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A MODULE-BASED AND UNIFIED APPROACH TO CHAOTIC CIRCUIT DESIGN AND ITS APPLICATIONS

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This paper proposes a module-based and unified approach to chaotic circuit design, where the description is based on the state equations without physical dimensions for simplicity of a general discussion. The main design process consists of transformation of state variables, transformation from differential to integral operations, and transformation of the time-scale. The designed circuit consists of anti-adder module integrator module, and inverter module. A novel 3-scroll Chua's circuit and a generalized Lorenz-like circuit are designed and implemented for verifying the effectiveness of this systematic circuit design methodology. Experimental observations are provided for confirmation. Comparing with the traditional circuit design methods, this new design approach has the following typical characteristics: (i) module-based and unified design; (ii) independent adjustment of system parameters; (iii) adjustment of distribution regions for the frequency spectra of chaotic signals; (iv) prominent observability.

Keywords: Chua's circuit; multi-scroll; module-based and unified circuit design; time-scale.

1. Introduction

Over the last four decades, chaos has been intensively investigated within the nonlinear science, information science, and engineering communities [Chen & Dong, 1998]. Aiming at real-world applications, nonlinear circuit design has become a key issue in chaos-based technologies.

Remarkably, Chua's circuit [Chua *et al.*, 1986; Kennedy, 1993; Zhong *et al.*, 2002] is a paradigm in the nonlinear circuit theory. Based on Chua's

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circuit, many modified nonlinear circuits have been designed and implemented [Elwakil & Kennedy, 2000], including the MCK circuit [Matsumoto *et al.*, 1986], revised Chua's circuit [Yin, 1997], and various multi-scroll circuits [Han *et al.*, 2005; Lü & Chen, 2006; Lü *et al.*, 2004b; Lü *et al.*, 2004c; Lü *et al.*, 2006; Lü *et al.*, 2003; Suykens & Vandewalle, 1993; Yalcin *et al.*, 2000; Yalcin *et al.*, 2002]. Most of these circuits are constructed by using capacitors, inductors and resistors along with Chua's diode. In this paper, we propose a novel circuit design method in which the basic design idea is very different from Chua's circuit but can realize the same chaotic dynamics and even much more.

As we know now, there are various design approaches of chaotic circuits by using various electronic or logic devices reported in the literature. For example, Elwakil *et al.* [2003[applied the so-called current feedback operational amplifiers (CFOAs) and digital logic operations to design the Lorenz-like circuit for implementing a four-wing butterfly attractor; Zhong and Tang [2002] designed the Chen circuit and Li *et al.* [2005] devised the hyperchaotic Chen circuit both based on the dimensionless state equations of the circuits. Most of the circuit design methods as above are not based on a common and unified framework and do not have the universality and compatibility. In the following, based on the dimensionless state equations of the circuit, a module-based and unified circuit design approach is then proposed. The designed circuit consists of three different functional blocks: anti-adder module, integrator module and inverter module. The main design process consists of transformation of state variables, transformation from differential to integral operations, and transformation of the time-scale. Comparing with the traditional circuit design methods, such as those of the Lorenz-like circuit [Elwakil *et al.*, 2003], Chen circuit [Zhong & Tang, 2002], and hyperchaotic Chen circuit [Li *et al.*, 2005], this new method has the following four typical characteristics: (i) module-based and unified design; (ii) independent adjustment of system parameters; (iii) adjustment of distribution regions for the frequency spectra of chaotic signals; (iv) prominent observability.

It should be especially pointed out that all state variables in the forms of the original or inverse variables input to the inverting terminals of the anti-adders and all noninverting terminals are connected to the earth in this proposed approach. However, in most traditional circuit design methods based on the dimensionless state equations of the circuits [Zhong *et al.*, 2002; Zhong & Tang, 2002; Li *et al.*, 2005], all state variables simultaneously input to the inverting and noninverting terminals. Therefore, all parameters in our method are independently adjustable, however, all parameters in the traditional approaches as above are coupled together and not independently adjustable. To verify the effectiveness of this new approach, a novel 3-scroll Chua's circuit and a generalized Lorenz-like circuit are designed and implemented with experimental observations provided for confirmation. Moreover, the proposed circuit design method can be easily and naturally generalized to the circuit designs of other chaotic circuits.

The rest of this paper is organized as follows. In Sec. 2, the new systematic circuit design approach is described and discussed. A novel 3-scroll Chua's circuit and a generalized Lorenz-like circuit are then designed and implemented in Secs. 3 and 4, respectively, with experimental observations reported. Finally, some conclusions are drawn in Sec. 5.

2. A Module-Based and Unified Circuit Design Approach

This section proposes a module-based and unified circuit design approach, in which the fundamental design principle differs from those of the traditional circuit design methods.

This approach is based on the dimensionless state equations of the circuit. The main procedure can be summarized into three key steps; that is, Step I: *transformation of state variables*; Step II: *transformation from differential to integral operations*; Step III: *transformation of the time-scale*.

To start, consider a general system of n-dimensional state equations described by

$$\begin{cases} \dfrac{dx_1}{d\tau} = \displaystyle\sum_{i=1}^{n} a_{1i}x_i + \sum_{j=1}^{n}\sum_{p=1}^{n} b_{jp}^{1}x_jx_p \\ \qquad + \cdots + f_1(x_1, x_2, \ldots, x_n) \\ \dfrac{dx_2}{d\tau} = \displaystyle\sum_{i=1}^{n} a_{2i}x_i + \sum_{j=1}^{n}\sum_{p=1}^{n} b_{jp}^{2}x_jx_p \\ \qquad + \cdots + f_2(x_1, x_2, \ldots, x_n) \\ \cdots\cdots\cdots \\ \dfrac{dx_n}{d\tau} = \displaystyle\sum_{i=1}^{n} a_{ni}x_i + \sum_{j=1}^{n}\sum_{p=1}^{n} b_{jp}^{n}x_jx_p \\ \qquad + \cdots + f_n(x_1, x_2, \ldots, x_n), \end{cases} \tag{1}$$

where the first sum represents the linear terms and the other sums are all cross products, with $f_i(x_1, x_2, \ldots, x_n)\,(1 \le i \le n)$ being the different types of nonlinear functions, which can be saturated function series, hysteresis function series, step wave, sawtooth wave, triangular wave, transconductor wave and exponent functions, and so on.

