Experimental Verification of 3-D Hysteresis Multi-Scroll Chaotic Attractors

Jinhu Lü
Institute of Systems Science
Academy of Mathematics and Systems Science
Chinese Academy of Sciences
Beijing 100080, China
Email: jhlu@iss.ac.cn

Simin Yu
College of Automation
Guangdong University of Technology
Guangzhou 510090, China

Henry Leung
Department of Electrical and Computer Engineering
University of Calgary
AB, T2N 1N4, Canada
Email: leungh@ucalgary.ca

Guanrong Chen
Department of Electronic Engineering
City University of Hong Kong
Hong Kong, China
Email: gchen@ee.cityu.edu.hk

Abstract— **This paper introduces a novel circuit design for experimental verification of 3-D hysteresis multi-scroll chaotic attractors. A block circuit diagram is shown for realizing 1-D $5 \sim 11-$scroll, 2-D $3 \times 5 \sim 11-$grid scroll, and 3-D $3 \times 3 \times 5 \sim 11-$grid scroll chaotic attractors by operating the switches. Moreover, this design provides a theoretical principle for hardware implementation of chaotic attractors in multi-directions with a large number of scrolls.**

I. Introduction

Recently, generating multi-scroll chaotic attractors by using some simple electronic circuits has been a topic with increasing interest [1-15]. Yalcin *et al.* [2] proposed a step function method for creating one-directional (1-D) $n-$scroll, two-directional (2-D) $n \times m-$grid scroll, and three-directional (3-D) $n \times m \times l-$grid scroll chaotic attractors. Lately, Lü *et al.* [3-4] presented a switching manifold approach for generating chaotic attractors with multiple-merged basins of attraction. Hysteresis can also create chaos [5-12]. Some theoretical analysis and synthesis of hysteresis chaos generators are reported in [5-7], and recently Lü *et al.* [9] introduced a hysteresis series method for creating 1-D $n-$scroll, 2-D $n \times m-$grid scroll, and 3-D $n \times m \times l-$grid scroll attractors, with a rigorously mathematical proof for the chaotic behaviors.

All the aforementioned multi-scroll chaotic attractors can be easily realized by *numerical simulations*. Yet, it is much more difficult to realize multi-scroll chaotic attractors by a physical *electronic circuit*. Yalcin *et al.* [15] experimentally confirmed the $3-$ and $5-$scroll chaotic attractors in a generalized Chua's circuit. Zhong *et al.* [14] also proposed a systematical circuit design for physically realizing up to as many as ten scrolls visible on the oscilloscope. Yalcin *et al.* [2] experimentally verified the maximum 2-D $3 \times 3-$grid scroll and 3-D $2 \times 2 \times 2-$grid scroll chaotic attractors.

It is very difficult to physically realize a nonlinear resistor which has an appropriate characteristic with many segments [2,14]. Moreover, the realization of a nonlinear resistor with multi-segment is essential for hardware implementation of a chaotic attractor in multi-directions with a large number of scrolls. However, physical conditions always limit or even prohibit such circuit realization. A novel block circuit diagram is designed in this paper for experimentally verifying the 3-D hysteresis multi-scroll chaotic attractors. It should be noted that it is the first time to report an experimental verification of 2-D $3 \times 11-$grid scroll and 3-D $3 \times 3 \times 11-$grid scroll chaotic attractors.

The rest of this paper is organized as follows. In Section II, the hysteresis multi-scroll chaotic system is briefly introduced. A novel block circuit diagram is then described in Section III for experimental verification of the 3-D hysteresis multi-scroll attractors. Conclusions are finally given in Section IV.

