Design and Implementation of Multi-directional Grid Multi-Torus Chaotic Attractors

Simin Yu
College of Automation
Guangdong University of Technology
Guangzhou 510090, China

Jinhu Lü
Key Laboratory of Systems and Control
Institute of Systems Science
Academy of Mathematics and Systems Science
Chinese Academy of Sciences
Beijing 100080, China
Email: jhlu@iss.ac.cn

Guanrong Chen
Department of Electronic Engineering
City University of Hong Kong
Hong Kong, China
Email: gchen@ee.cityu.edu.hk

***Abstract*— This paper introduces a novel four-order system, which can generate one-directional (1-D) $n-$torus, two-directional (2-D) $n \times m-$torus, three-directional (3-D) $n \times m \times l-$torus, four-directional (4-D) $n \times m \times l \times p-$torus chaotic attractors. Furthermore, a novel block circuit diagram is designed for the hardware implementation of multi-directional grid multi-torus chaotic attractors. This is the first time in the literature to experimentally verify a $5 \times 5 \times 3 \times 3-$torus chaotic attractors.**

I. INTRODUCTION

Over the last two decades, the design and circuit implementation of chaotic oscillators have been a subject of increasing interest due to their applications in various chaos-based technologies and information systems [1]. In particular, the theoretical design and hardware implementation of various complex multi-scroll chaotic attractors have seen a rapid development [1-11]. Suykens and Vandewalle firstly introduced a family of $n-$double scroll chaotic attractors [2]. A switching manifold method for creating chaotic attractors with multiple-merged basins of attraction was proposed by Lü *et al.* in [5]. Yalcin *et al.* presented a family of scroll grid attractors by using a step function approach, including 1-D $n-$scroll, 2-D $n \times m-$grid scroll, and 3-D $n \times m \times l-$grid scroll chaotic attractors [3]. Lü *et al.* [6-8] introduced the hysteresis and saturated functions series methods for generating 1-D $n-$scroll, 2-D $n \times m-$grid scroll, and 3-D $n \times m \times l-$grid scroll chaotic attractors, with a rigorously mathematical proof and a physical realization for the chaotic behaviors. Last but not least, Yu *et al.* [10] proposed a general jerk circuit approach for creating various types of $n-$scroll chaotic attractors.

As is known today, a stable torus is observed as a result after the system meets the super-critical Neimark-Sacker bifurcation for a limit cycle [11]. The tori are easily observed in two-dimensional and periodically forced dynamical systems. However, there is only a few publications on stable tori in three-dimensional autonomous systems. Moreover, the physical circuit implementations of various tori attractors are quite difficult. In this paper, we propose a simple four-order system for creating 1-D $n-$torus, 2-D $n \times m-$torus, 3-D $n \times m \times l-$torus, and 4-D $n \times m \times l \times p-$torus chaotic attractors. Also, a simple block circuit diagram is constructed for experimentally verifying these multi-directional grid multi-torus chaotic attractors.

The rest of this paper is organized as follows. In Section II, a novel four-order system is introduced for creating multi-directional grid multi-torus chaotic attractors. A simple block circuit diagram is then designed in Section III, for the hardware implementations of the multi-directional grid multi-torus chaotic attractors. Conclusions are finally drawn in Section IV.

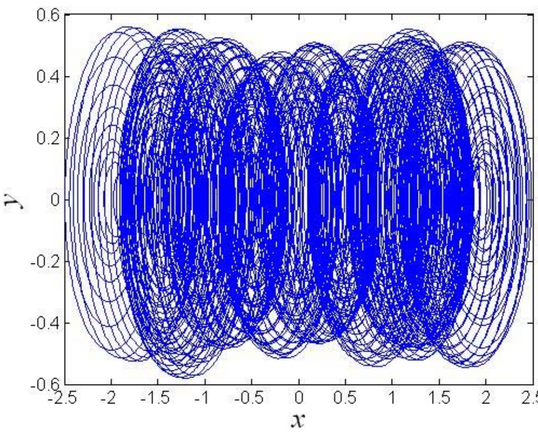

Fig. 1. $8-$torus chaotic attractor.