It is well known that the dynamic regions of most electronic devices are very limited. For example, the linear dynamic region of the operational amplifier TL082 with electrical source of $\pm 15\,\mathrm{V}$ is only within $\pm 13.5\,\mathrm{V}$. However, for many chaotic systems, the dynamic regions of the state variables in the dimensionless state equations are far exceeding the linear dynamic regions of the operational amplifiers. To physically realize these chaotic systems, one has to compress the dynamic regions of all the state variables into the linear dynamic regions of the operational amplifiers. To do so, one may let $x_i' = kx_i (1 \le i \le N)$, where $k \le 1$ is the compressed ratio. Then, according to Eq. (1), one has

$$\begin{cases} \dfrac{dx_1'}{d\tau} = \displaystyle\sum_{i=1}^{n} a_{1i}x_i' + \frac{1}{k}\sum_{j=1}^{n}\sum_{p=1}^{n} b_{jp}^{1}x_j'x_p' \\ \qquad + \cdots + kf_1\left(\dfrac{x_1'}{k}, \dfrac{x_2'}{k}, \ldots, \dfrac{x_n'}{k}\right) \\ \dfrac{dx_2'}{d\tau} = \displaystyle\sum_{i=1}^{n} a_{2i}x_i' + \frac{1}{k}\sum_{j=1}^{n}\sum_{p=1}^{n} b_{jp}^{2}x_j'x_p' \\ \qquad + \cdots + kf_2\left(\dfrac{x_1'}{k}, \dfrac{x_2'}{k}, \ldots, \dfrac{x_n'}{k}\right) \\ \cdots\cdots\cdots \\ \dfrac{dx_n'}{d\tau} = \displaystyle\sum_{i=1}^{n} a_{ni}x_i' + \frac{1}{k}\sum_{j=1}^{n}\sum_{p=1}^{n} b_{jp}^{n}x_j'x_p' \\ \qquad + \cdots + kf_n\left(\dfrac{x_1'}{k}, \dfrac{x_2'}{k}, \ldots, \dfrac{x_n'}{k}\right). \end{cases} \tag{2}$$

It is quite easy to obtain the integral form of Eq. (2), based on which one can further get the final circuit equation by using a transformation of the time-scale. According to the above circuit equation, one can easily design a block circuit diagram, which includes three main modules: *anti-adders module integrators module* and *inverters module*.

In the following, this circuit design approach is illustrated by working out two typical examples: a novel 3-scroll Chua's circuit and a generalized Lorenz-like system.

3. A Novel 3-Scroll Chua's Circuit

It is known that the piecewise linear function in the original Chua's circuit can be replaced by other nonlinear functions, such as the sine function [Tang *et al.*, 2001], exponent function [Abdomerovic *et al.*, 2000], and polynomial function [Tang & Man, 1998; Zhong, 1994], so as to generate various double-scroll or even multi-scroll chaotic attractors from the circuit [Yu *et al.*, 2004a; Yu *et al.*, 2004b; Yu *et al.*, 2005]. Here, we are especially interested in polynomial characteristic functions, such as $ax+bx|x|$, $ax+bx^3$, $a+bx+cx^2+dx^3$, etc. Notice that all these polynomial characteristic functions can only generate double-scroll Chua's attractors. To create more scrolls in Chua's circuit, the polynomial characteristic function is first modified to be $ax+bx|x|+cx^3$. Thus, Chua's circuit equation becomes

$$\begin{cases} \dfrac{dx}{d\tau} = \alpha(y - h(x)) \\ \dfrac{dy}{d\tau} = x - y + z \\ \dfrac{dz}{d\tau} = -\beta y, \end{cases} \tag{3}$$

where $\alpha = 12.8, \beta = 19.1, h(x) = ax + bx|x| + cx^3$. When $a = 0.472, b = -1, c = 0.47$, the polynomial characteristic curve is shown in Fig. 1. Here, five equilibria, four turning points and five characteristic regions are denoted by $x_i (0 \le i \le 4)$, $e_i (1 \le i \le 4)$ and $D_i (0 \le i \le 4)$, respectively. Figure 2 shows the numerical simulation results of the

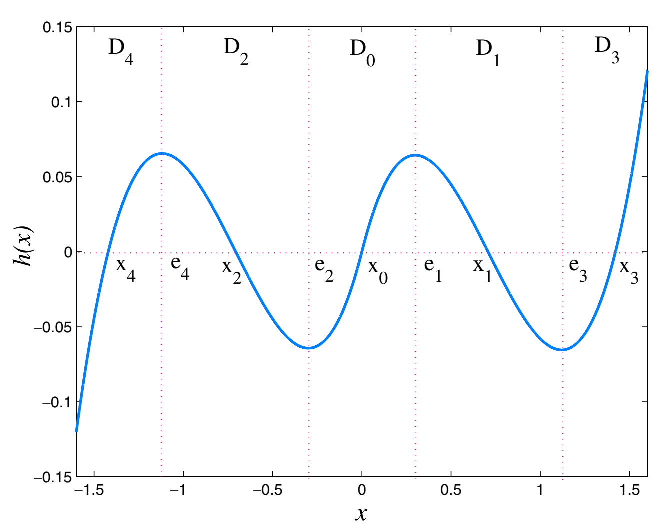
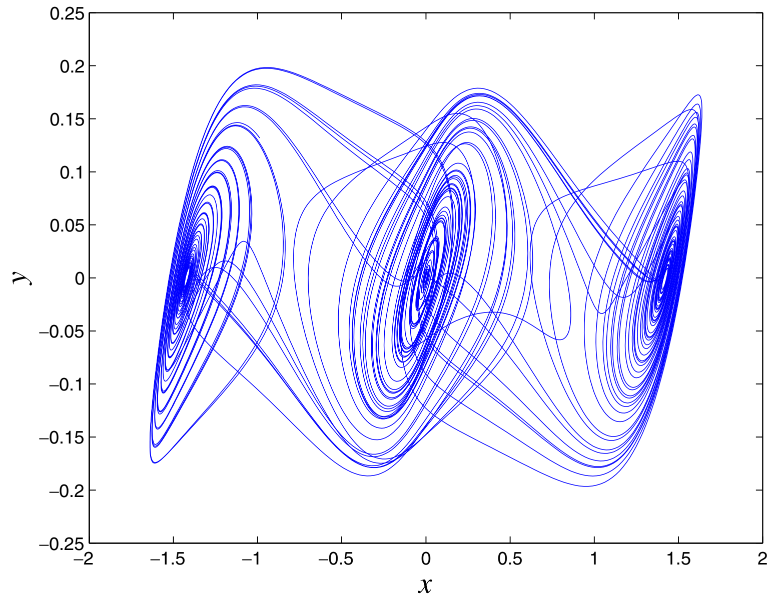
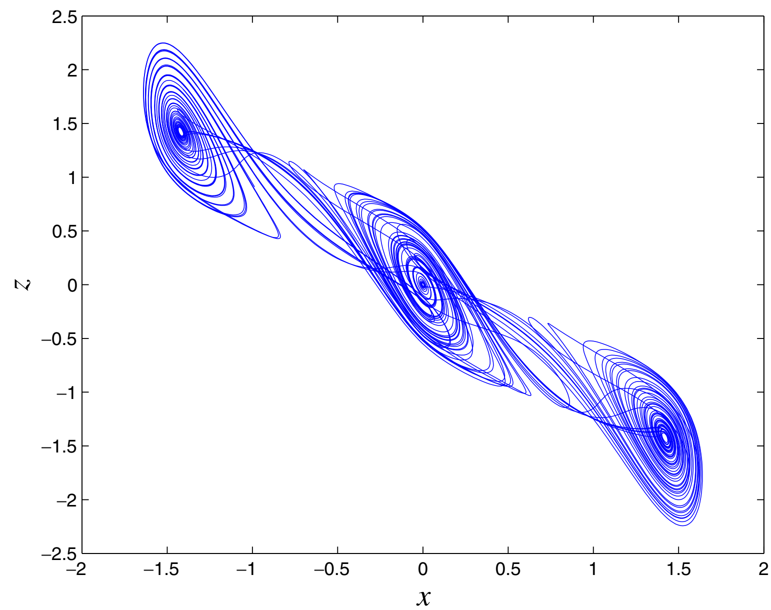


Fig. 1. $h(x)$ and its five characteristic regions.



