \end{aligned} \tag{3.34}$$

By the assumption in Theorem 2.2, it follows that

$$x_2(\xi_2) \leq \ln\left[\frac{\overline{e}}{\overline{a}\,\overline{d}}\left[\overline{r}\mathrm{e}^{2\overline{r}\omega}\alpha\left(1-\frac{\mathrm{e}^{-2\overline{r}\omega}\alpha}{k}\right)-\frac{\overline{A}}{C\overline{E}}\left[\left(\overline{E}-B\overline{D}\right)\mathrm{e}^{-2\overline{r}\omega}\alpha-\overline{D}\right]\mathrm{e}^{-2\overline{D}\omega}\right]\right]. \tag{3.35}$$

By (2.12) and (3.35), we have

$$\begin{aligned} x_2(t) &\leq x_2(\xi_2) + \int_0^{\omega} |x_2'(t)|\mathrm{d}t \\ &\leq \ln\left[\frac{\overline{e}}{\overline{a}\,\overline{d}}\left[\overline{r}\mathrm{e}^{2\overline{r}\omega}\alpha\left(1-\frac{\mathrm{e}^{-2\overline{r}\omega}\alpha}{k}\right)-\frac{\overline{A}}{C\overline{E}}\left[\left(\overline{E}-B\overline{D}\right)\mathrm{e}^{-2\overline{r}\omega}\alpha-\overline{D}\right]\mathrm{e}^{-2\overline{D}\omega}\right]\right]+2\overline{d}\omega. \end{aligned} \tag{3.36}$$

Therefore, by (3.29) and (3.36), we have

$$\max_{t\in[0,\omega]}|x_2(t)| \leq \max\left\{\left|\ln\left[\frac{\overline{e}}{\overline{a}\,\overline{d}}\left[\overline{r}\mathrm{e}^{-2\overline{r}\omega}\alpha\left(1-\frac{\mathrm{e}^{2\overline{r}\omega}\alpha}{k}\right)-\frac{\overline{A}}{C\overline{E}}\left[\left(\overline{E}-B\overline{D}\right)\mathrm{e}^{2\overline{r}\omega}\alpha-\overline{D}\right]\mathrm{e}^{2\overline{D}\omega}\right]\right]-2\overline{d}\omega\right|,\right.$$

$$\left|\ln\left[\frac{\overline{e}}{\overline{a}\,\overline{d}}\left[\overline{r}\mathrm{e}^{2\overline{r}\omega}\alpha\left(1-\frac{\mathrm{e}^{-2\overline{r}\omega}\alpha}{k}\right)-\frac{\overline{A}}{C\overline{E}}\left[\left(\overline{E}-B\overline{D}\right)\mathrm{e}^{-2\overline{r}\omega}\alpha-\overline{D}\right]\mathrm{e}^{-2\overline{D}\omega}\right]\right]+2\overline{d}\omega\right|\right\}. \tag{3.37}$$

This completes the proof of Lemma 2.5. □

4. Conclusions

It is usually observed that population densities in the real world tend to fluctuate. Therefore, modelling population interactions and understanding this oscillatory phenomenon are a very basic and important ecological problem. Although much progress has been made in the study of modelling and understanding three species predator–prey systems, models such as (1.1)–(1.3), have been largely discussed by assuming that the environment is constant, which is indeed rarely the case in real life. Naturally, more realistic and interesting models with three species interactions should take into account both the seasonality of the changing environment and the effects of time delays. Therefore, it is interesting and important to study systems with periodic delays (1.4).

In this paper, based on the powerful and effective coincidence degree theory, the existence of positive periodic solutions for predator–prey systems with periodic delays (1.4) is obtained under suitable conditions. Contrary to the principle of competitive exclusion, our results demonstrate that in the case of two predators competing for a single prey species, by suitably modelling three species predator prey system, two predators and their prey can coexist along a positive periodic solution. Also, our results generalize corresponding results in [2].

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