Topology Identification of An Uncertain General Complex Dynamical Network

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***Abstract*— In real-world complex networks, there exists many uncertain information, such as uncertain topological structures and uncertain system parameters. Without question, the topology identification and parameter identification are two traditionally challenging questions in complex networks. Based on the adaptive observers, our approach can identify the topological structures and system parameters of the uncertain complex dynamical networks together. In particular, our method is also very effective for the complex networks with different node dynamics. Moreover, the proposed approach can be used to monitor the online evolution of network topological structures and system parameters. Finally, several typical simulations are used to verify the effectiveness of the proposed approach.**

I. Introduction

Complex networks exist in numerous natural and man-made systems, including food webs, cellular and metabolic networks, electrical power grids, computer networks, the World Wide Web, social networks, and so on [1-3]. Over the last decade, the study of complex networks pervades through almost all scientific disciplines, from computer science to sociology, from cell biology to ecology, to name just a few [4-6].

Nowadays, the research on complex networks has been focused on the topological structures as well as on how their topological properties, such as clustering coefficients, connectivity distributions, and average network distances, influence their dynamical behaviors [2-4]. It is well known that the macroscopic behavior of a network is often determined by both its dynamical rules governing the nodes and the flow occurring along the links. Therefore, on the contrary, a natural question to follow is whether or not one can estimate its topological structure by using its nodes dynamical behaviors? This paper will give a positive answer to this question.

As a matter of fact, in the real-world complex networks, the exact network topological structures are sometimes unknown or uncertain. In view of their characteristics, identifying their topological structures become a key problem in many disciplines, such as the protein-protein or protein-DNA (Deoxyribonucleic Acid) interactions in the regulation of various cellular processes [4]. As we know now, the protein-DNA interactions often play the pivotal roles in many cell processes, such as DNA replication, modification, repair and RNA (Ribonucleic Acid) transcription [4]. Another example is biological neural networks. In addition to modeling the neurons, the exact topological structure of a biological neural network also plays an important role in the certain function. However, due to the complex nature of the real-world networks, such as nonlinearity and high dimension, it is very difficult to identify their topological structures correctly.

Recently, there are some results reported on the identification of network topological structures [7-9]. However, most of the aforementioned approaches are only valid for the known system parameters. In fact, in many practical situations, the system parameters often cannot be exactly known beforehand [1,4]. Therefore, it is very necessary to propose an effective method for identifying the network topological structures and system parameters together. Based on the synchronization-based estimation theory and adaptive control techniques, an effective approach is then proposed to identify the network topological structure and system parameters of an uncertain complex dynamical network together in this paper. In particular, this method is also valid for the network with different dynamical nodes and the switching of system parameters. Moreover, it is an online method and can monitor the dynamical evolution of network topological structures and system parameters.

The rest of the paper is organized as follows. Sections II and III discuss the identifications of topological structure and system parameters of an uncertain complex dynamical network with the identical nodes and different nodes, respectively. Several numerical examples are then given to verify the effectiveness of the proposed method in Section IV. Section V gives the concluding remarks.

II. An Uncertain General Complex Dynamical Network with Identical Dynamical Nodes

Consider the node system $\dot{x} = \bar{f}(x)$, which can be rewritten in the form of

$$\dot{\mathbf{x}} = f(\mathbf{x}) + F(\mathbf{x})\,\alpha, \tag{1}$$

where $\mathbf{x} \in \mathbf{R}^n$ is the state vector, $\alpha \in \mathbf{R}^m$ is the unknown system parameter vector, $f(x)$ is an $n \times 1$ matrix, $F(x)$ is an

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$n \times m$ matrix satisfying $F_{ij}(\mathbf{x}) \in L_\infty$.