II. MULTI-DIRECTIONAL GRID MULTI-TORUS CHAOTIC SYSTEM

In the following, we propose a four-order multi-directional grid multi-torus chaotic system, which is described by

$$\begin{cases} \frac{dx}{dt} &= y - f_2(y) \\ \frac{dy}{dt} &= z - f_3(z) \\ \frac{dz}{dt} &= w - f_4(w) \\ \frac{dw}{dt} &= -\alpha x - \beta y - \gamma z - \delta w + \alpha f_1(x) \\ & \quad + \beta f_2(y) + \gamma f_3(z) + \delta f_4(w), \end{cases} \quad (1)$$

where $\alpha, \beta, \gamma, \delta$ are the system parameters and $f_1(x), f_2(y), f_3(z), f_4(w)$ are the step function series or

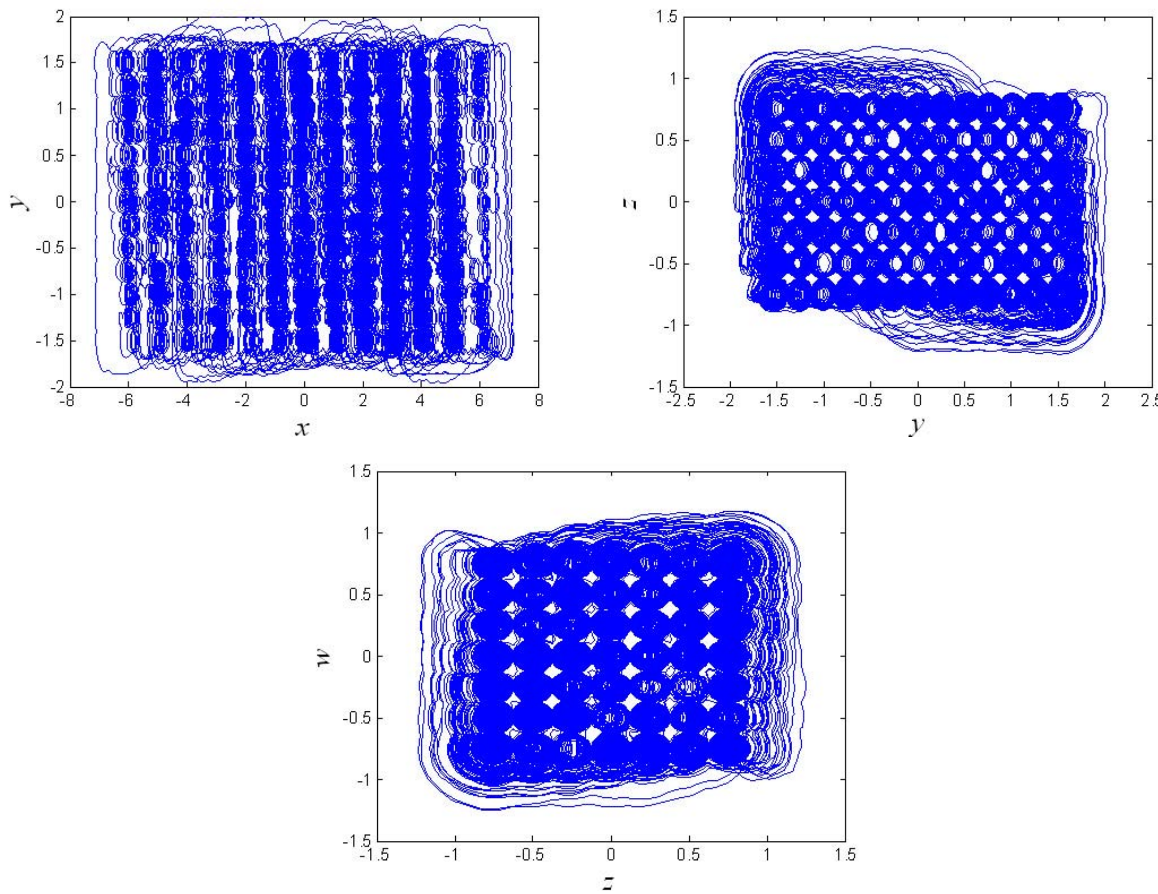

Fig. 2. Various plane projections of $13 \times 13 \times 7 \times 7-$torus chaotic attractor. (a) $x - y$ plane; (b) $y - z$ plane; (c) $z - w$ plane.

