Experimental Confirmation of $n$−scroll Hyperchaotic Attractors

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Abstract—A systematic circuit design approach is proposed for experimental verification of hyperchaotic 2, 3, 4−scroll attractors from a generalized Matsumoto-Chua-Kobayashi (MCK) circuit. The recursive formulas for system parameters are rigorously derived for improving the hardware implementation.

I. INTRODUCTION

Hyperchaos was first observed from a real physical system by Matsumoto, Chua and Kobayashi in [1]. Then, Yalcin et al. [2] introduced some hyperchaotic $n$−double-scroll chaotic attractors by adding breakpoints in the piecewise-linear (PWL) characteristic of the MCK circuit and confirmed the hyperchaotic 4− and 6−scroll attractors by computer simulations. Yu et al. [3] proposed hyperchaotic $n$−scroll attractors and realized hyperchaotic $3 \sim 10$−scroll attractors by computer simulations. Itoh et al. [4] investigated the impulsive synchronization of a hyperchaotic double-scroll attractor and its application to spread-spectrum communication systems. It has been known that it is generally difficult to implement multi-scroll chaotic and hyperchaotic attractors by a physical electronic circuit. Yalcin et al. [5] experimentally confirmed 3− and 5−scroll chaotic attractors in a generalized Chua’s circuit, while Zhong et al. [6] proposed a systematic circuit design method for physically implementing up to as many as ten scrolls visible on the oscilloscope. Han et al. [7] constructed a double-hysteresis building block to physically realize a 9−scroll chaotic attractor. There are some other approaches reported in the literature for the design and circuit implementation of multi-scroll chaotic attractors [8−14]. It is generally quite difficult to physically build a nonlinear resistor having an appropriate characteristic with many segments. In this effort, Lü et al. [13] designed a novel circuit diagram to physically verify the multi-directional multi-scroll chaotic attractors. The main obstacle is that the device must have a very wide dynamic range [3,6], however physical conditions always limit or even prohibit such circuit realization [6]. Recently, Lü and Chen [14] reviewed the main advances of multi-scroll chaos generation.

In this paper, we describe the design of a novel block circuit diagram to experimentally confirm hyperchaotic $n$−scroll attractors. This is the first time in the literature to report an experimental verification of hyperchaotic 3− and 4−scroll attractors. Moreover, the derived recursive formulas for system parameters provide a theoretical basis for physical realization of hyperchaotic attractors with a large number of scrolls.

The rest of the paper is organized as follows. In Section II, a general MCK circuit is briefly described. Then, a novel block circuit diagram is designed for hardware implementation of hyperchaotic 2, 3, 4−scroll attractors, and its dynamic equation is rigorously derived in Section III. Conclusions are finally drawn in Section IV.

II. A GENERALIZED MCK CIRCUIT

The dimensionless state equation of the hyperchaotic MCK circuit is described by [1]

$$\begin{align*}
\frac{dx}{dt} & = \alpha[g(y - x) - z] \\
\frac{dy}{dt} & = \beta[-g(y - x) - w] \\
\frac{dz}{dt} & = \gamma_0(x + z) \\
\frac{dw}{dt} & = \gamma y,
\end{align*}$$

(1)

where $g(y - x) = m_1(y - x) + 0.5(m_0 - m_1)||y - x + 1| - |y - x - 1||$. When $\alpha = 2$, $\beta = 20$, $\gamma_0 = 1$, $\gamma = 1.5$, $m_0 = -0.2$, $m_1 = 3$, system (1) has a hyperchaotic double-scroll attractor with Lyapunov exponents $\lambda_1 = 0.24$, $\lambda_2 = 0.06$, $\lambda_3 = 0$, $\lambda_4 = -53.8$.

