

Multi-target localization and circumnavigation by a single agent using bearing measurements[‡]

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SUMMARY

This paper considers the problem of localization and circumnavigation of a group of targets, which are either stationary or moving slowly with unknown speed, by a single agent. An estimator is proposed, initially for the stationary target case, to localize the targets and the center of mass of them as well as a control law that forces the agent to move on a circular trajectory around the center of mass of the targets such that both the estimator and the controller are exponentially stable. Then the case where the targets might experience slow but possibly steady movements is studied. The system inputs include the agent's position and the bearing angles to the targets. The performance of the proposed algorithms is verified through simulations. Copyright © 2014 John Wiley & Sons, Ltd.

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1. INTRODUCTION

A common surveillance problem is to arrange for one or possibly several agents to navigate around a single or a group of targets on a circular trajectory of prescribed radius. A simple scenario for such a problem is that there is a single agent whose goal is to circle around a stationary target with known position. The task is to find a control law that causes the agent to move to and then around a circle in an agreed sense (i.e., clockwise or counterclockwise) with prescribed radius centered on the target. There are various ways in which the problem can be made more complex, for example, when there is a group of agents/targets or when the target(s) is moving. Prior literature dealing with such problems includes, but is not limited to, [1–6].

When the target(s) has an unknown initial position, an estimator of the target position as well as a control algorithm that forces the agent to move on the desired circular trajectory are required for the surveillance task. In such problems, ideas of adaptive control or dual control can appear and with certain controls, no estimation is in fact possible; this sort of phenomenon is associated with dual control, and it is the persistence of excitation concept of adaptive control [7, 8], which clarifies what needs to happen. Simultaneous control and estimation are required, and the quality of estimation can heavily depend on the control used.

Problems of this type with single-agent or multi-agent collaborative circumnavigation algorithms have recently been studied. In [9–11], localization and circumnavigation of a moving target have

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‡An abbreviated conference version of this paper has been presented in [12].

been studied when the agent(s) can measure the relative position of the target, that is, its range and bearing. In some applications, however, it is preferred to employ localization and circumnavigation algorithms that require less sensed knowledge about the target so that the proposed algorithm can be used to control a UAV with limited payload capacity (and thus limited sensing capability). There have been some research efforts to study such localization and circumnavigation problems using distance-only measurements [13–15], bearing-only measurements [16, 17], and received signal strength (RSS) measurement [18]. In the scenarios where the agent has to maintain radio silence for the fear that its position will be detected, it is usually preferred not to use distance measurements. This is because of the fact that distance measurement techniques are usually active methods in which the agent must transmit signals. In contrast, RSS measurement techniques and usually bearing measurement techniques are passive methods. RSS-based localization techniques measure the strength of the received signal and use a log-normal radio propagation model to estimate the distance to the target. The path loss exponent is a key parameter in the log-normal model, which depends on the environment in which the sensor is deployed. The problem with this method is that an accurate knowledge of the path loss exponent is required in order to convert signal strength measurements to range, and it can be difficult to obtain [19].

The problem of bearings-only target localization has been studied in the literature using statistical estimators such as extended Kalman filter (EKF) and unscented Kalman filter [20, 21]. It is assumed in such estimators that the agent knows the system model as well as the noise model. Although these estimators work well when the agent knows the motion characteristics of the target, they cannot be used when the target can move freely while the agent is not aware of the target motion. Many of the current results in the literature assumed that the target is either stationary or moving with a known constant velocity. We however consider the scenario that the target is allowed to move on any directions and the agent does not know the motion characteristics of the target.

In this paper, we investigate the case where there is a single agent but multiple targets, which can be either stationary or moving. We assume that the agent has a single integrator model and propose estimation and control algorithms using bearing measurements for determining the estimated positions of the targets and making the agent circle around them. The algorithms proposed in this paper are inspired by the algorithms in [16], which are for the single target case. However, the algorithms proposed here are not trivial extensions of the single target case where the target itself acts as the center of the circle around which the agent moves. In the multiple targets case, the center of the circle is not naturally or automatically defined; therefore we need to add a center estimation algorithm. This changes the structure of the control algorithm and adds another layer of conceptual and computational complexity to the problem.

As noted, the algorithms we propose in this paper use bearing but not range measurements and, as is generally desirable and usual, avoid using derivatives of measurements, that is, do not measure angular velocity, to avoid high-frequency noise effect. In the stationary target case, the estimation and control algorithms exhibit exponentially fast convergence. In terms of robustness against noise and system uncertainties, although filtering algorithms like EKF can be applied when the measurements are noisy or when the target is moving slowly, the proposed method, without using any filtering algorithm, can tolerate measurement noise and slow movement of the target. This results from the fact that exponentially stable systems are robust against many types of system uncertainties.

The rest of this paper is structured as follows. In Section 2, the problem is formally defined, and the proposed solution is provided in Section 3. Section 4 contains results from Matlab simulations demonstrating the feasibility of the proposed algorithm. Finally, conclusions and proposals for future work are presented in Section 5.