II. Hysteresis Multi-Scroll Chaotic Attractors

Recently, Lü *et al.* [9] introduced a three-dimensional hysteresis multi-scroll chaotic system, which is described by

$$\dot{X} = AX + B\,\theta(X), \tag{1}$$

where $X = (x, y, z)^T$ is the state vector, $B = -A$, and

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & -c \end{pmatrix}.$$

There are three different cases to consider, as follows:

(1) 1-D hysteresis $n-$scroll attractors:

$$\theta(X) = \begin{pmatrix} h(x, p_1, q_1) \\ 0 \\ 0 \end{pmatrix}, \tag{2}$$

where the hysteresis series function $h(x, p_1, q_1)$ is defined by

$$h(x,p_1,q_1) = \begin{cases} -p_1 & \text{if} \quad x < -p_1 + 1 \\ i & \text{if} \quad \begin{array}{l} i-1 < x < i+1 \\ i = -p_1+1, \cdots, q_1 - 1 \end{array} \\ q_1 & \text{if} \quad x > q_1 - 1. \end{cases} \tag{3}$$

(2) 2-D hysteresis $n \times m-$grid scroll attractors:

$$\theta(X) = \begin{pmatrix} h(x, p_1, q_1) \\ h(y, p_2, q_2) \\ 0 \end{pmatrix}, \tag{4}$$

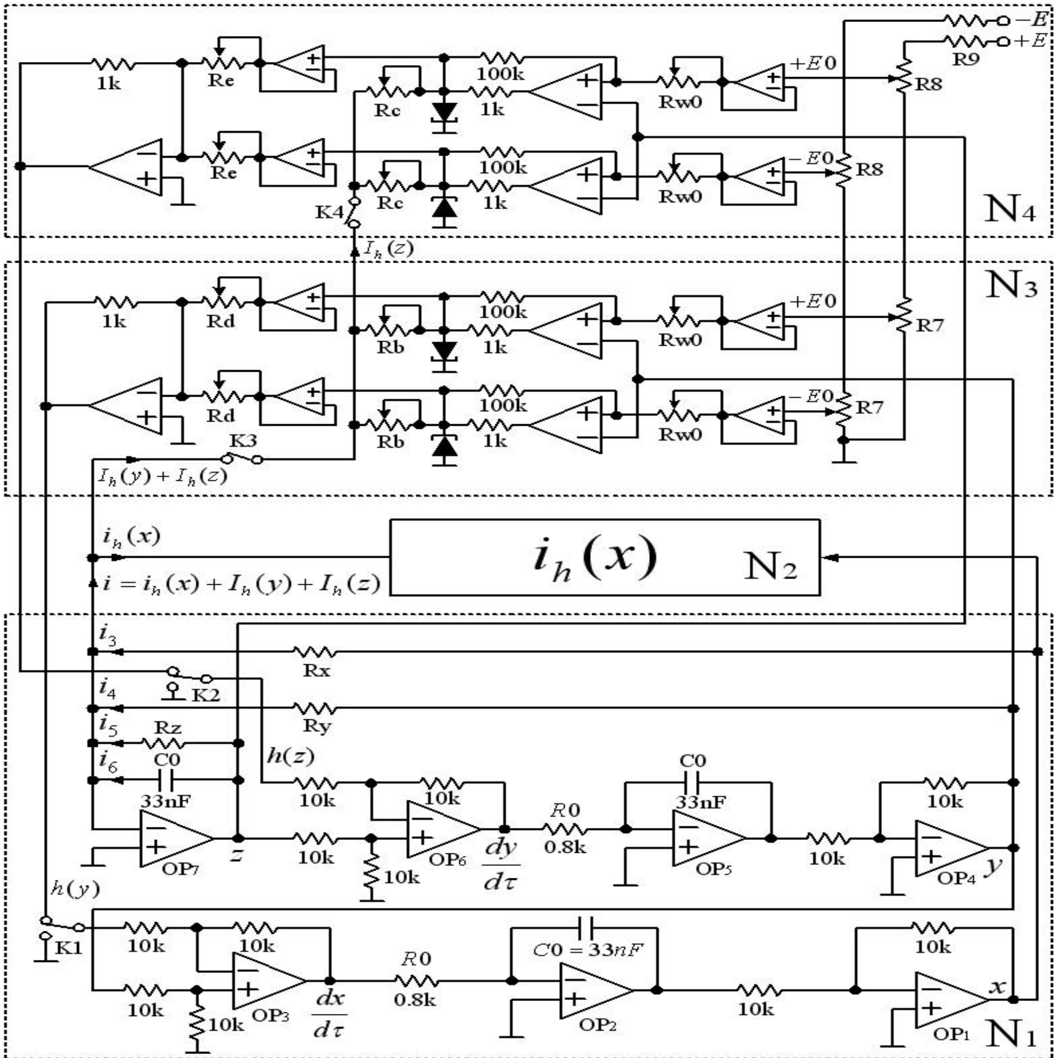

Fig. 1. Circuit diagram for 3-D hysteresis multi-scroll chaotic attractors.