Assume that a complex dynamical network consisting of N identical nodes with general couplings, which is described by

$$\dot{\mathbf{x}}_i = f(\mathbf{x}_i) + F(\mathbf{x}_i)\,\alpha + \sum_{j=1}^{N} a_{ij} h_j(\mathbf{x}_j), \quad (2)$$

where $1 \le i \le N$, $\mathbf{x}_i = (x_{i1}, x_{i2}, \cdots, x_{in})^T \in \mathbf{R}^N$ is the state vector of the $i-$th node, node dynamics is $\dot{x} = \bar{f}(\mathbf{x})$. $\mathbf{A} = (a_{ij}) \in \mathbf{R}^{N\times N}$ is the unknown or uncertain coupling configuration matrix. If there exists a link from node i to node j $(j \ne i)$, then $a_{ij} \ne 0$; otherwise, $a_{ij} = 0$.

Here the inner coupling function $h_j(\cdot) : \mathbf{R}^n \to \mathbf{R}^n$ is a general function satisfying Lipschitz condition. And the coupling configuration matrix $\mathbf{A}$ need not to be symmetric, irreducible and diffusive. Assume that α is the unknown parameter vector of the node dynamics. To identify the topological structure matrix $\mathbf{A}$ and the system parameter vector α, two useful assumptions are introduced in the following.

Assumption 1 (A1). *Suppose that there exists a nonnegative constant L_1 satisfying:*

$$\|\bar{f}(\boldsymbol{x}_i) - \bar{f}(\boldsymbol{y}_i)\| \le L_1 \|\boldsymbol{x}_i - \boldsymbol{y}_i\|, \quad i = 1, \cdots, N. \quad (3)$$

Assumption 2 (A2). *Suppose that there exist nonnegative constants L_{2j} $(1 \le j \le N)$ satisfying:*

$$\|h_j(\boldsymbol{x}_j) - h_j(\boldsymbol{y}_j)\| \le L_{2j} \|\boldsymbol{x}_j - \boldsymbol{y}_j\|, \quad j = 1, \cdots, N. \quad (4)$$

For simplification, $\|\cdot\|$ denotes $2-$norm and $L_2 = \max\limits_{1\le j\le N}\{L_{2j}\}$.

Consider another uncertain general complex dynamical network, which is characterized by

$$\dot{\hat{\mathbf{x}}}_i = f(\hat{\mathbf{x}}_i) + F(\hat{\mathbf{x}}_i)\,\hat{\alpha} + \sum_{j=1}^{N} \hat{a}_{ij} h_j(\hat{\mathbf{x}}_j) + \mathbf{u}_i, \quad (5)$$

where $1 \le i \le N$, $\hat{\mathbf{x}}_i = (\hat{x}_{i1}, \hat{x}_{i2}, \cdots, \hat{x}_{in})^T \in \mathbf{R}^N$ is the state vector of the $i-$th node, $(u)_i \in \mathbf{R}^n$ is the control inputs, and $\hat{a}_{ij}$ is the estimation of a_{ij}, and the parameter vector $\hat{\alpha}$ is the estimation of the unknown or uncertain vector α.

Denote $\tilde{\mathbf{x}}_i = \hat{\mathbf{x}}_i - \mathbf{x}_i$, $\tilde{a}_{ij} = \hat{a}_{ij} - a_{ij}$, $\tilde{\alpha}_{ij} = \hat{\alpha}_{ij} - \alpha$. Thus the error system is given by

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}_i \;=\; & f(\hat{\mathbf{x}}_i) - f(\mathbf{x}_i) + F(\hat{\mathbf{x}}_i)\hat{\alpha} - F(\mathbf{x}_i)\alpha \\ & + \sum_{j=1}^{N} \hat{a}_{ij} h_j(\hat{\mathbf{x}}_j) - \sum_{j=1}^{N} a_{ij} h_j(\mathbf{x}_j) - k_i \tilde{\mathbf{x}}_i, \end{aligned} \quad (6)$$

where $1 \le i \le N$.