2. PROBLEM STATEMENT

Suppose there are n targets with unknown positions $\mathbf{p}_{T_i}(t) \in \mathbb{R}^2$, $i \in 1, 2, 3, \dots, n$ at time t and there is also an agent moving on a known trajectory $\mathbf{p}_A(s) \in \mathbb{R}^2$ for $s \leq t$. *Until further notice*, we shall assume all $\mathbf{p}_{T_i}(t)$ are constant. Both the targets and the agent are assumed to be modellable as

points. Let $\varphi_i(t)$, $i = 1, 2, \dots, n$ be the unit vectors in the direction of the line going from $p_A(t)$ to $p_{T_i}(t)$, that is,

$$\varphi_i(t) = \frac{p_{T_i}(t) - p_A(t)}{\|p_{T_i}(t) - p_A(t)\|} \quad (1)$$

and let $\bar{\varphi}_i(t)$ be the unit vector obtained by $\pi/2$ clockwise rotation of $\varphi_i(t)$. Let $p_T(t)$ be the point around which the agent is seeking to move. We call this point the *virtual target* and define it as

$$p_T(t) = \frac{1}{n} \sum_i p_{T_i}(t) \quad (2)$$

Also, let $\rho(t)$ be

$$\rho(t) = \|p_A(t) - p_T(t)\| \quad (3)$$

and $\rho_d(t)$ be the desired radius of a circle with $p_T(t)$ as the center on which the agent should move. The value of $\rho_d(t)$ will be estimated by the agent and should be such that the circle encloses all targets. Of course, just as the agent does not initially know the positions of the targets, it does not initially know the position of the virtual target and the radius of the circle. The case $s = t$ where $n = 5$ is depicted in Figure 1. Let $\hat{p}_{T_i}(t)$ be the estimated position of target i and $\hat{p}_T(t)$ be the estimate of $p_T(t)$ at time t . The discussion on how to calculate $\hat{p}_{T_i}(t)$ and $\hat{p}_T(t)$ appears in the next section. We suppose that the agent can measure the bearing angles to all targets and define $\rho_d(t)$ as

$$\rho_d(t) = \max_i \|\hat{p}_T(t) - \hat{p}_{T_i}(t)\| + d \quad (4)$$

where $d > 0$ is a constant scalar. The constant d makes the desired circle be larger than the maximum distance of the targets to the center of the circle so that all targets are inside the circle and not on the perimeter.

There are two tasks to be performed by the agent. The first is to estimate the position of the virtual target $p_T(t)$ and the radius $\rho_d(t)$ such that the estimation error given in (5) converges exponentially

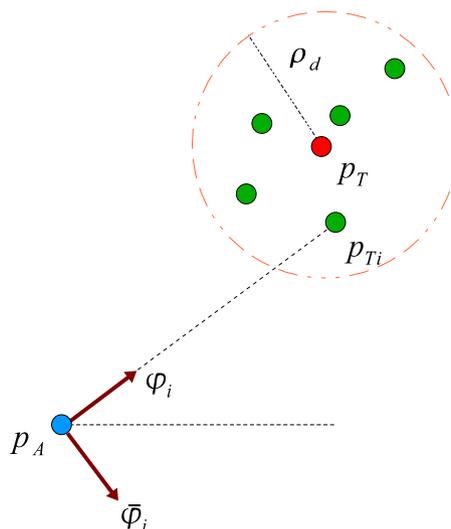


Figure 1. An illustration of the problem and the relationship between variables.

fast to zero when the targets are stationary and to a small neighborhood of zero when the targets are moving slowly:

$$\tilde{\mathbf{p}}_T(t) = \hat{\mathbf{p}}_T(t) - \mathbf{p}_T(t) \tag{5}$$

Note that when the target positions $\mathbf{p}_{T_i}(t)$ are constant, $\mathbf{p}_T(t)$ will also be taken as constant. The second task is to move toward and then on the desired circle around the virtual target such $\rho(t) - \rho_d(t)$ converges to zero exponentially fast in the stationary target case and to a neighborhood of zero in the moving target case.

3. PROPOSED SOLUTION

We previously studied the case where there was only one target and one agent [16]. It is shown in [16] that both estimator and controller are exponentially stable. Before considering the case where there is more than one target, we first recall the estimator and controller for the case where there is only one target located at $\mathbf{p}_{T_1}(t)$:

$$\dot{\hat{\mathbf{p}}}_{T_1}(t) = k_{est} (\mathbf{I} - \boldsymbol{\varphi}_1(t)\boldsymbol{\varphi}_1^\top(t)) (\mathbf{p}_A(t) - \hat{\mathbf{p}}_{T_1}(t)) \tag{6}$$

and

$$\dot{\mathbf{p}}_A(t) = \mathbf{u}(t) = (\hat{\rho}_1(t) - \rho_d(t)) \boldsymbol{\varphi}_1(t) + \alpha \bar{\boldsymbol{\varphi}}_1(t) \tag{7}$$

where $\hat{\mathbf{p}}_{T_1}(t)$ is the estimate of $\mathbf{p}_{T_1}(t)$ at time t , \mathbf{I} is the 2×2 identity matrix, $\hat{\rho}_1(t) = \|\mathbf{p}_A(t) - \hat{\mathbf{p}}_{T_1}(t)\|$, $\mathbf{u}(t)$ is the control input, and k_{est} and α are positive constants. When the estimator converges, $\hat{\rho}_1(t) \rightarrow \rho_1(t) = \|\mathbf{p}_A(t) - \mathbf{p}_{T_1}(t)\|$ and according to (7), the agent moves toward the desired circle if $\hat{\rho}_1(t) \neq \rho_d(t)$. Once it reaches the circle, it moves with the tangential speed of α around the target. Note that these equations give exponential convergence when $\mathbf{p}_{T_1}(t)$ is constant, and robust behavior for slow motion of the target.

