A Novel Multiscroll Chaotic System and Its Realization

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***Abstract*— This paper proposes a novel multiscroll chaotic system, which is different from Chua's circuit and all its variants in most aspects of the algebraic form, circuit design, and geometrical structure of the attractor. In particular, the multiscroll attractor of this new system is more complex than that of the generalized Chua's circuit when they both have the same number of scrolls. The dynamical behaviors of the new system are then analyzed, including the bifurcation diagram and the Lyapunov exponent spectra. Moreover, a module-based circuit diagram is designed for realizing various multiscroll attractors. Finally, experimental circuits are implemented with physical observations reported.**

I. Introduction

Chua's circuit is an important paradigm in nonlinear circuit theory [1]. Inspired by Chua's chaotic double-scroll circuit, many extended nonlinear circuits have been designed and implemented, including various multiscroll circuits, in the past decades [2-14].

In retrospect, Suykens *et al.* [2,5] proposed some families of circuits that can generate $n-$double scroll chaotic attractors. Yalcin *et al.* [6] presented a family of systems with grid-scroll chaotic attractors. Lü *et al.* [7-9] proposed an approach using hysteresis and saturated functional series for creating 1-D $n-$scroll, 2-D $n \times m-$grid scroll, and 3-D $n \times m \times l-$grid scroll chaotic attractors, with rigorously mathematical proofs and physical realizations. Yu *et al.* [10] introduced a general jerk circuit for generating various types of $n-$scroll chaotic attractors. Very recently, Lü and Chen provided a detailed review [3] of the main advances in theories, methods, implementations, and applications of multiscroll chaos generation. Moreover, some new progress can be found in [11,12].

Notice that most of the aforementioned works use circuits constructed by using capacitors, inductors, and resistors, along with Chua's diode, which belong to the typical $\pi-$type circuits. From a different point of view, this paper introduces a novel multiscroll chaotic system, which is different from Chua's circuit and all its variants, in most aspects of the algebraic form, circuit design, and geometrical structure of the attractor. It appears that the multiscroll chaotic attractor of this new system is more complex than that of the generalized Chua's circuit even when they have the same number of scrolls. This is because that this new attractor is nesting, however, Chua's multiscroll attractor is not nesting.

The paper is organized as follows. Section II introduces the new multiscroll chaotic system, with its dynamical behaviors analyzed in Section III. Section IV designs a module-based circuit diagram for implementing the multiscroll chaotic attractors. Section V concludes the paper.

II. A Novel Multiscroll Chaotic System

The proposed multiscroll chaotic system is described by

$$\begin{cases} \frac{dx}{d\tau} = \beta\, y - x - \alpha\, f(x) \\ \frac{dy}{d\tau} = \beta\, x - \gamma\, z \\ \frac{dz}{d\tau} = \xi\, y - z\,, \end{cases} \tag{1}$$

where $\alpha = 4.2$, $\beta = 6.7$, $\gamma = 4.0$, and ξ is a parameter. When $\xi = 18.5$ and $f(x) = x - (|x+1| - |x-1|)$, system (1) has a double-scroll chaotic attractor. When

$$f(x) = f_1(x) = x + \sum_{n=1}^{2N-1} (-1)^n [|x+(2n-1)| - |x-(2n-1)|] \tag{2}$$

or

$$f(x) = f_2(x) = x + \sum_{n=1}^{2N} (-1)^{n-1} [|x+(2n-1)| - |x-(2n-1)|]\,, \tag{3}$$

system (1) can generate a $2N$ or $2N+1$-scroll attractor. For example, system (1) with (2) can create a $10-$scroll chaotic attractor, as shown in Fig. 1 (a), with $N = 5$, $\xi = 11.6$; system (1) with (3) can generate a $9-$scroll chaotic attractor, as shown in Fig. 1 (b), with $N = 4$, $\xi = 11.7$.