Based on the assumptions **A1** and **A2**, one gets the following theorems.

Theorem 1. *Suppose that **A1** and **A2** hold. Then the unknown or uncertain coupling configuration matrix $\mathbf{A}$ of the uncertain general complex dynamical network (2) and the parameter vector α can be identified by the estimated values $\hat{\mathbf{A}}$ and $\hat{\alpha}$, respectively, using the following response network:*

$$\begin{cases} \dot{\hat{\boldsymbol{x}}}_i &= f(\hat{\boldsymbol{x}}_i) + F(\hat{\boldsymbol{x}}_i)\hat{\alpha} + \sum\limits_{j=1}^{N} \hat{a}_{ij} h_j(\hat{\boldsymbol{x}}_j) + \boldsymbol{u}_i \\ \boldsymbol{u}_i &= -k_i \tilde{\boldsymbol{x}}_i,\ \dot{k}_i = d_i \|\tilde{\boldsymbol{x}}_i\|^2 \\ \dot{\hat{\alpha}} &= -\sum\limits_{i=1}^{N} F^T(\hat{\boldsymbol{x}}_i)\tilde{\boldsymbol{x}}_i,\ \dot{\hat{a}}_{ij} = \tilde{\boldsymbol{x}}_i^T h_j(\hat{\boldsymbol{x}}_j), \end{cases} \quad (7)$$

where $1 \le i, j \le N$ and d_i are positive constants.

Proof: Construct the Lyapunov function

$$\begin{aligned} 2V \;=\; & \sum_{i=1}^{N} \tilde{\mathbf{x}}_i^T \tilde{\mathbf{x}}_i + \sum_{i=1}^{N}\sum_{j=1}^{N} (\hat{a}_{ij} - a_{ij})^2 + \tilde{\alpha}^T \tilde{\alpha} \\ & + \sum_{i=1}^{N} \tfrac{1}{d_i}(k_i - k^*)^2, \end{aligned}$$

where k^* is a sufficiently large positive constant to be determined. Denote $\max\{L_1, L_3\} = L$. Thus one has

$$\begin{aligned} \dot{V} \;=\; & \sum_{i=1}^{N} \tilde{\mathbf{x}}_i^T \dot{\tilde{\mathbf{x}}}_i + \sum_{i=1}^{N}\sum_{j=1}^{N} \tilde{a}_{ij}\dot{\hat{a}}_{ij} + \tilde{\alpha}^T \dot{\tilde{\alpha}} \\ & + \sum_{i=1}^{N} \tfrac{1}{d_i}(k_i - k^*)\dot{k}_i \\ \le\; & \left(L_1 + L_2 \lambda_{\max}\left(\tfrac{\mathbf{A}+\mathbf{A}^T}{2}\right) - k^*\right) \tilde{\mathbf{x}}^T \tilde{\mathbf{x}}, \end{aligned}$$

where $\lambda_{\max}\left(\frac{\mathbf{A}+\mathbf{A}^T}{2}\right)$ is the maximal eigenvalue of the symmetric matrix $\frac{\mathbf{A}+\mathbf{A}^T}{2}$. Denote $\tilde{\mathbf{x}} = (\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, ..., \tilde{\mathbf{x}}_N)^T$.

Let $k^* = L_1 + L_2 \lambda_{\max}\left(\frac{\mathbf{A}+\mathbf{A}^T}{2}\right) + 1$. One obtains