When there is more than one target, the first step is to find the virtual target $\mathbf{p}_T(t)$ around which the agent is going to move and the radius of the circle, $\rho_d(t)$.

3.1. Stationary targets

We continue to assume that the targets are all stationary. We will later consider the case where the targets move slowly in Section 3.2. Our first approach is to estimate all target positions at the same time and then use the individual estimates $\hat{\mathbf{p}}_{T_i}(t)$ to calculate $\hat{\mathbf{p}}_T(t)$ as

$$\hat{\mathbf{p}}_T(t) = \frac{1}{n} \sum_i \hat{\mathbf{p}}_{T_i}(t) \tag{8}$$

Note that when all estimators converge, $\hat{\mathbf{p}}_T(t)$ also converges to \mathbf{p}_T . Similarly to (6), the estimator for target i can be written as

$$\dot{\hat{\mathbf{p}}}_{T_i}(t) = k_{est} (\mathbf{I} - \boldsymbol{\varphi}_i(t)\boldsymbol{\varphi}_i^\top(t)) (\mathbf{p}_A(t) - \hat{\mathbf{p}}_{T_i}(t)) \tag{9}$$

and after some calculations, the estimation error dynamics can be written as

$$\begin{aligned} \dot{\tilde{\mathbf{p}}}_{T_i}(t) &= -k_{est} (\mathbf{I} - \boldsymbol{\varphi}_i(t)\boldsymbol{\varphi}_i^\top(t)) \tilde{\mathbf{p}}_{T_i}(t) \\ &= -k_{est} \bar{\boldsymbol{\varphi}}_i(t) \bar{\boldsymbol{\varphi}}_i^\top(t) \tilde{\mathbf{p}}_{T_i}(t) \end{aligned} \tag{10}$$

where $\tilde{\mathbf{p}}_{T_i}(t) = \hat{\mathbf{p}}_{T_i}(t) - \mathbf{p}_{T_i}(t)$. Let $\hat{\rho}(t)$ and the unit vector $\boldsymbol{\varphi}(t)$ be defined as

$$\hat{\rho}(t) = \|\mathbf{p}_A(t) - \hat{\mathbf{p}}_T(t)\| \tag{11}$$

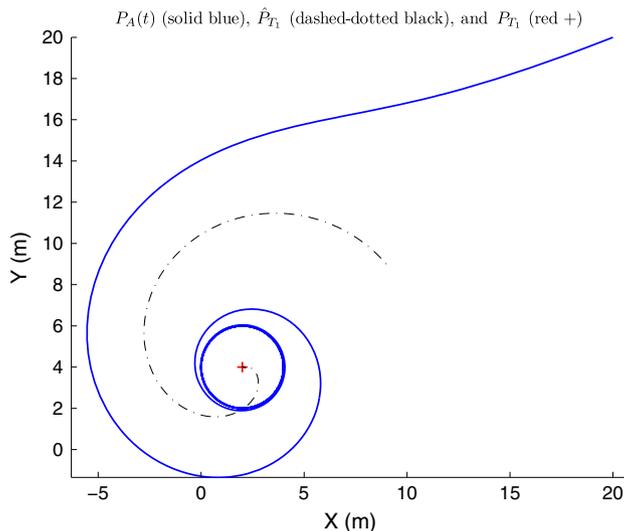


Figure 2. Simulation of a single target case when the controller makes the agent move around the target.

$$\varphi(t) = \frac{\hat{p}_T(t) - p_A(t)}{\|\hat{p}_T(t) - p_A(t)\|} \tag{12}$$

Then the controller for the multi-target case can be written as

$$\dot{p}_A(t) = u(t) = (\hat{\rho}(t) - \rho_d(t)) \varphi(t) + \alpha \bar{\varphi}(t) \tag{13}$$

where $\bar{\varphi}(t)$ is the unit vector obtained by $\pi/2$ clockwise rotation of $\varphi(t)$. Note that the unit vector $\varphi(t)$ is not a measurement of the bearing angle of an actual target, but represents the bearing angle of $\hat{p}_T(t)$, which is the estimated position of the virtual target p_T . The reason why we use the bearing angle of $\hat{p}_T(t)$ rather than the bearing angle to an actual target is that we do not have any measurement from the center of the circle around the targets (virtual target) in the multi-target case.

There is an important distinction now which should be made. For the single target case, the unit vector $\varphi(t)$ in (7) always points toward the target, but in the multi-target case explained earlier, it points to $\hat{p}_T(t)$ rather than $p_T(t)$. Hence, in the multi-target case with the additional level of complexity, it is harder to estimate the circle center. This can be seen by comparing Figures 2 and 3. In Figure 2, we simulated a single target case and used the estimator (6) and the controller (7). In this case, the unit vector $\varphi_1(t)$ used in both the estimator and controller points toward the target. In Figure 3, we used the same initial conditions for the agent and the target estimate, and simulated the case where the estimator (9) and the controller (13) are used, and there is only a single target. The value of $\rho_d(t)$ in both cases is set to 2 as there is only one target and there is no need to estimate the desired radius of the circle. Thus, the only difference is that in Figure 2, we used the controller (7) in which the unit vector $\varphi_1(t)$ is used and points toward the target, but in Figure 3 we used the controller (13) in which the unit vector $\varphi(t)$ is used and points toward the target estimate.