(a) $x - y$ plane



(b) $x - z$ plane

Fig. 2. Plane projection of the 3-scroll Chua's attractor.

3-scroll Chua's attractor with Lyapunov exponents: $\lambda_1 = 0.2, \lambda_2 = 0, \lambda_3 = -5.56$.

Both theoretical analysis and numerical simulations show that there are various bifurcation phenomena in system (3). Figures 3(a)–3(d) show the bifurcation diagrams versus parameter a with $\alpha = 12.8,\ \beta = 19.1,\ b = -1,\ c = 0.47$, parameter b with $\alpha = 12.8,\ \beta = 19.1,\ a = 0.472,\ c = 0.47$, parameter c with $\alpha = 12.8,\ \beta = 19.1,\ a = 0.472$, $b = -1$, and parameter α with $\beta = 19.1,\ a = 0.472$, $b = -1,\ c = 0.47$, respectively. According to Fig. 3, there are some continuous chaotic regions for parameters α, a, b, c, which are very useful for hardware implementation.

When $a = 0.472, b = -1, c = 0.47$, system (3) has five equilibria: $(x_i, 0, -x_i)(0 \leq i \leq 4)$, where

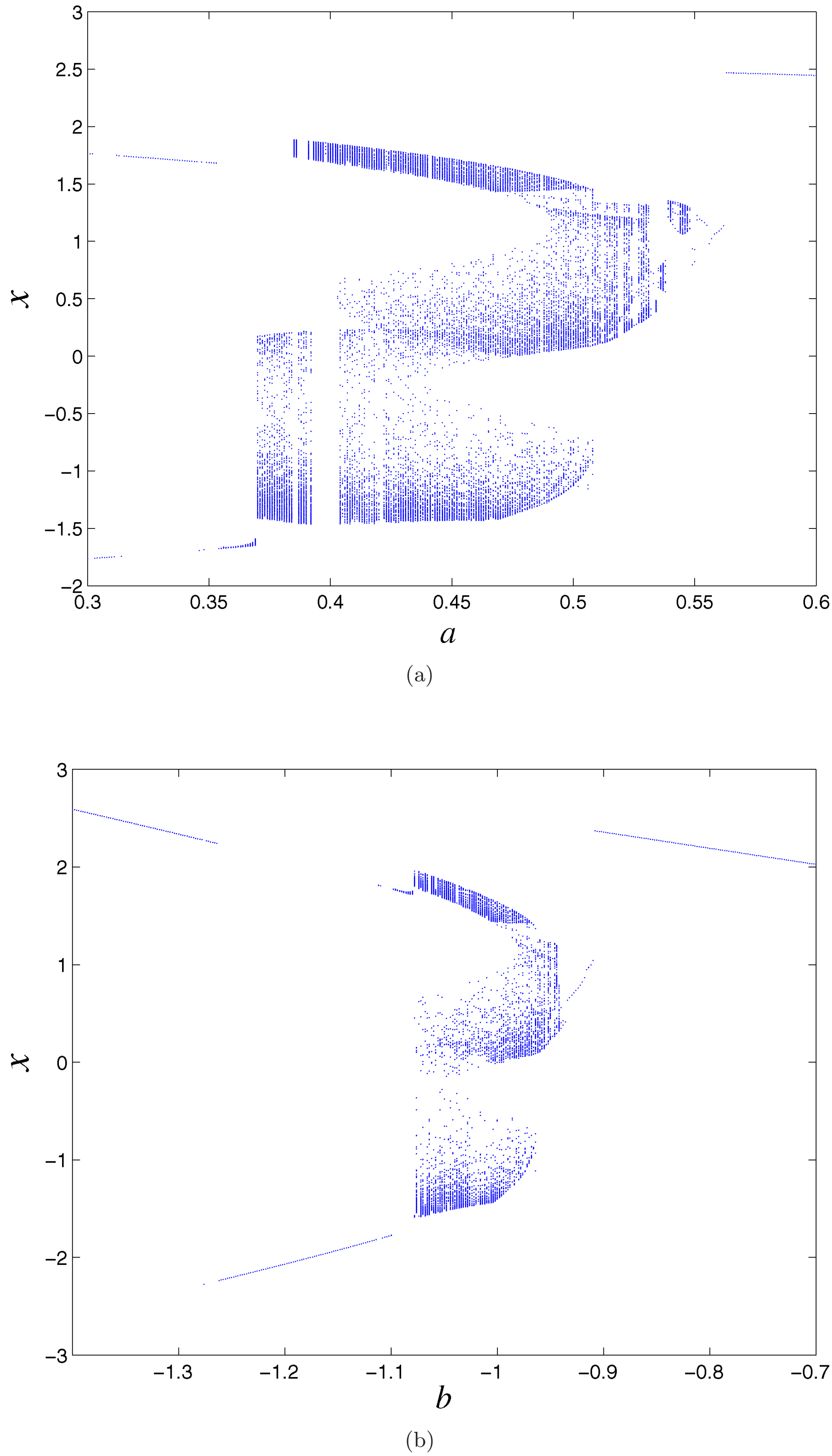
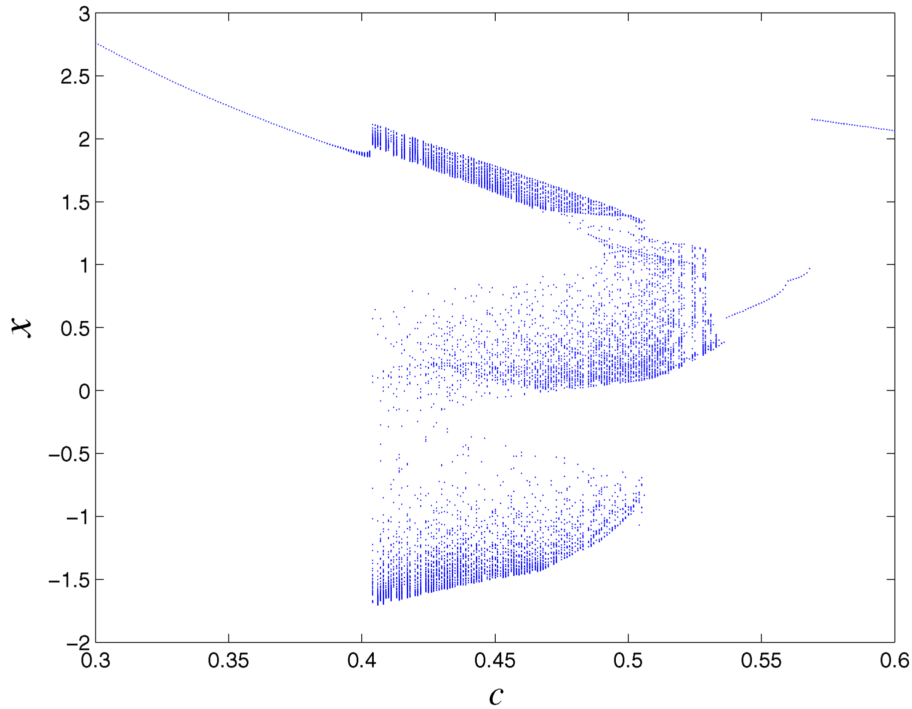
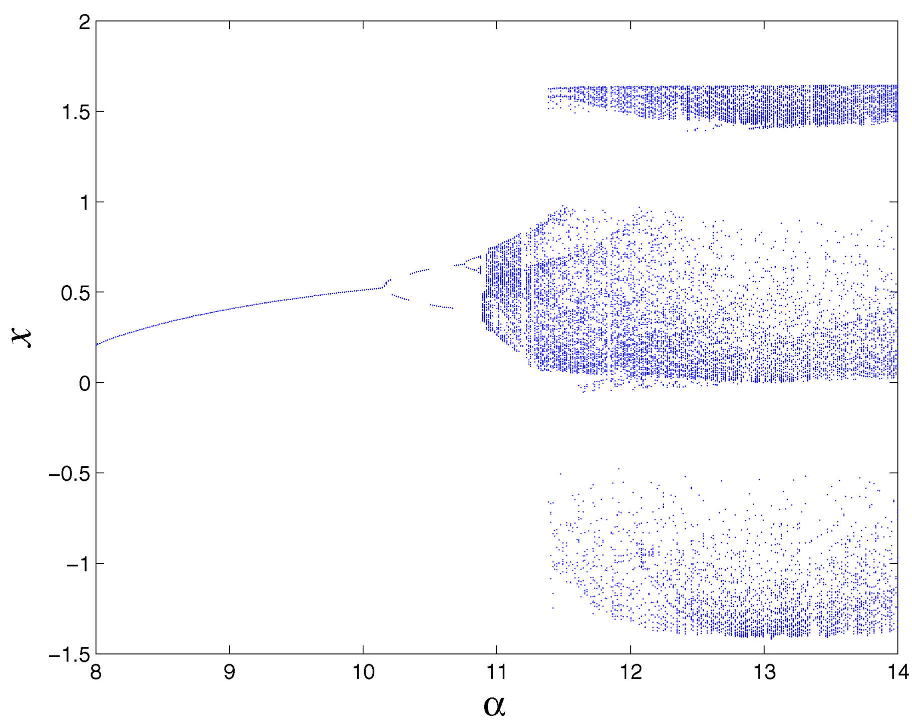