$$\dot{V}|_{(6)(7)} \le -\tilde{\mathbf{x}}^T \tilde{\mathbf{x}}.$$

Obviously, $M = \{\dot{V}|_{(6)(7)} = 0\} = \{\tilde{\mathbf{x}} = \mathbf{0}\}$. From (6), the largest invariant set E contained in M is $E = \{\tilde{\mathbf{x}} = \mathbf{0}, \hat{\mathbf{A}} = \mathbf{A}, \hat{\alpha} = \alpha, k = k^*\}$. In fact, if one of the following equalities cannot hold: $\hat{\mathbf{A}} = \mathbf{A}, \hat{\alpha} = \alpha$, then $\tilde{\mathbf{x}} = \mathbf{0}$ cannot be a fixed point of system (6) at all. That is, one cannot conclude that the solution $\tilde{\mathbf{x}}$ is equal to $\mathbf{0}$ for $t > 0$ when the initial values $\tilde{\mathbf{x}}(0) = \mathbf{0}$. According to the LaSalle's invariance principle, starting from arbitrary initial values of system (6), the trajectory asymptotically converges to the set E, i.e., $\tilde{\mathbf{x}} \to \mathbf{0}$, $\hat{\mathbf{A}} \to \mathbf{A}$, and $\hat{\alpha} \to \alpha$ as $t \to \infty$. It indicates that the topological structure matrix $\mathbf{A}$ and the system parameter vector α can be effectively estimated using the updating laws (7).

Remark 1. It is well known that the proof of **Theorem 1** should be based on LaSalle's invariance principle since the Lyapunov direct method can only guarantee the stability in the sense of Lyapunov but not asymptotic stability [10]. Moreover, the inner coupling functions $\mathrm{h}_j(\cdot)$ are not necessarily linear, and the coupling configuration matrix $\mathbf{A}$ need not to be symmetric, irreducible, even diffusive as well.

III. An Uncertain General Complex Dynamical Network with Different Nodes

Let node dynamics be $\dot{\mathbf{x}} = \bar{f}(\mathbf{x})$ or $\dot{\mathbf{y}} = \bar{\mathbf{g}}(\mathbf{y})$. And it can be rewritten in the form of $\dot{\mathbf{x}} = \mathrm{f}(\mathbf{x}) + \mathrm{F}(\mathbf{x})\,\alpha$or

$$\dot{\mathbf{y}} = \mathrm{g}(\mathbf{y}) + \mathrm{G}(\mathbf{y})\,\beta, \quad (8)$$

where $\mathbf{y} \in \mathbf{R}^n$ is the state vector, $\beta \in \mathbf{R}^q$ is the unknown parameter vector, $\mathrm{g}(\mathbf{y})$ is an $n \times 1$ matrix, $\mathrm{G}(\mathbf{y})$ is an $n \times q$ matrix satisfying $\mathrm{G}_{ij}(\mathbf{y}) \in L_\infty$ for $\mathbf{y} \in \mathbf{R}^n$.

Similarly, consider another uncertain general complex dynamical network with different node dynamics, which is described by

$$\begin{cases} \dot{\mathbf{x}}_i &= \mathrm{f}(\mathbf{x}_i) + \mathrm{F}(\mathbf{x}_i)\alpha + \sum_{j=1}^{N} a_{ij}\mathrm{h}_j(\mathbf{x}_j) \\ & i = 1, ..., l, (l < N) \\ \dot{\mathbf{x}}_i &= \mathrm{g}(\mathbf{x}_i) + \mathrm{G}(\mathbf{x}_i)\beta + \sum_{j=1}^{N} a_{ij}\mathrm{h}_j(\mathbf{x}_j) \\ & i = l+1, ..., N. \end{cases} \tag{9}$$

In this model, node dynamics is $\dot{\mathbf{x}} = \bar{f}(\mathbf{x})$ or $\dot{\mathbf{y}} = \bar{\mathbf{g}}(\mathbf{y})$. Both α and β are the unknown parameter vectors of the different dynamical systems. Similarly, one has the following hypotheses.