It can be seen that the motion of the agent changes a lot when the controller changes from (7) to (13). Furthermore, the motion of the virtual target estimate also persists for longer, or is a bigger motion (i.e., the estimate of the center of the target circle wanders more before convergence).

In what follows, we show that by using the estimator (9) and the controller (13), the estimation error $\tilde{p}_T(t)$ converges to zero exponentially fast. To this end, we recall the following definition and proposition [7].

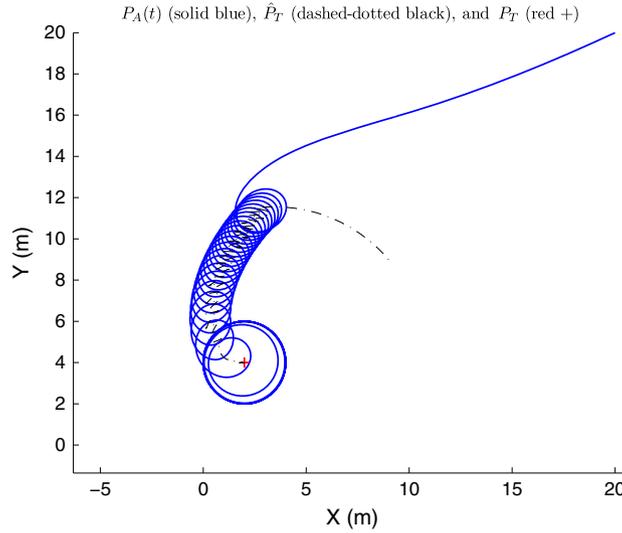


Figure 3. Simulation of a single target case when the controller makes the agent move around the target estimate.

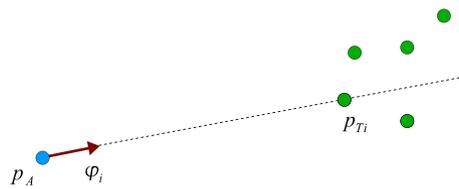


Figure 4. An example of the trajectory of the agent along which $\varphi_i(t)$ and also $\bar{\varphi}_i(t)$ are not persistently exciting.

Definition 1

$w(t) : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times r} \forall n, r \geq 1$ is persistently exciting if there exist some positive $\alpha_1, \alpha_2, \delta$ such that

$$\alpha_1 \mathbf{I} \leq \int_{t_0}^{t_0+\delta} w(t)w^\top(t)dt \leq \alpha_2 \mathbf{I} \quad \text{for all } t_0 \geq 0 \quad (14)$$

Proposition 1

Consider the differential equation

$$\dot{x} = -w(t)w^\top(t)x \quad (15)$$

where $w(t) : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times r} \forall n, r \geq 1$ is a regulated matrix function (i.e., one-sided limits exist for all $t \in \mathbb{R}_+$). Then (15) is exponentially asymptotically stable if and only if $w(t)$ is persistently exciting.

The persistence of excitation condition in (14) requires that $w(t)$ rotates sufficiently in space such that the integral of the matrix $w(t)w^\top(t)$ is uniformly positive definite over any interval of some length δ [7, 8]. Consider now the estimation error equation of target i in (10). If the unit vector $\bar{\varphi}_i(t)$ (or the unit vector $\varphi_i(t)$) rotates sufficiently in space, that is, if the bearing angle to target i changes sufficiently fast, then according to Proposition 1, \tilde{p}_{T_i} converges to zero exponentially fast. The only situation where $\bar{\varphi}_i(t)$ does not rotate sufficiently is when the agent moves on a straight line toward the target i or when it converges to such a straight line. Consider the case shown in Figure 4. If the agent moves on the dotted line shown in Figure 4, $\varphi_i(t)$ does not change and is not

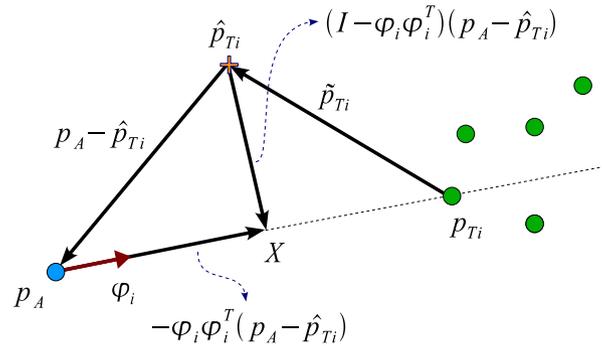


Figure 5. An illustration on how the estimator in (9) works.

persistently exciting. But the other unit vectors $\varphi_j(t)$ to other targets rotate, and therefore, other $\tilde{p}_{T_j}(t)$ converge to zero exponentially fast. The following theorem shows that by using the estimator (9) and the controller (13), the agent cannot move on such a straight line as shown in Figure 4, and all of the unit vectors pointing from the agent to the targets are persistently exciting.

Theorem 1

Adopt the notation in the preceding text and assume that all targets are stationary. Then by using the estimator (9) and the controller (13), the estimation error for each target converges to zero exponentially fast and consequentially $\tilde{p}_T(t)$ has the same property.