Fig. 3. Bifurcation diagrams of parameters a, b, c, α.



(c)



(d)

Fig. 3. (*Continued*)

$x_0 = 0, x_{1,2} = \pm 0.7068, x_{3,4} = \pm 1.4209$. Moreover, the Jacobians of system (3) evaluated at its equilibria $(x_i, 0, -x_i)(0 \le i \le 4)$ are given by

$$J(x_i) = \begin{pmatrix} -\alpha \left.\dfrac{dh(x)}{dx}\right|_{x=x_i} & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix}$$

for $0 \le i \le 4$. Their corresponding eigenvalues are: $\gamma_0 = -7.4606$, $\sigma_0 \pm j\omega_0 = 0.2095 \pm j3.9273$, $\gamma_{1,2} = 4.3504$, $\sigma_{1,2} \pm j\omega_{1,2} = -1.1570 \pm j3.4629$, and $\gamma_{3,4} = -7.5171$, $\sigma_{3,4} \pm j\omega_{3,4} = 0.2066 \pm j3.9328$, respectively. Obviously, $x_0,\ x_3,\ x_4$ are the equilibria with index 2, which can generate scrolls in the attractor; on the other hand, x_1, x_2 are the equilibria with index 1, which can connect neighboring scrolls in the attractor.

In the following, a circuit diagram is designed to physically realize the above 3-scroll Chua's attractor. According to the circuit design method introduced in Sec. 2, one firstly gets the corresponding Eq. (3) after transformation of state variables, as follows:

$$\begin{cases} \dfrac{du}{d\tau} = \alpha\left[v - \left(au + \dfrac{b}{k}u|u| + \dfrac{c}{k^2}u^3\right)\right] \\ \dfrac{dv}{d\tau} = u - v + w \\ \dfrac{dw}{d\tau} = -\beta v, \end{cases} \tag{4}$$

where k is the compressed ratio. From Fig. 2, one can see that the dynamic regions of all the state variables of system (3) belong to the linear dynamic regions of the operational amplifiers. In this case, one may simply let $k = 1$.

Thus, the integral form of system (4) is given by

$$\begin{cases} u = \displaystyle\int [-a_{11}(-v) - a_{12}h(u)]d\tau \\ v = \displaystyle\int [-a_{21}(-u) - a_{22}v - a_{23}(-w)]d\tau \\ w = \displaystyle\int -a_{31}v d\tau, \end{cases} \tag{5}$$

where $a_{ij}(1 \le i, j \le 3)$ are system parameters and $a_{11} = a_{12} = \alpha = 12.8$, $a_{13} = a_{32} = a_{33} = 0$, $a_{21} = a_{22} = a_{23} = 1$, $a_{31} = \beta = 19.1$. $h(u) = au + (b/k)u|u| + (c/k^2)u^3$, $a = 0.472, b = -1$, $c = 0.47$, $k = 1$. Based on system (5), one can design a block circuit diagram to physically realize the 3-scroll Chua's attractor, as shown in Fig. 4.

All operational amplifiers shown in Fig. 4 are TL082, the supply voltages of the positive and negative electrical sources are ± 15 V. Moreover, for convenient adjustment and higher precision, all resistors are precisely adjustable resistors or potentiometers. In addition, in Fig. 4, the operational amplifiers OP1, OP4, OP7 are anti-adder modules; the operational amplifiers OP2, OP5, OP8 are anti-integrator modules; the operational amplifiers OP3, OP6, OP9 are inverter modules. Figures 4(a) and 4(b) are the basic Chua's circuit and polynomial signal generator for $h(u) = au + (b/k)u|u| + (c/k^2)u^3$, respectively. Here, the absolute-value circuit is shown in the dashed-line block of Fig. 4(b). From the generalized superposition principle, the relationship between input and output of the absolute-value circuit satisfies $u_0 = -u - 2u_1$. For $u < 0$, the diodes D_1 and D_2 are connected and $u_1 = 0$. Thus, $u_0 = -u > 0$. For $u > 0$, the diodes D_1 and D_2 are disconnected and $u_1 = -u$. Thus, $u_0 = -u + 2u = u > 0$. Therefore, $u_0 = |u|$.

According to Fig. 4(b), the relationship between the input and the output of the polynomial signal generator is described by

$$h(u) = \frac{R_n}{R_a}u - \frac{R_n}{10R_b}u|u| + \frac{R_n}{100R_c}u^3 \tag{6}$$

where $a = R_n/R_a$, $b/k = -(R_n/10R_b)$, $c/k^2 = R_n/100R_c$, $k = 1$. When R_n is fixed, one can adjust the system parameters $a, b/k, c/k^2$ of polynomial $h(u)$ by adjusting the resistors R_a, R_b, R_c, respectively.

It follows from Fig. 4(a) that the state equation of the nonlinear circuit is given by

$$\begin{cases} u = \dfrac{1}{R_0C_0}\displaystyle\int\left[-\dfrac{R_f}{R_{11}}(-v) - \dfrac{R_f}{R_{12}}h(u)\right]dt \\ v = \dfrac{1}{R_0C_0}\displaystyle\int\left[-\dfrac{R_f}{R_{21}}(-u) - \dfrac{R_f}{R_{22}}v - \dfrac{R_f}{R_{23}}(-w)\right]dt \\ w = \dfrac{1}{R_0C_0}\displaystyle\int\left[-\dfrac{R_f}{R_{31}}v\right]dt. \end{cases} \tag{7}$$

Let $\tau = t/R_0C_0$. Here, $1/R_0C_0$ is the integral constant of the integrators shown in Fig. 4(a), which is also the transformation factor of the time-scale.