Assumption 3 (A3). *Suppose that there exist two nonnegative constants L_1 and L_3 satisfying*

$$\begin{array}{ll} \|\bar{f}(\mathbf{x}_i) - \bar{f}(\mathbf{y}_i)\| \leq L_3\|\mathbf{x}_i - \mathbf{y}_i\|, & i = 1, ..., l \\ \|\bar{g}(\mathbf{x}_i) - \bar{g}(\mathbf{y}_i)\| \leq L_4\|\mathbf{x}_i - \mathbf{y}_i\|, & i = l+1, ..., N, \end{array}$$

where $\mathbf{x}_i, \mathbf{y}_i$ are the time-varying vectors and $\|\cdot\|$ is 2-norm.

Take (9) as the drive system, then the response system is described by

$$\begin{cases} \dot{\hat{\mathbf{x}}}_i &= \mathrm{f}(\hat{\mathbf{x}}_i) + \mathrm{F}(\hat{\mathbf{x}}_i)\hat{\alpha} + \sum_{j=1}^{N} \hat{a}_{ij}\mathrm{h}_j(\hat{\mathbf{x}}_j) + \mathbf{u}_i \\ & i = 1, ..., l, (l < N) \\ \dot{\hat{\mathbf{x}}}_i &= \mathrm{g}(\hat{\mathbf{x}}_i) + \mathrm{G}(\hat{\mathbf{x}}_i)\hat{\beta} + \sum_{j=1}^{N} \hat{a}_{ij}\mathrm{h}_j(\hat{\mathbf{x}}_j) + \mathbf{u}_i \\ & i = l+1, ..., N. \end{cases} \tag{10}$$

Thus the error system is given by

$$\begin{cases} \dot{\tilde{\mathbf{x}}}_i = \mathrm{f}(\hat{\mathbf{x}}_i) - \mathrm{f}(\mathbf{x}_i) + \mathrm{F}(\hat{\mathbf{x}}_i)\hat{\alpha} - \mathrm{F}(\mathbf{x}_i)\alpha + \sum_{j=1}^{N} \hat{a}_{ij}\mathrm{h}_j(\hat{\mathbf{x}}_j) \\ \quad - \sum_{j=1}^{N} a_{ij}\mathrm{h}_j(\mathbf{x}_j) - k_i\tilde{\mathbf{x}}_i, i = 1, ..., l, (l < N) \\ \dot{\tilde{\mathbf{x}}}_i = \mathrm{g}(\hat{\mathbf{x}}_i) - \mathrm{g}(\mathbf{x}_i) + \mathrm{G}(\hat{\mathbf{x}}_i)\hat{\beta} - \mathrm{G}(\mathbf{x}_i)\beta + \sum_{j=1}^{N} \hat{a}_{ij}\mathrm{h}_j(\hat{\mathbf{x}}_j) \\ \quad - \sum_{j=1}^{N} a_{ij}\mathrm{h}_j(\mathbf{x}_j) - k_i\tilde{\mathbf{x}}_i, i = l+1, ..., N, \end{cases} \tag{11}$$

Similarly, one has the following adaptive control theorem.

Theorem 2. *Suppose that **A2** and **A3** hold. Then the unknown or uncertain coupling configuration matrix **A** of the uncertain general complex dynamical network (9) and the parameter vector α, β can be identified by the estimated values $\hat{\mathbf{A}}$, $\hat{\alpha}$ and $\hat{\beta}$, respectively, using the following response network:*