where the hysteresis series functions $h(x, p_1, q_1)$ and $h(y, p_2, q_2)$ are defined by (3).

(3) 3-D hysteresis $n \times m \times l-$grid scroll attractors:

$$\theta(X) = \begin{pmatrix} h(x, p_1, q_1) \\ h(y, p_2, q_2) \\ h(z, p_3, q_3) \end{pmatrix}, \quad (5)$$

where the hysteresis series functions $h(x, p_1, q_1)$, $h(y, p_2, q_2)$, and $h(z, p_3, q_3)$ are defined by (3).

System (1) with (2) can generate a 1-D $(p_1 + q_1 + 1)-$scroll chaotic attractor for some suitable parameters a, b, c; system (1) with (4) can create a 2-D $(p_1 + q_1 + 1) \times (p_2 + q_2 + 1)-$grid scroll chaotic attractor for some suitable parameters a, b, c; system (1) with (5) can generate a 3-D $(p_1 + q_1 + 1) \times (p_2 + q_2 + 1) \times (p_3 + q_3 + 1)-$grid scroll chaotic attractor for some suitable parameters a, b, c. Moreover, Lü *et al.* [9] constructed a two-dimensional Poincaré return map to verify the chaotic behaviors of the generated hysteresis multi-scroll attractors via a rigorous *theoretical approach*. However, experimental observations are still lacking. For this purpose, a novel circuit diagram is designed below, to generate the hysteresis multi-scroll chaotic attractors by means of a physical *electronic circuit*.

III. Experimental Verification of Hysteresis Multi-Scroll Attractors

Figure 1 shows the circuit diagram for the hysteresis multi-scroll attractors. Figure 2 shows the circuit diagram of the hysteresis generator.

Table 1
On-off of switches $K1 \sim K4$ and the number of directions for the multi-scroll attractors.

$K1$	$K2$	$K3$	$K4$	Number of directions
off	off	off	off	1
on	off	on	off	2
on	on	on	on	3

As in Table 1, when the switches $K1$, $K2$, $K3$, $K4$ are switched off, Fig. 1 can generate the 1-D $5 \sim 11-$scroll