Then, one obtains

$$\begin{cases} u = \displaystyle\int \left[-\frac{R_f}{R_{11}}(-v) - \frac{R_f}{R_{12}} h(u) \right] d\tau \\ v = \displaystyle\int \left[-\frac{R_f}{R_{21}}(-u) - \frac{R_f}{R_{22}} v - \frac{R_f}{R_{23}}(-w) \right] d\tau \\ w = \displaystyle\int \left[-\frac{R_f}{R_{31}} v \right] d\tau. \end{cases} \tag{8}$$

According to (5) and (8), all system parameters are given by $a_{11} = R_f/R_{11}$, $a_{12} = R_f/R_{12}$, $a_{21} = R_f/R_{21}$, $a_{22} = R_f/R_{22}$, $a_{23} = R_f/R_{23}$, $a_{31} = R_f/R_{31}$. Let $R_f = 100k$, $R_{11} = 7.8k$, $R_{12} = 7.8k$, $R_{21} = 100k$, $R_{22} = 100k$, $R_{23} = 100k$, $R_{31} = 5.3k$. Then, one has $a_{11} = a_{12} = 12.8$, $a_{21} = a_{22} = a_{23} = 1$, $a_{31} = 19.1$. Since there are three anti-adder modules in Fig. 4, for a fixed R_f, one can independently adjust various system parameters, $a_{ij}(1 \leq i, j \leq 3)$, by tuning the corresponding resistors $R_{ij}(1 \leq i, j \leq 3)$ in Fig. 4, respectively. Therefore, this independently adjustable characteristic is one of the useful features of the modular circuit design. Figure 5 shows the experimental observations of the 3-scroll Chua's attractor.

From (7), the transformation factor of the time-scale is completely determined by the integral resistor R_0 and integral capacitance C_0. Comparing with the traditional design methods, such as that of Chua's circuit, this module-based and unified design approach can change the distribution region of the frequency spectrum of a chaotic signal as required by tuning integral resistor R_0 or integral capacitance C_0 for real-world applications. That is, when R_0 (or C_0) is decreasing, one can

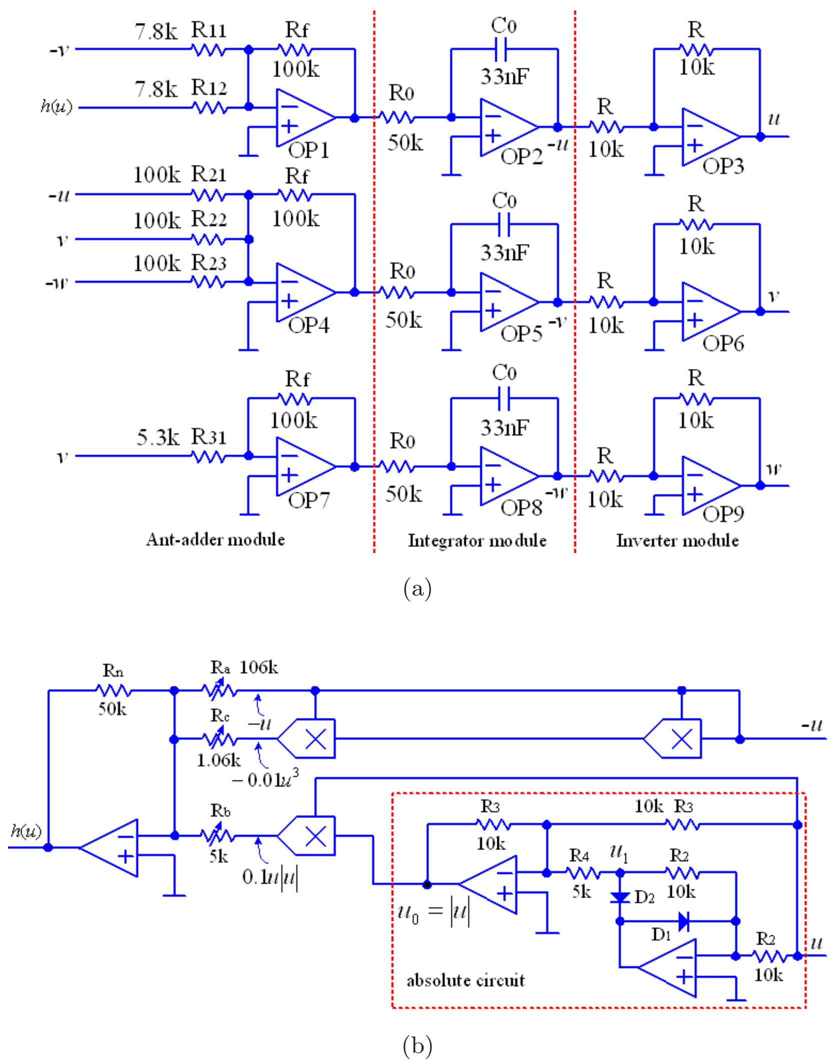
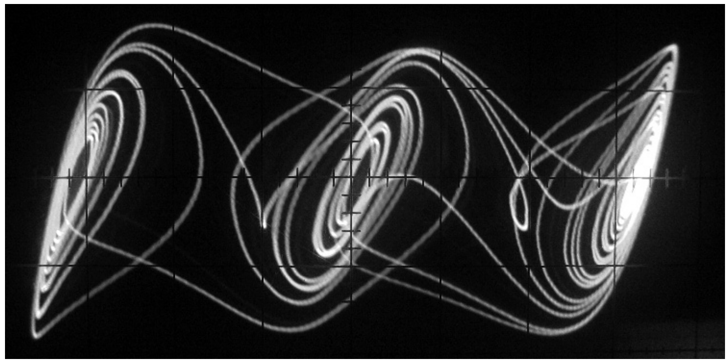
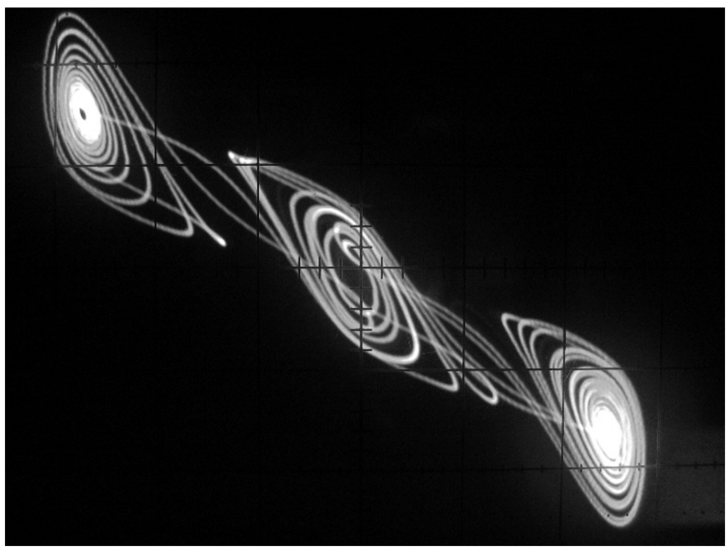


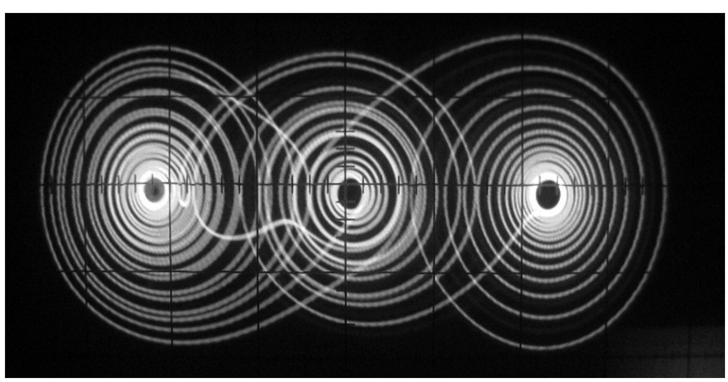
Fig. 4. Circuit diagram for realizing the 3-scroll Chua's attractor.