$$\begin{cases} \dot{\hat{\mathbf{x}}}_i &= \mathrm{f}(\hat{\mathbf{x}}_i) + \mathrm{F}(\hat{\mathbf{x}}_i)\hat{\alpha} + \sum_{j=1}^{N} \hat{a}_{ij}\mathrm{h}_j(\hat{\mathbf{x}}_j) + \mathbf{u}_i \\ & i = 1, ..., l, (l < N) \\ \dot{\hat{\mathbf{x}}}_i &= \mathrm{g}(\hat{\mathbf{x}}_i) + \mathrm{G}(\hat{\mathbf{x}}_i)\hat{\beta} + \sum_{j=1}^{N} \hat{a}_{ij}\mathrm{h}_j(\hat{\mathbf{x}}_j) + \mathbf{u}_i \\ & i = l+1, ..., N, \\ \mathbf{u}_i &= -k_i(\hat{\mathbf{x}}_i - \mathbf{x}_i), \dot{k}_i = d_i\|\tilde{\mathbf{x}}_i\|^2 \\ \dot{\hat{\alpha}} &= -\sum_{i=1}^{l} \mathrm{F}^T(\hat{\mathbf{x}}_i)\tilde{\mathbf{x}}_i, \dot{\hat{\beta}} = -\sum_{i=l+1}^{N} \mathrm{G}^T(\hat{\mathbf{x}}_i)\tilde{\mathbf{x}}_i, \\ & \dot{\hat{a}}_{ij} = -\tilde{\mathbf{x}}_i^T\mathrm{h}_j(\hat{\mathbf{x}}_j) \end{cases} \tag{12}$$

where $1 \leq i, j \leq N$ and d_i are any positive constants.

Proof. Similarly, construct the Lyapunov function candidate as follows:

$$\begin{aligned} 2V &= \sum_{i=1}^{N} \tilde{\mathbf{x}}_i^T\tilde{\mathbf{x}}_i + \sum_{i=1}^{N}\sum_{j=1}^{N} (\hat{a}_{ij} - a_{ij})^2 + \tilde{\alpha}^T\tilde{\alpha} \\ &\quad + \tilde{\beta}^T\tilde{\beta} + \sum_{i=1}^{N} \tfrac{1}{d_i}(k_i - k^*)^2, \end{aligned}$$

where k^* is a sufficiently large positive constant to be determined. Denote $\max\{L_1, L_3\} = L$. Then one obtains

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N} \tilde{\mathbf{x}}_i^T\dot{\tilde{\mathbf{x}}}_i + \sum_{i=1}^{N}\sum_{j=1}^{N} \tilde{a}_{ij}\dot{\hat{a}}_{ij} + \tilde{\alpha}^T\dot{\tilde{\alpha}} + \tilde{\beta}^T\dot{\tilde{\beta}} \\ &\quad + \sum_{i=1}^{N} \tfrac{1}{d_i}(k_i - k^*)\dot{k}_i \\ &\leq \left(L + L_2\lambda_{\max}\left(\tfrac{\mathbf{A}+\mathbf{A}^T}{2}\right) - k^*\right)\tilde{\mathbf{x}}^T\tilde{\mathbf{x}} \end{aligned}$$

where $\lambda_{\max}\left(\frac{\mathbf{A}+\mathbf{A}^T}{2}\right)$ is the maximal eigenvalue of the symmetric matrix $\frac{\mathbf{A}+\mathbf{A}^T}{2}$. Denote $\tilde{\mathbf{x}} = (\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, ..., \tilde{\mathbf{x}}_N)^T$.

Let $k^* = L + L_2\lambda_{\max}\left(\frac{\mathbf{A}+\mathbf{A}^T}{2}\right) + 1$. Thus one gets

$$\dot{V}|_{(11)(12)} \leq -\tilde{\mathbf{x}}^T.\tilde{\mathbf{x}}$$

The rest of the proof is similar to that of Theorem 1 and omitted here. According to the LaSalle's invariance principle, starting from arbitrary initial values of (11), $\tilde{\mathbf{x}} \to \mathbf{0}, \hat{\mathbf{A}} \to \mathbf{A}$, and $\hat{\alpha} \to \alpha, \hat{\beta} \to \beta$ as $t \to \infty$.

Remark 2. Theorem 2 can be easily extended to the network with multiple different node dynamics. It is shown that using similar adaptive feedback control approach, the exact topological structure and system parameters of network (9) can be identified together.

IV. Numerical Simulations

Numerous numerical simulations show the effectiveness of the proposed method. Here we only use a simple example to explain the main working principles.