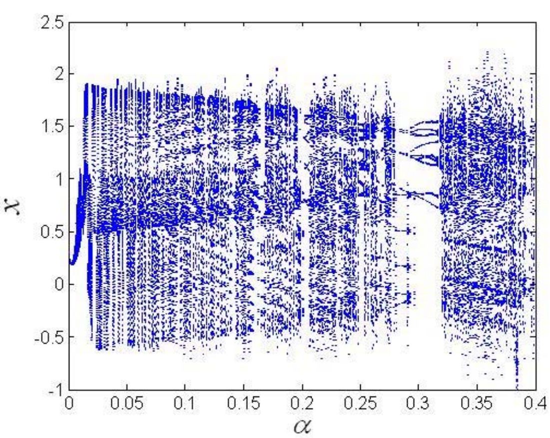

Fig. 3. Bifurcation diagram of parameter α.

saturated function series. For $f_1(x) \neq 0$, $f_2(y) = f_3(z) = f_4(w) = 0$, system can generate 1-D multi-torus chaotic attractors; for $f_1(x) \neq 0$, $f_2(y) \neq 0$, $f_3(z) = f_4(w) = 0$, system can generate 2-D multi-torus chaotic attractors; for $f_1(x) \neq 0$, $f_2(y) \neq 0$, $f_3(z) \neq 0$, $f_4(w) = 0$, system can generate 3-D multi-torus chaotic attractors; for $f_1(x) \neq 0$, $f_2(y) \neq 0$, $f_3(z) \neq 0$, $f_4(w) \neq 0$, system can generate 4-D multi-torus chaotic attractors.

When $f_1(x) = A_1 \sum_{i=1}^{2} \{\text{sgn}[x - (2i-1)A_1] + \text{sgn}[x + (2i-1)A_1]\}$ and $f_2(y) = f_3(z) = f_4(w) = 0$, system (1) has a $8-$torus chaotic attractor for $\alpha = 0.045, \beta = \gamma = \delta = 1$, $A_1 = 0.5$ as shown in Fig. 1. When $f_1(x) = A_1 \sum_{i=1}^{I} \{\text{sgn}[x - (2i-1)A_1] + \text{sgn}[x + (2i-1)A_1]\}$, $f_2(y) = A_2 \sum_{j=1}^{J} \{\text{sgn}[y - (2j-1)A_2] + \text{sgn}[y + (2j-1)A_2]\}$, $f_3(z) = A_3 \sum_{k=1}^{K} \{\text{sgn}[z - (2k-1)A_3] + \text{sgn}[z + (2k-1)A_3]\}$, $f_4(w) = A_4 \sum_{q=1}^{Q} \{\text{sgn}[w - (2q-1)A_4] + \text{sgn}[w + (2q-1)A_4]\}$, system (1) can create various 1-D $n-$torus, 2-D $n \times m-$torus, 3-D $n \times m \times l-$torus, 4-D $n \times m \times l \times p-$torus chaotic attractors. For example, has a $13 \times 13 \times 7 \times 7-$torus chaotic attractor for $\alpha = 0.1$, $\beta = \gamma = \delta = 1$, $I = J = 6$, $K = Q = 3$, $A_1 = 0.5$, $A_2 = A_3 = A_4 = \frac{A_1}{4}$ as shown in Fig. 2.