For comparison, recall that Suykens *et al.* [2,5] proposed a generalized Chua's circuit, described by

$$\begin{cases} \frac{dx}{d\tau} = \alpha[y - f(x)] \\ \frac{dy}{d\tau} = x - y + z \\ \frac{dz}{d\tau} = -\beta\, y, \end{cases} \tag{4}$$

where $\alpha = 10$, $\beta = 15$. When

$$f(x) = m_{2n-1} x + \frac{1}{2} \sum_{i=1}^{2n-1} (m_{i-1} - m_i)(|x+c_i| - |x-c_i|), \tag{5}$$

system (4) can create an $n-$scroll chaotic attractor. For $\alpha = 9$, $\beta = 14.286$, $m = (\frac{0.9}{7}, -\frac{3}{7}, \frac{3.5}{7}, -\frac{2.4}{7}, \frac{2.52}{7}, -\frac{1.68}{7}, \frac{2.52}{7},$

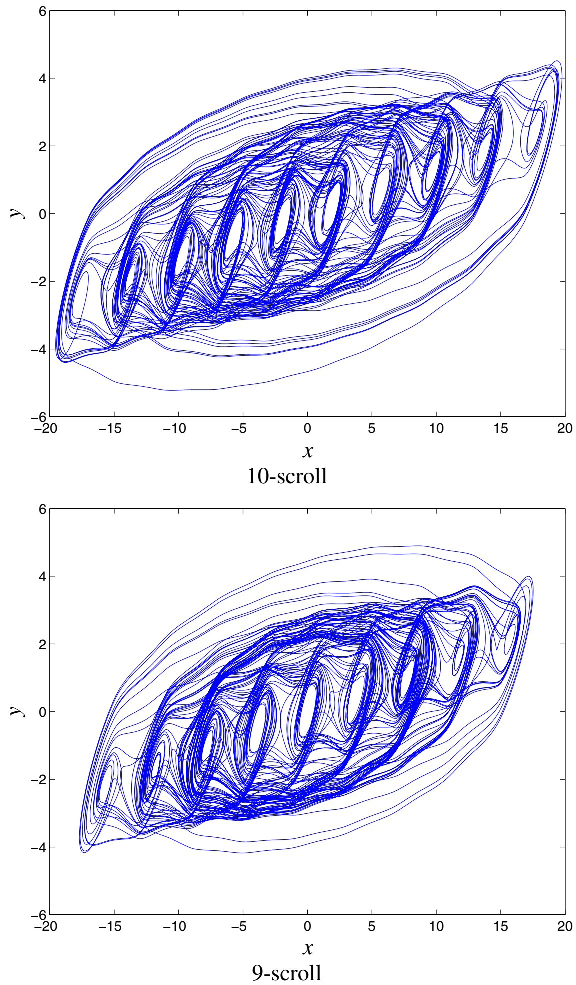


Fig. 1. Multiscroll chaotic attractors of system (1).

$-\frac{1.68}{7}$), $c = (1,\, 2.15,\, 3.6,\, 6.2,\, 9,\, 14,\, 25)$, system (4) with (5) has a $7-$scroll chaotic attractor, as shown in Fig. 2.

Remark: In the algebraic form, system (1) is different from Chua's circuit and all its variants, including the generalized Chua's circuit (4), since the second equation of system (1) only has two items $(x,\, z)$ but that of Chua's circuit has three (x, y, z). In terms of the geometrical structure of the multiscroll attractor, the attractor of system (1) as shown in Fig. 1 is more complex than that of the generalized Chua's circuit (4), shown in Fig. 2. This is because that the attractors of system (1) is nesting, however, those of system (4) is not nesting.

III. Dynamical Behaviors of the Multiscroll Chaotic System

Let $\dot{x} = 0$, $\dot{y} = 0$, $\dot{z} = 0$ in system (1). Then, the system equilibria satisfy the following equations:

$$\begin{cases} f(x) = (\frac{\beta^2}{\gamma\xi} - 1)\frac{x}{\alpha} \\ y = \frac{\beta\, x}{\gamma\xi} \\ z = \frac{\beta\, x}{\gamma}. \end{cases} \tag{6}$$