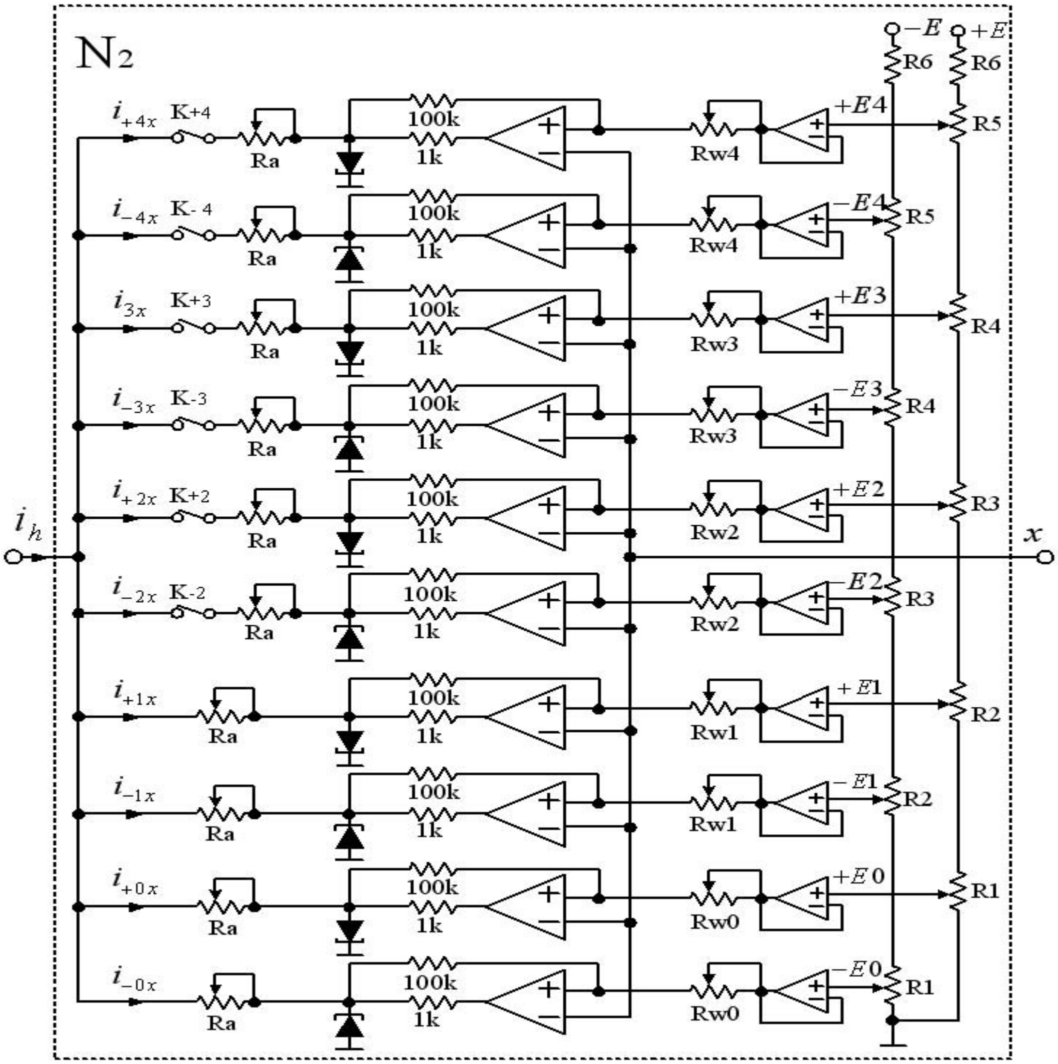

Fig. 2. Circuit diagram of hysteresis series generator.

chaotic attractors by switching the switches $K_{\pm n}(n = 2, 3, 4)$ in Fig. 2 based on Table 2; when the switches $K1$, $K3$ are switched on and the switches $K2$, $K4$ are switched off, Fig. 1 can create the 2-D $3 \times 5 \sim 11-$grid scroll chaotic attractors by switching the switches $K_{\pm n}(n = 2, 3, 4)$ in Fig. 2 based on Table 2; when the switches $K1$, $K2$, $K3$, $K4$ are switched on, Fig. 1 can generate the 3-D $3 \times 3 \times 5 \sim 11-$grid scroll chaotic attractors by switching the switches $K_{\pm n}(n = 2,\ 3,\ 4)$ shown in Fig. 2, based on Table 2.

Figure 3 shows the experimental observations of the 1-D $11-$scroll and 2-D $3 \times 11-$grid scroll chaotic attractors. Figure 4 shows experimental observations of a 3-D $3 \times 3 \times 11-$grid scroll chaotic attractor.

Remarks:

(1) Notice that the real measurement values of $E_n\,(0 \le n \le 4)$ in the circuit experiment may have a small departure from the theoretically calculated values, due to the discreteness of the real circuit parameters and the measurement errors. The differences can be corrected via a small adjustment of the resistors $R_{w0} \sim R_{w4}$ and $R_1 \sim R_9$ in the circuit implementation.

Table 2
On-off of switches $K_{\pm n}(n = 2, 3, 4)$ and the number of scrolls N.