(a) $x - y$ plane, where $x = 0.5\,\mathrm{V/div}$, $y = 1.0\,\mathrm{V/div}$



(b) $x - z$ plane, where $x = 1\,\mathrm{V/div}$, $z = 0.6\,\mathrm{V/div}$

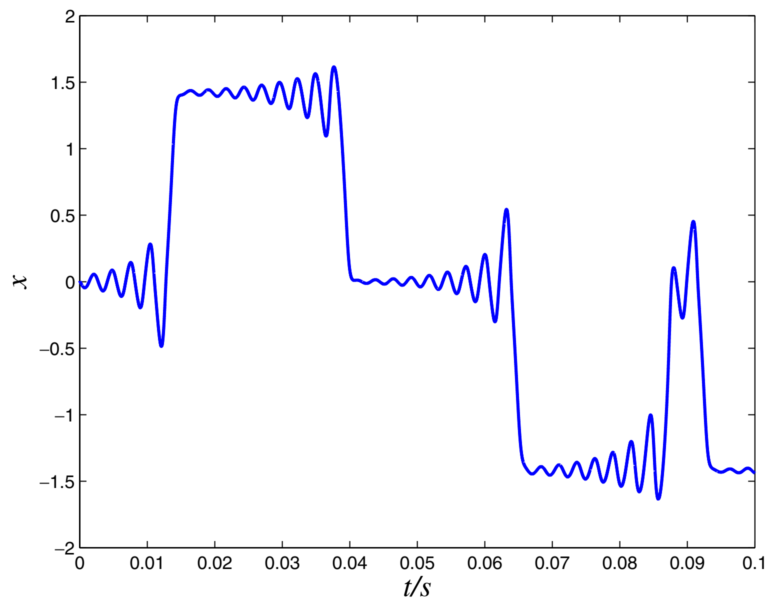


(c) $y - z$ plane, where $y = 1\,\mathrm{V/div}$, $z = 0.6\,\mathrm{V/div}$

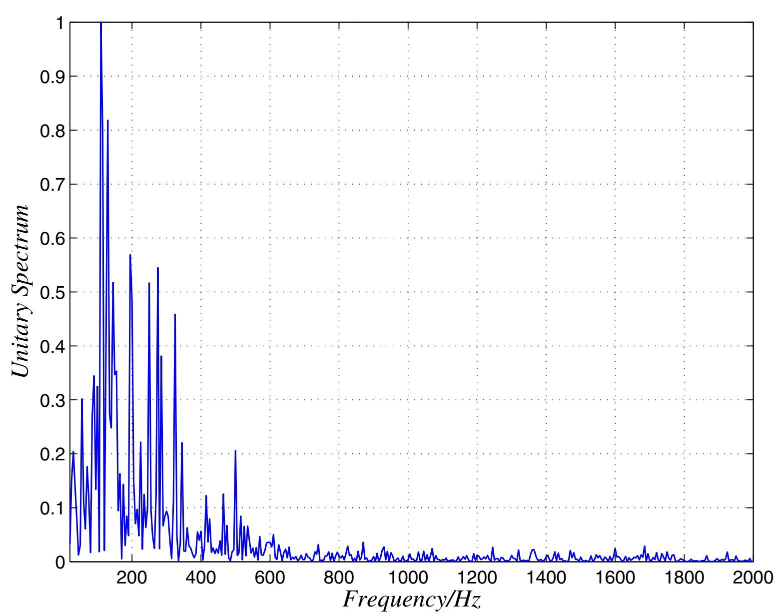
Fig. 5. Experimental observations of the 3-scroll Chua's attractor.

extend the distribution region of the frequency spectrum of the high-frequency end of a chaotic signal. However, when R_0 (or C_0) is increasing, one can reduce the distribution region of the frequency spectrum of the high-frequency end of a chaotic signal. Therefore, comparing with the fixed capacitance and inductance in Chua's circuit, this adjusting characteristic is very useful for real circuit design and practical engineering applications.

Here, two typical examples are used to show the effectiveness of this proposed design method. Figure 6 shows the waveforms and power spectrums of the time domain of the variable x for two different cases: (I) $C_0 = 33\,nF$, $R_0 = 50\,k\Omega$ and (II) $C_0 = 33\,nF$, $R_0 = 10\,k\Omega$, respectively, where the other parameters are given in Fig. 4. Our experimental observations are consistent with the numerical observations in Fig. 6.