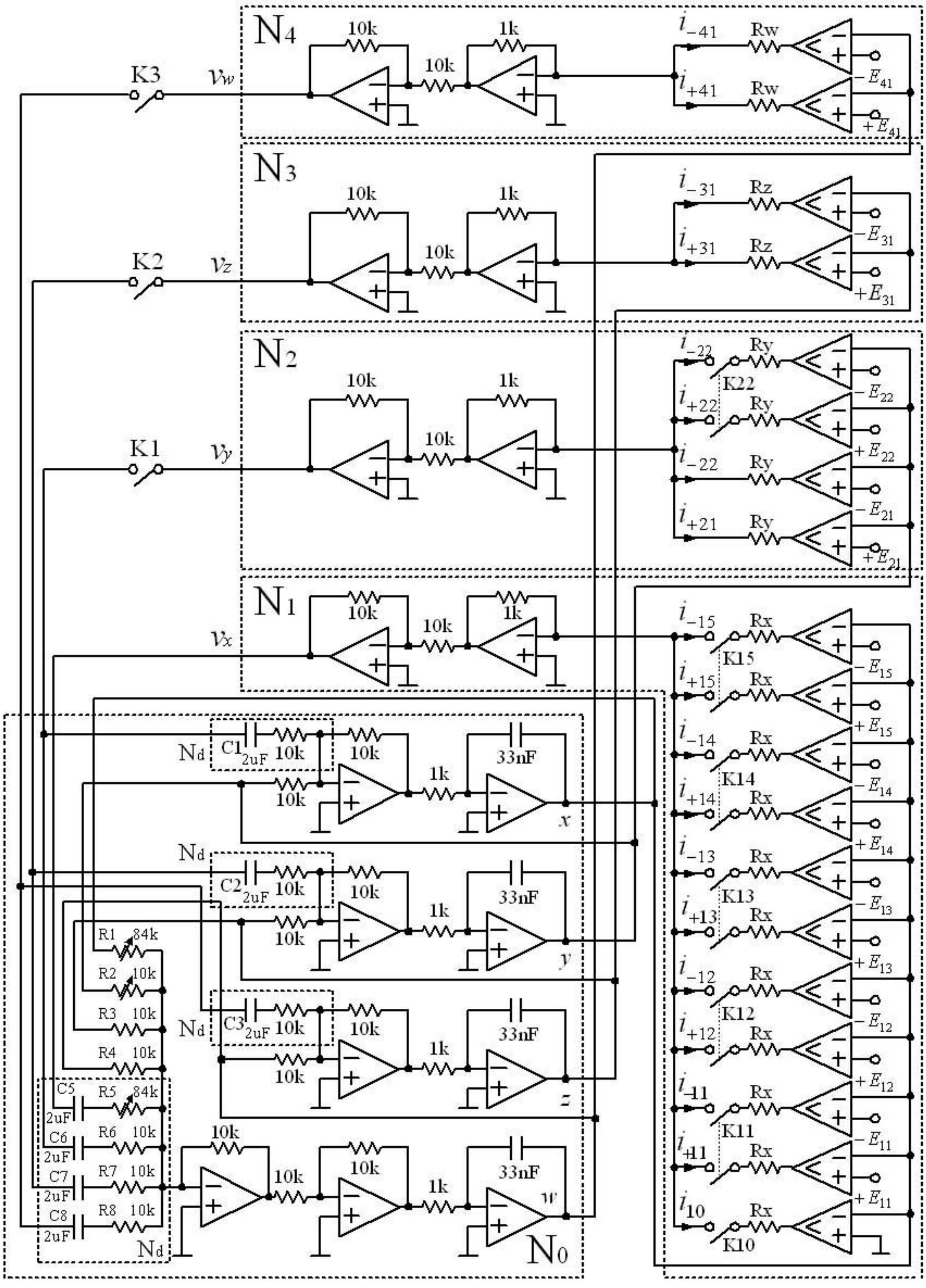

Fig. 4. Circuit diagram for implementing multi-directional multi-torus chaotic attractors.