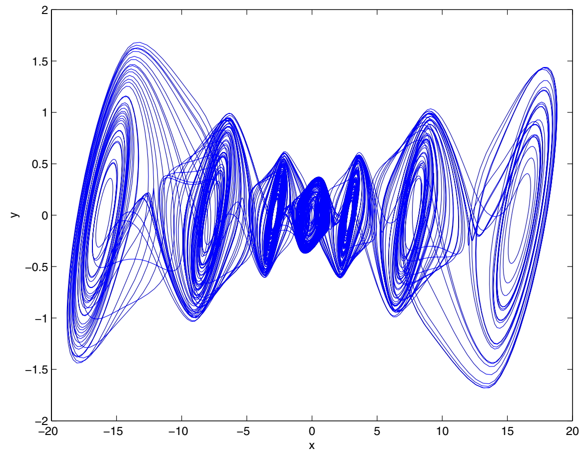


Fig. 2. 7-scroll chaotic attractor of the generalized Chua's circuit (2).

According to (2) and (3), the equilibria are obtained as follows: for the case of an even number of scrolls,

$$\begin{cases} x^{\pm n} = \pm\frac{(-1)^{n+1}2\alpha n}{(1-\frac{\beta^2}{\gamma\xi})+(-1)^{n+1}\alpha} \\ y^{\pm n} = \frac{\beta}{\gamma\xi}x^{\pm n} \\ z^{\pm n} = \frac{\beta}{\gamma}x^{\pm n} \end{cases} \tag{7}$$

and for the case of an odd number of scrolls,

$$\begin{cases} x^{\pm n} = \pm\frac{(-1)^{n}2\alpha n}{(1-\frac{\beta^2}{\gamma\xi})+(-1)^{n}\alpha} \\ y^{\pm n} = \frac{\beta}{\gamma\xi}x^{\pm n} \\ z^{\pm n} = \frac{\beta}{\gamma}x^{\pm n} \end{cases} \tag{8}$$

where $0 \le n \le 2N - 1$ and $\Omega^{\pm n}(x^{\pm n}, y^{\pm n}, z^{\pm n})$ denotes the equilibria in the positive and negative x-axis, respectively. Clearly, all equilibria are located in a inclining line. However, all equilibria of the generalized Chua's circuit are located in the x-axis.

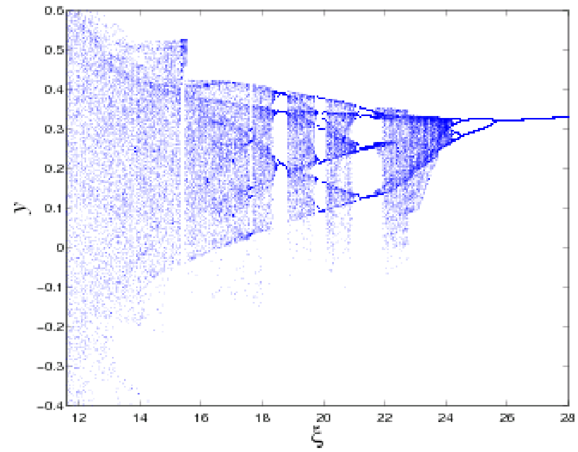
Figures 3 (a) and (b) show the bifurcation diagram and the spectrum of largest Lyapunov exponents of system (1) versus the parameter ξ. There exists a typical period-doubling bifurcation phenomenon in Fig. 3 (a). From Fig. 3 (b), the largest Lyapunov exponent of system (1) within the range [11.6,28] is 1.2, which is obviously greater than that of Chua's circuit, which is only 0.3.

IV. Circuit Implementation of Multiscroll Chaotic Attractors

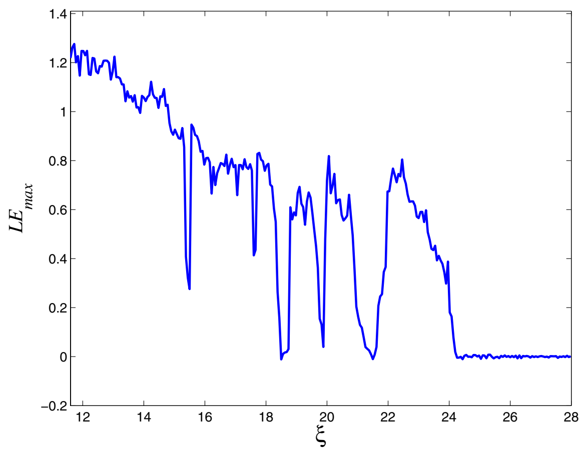
In this section, a module-based circuit diagram is designed for realizing the multiscroll chaotic attractors of system (1) with (2) or (3), as shown by Figs. 4 and 5.