To generate hyperchaotic $n$−scroll attractors from (1), we first generalize the characteristic function $g(y - x)$, given in [3], as follows:

$$g(y - x) = m_{N-1}(y - x) + 0.5 \sum_{i=1}^{N-1} (m_{i-1} - m_i)(|y - x + x_i| - |y - x - x_i|).$$

(2)

The recursive formulas of positive switching points $x_i(i =
where \( m_i (0 \leq i \leq N - 1) \) are the slopes of the segments and \( k_i = \frac{x_{i+1}-x_i}{x_i-x_i} (1 \leq i \leq N - 2) \), in which \( x^E_i (1 \leq i \leq N - 2) \) are the positive equilibrium points of \( g(x) \).

To control the hyperchaotic signal into the region of the operational amplifier, we may assume that \( x_1 < 1 \). Here, we suppose that \( x_1 = 0.5 \). From (3), we determine the system parameters as follows: (i) when \( N = 2, m_0 = -0.2, m_1 = 3 \), system (1) with (2) has a hyperchaotic double-scroll attractor; (ii) when \( N = 3, m_0 = 3, m_1 = -0.8, m_2 = 3, x_2 = 1.8333 \), system (1) with (2) has a hyperchaotic 3-scroll attractor; (iii) when \( N = 4, m_0 = m_2 = -0.7, m_1 = m_3 = 2.9, x_2 = 1.5289, x_3 = 3.0239 \), system (1) with (2) has a hyperchaotic 4-scroll attractor.

III. CIRCUIT DESIGN AND IMPLEMENTATION

In this section, a circuit diagram is constructed to experimentally verify the hyperchaotic 2, 3, 4-scroll attractors. Also, the dynamic equation is rigorously derived from the circuit diagram shown in Fig. 1.

A. Circuit diagram and its dynamic equation

Fig. 1 shows the circuit diagram, where \( N_1 \) is the generator of the negative resistor \(-R\), and \( N_2 \) is the multi-PWL function generator satisfying \( I_N = f(v_{C_2} - v_{C_1}) \). All operational amplifiers are selected as Type TL082. The voltage of the electric source is \( E = 15V \). Thus, the saturating voltages of the operation amplifiers are \( E_{sat} = 14.3V \).

According to Fig. 1, the circuit equation is derived as follows:

\[
\begin{align*}
C_1 \frac{dv_{C1}}{dt} & = f(v_{C2} - v_{C1}) - i_{L1} \\
C_2 \frac{dv_{C2}}{dt} & = -f(v_{C2} - v_{C1}) - i_{L2} \\
L_1 \frac{di_{L1}}{dt} & = v_{C1} + Ri_{L1} \\
L_2 \frac{di_{L2}}{dt} & = v_{C2},
\end{align*}
\]

where

\[
f(v_{C2} - v_{C1}) = G_{N-1}(v_{C2} - v_{C1}) + 0.5 \sum_{i=1}^{N-1} (G_{i-1} - G_i)(|v_{C2} - v_{C1} + E_i| - |v_{C2} - v_{C1} - E_i|)
\]

is a piecewise-linear characteristic function.

Comparing systems (1) with (4), we get the transformation relationship of parameters as follows:

\[
\begin{align*}
\tau_0 = 2RC_1, & \quad \tau = \frac{1}{\gamma_0}, \quad \alpha = 2, \beta = \frac{2C_2}{\gamma_0} = 20 \\
\gamma_0 = \frac{2RC_1}{C_1}, & \quad \gamma = \frac{2C_2}{\gamma_0} = 1.5 \\
x = \frac{v_{C1}}{V_{BP}}, & \quad y = \frac{v_{C2}}{V_{BP}}, \quad z = \frac{i_{L1}}{V_{BP}}, \quad w = \frac{i_{L2}}{V_{BP}} \\
V_{BP} = 1V, & \quad G_i = m_iG(i = 0, 1, 2, \cdots), \quad G = \frac{1}{\tau_0} \\
g(y-x) = Rf(v_{C2} - v_{C1}),
\end{align*}
\]

where \( V_{BP} = 1V, \frac{1}{\tau_0} = \frac{1}{2RC_1} \) is the time-scale transformation factor.