Proof

If we show that all $\tilde{p}_{T_i}(t)$ converge to zero exponentially fast, then we can conclude that $\tilde{p}_T(t)$ also converges to zero exponentially fast. We prove by contradiction and assume that at least for target i , $\varphi_i(t)$ is not persistently exciting and therefore $\tilde{p}_{T_i}(t)$ does not converge to zero exponentially fast. Consider Proposition 1 and let $w(t)$ in (14) be $w(t) = \bar{\varphi}_i(t)$. Note that $\bar{\varphi}_i(t)\bar{\varphi}_i(t)^\top$ is singular for all t and the persistency of excitation condition in (14) requires that $\bar{\varphi}_i(t)$ rotates sufficiently in space that the integral of the matrix $\bar{\varphi}_i(t)\bar{\varphi}_i(t)^\top$ is uniformly positive definite over *any interval* of some length $\delta > 0$. So we consider that the motion of the agent is such that $\bar{\varphi}_i(t)$ does not change or changes very slowly that there is no $\delta > 0$ such that the integral of the matrix $\bar{\varphi}_i(t)\bar{\varphi}_i(t)^\top$ over δ is uniformly positive definite[§].

Note that the estimated position of target i always converges to a constant value (which might not be the correct position of target i) exponentially fast. This can be seen from (9) and Figure 5, as the estimator always forces the estimated position of target i to go to the point X shown in Figure 5, which is on the line passing through the agent and target i .

So the estimated position of the virtual target that is the average of the estimated position of all targets converges to a constant value exponentially fast. Then according to (13), the agent tries to move around the estimated position of the virtual target and has a nonzero tangential velocity equal to α . Thus, the agent cannot keep moving on the straight line shown in Figure 4, and therefore, $\bar{\varphi}_i(t)$ also rotates, which ensures that the estimated position of target i converges to its actual position. \square

Having established that the estimation process proceeds satisfactorily, it remains to demonstrate that the control law achieves the required objective.

Theorem 2

Using the estimator (9) and the controller (13), $\rho(t) - \rho_d(t)$ converges to zero exponentially fast.

[§]This means that agent i moves on a straight line toward the target or converges to such a straight line.

Proof

Considering (11), (12), and (13), one has

$$\begin{aligned} \dot{\hat{\rho}}(t) &= \frac{\left(\dot{\mathbf{p}}_A(t) - \dot{\hat{\mathbf{p}}}_T(t)\right)^\top (\mathbf{p}_A(t) - \hat{\mathbf{p}}_T(t))}{\hat{\rho}(t)} \\ &= -\left(\dot{\mathbf{p}}_A(t) - \dot{\hat{\mathbf{p}}}_T(t)\right)^\top \boldsymbol{\varphi}(t) \\ &= -\dot{\hat{\rho}}(t) + \rho_d(t) + \dot{\hat{\mathbf{p}}}_T(t)^\top \boldsymbol{\varphi}(t) \end{aligned} \tag{16}$$

Because the targets are stationary, $\dot{\hat{\mathbf{p}}}_{T_i}(t) = \dot{\tilde{\mathbf{p}}}_{T_i}(t)$ and thus the last term on the right-hand side of the aforementioned equation is $-\frac{k_{est}}{n} \sum_i (\tilde{\boldsymbol{\varphi}}_i(t) \tilde{\boldsymbol{\varphi}}_i^\top(t) \tilde{\mathbf{p}}_{T_i}(t))^\top \boldsymbol{\varphi}(t)$. Because all $\tilde{\mathbf{p}}_{T_i}(t)$ converge to zero exponentially fast (Theorem 1) and therefore $\rho_d(t)$ converges exponentially fast to a constant value, then in the light of (16), $\hat{\rho}(t) - \rho_d(t)$ converges to zero exponentially fast.

Consider now a triangle with vertices at $\mathbf{p}_A(t)$, $\mathbf{p}_T(t)$, and $\hat{\mathbf{p}}_T(t)$. Then by the triangle inequality, one has

$$\rho(t) \leq \hat{\rho}(t) + \|\tilde{\mathbf{p}}_T(t)\| \tag{17}$$

Because $\hat{\rho}(t)$ and $\|\tilde{\mathbf{p}}_T(t)\|$ converge, respectively, to $\lim_{t \rightarrow \infty} \rho_d(t)$ and zero exponentially fast, $\rho(t)$ also converges to $\lim_{t \rightarrow \infty} \rho_d(t)$ exponentially fast. \square

Remark 1

Although we assumed that the bearing angles to all targets are available to the agent at all time, $\hat{\mathbf{p}}_T(t)$ and $\rho_d(t)$ still converge to their desired values if the agent is not able to measure the bearing angle to one or some of the targets for some period. In this case, the convergence rate might be slower. An example of this scenario is presented in Section 4.

3.2. Slowly moving targets

We have discussed in the previous subsection how the proposed algorithm works when $\mathbf{p}_{T_i}(t)$, $\forall i = 1, \dots, n$ are assumed constant. Now we would like to show that when the targets are moving slowly, the estimation error $\tilde{\mathbf{p}}_T(t)$ converges to a neighborhood of zero. To this end, we recall the following proposition (Theorem 8.3 in [22]):

Proposition 2

If the coefficient matrix $\mathbf{A}(t)$ is continuous for all $t \in [0, \infty)$ and constants $a > 0$, $b > 0$ exist such that for every solution of the homogeneous differential equation

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t)$$

one has

$$\|\mathbf{x}(t)\| \leq b\|\mathbf{x}(t_0)\|e^{-a(t-t_0)}, \quad 0 \leq t_0 < t < \infty$$

then for each $\mathbf{f}(t)$ bounded and continuous on $[0, \infty)$, every solution of the nonhomogeneous equation

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{f}(t), \quad \mathbf{x}(t_0) = \mathbf{0}$$

is also bounded for $t \in [0, \infty)$.