K_{+2}	K_{-2}	K_{+3}	K_{-3}	K_{+4}	K_{-4}	N
off	off	off	off	off	off	5
on	off	off	off	off	off	6
on	on	off	off	off	off	7
on	on	on	off	off	off	8
on	on	on	on	off	off	9
on	on	on	on	on	off	10
on	on	on	on	on	on	11

(2) Based on Tables 1 and 2, we can arbitrarily control the switches $Ki\,(1 \le i \le 4)$ shown in Fig. 1 and the switches $K_{\pm n}(n = 2,\ 3,\ 4)$ shown in Fig. 2, to arbitrarily design various 1-D $n-$scroll, 2-D $n \times m-$grid scroll, and 3-D $n \times m \times l-$grid scroll chaotic attractors.

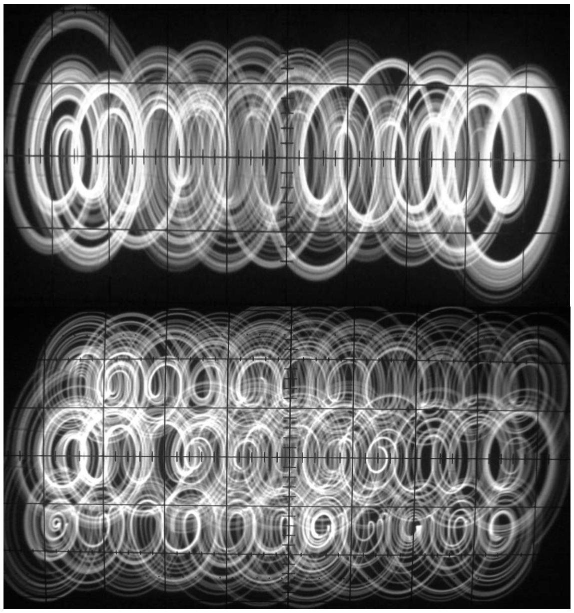

Fig. 3. Experimental observations of hysteresis multi-scroll attractors in the $x - y$ plane. From up to down: (a) 1-D $11-$scroll, where $x = 1.3V/div$, $y = 0.75V/div$; (b) 2-D $3 \times 11-$grid scroll, where $x = 1.44V/div$, $y = 0.8V/div$.

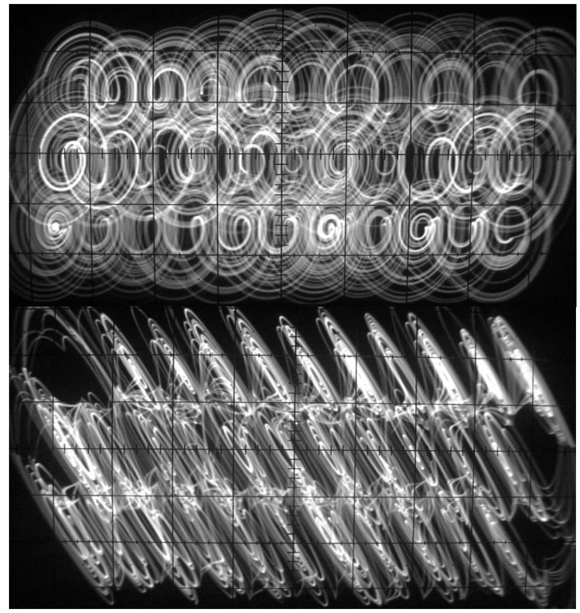

Fig. 4. Experimental observations of 3-D $3 \times 3 \times 11-$grid scroll attractors. From up to down: (a) $x - y$ plane, where $x = 1.44V/div$, $y = 0.8V/div$; (b) $x - z$ plane, where $x = 1.44V/div$, $z = 0.8V/div$.

IV. Conclusions

This paper has reported a design of a novel block circuit diagram for hardware implementation of 1-D $5 \sim 11-$scroll, 2-D $3 \times 5 \sim 11-$grid scroll, and 3-D $3 \times 3 \times 5 \sim 11-$grid scroll chaotic attractors by different switchings. This design idea provides a theoretical principle for physical realization of a chaotic attractor in multi-directions with a large number of scrolls. It should be pointed out that this is the first time to report the experimental verification of 2-D $3 \times 11-$grid scroll and 3-D $3 \times 3 \times 11-$grid scroll chaotic attractors. Moreover, the hardware implementation of reliable nonlinear circuits for generating various complex chaotic signals provides a basis for future applications of chaos-based information systems.