Let $f_1 = A_1 \sum_{i=1}^{4}\{\text{sgn}[x-(2i-1)A_1] + \text{sgn}[x+(2i-1)A_1]\}$, $f_2(y) = f_3(z) = f_4(w) = 0$, $A_1 = 1$, $A_2 = A_3 = A_4 = \frac{A_1}{4}$, $\beta = \gamma = \delta = 1$. Then the bifurcation diagram of parameter α is shown in Fig. 3. When $\alpha = 0.1$,

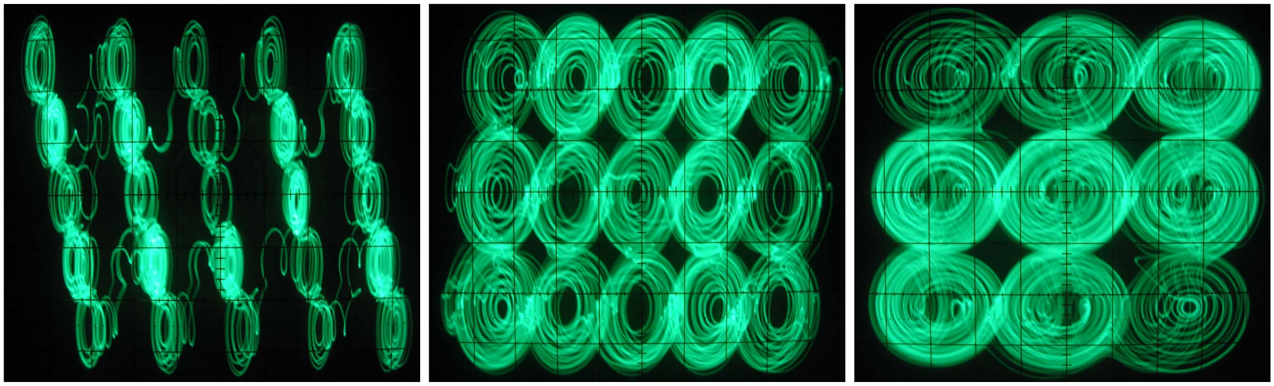

Fig. 5. Experimental observations of 4-D $5 \times 5 \times 3 \times 3$ torus chaotic attractors. (a) $x - y$ plane, where $x = 0.6V/div$, $y = 0.2V/div$; (b) $y - z$ plane, where $y = 0.16V/div$, $z = 0.1V/div$; (c) $z - w$ plane, where $z = 0.12V/div$, $w = 0.12V/div$.

system (1) is chaotic and its Lyapunov exponents are given by $LE_1 = 0.015$, $LE_2 = 0$, $LE_3 = -0.03$, $LE_4 = -0.98$.

III. Circuit Implementation and Experimental Observations

Based on the operational principles of multi-torus chaotic attractors, from (1), one can construct a circuit diagram to realize various multi-scroll chaotic attractors.

Figure 4 shows such a circuit diagram. This circuit diagram includes seven different parts; that is, Part I: *basic four-order grid multi-torus circuit* N_0; Part II: *capacitance coupling sub-circuit* N_d; Part III: *generator* N_1 *of step function series in* $x-$*direction*; Part IV: *generator* N_2 *of step function series in* $y-$*direction*; Part V: *generator* N_3 *of step function series in* $z-$*direction*; Part VI: *generator* N_4 *of step function series in* $w-$*direction*; Part VII: *switch linkages, including* K_1, K_2, K_3, K_{10}, K_{11}, K_{12}, K_{13}, K_{14}, K_{15}, K_{22}. Assume that the supply voltages and saturated voltages of all operational amplifiers are $V = \pm 15\,V$ and $V_{sat} = \pm 13.5\,V$, respectively. Furthermore, switch linkages K_1, K_2, K_3 control the number of directions for the four-order grid multi-torus chaotic attractors. However, switch linkages K_{10}, K_{11}, K_{12}, K_{13}, K_{14}, K_{15} control the number of tori in $x-$direction and switch linkage K_{22} controls the number of tori in $y-$direction for four-order grid multi-torus chaotic attractors, respectively.