The procedure consists of four steps: Step I: *proportional compression transformation of state variables*; Step II: *transformation from differential to integral operations*; Step III: *transformation of the time-scale*; Step IV: *Module-based design of nonlinear functions*.

Select suitable a proportional compression ratio, $\frac{1}{5}$, and then carry out the proportional compression transformation of the



(a) Bifurcation diagram



(b) Spectrum of largest Lyapunov exponents

Fig. 3. Dynamical analysis of the multiscroll chaotic system (1).

system state variables, on (1), (2) and (3). Clearly, system (1) keeps invariant, but (2) and (3) are transformed into the following forms:

$$f(x)=\begin{cases} x+\sum\limits_{n=1}^{2N-1}(-1)^n\left(|x+\frac{2n-1}{5}|-|x-\frac{2n-1}{5}|\right)\\ x+\sum\limits_{n=1}^{2N}(-1)^{n-1}\left(|x+\frac{2n-1}{5}|-|x-\frac{2n-1}{5}|\right), \end{cases} \tag{9}$$

where the first and second equations are the expressions of $f(x)$ for the cases of even and odd numbers of scrolls, respectively.

Let $\tau=\frac{t}{\tau_0}$ and carry out the transformation of the time-scale as follows:

$$\begin{cases} x=-\left(-\frac{1}{R_0C_0}\right)\int[-\beta(-y)-x-\alpha f(x)]dt\\ y=-\left(-\frac{1}{R_0C_0}\right)\int[-\beta(-x)-\gamma z]dt\\ z=-\left(-\frac{1}{R_0C_0}\right)\int(-\xi(-y)-z)dt, \end{cases} \tag{10}$$

where $\tau_0=R_0$ is the transformation factor of the time-scale and C_0 is the integral constant of the integrators. If R_0 and C_0 are increased, then the phase portraits become more and more sparse; if R_0 and C_0 are decreased, then the phase portraits become more and more dense.

All operational amplifiers shown in Figs. 4 and 5 are TL082. Assume that the supply voltages and saturated voltages of all operational amplifiers are $\pm E=\pm 15\,V$ and $\pm E_{sat}=\pm 13.5\,V$, respectively. For convenient adjustment and higher precision, all resistors are precisely adjustable resistors or potentiometers.

To generate an even number of scrolls from circuits shown in Figs. 4 and 5, all resistors are selected as follows: $R_{i1}=1k\Omega\,(2\le i\le 19)$, $R_{22}=3.97k\Omega$, $R_{42}=3.5k\Omega$, $R_{62}=5.19k\Omega$, $R_{82}=5.14k\Omega$, $R_{102}=7.5k\Omega$, $R_{122}=8.64k\Omega$, $R_{142}=13.5k\Omega$, $R_{162}=21.5k\Omega$, $R_{182}=67.5k\Omega$, $R_{32}=0.99k\Omega$, $R_{52}=1.25k\Omega$, $R_{72}=1.6k\Omega$, $R_{92}=2.06k\Omega$, $R_{112}=2.75k\Omega$, $R_{132}=3.82k\Omega$, $R_{152}=5.75k\Omega$, $R_{172}=10.25k\Omega$, $R_{192}=32.75k\Omega$.

To generate an odd number of scrolls from circuits shown in Figs. 4 and 5, all resistors are selected as follows: $R_{i1}=1k\Omega\,(2\le i\le 19)$, $R_{22}=4.5k\Omega$, $R_{42}=4.19k\Omega$, $R_{62}=6.14k\Omega$, $R_{82}=6.5k\Omega$, $R_{102}=9.64k\Omega$, $R_{122}=12.5k\Omega$, $R_{142}=22.5k\Omega$, $R_{162}=66.5k\Omega$, $R_{32}=1.25k\Omega$, $R_{52}=1.60k\Omega$, $R_{72}=2.07k\Omega$, $R_{92}=2.75k\Omega$, $R_{112}=3.82k\Omega$, $R_{132}=5.75k\Omega$, $R_{152}=10.25k\Omega$, $R_{172}=32.75k\Omega$.