As we know, Lü system is described by

$$\dot{\mathbf{x}} = f(\mathbf{x}) + F(\mathbf{x})\alpha, \tag{13}$$

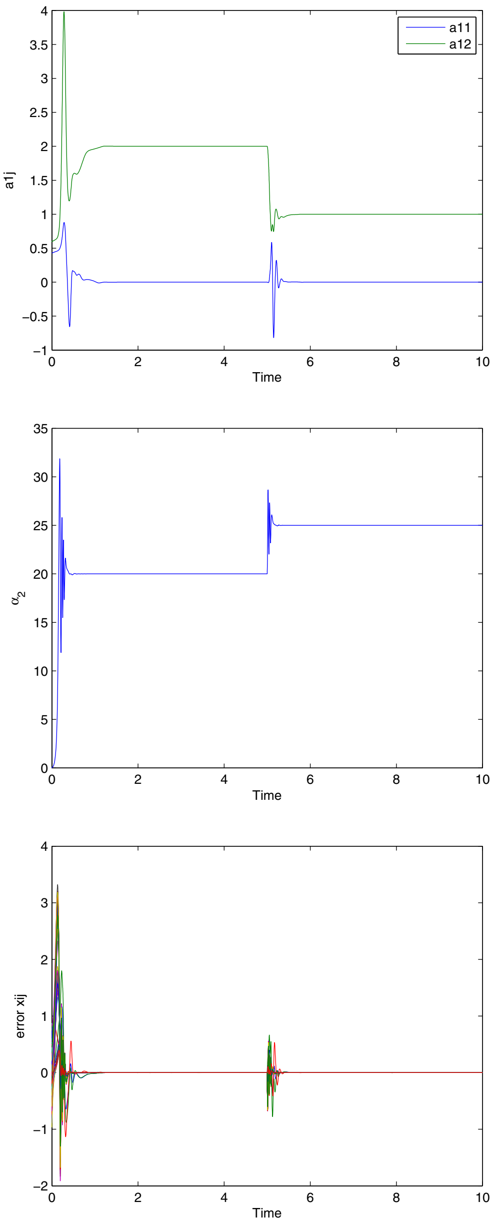

Fig. 1. (a) Topological structure identification of network (2) with 50 nodes. (b) System parameters identification of network (2) with 50 nodes. (c) Synchronous errors $\tilde{x}_{ij}$ for $1 \leq i \leq 50$ and $1 \leq j \leq 3$.

where

$$f(\mathbf{x}) = \begin{pmatrix} 0 \\ -xz \\ xy \end{pmatrix}, \; F(\mathbf{x}) = \begin{pmatrix} y-x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & -z \end{pmatrix},$$

$\mathbf{x} = (x, y, z)^T$ is the state vector and $\alpha = (\alpha_1, \alpha_2, \alpha_3)^T$ is the system parameters. Considering the network consists of 50 identical Lü systems and its inner-coupled functions are taken as $h_j(x_j) = x_j$ for $1 \leq j \leq 50$. For simplification, assume that the system parameter α_2 and the network topological structure parameters a_{11} and a_{12} are unknown.

Let $\alpha_1 = 36$, $\alpha_3 = 3$. In the numerical simulation, α_2 switches from 20 to 25, a_{11} switches from 2 to 1, and a_{12} is set up to 0. Figure 1 shows the identification results. Obviously, all identifying errors are quick convergent to zero.

V. Conclusion

We have proposed an adaptive feedback control method to identify the exactly topological structure of an uncertain complex dynamical network. In particular, this approach can identify the topological structure and system parameters of an uncertain complex dynamical network together. Based on the Lyapunov stability theorem, two useful identifying criteria were attained in this paper. Moreover, this method is also valid for the network with different dynamical nodes and switchings of system parameters. Numerical simulations are also given to show the effectiveness of the identifications of topological structures and system parameters. It sheds light on the future real-world application.

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