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References

[1] J. A. K. Suykens and J. Vandewalle, "Generation of $n-$double scrolls ($n = 1, 2, 3, 4, \cdots$)," *IEEE Trans. Circuits Syst. I*, vol. 40, no. 11, pp. 861-867, Nov. 1993.

[2] M. E. Yalcin, J. A. K. Suykens, J. Vandewalle, and S. Ozoguz, "Families of scroll grid attractors," *Int. J. Bifurcation Chaos*, vol. 12, no. 1, pp. 23-41, Jan. 2002.

[3] J. Lü, T. Zhou, G. Chen, and X. Yang, "Generating chaos with a switching piecewise-linear controller," *Chaos*, vol. 12, no. 2, pp. 344-349, Jun. 2002.

[4] J. Lü, X. Yu, and G. Chen, "Generating chaotic attractors with multiple merged basins of attraction: A switching piecewise-linear control approach," *IEEE Trans. Circuits Syst. I*, vol. 50, no. 2, pp. 198-207, Feb. 2003.

[5] J. E. Varrientos and E. Sanchez-Sinencio, "A 4-D chaotic oscillator based on a differential hysteresis comparator," *IEEE Trans. Circuits Syst. I*, vol. 45, no. 1, pp. 3-10, Jan. 1998.

[6] M. Kataoka and T. Saito, "A 2-port VCCS chaotic oscillator and quad screw attractor," *IEEE Trans. Circuits Syst. I*, vol. 48, no. 2, pp. 221-225, Feb. 2001.

[7] T. Saito, "An approach toward higher dimensional hysteresis chaos generators," *IEEE Trans. Circuits Syst. I*, vol. 37, no. 3, pp. 399-409, Mar. 1990.

[8] A. S. Elwakil and M. P. Kennedy, "Systematic realization of a class of hysteresis chaotic oscillators," *Int. J. Circuit Theor. Appl.*, vol. 28, no. 4, pp. 319-334, Jul./Aug. 2000.

[9] J. Lü, F. Han, X. Yu, and G. Chen, "Generating 3-D multi-scroll chaotic attractors: A hysteresis series switching method," *Automatica*, vol. 40, no. 10, pp. 1677-1687, Nov. 2004.

[10] J. Lü, G. Chen, X. Yu, and H. Leung, "Design and analysis of multi-scroll chaotic attractors from saturated function series," *IEEE Trans. Circuits Syst. I*, vol. 51, no. 12, pp. 2476-2490, Dec. 2004.

[11] J. Lü, X. Yu, and G. Chen, "Switching control for multi-scroll chaos generation: An overview," *Proceedings of Physics and Control*, vol. 2, pp. 420-428, Saint Petersburg, Russia, 20-22 Aug. 2003.

[12] J. Lü, G. Chen, X. Yu, and H. Leung, "Generating multi-scroll chaotic attractors via switching control," *Proceedings of the 5th Asian Control Conference*, Melbourne, Australia, pp. 1763-1771, 20-23 Jul. 2004.

[13] S. M. Yu, S. S. Qiu, and Q. H. Lin, "New results of study on generating multiple-scroll chaotic attractors," *Science in China Series F*, vol. 46, no. 2, pp. 104-115, Feb. 2003.

[14] G. Zhong, K. F. Man, and G. Chen, "A systematic approach to generating $n-$scroll attractors," *Int. J. Bifurcation Chaos*, vol. 12, no. 12, pp. 2907-2915, Dec. 2002.

[15] M. E. Yalcin, J. A. K. Suykens, and J. Vandewalle, "Experimental confirmation of $3-$ and $5-$scroll attractors from a generalized Chua's circuit," *IEEE Trans. Circuits Syst. I*, vol. 47, no. 3, pp. 425-429, Mar. 2000.