When S_0 is switched off, $S_i\,(1\le i\le 4)$ are switched on, and $R=8.55k\Omega$, the circuit in Figs. 4 and 5 can generate a $9-$scroll chaotic attractor as shown in Fig. 6 (a). When $S_i\,(0\le i\le 4)$ are switched on and $R=8.4k\Omega$, the circuit in Figs. 4 and 5 can generate a $10-$scroll chaotic attractor as shown in Fig. 6 (b).

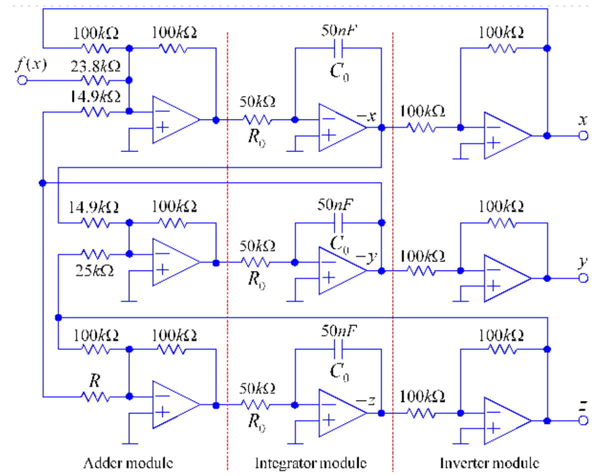


Fig. 4. Module-based circuit diagram for realizing multiscroll chaotic attractors of system (1).

V. Conclusions

This paper has proposed a novel multiscroll chaotic system, which is different from Chua's circuit and all its variants in most aspects of the algebraic form, circuit design, and geometrical structure of the multiscroll attractor. It has been observed that the multiscroll chaotic attractor of this new

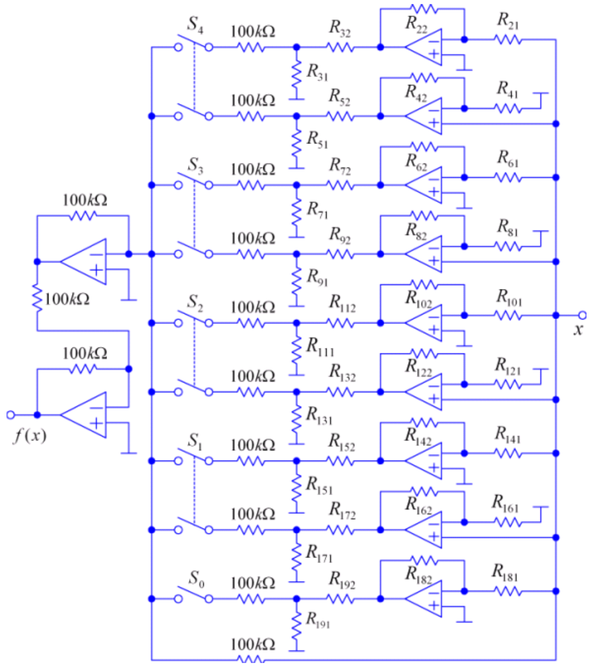
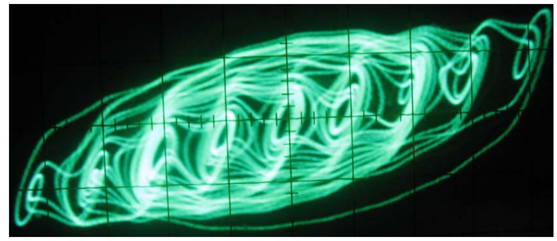
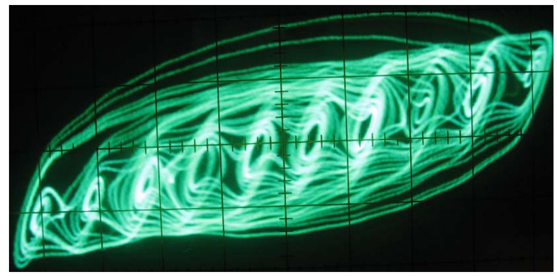


Fig. 5. Circuit diagram for realizing the nonlinearity $f(x)$.