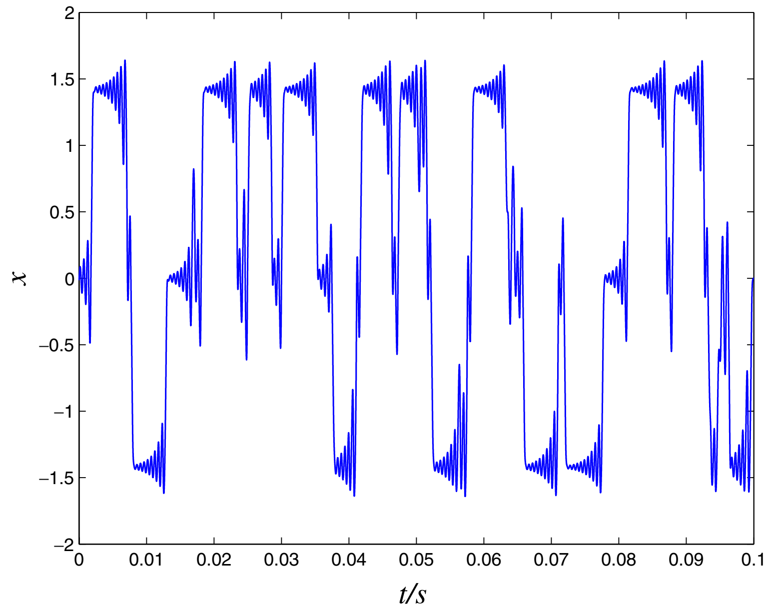


(a) Waveform of x, $C_0 = 33\,nF, R_0 = 50\,k\Omega$

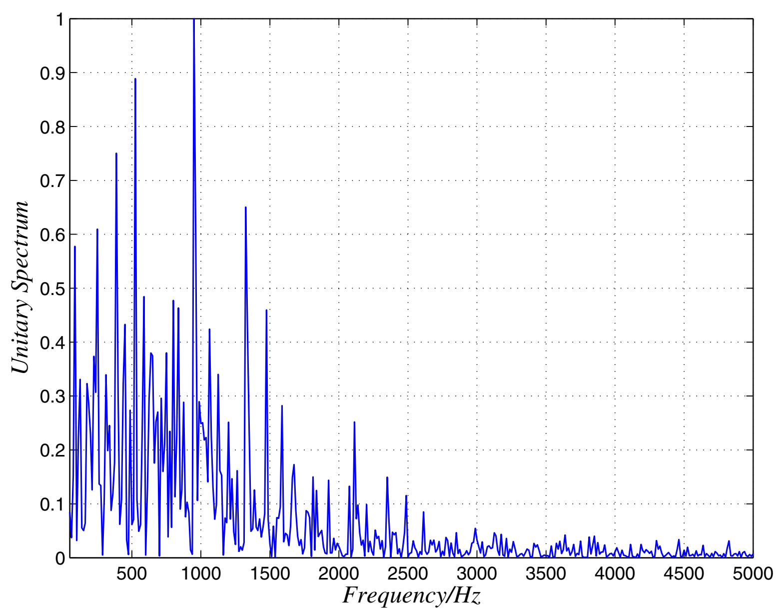


(b) Power spectrum of x, $C_0 = 33\,nF, R_0 = 50\,k\Omega$

Fig. 6. Numerical simulations of the waveforms and power spectrums of variable x.



(c) Waveform of x, $C_0 = 33\,nF, R_0 = 10\,k\Omega$



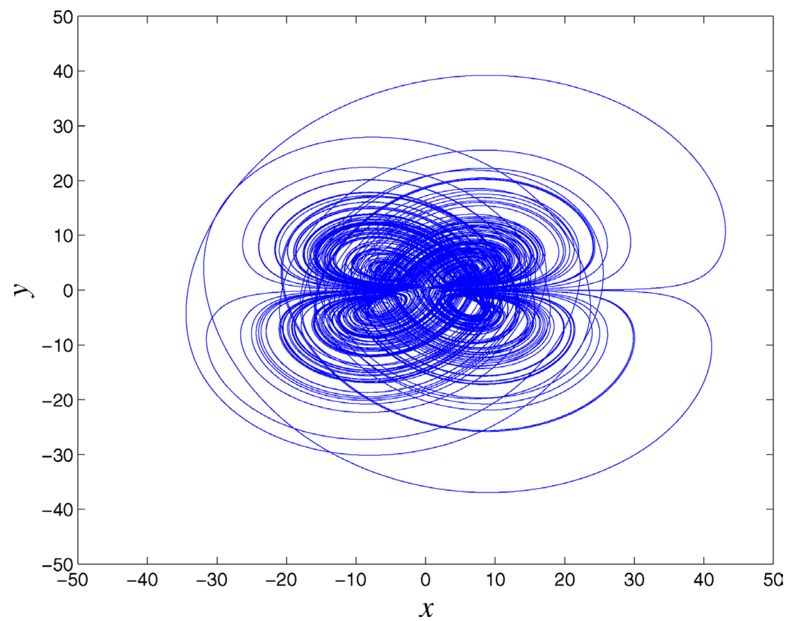
(d) Power spectrum of x, $C_0 = 33\,nF, R_0 = 10\,k\Omega$

Fig. 6. (*Continued*)

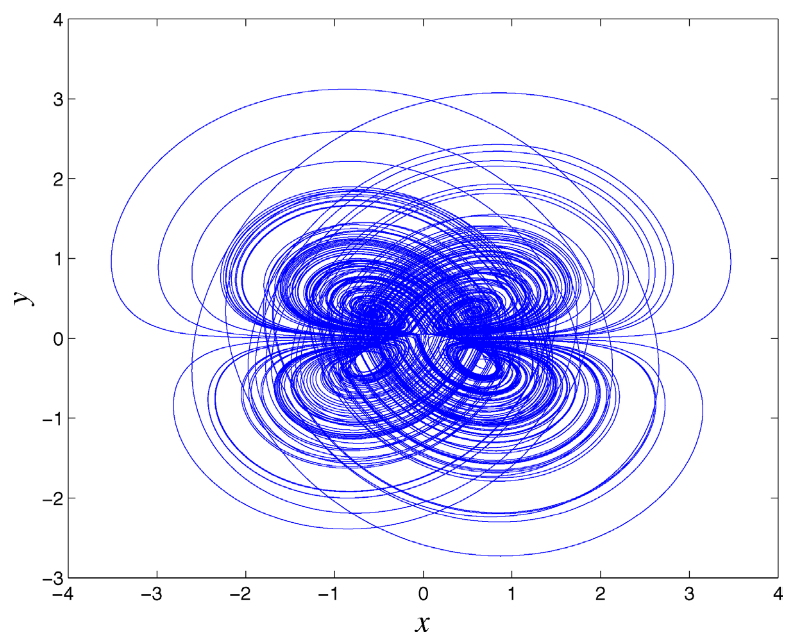
It is well known that almost all design methods of chaotic circuit are not beauideal. Comparing with the traditional design approaches, such as that of Chua's circuit, the main disadvantage of the proposed design method is that it is likely to need more electronic devices. More electronic devices may increase the total hardware errors. However, one can minimize the total hardware errors by using the electronic devices with high precision.

4. Circuit Implementation of a Generalized Lorenz-like System

A generalized Lorenz-like system was proposed in [Lü & Chen, 2002; Lü *et al.*, 2002], which is



(a) Original attractor



(b) Compressed attractor

Fig. 7. Numerical simulations of the generalized Lorenz-like system.

described by

$$\begin{cases} \dfrac{dx}{d\tau} = \dfrac{ab}{a+b}x - yz \\ \dfrac{dy}{d\tau} = -ay + xz + d \\ \dfrac{dz}{d\tau} = -bz + xy. \end{cases} \tag{9}$$

When $a = 10, b = 5, d = 5$, system (9) has a Lorenz-like chaotic attractor [Lü *et al.*, 2004a], as shown in Fig. 7(a).

Obviously, the dynamic regions of the state variables x, y, z of system (8) are far exceeding the linearly dynamic regions of the operational amplifiers. To physically realize system (9), one has to compress the dynamic regions of the state variables x, y, z. To do so, let $u = kx, v = ky, w = kz$, where

$k = 0.1$. Then, one gets

$$\begin{cases} \dfrac{du}{d\tau} = \dfrac{ab}{a+b}u - vw \\ \dfrac{dv}{d\tau} = -av + 10uw + 10d \\ \dfrac{dw}{d\tau} = -bw + 10uv. \end{cases} \tag{10}$$

When $a = 10, b = 5, d = 5$, system (10) has a chaotic attractor as shown in Fig. 7(b). Obviously, the dynamic regions of the state variables x, y, z of system (10) are compressed to be within the linearly dynamic regions of the operational amplifiers.

Similarly, based on the circuit design principle proposed in Sec. 2, one can construct a circuit diagram, as shown in Fig. 8, to physically realize system (10). Figure 9 shows the experimental observations of the 4-scroll generalized Lorenz-like chaotic attractor.