It is also shown in [22] that if $\|f(t)\| \leq K_f < \infty$, for some positive constant K_f , then the solution of the perturbed system satisfies

$$\|x(t)\| \leq b\|x(t_0)\|e^{-a(t-t_0)} + \frac{bK_f}{a} (1 - e^{-a(t-t_0)}) \tag{18}$$

We make the following assumption on the motion of the targets.

Assumption 1

The trajectories of the targets are such that $\|\dot{p}_{T_i}(t)\| \forall i = 1, \dots, n$ are bounded and piecewise continuous for $t \geq 0$ and there exists a sufficiently small ε such that $\|\dot{p}_{T_i}(t)\| < \varepsilon$. Furthermore, there exists some positive scalar $D > 0$ such that $\max_i \|p_T(t) - p_{T_i}(t)\| \leq D$ for all $t \geq 0$.

The condition $\max_i \|p_T(t) - p_{T_i}(t)\| \leq D$ guarantees that the distance between the targets is bounded, which is a necessary condition for the agent to circumnavigate the targets. We show in the following theorem that the estimation and control error converge to a neighborhood of zero when the targets are moving such that the aforementioned assumption holds.

Theorem 3

Adopt the notation earlier and suppose Assumption 1 holds. Then by using estimator (9) and controller (13), the estimation error $\tilde{p}_T(t)$ and the control error $\rho(t) - \rho_d(t)$ converge to neighborhoods of zero exponentially fast.

Proof

The key assumption in this theorem is that the speed of the targets is sufficiently low and the distance between any two targets is less than $2D$ for all $t \geq 0$. This assumption should always hold as the agent should always move significantly faster than the targets and the distance between any two targets should be bounded so that the agent can circumnavigate the targets. Now when the targets are moving, the estimation error dynamics for target i changes from (10) to

$$\dot{\tilde{p}}_{T_i}(t) = -k_{est}\bar{\varphi}_i(t)\bar{\varphi}_i^\top(t)\tilde{p}_{T_i}(t) - \dot{p}_{T_i}(t) \tag{19}$$

Based on the results in the stationary target case, we know that (10) is asymptotically exponentially stable. Because $\dot{p}_{T_i}(t)$ is bounded and can be regarded as a non-vanishing perturbation applied to an exponentially stable system, we can conclude according to Proposition 2 that $\tilde{p}_{T_i}(t)$ converges exponentially fast to a neighborhood of the zero. Thus $\tilde{p}_T(t)$ also converges to a neighborhood of zero exponentially fast and the size of this neighborhood is proportional to the maximum speed of the targets.

When the targets are non-stationary, $\hat{\rho}(t)$ in (16) also changes from $\dot{\hat{\rho}}(t) = -\hat{\rho}(t) + \rho_d(t) + \dot{\tilde{p}}_T(t)^\top \varphi(t)$ to

$$\dot{\hat{\rho}}(t) = -\hat{\rho}(t) + \rho_d(t) + \dot{\tilde{p}}_T(t)^\top \varphi(t) + \dot{p}_T(t)^\top \varphi(t). \tag{20}$$

Because $\rho_d(t)$ is bounded, it can be shown that $\hat{\rho}(t) - \rho_d(t)$ converges to a neighborhood of zero exponentially fast. Then according to (17), $\rho(t) - \rho_d(t)$ also converges to a neighborhood of zero exponentially fast. □

4. SIMULATIONS

In this section, we consider two different scenarios corresponding to whether the targets are stationary or moving. First, we consider the case where there are three stationary targets at $p_{T_1} = [2, 4]^\top$, $p_{T_2} = [1, 2]^\top$, and $p_{T_3} = [3, 3]^\top$. We assume that the initial target estimates are $\hat{p}_{T_1}(0) = [3, 2]^\top$, $\hat{p}_{T_2}(0) = [3, 0]^\top$, and $\hat{p}_{T_3}(0) = [7, 3]^\top$. We also assume that $\alpha = 5$, $k_{est} = 5$, and d in (4) is

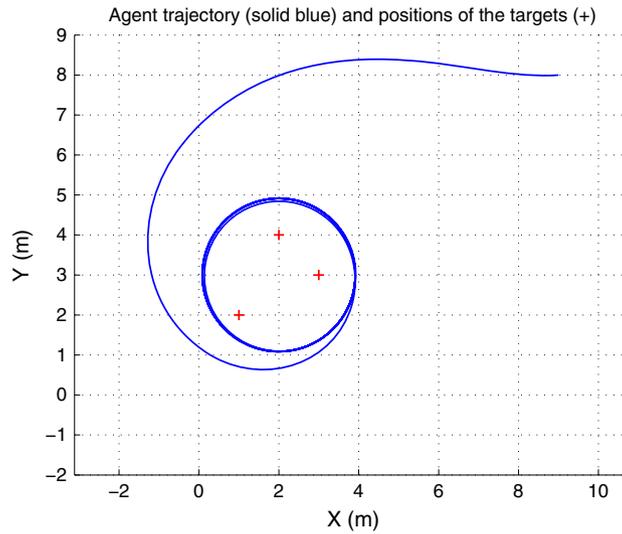


Figure 6. Agent trajectory in $X - Y$ plane, for the case where the targets are stationary.