(a) 9-scroll



(b) 10-scroll

Fig. 6. Experimental observations of multiscroll attractors of system (1).

system is more complex than that of the generalized Chua's circuit even when they have the same number of scrolls. Moreover, the dynamical behaviors, such as bifurcation and Lyapunov exponents, of this new system has been investigated. Furthermore, using a module-based approach, a simple circuit diagram has been constructed for implementing the multiscroll chaotic attractors. Finally, experimental circuits have been implemented, wit physical observations reported. The simple novel system will provide a new possibility for both theoretical studies and technical applications in the near future.

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References

[1] L. O. Chua, M. Komuro, and T. Matsumoto, "The double scroll family," *IEEE Trans. Circuits Syst.*, vol. 33, no. 11, pp. 1072-1118, Nov. 1986.

[2] J. A. K. Suykens and J. Vandewalle, "Generation of n-double scrolls ($n = 1, 2, 3, \cdots$)," *IEEE Trans. Circuits Syst. I*, vol. 40, no. 11, pp. 861-867, Nov. 1993.

[3] J. Lü and G. Chen, "Multi-scroll chaos generation: Theories, methods and applications," *Int. J. Bifurcation and Chaos*, vol. 16, no. 4, pp. 775-858, Apr. 2006.

[4] J. Lü and G. Chen, "A new chaotic attractor coined," *Int. J. Bifurcation and Chaos*, vol. 12, no. 3, pp. 659-661, Mar. 2002.

[5] J. A. K. Suykens, A. Huang, and L. O. Chua, "A family of n-scroll attractors from a generalized Chua's circuit," *Int. J. Electron. Commun.*, vol. 51, pp. 131 C 138, 1997.

[6] M. E. Yalcin, J. A. K. Suykens, J. Vandewalle, and S. Ozoguz, "Families of scroll grid attractors," *Int. J. Bifurcation Chaos*, vol. 12, no. 1, pp. 23-41, Jan. 2002.

[7] J. Lü, F. Han, X. Yu, and G. Chen, "Generating 3-D multi-scroll chaotic attractors: A hysteresis series switching method," *Automatica*, vol. 40, no. 10, pp. 1677 C 1687, Oct. 2004.

[8] J. Lü, G. Chen, X. Yu, and H. Leung, "Design and analysis of multiscroll chaotic attractors from saturated function series," *IEEE Trans. Circuits Syst. I*, vol. 51, no. 12, pp. 2476 C 2490, Dec. 2004.

[9] J. Lü, S. Yu, H. Leung, and G. Chen, "Experimental verification of multidirectional multiscroll chaotic attractors," *IEEE Trans. Circuits Syst. I*, vol. 53, no. 1, pp. 149-165, Jan. 2006.

[10] S. Yu, J. Lü, H. Leung, and G. Chen, "Design and implementation of n-scroll chaotic attractors from a general jerk circuit," *IEEE Trans. Circuits Syst. I*, vol. 52, no. 7, pp. 1459-1477, Jul. 2005.

[11] S. Yu, J. Lü, K. S. Tang, and G. Chen, "A general multi-scroll Lorenz system family and its realization via digital signal processors," *Chaos*, vol. 16, no. 3, 033126, Sep. 2006.

[12] S. Yu, J. Lü, and G. Chen, "Theoretical design and circuit implementation of multi-directional multi-torus chaotic attractors," *IEEE Trans. Circuits Syst. I*, vol. 54, no. 9, pp. 2087-2098, Sep. 2007.

[13] A. G. Radwan, A. M. Soliman, and A. S. Elwakil, "1-D digitally-controlled multi-scroll chaos generator," *Int. J. Bifurcation and Chaos*, vol. 17, no. 1, pp. 227-242, Jan. 2007.

[14] P. Arena, S. De Fiore, L. Fortuna, M. Frasca, L. Patane, and G. Vagliasindi, "Weak chaos control for action-oriented perception: Real time implementation via FPGA," *Proc. 1st IEEE/RAS-EMBS Int. Conf. Biomedic. Robot. Biomechatron.*, pp. 555-560, Feb. 2006.