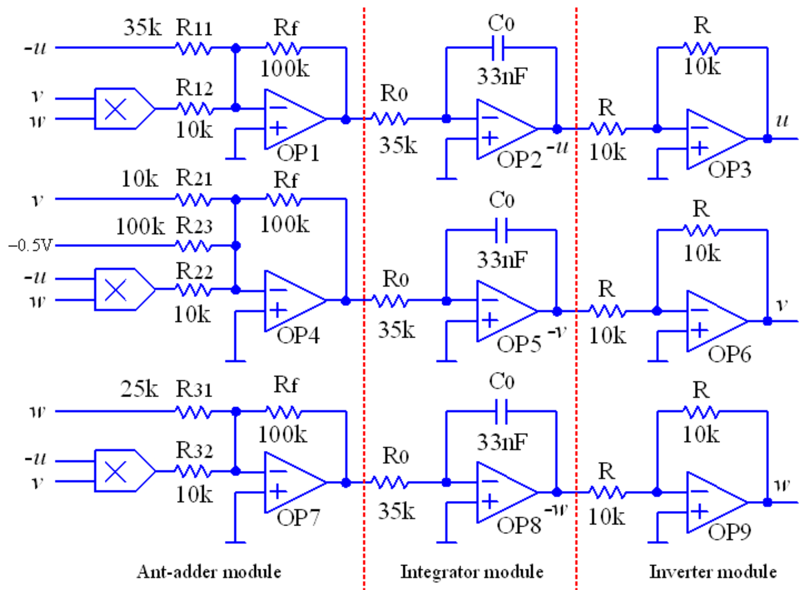
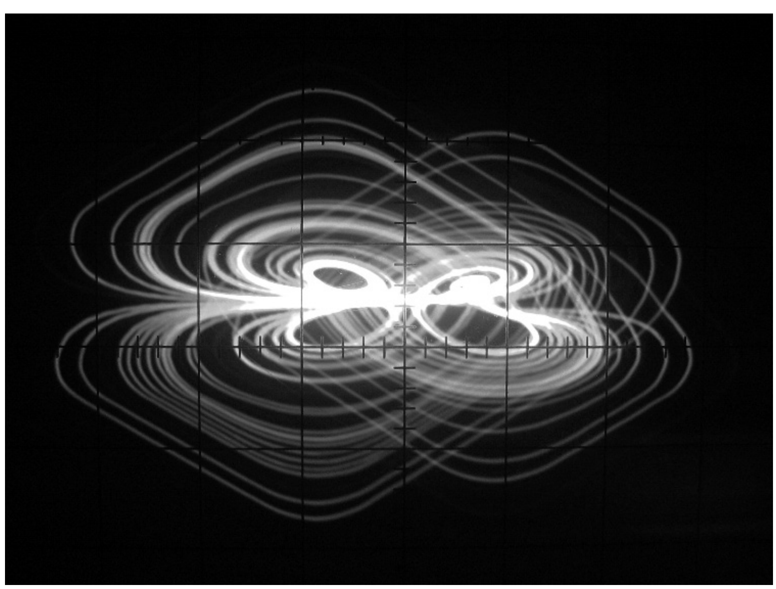
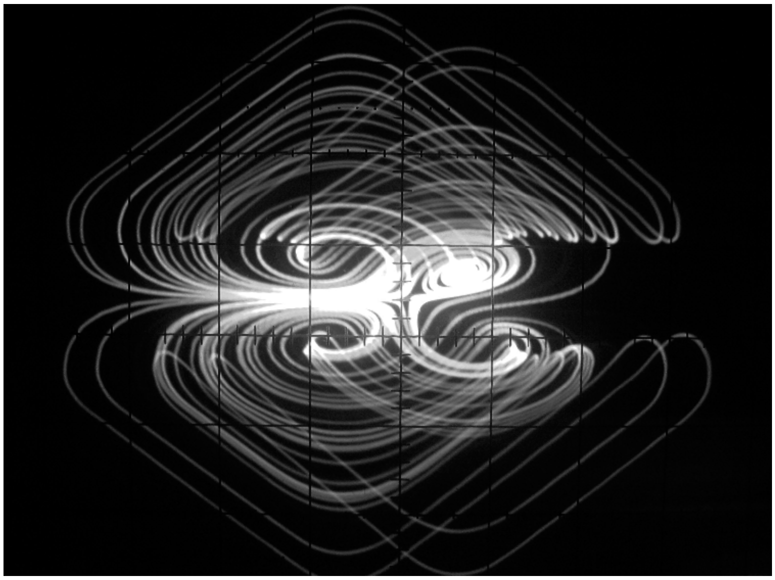


Fig. 8. Circuit diagram for realizing the generalized Lorenz-like system.

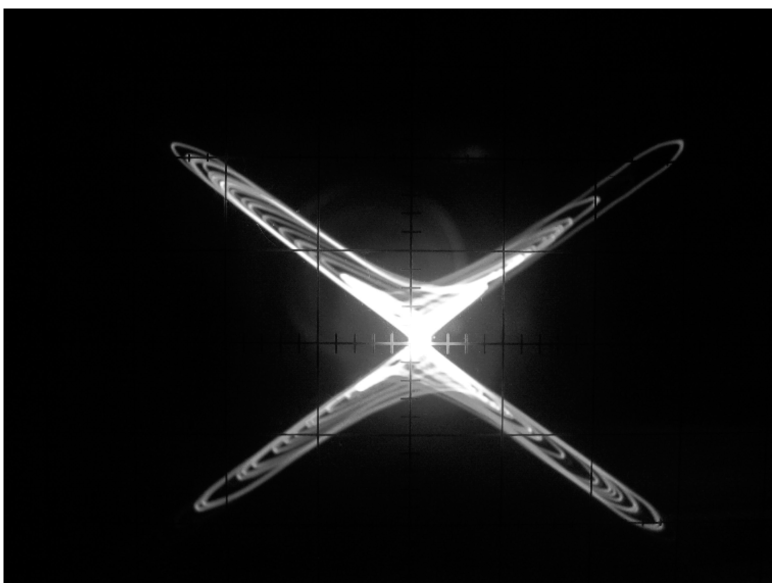


(a) $x - y$ plane, $x = 1.0\,\text{V/div}$, $y = 0.6\,\text{V/div}$

Fig. 9. Experimental observations of the generalized Lorenz-like system.



(b) $x - z$ plane, $x = 1.0\,\mathrm{V/div}$, $z = 1.0\,\mathrm{V/div}$



(c) $y - z$ plane, where $y = 1.0\,\mathrm{V/div}$, $z = 1.2\,\mathrm{V/div}$

Fig. 9. (*Continued*)

5. Conclusions

We have introduced a module-based and unified approach to chaotic circuit design. This method is based on the dimensionless state equations, and the main design process consists of transformation of state variables (which extends the parameter ranges in hardware implementation of nonlinear circuits), transformation from differential to integral operations, and transformation of the time-scale. The designed circuit includes three different function blocks: anti-adder module integrator module and inverter module. Comparing with the traditional circuit design methods, this systematic approach has the following four typical characteristics: (i) module-based and unified design; (ii) independent adjustment of system parameters; (iii) adjustment of distribution regions for the frequency spectra of chaotic signals; (iv) prominent observability. To measure the current of the inductance of Chua's circuit, an additional circuit is needed to perform the current-voltage transformation. Moreover, a novel 3-scroll Chua's circuit and a generalized Lorenz-like circuit have been designed and implemented to verify the effectiveness of the new design methodology, furthermore confirmed by experimental observations. It is believed that this module-based and unified circuit design approach will have good applications in practice.

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