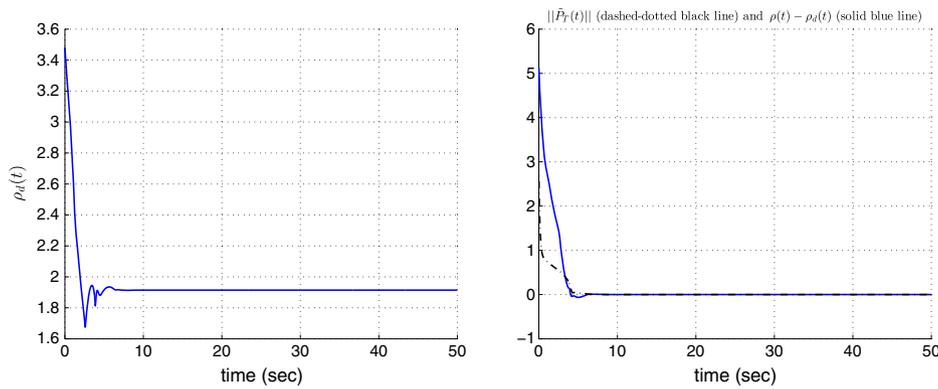


Figure 7. The left-hand side figure shows the estimated radius of the circle $\rho_d(t)$, and the right-hand side figure shows $\|\hat{\mathbf{p}}_T(t)\|$ and $\rho(t) - \rho_d(t)$ for the case where the targets are stationary.

0.5. Simulation results for the case where the bearing angles to all targets are available to the agent for all $t > 0$ are shown in Figures 6 and 7. It can be seen that the estimation error exponentially converges to zero and the circling radius exponentially converges to $\rho_d = 1.91$ m. We then suppose that the measurements are taken in segments such that the agent can only measure one bearing angle at a time and will switch between the targets at intervals of $\tau = 0.05$ s. Thus, the estimated position of only one of the targets is updated at any time. The results are shown in Figures 8 and 9. If we compare the results of segmented and continuous measurement cases, we see that in both cases the estimation and control errors converge exponentially to zero; however, in the segmented measurement case, the convergence is slower.

We then consider the case where the bearing angles to the targets are perturbed by normally distributed random noises. We simulate two different scenarios by applying noises with two different standard deviations to the bearing measurements. We assume the noises are zero mean with standard deviation of 0.1 and 0.2. Simulation results for these cases are shown in Figure 10. It can be seen that the errors go to neighborhoods of zero and that the size of these neighborhoods is larger when the standard deviation of noise is larger.

Now we consider the case where the targets are moving slowly such that $\mathbf{p}_{T_1} = [2 + .025t, 4 + \sin(.03t) + .025t]^T$, $\mathbf{p}_{T_2} = [1 + .03t, 2 + \sin(.025t) + .03t]^T$, and $\mathbf{p}_{T_3} = [3 + .025t, 3 +$

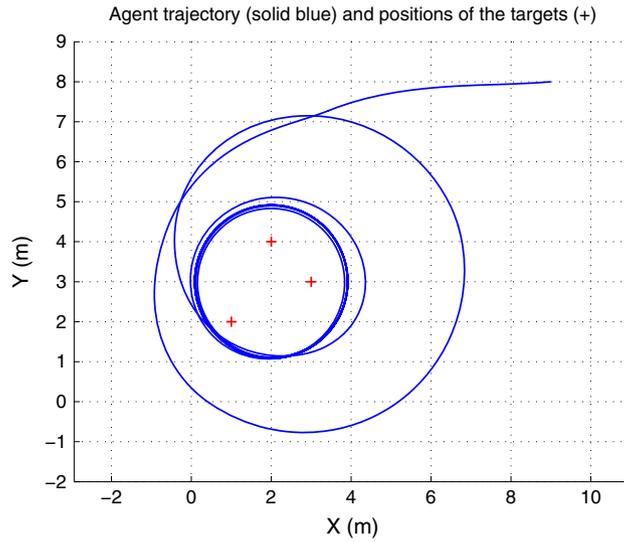


Figure 8. Agent trajectory in $X - Y$ plane, for the case where the targets are stationary and the bearing measurements are taken in segments.

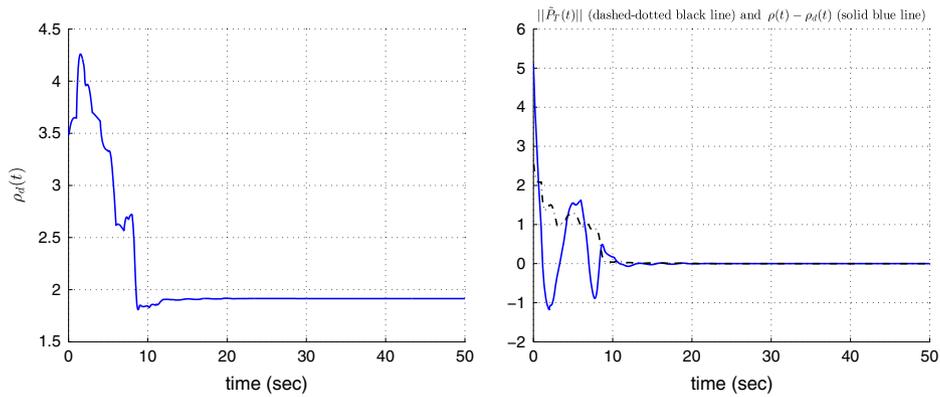


Figure 9. The left-hand side figure shows the estimated radius of the circle $\rho_d(t)$, and the right-hand side figure shows $\|\tilde{P}_T(t)\|$ and $\rho(t) - \rho_d(t)$ for the case where the targets are stationary and the bearing measurements are taken in segments.