From (5), we have the parameters:

\[
\begin{align*}
L_1 & = 9mH, \quad L_2 = 6mH, \quad C_1 = 50nF, \quad C_2 = 5nF, \quad R = 300\Omega. \quad \text{Then, we can get the theoretical values of the resistors based on the parameters given in Section II as follows:}
\end{align*}
\]

(1) For hyperchaotic 2-scroll attractor:

\[
\begin{align*}
G_0 & = \frac{m_0}{R} = -0.67mS, \quad G_1 = \frac{m_1}{R} = 10mS, \\
E_1 & = x_1V_{BP}, \quad r_1 = \frac{R_{13}}{R_{11}} = G_1R_2 - 1 = 1.00, \\
r_2 & = \frac{R_{23}}{R_{21}} = \frac{E_2}{E_1} = 28.6, \\
r_3 & = \frac{R_{34}}{R_{31}} = -\frac{R^2_{23}}{R_3} - 1 = 12.4.
\end{align*}
\]

(2) For hyperchaotic 3-scroll attractor:

\[
\begin{align*}
G_0 & = \frac{m_0}{R} = 10mS, \quad G_1 = \frac{m_1}{R} = -2.7mS, \\
G_2 & = \frac{m_2}{R} = 10mS, \quad E_1 = x_1V_{BP}(i = 1, 2), \\
r_1 & = \frac{R_{13}}{R_{11}} = G_1R_2 - 1 = 1.00, \\
r_2 & = \frac{R_{23}}{R_{21}} = \frac{E_2}{E_1} = 7.80, \\
r_3 & = \frac{R_{34}}{R_{31}} = \frac{R^2_{23}}{R_3} - 1 = 2.08, \\
r_4 & = \frac{R_{45}}{R_{41}} = \frac{E_4}{E_3} - 1 = 27.60, \\
r_5 & = \frac{R_{56}}{R_{51}} = -\frac{1+r_3}{R_2(G_1-G_0)} - 1 = 10.29.
\end{align*}
\]

(3) For hyperchaotic 4-scroll attractor:

\[
\begin{align*}
G_0 & = \frac{m_0}{R} = -2.3mS, \quad G_1 = \frac{m_1}{R} = 9.7mS, \\
G_2 & = \frac{m_2}{R} = -2.3mS, \quad G_3 = \frac{m_3}{R} = 9.7mS, \\
E_1 & = x_1V_{BP}(i = 1, 2, 3), \quad r_1 = \frac{R_{13}}{R_{11}} = G_3R_2 - 1 = 0.93, \\
r_2 & = \frac{R_{23}}{R_{21}} = \frac{E_2}{E_1} = 4.73, \\
r_3 & = \frac{R_{34}}{R_{31}} = \frac{R^2_{23}}{R_3} - 1 = 0.97, \\
r_4 & = \frac{R_{45}}{R_{41}} = \frac{E_4}{E_3} - 1 = 8.35, \\
r_5 & = \frac{R_{56}}{R_{51}} = -\frac{1+r_3}{R_2(G_1-G_0)} - 1 = 2.90, \\
r_6 & = \frac{R_{67}}{R_{61}} = \frac{E_6}{E_5} = 28.6, \\
r_7 & = \frac{R_{78}}{R_{71}} = \frac{R^2_{67}}{R_3} - 1 = 10.90.
\end{align*}
\]
switched off, the circuit diagram can create a hyperchaotic double-scroll attractor; when $K_1$, $K_2$, $K_3$ are switched on and $K_4$ is switched off, the circuit diagram can generate a hyperchaotic 3-scroll attractor, as shown in Fig. 2 (a); when $K_1$, $K_2$, $K_3$, $K_4$ are switched on, the circuit diagram can create a hyperchaotic 4-scroll attractor, as shown in Fig. 2 (b).

IV. CONCLUSIONS

This brief paper has proposed a novel block circuit diagram for hardware implementation of hyperchaotic 2, 3, 4-scroll attractors in a generalized MCK circuit. In addition, the derived recursive formulas for system parameters provide a theoretical basis for physical realization of the hyperchaotic attractors with a large number of scrolls.

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REFERENCES

Fig. 2. Experimental observations of hyperchaotic \( n \)-scroll attractors.