Assume that K_1, K_2, K_3, K_{11}, K_{12}, K_{22} are switched on and K_{10}, K_{13}, K_{14}, K_{15} are switched off. Then the circuit diagram can generate a $5 \times 5 \times 3 \times 3-$torus chaotic attractor for parameters $R_x = 13.5kQ, R_y = R_z = R_w = 54kQ$, $E_{11} = 1.00, V\, E_{12} = 3.00V$, $E_{21} = E_{31} = E_{41} = 0.25V$, $E_{22} = 0.75$. The as shown in Fig. 5.

IV. Conclusions

We have introduced a novel four-order system for generating 1-D $n-$torus, 2-D $n \times m-$torus, 3-D $n \times m \times l-$torus, 4-D $n \times m \times l \times p-$torus chaotic attractors. Moreover, a novel block circuit diagram is constructed for physically realizing multi-directional grid multi-torus chaotic attractors. Also, it is the first time in the literature to report the hardware implementation of a $5 \times 5 \times 3 \times 3-$torus chaotic attractors.

Acknowledgment

This work was supported by the National Natural Science Foundation of China under Grants No.60304017, No.20336040 and No.60572073, the Scientific Research Startup Special Foundation on Excellent PhD Thesis and Presidential Award of Chinese Academy of Sciences, Natural Science Foundation of Guangdong Province under Grants No.32469 and No.5001818, Science and Technology Program of Guangzhou City under Grant No.2004J1-C0291.

References

[1] J. Lü and G. Chen, "Multi-scroll chaos generation: Theories, methods and applications," *Int. J. Bifurcation Chaos*, vol. 16, in press, 2006.

[2] J. A. K. Suykens and J. Vandewalle, "Generation of $n-$double scrolls ($n = 1, 2, 3, 4, \cdots$)," *IEEE Trans. Circuits Syst. I*, vol. 40, no. 11, pp. 861-867, Nov. 1993.

[3] M. E. Yalcin, J. A. K. Suykens, J. Vandewalle, and S. Ozoguz, "Families of scroll grid attractors," *Int. J. Bifurcation Chaos*, vol. 12, no. 1, pp. 23-41, Jan. 2002.

[4] A. S. Elwakil, "Integrator-based circuit-independent chaotic oscillator structure," *Chaos*, vol. 14, no. 2, pp. 364-369, Jun. 2004.

[5] J. Lü, X. Yu, and G. Chen, "Generating chaotic attractors with multiple merged basins of attraction: A switching piecewise-linear control approach," *IEEE Trans. Circuits Syst. I*, vol. 50, no. 2, pp. 198-207, Feb. 2003.

[6] J. Lü, F. Han, X. Yu, and G. Chen, "Generating 3-D multi-scroll chaotic attractors: A hysteresis series switching method," *Automatica*, vol. 40, no. 10, pp. 1677-1687, Oct. 2004.

[7] J. Lü, G. Chen, X. Yu, and H. Leung, "Design and analysis of multi-scroll chaotic attractors from saturated function series," *IEEE Trans. Circuits Syst. I*, vol. 51, no. 12, pp. 2476-2490, Dec. 2004.

[8] J. Lü, S. M. Yu, H. Leung, and G. Chen, "Experimental verification of multi-directional multi-scroll chaotic attractors," *IEEE Trans. Circuits Syst. I*, vol. 53, no. 1, pp. 149-165, Jan. 2006.

[9] S. M. Yu, Q. H. Lin, and S. S. Qiu, "A family of multiple folded torus chaotic attractors," *Acta Physica Sinica*, vol. 53, no. 7, pp. 2084-2088, Jul. 2004. (in Chinese)

[10] S. M. Yu, J. Lü, H. Leung, and G. Chen, "Design and implementation of $n-$scroll chaotic attractors from a general jerk circuit," *IEEE Trans. Circuits Syst. I*, vol. 52, no. 7, pp. 1459-1476, Jul. 2005.

[11] Y. Nishiuchi, T. Ueta and H. Kawakami, "Stable torus and its bifurcation phenomena in a simple three-dimensional autonomous circuit," *Chaos, Solitons & Fractals*, vol. 27, no. 4, pp. 941-951, Feb. 2006.