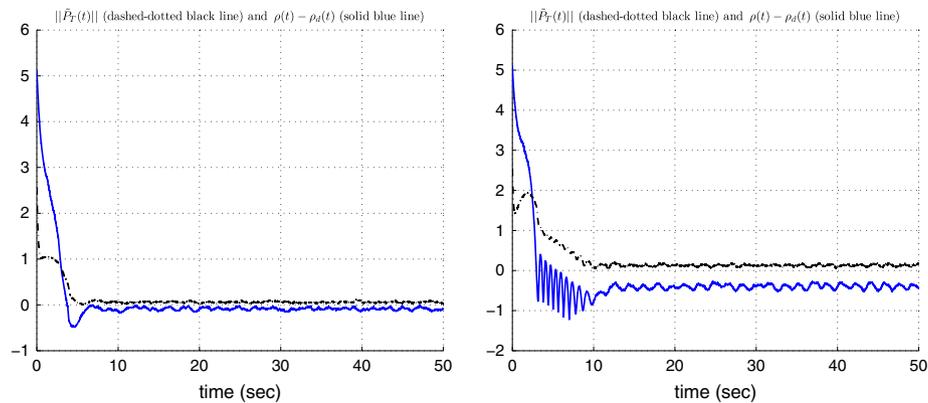


Figure 10. Simulation results for the case where the bearing measurements are noisy. The left-hand side figure shows the case where the standard deviation of noise is 0.1, while the right-hand side figure is for case where the standard deviation is 0.2.

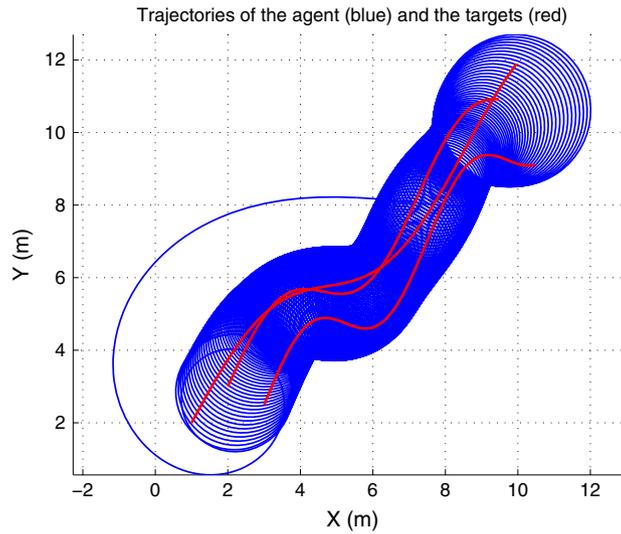


Figure 11. Agent trajectory in $X - Y$ plane for the case where the targets are moving.

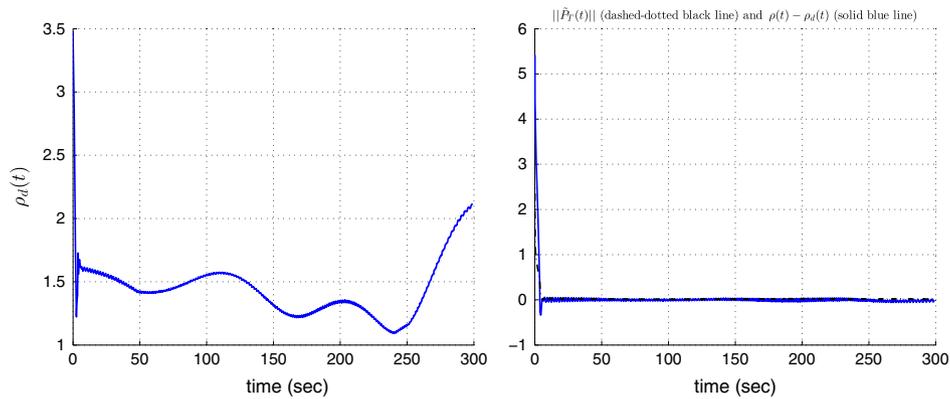


Figure 12. The left-hand side figure shows the estimated radius of the circle $\rho_d(t)$, and the right-hand side figure shows $\|\tilde{p}_T(t)\|$ and $\rho(t) - \rho_d(t)$ for the case where the targets are stationary and the bearing measurements are taken in segments.

$\sin(.035t) + .025t]^T$. Simulation results for this case are shown in Figures 11 and 12. Note that the desired radius of the circle in Figure 12 changes by time as the targets move in different directions with different speed and thus the radius of the circle changes by time. It can be seen that the estimation and control error do not converge to zero, but converge to neighborhoods of zero.

5. CONCLUSION AND FUTURE WORK

In this paper, we considered the localization and circumnavigation problem of multiple targets. We proposed estimator and control algorithms and showed that with stationary targets, the estimation error and the control error converge to zero exponentially fast. It is shown that for the moving target case, the estimator and controller can tolerate slow motion of targets with only modest effects on accuracy. It also appears that the larger the speed of the targets, the larger the estimation error.

Future directions of research include general collision avoidance; considering more realistic agent models; having three or more agents forming a polygon formation circling a target/multiple targets; estimating the speed of targets if they are moving at an unknown constant speed; and using methods like EKF or IPDA-FR [23] to further reduce the effect of noise and to increase accuracy.

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