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Adaptive synchronization of uncertain Rössler hyperchaotic system based on parameter identification

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Abstract

In this Letter, an approach of adaptive synchronization and parameters identification of uncertain Rössler hyperchaotic system is proposed. The suggested tool proves to be globally and asymptotically stable by means of Lyapunov method. With this new and effective method, parameters identification and synchronization of Rössler hyperchaotic with all the system parameters unknown, can be achieved simultaneously. Theoretical proof and numerical simulation demonstrate the effectiveness and feasibility of the proposed technique.

PACS: 05.45.+b

Keywords: Synchronization; Parameters identification; Hyperchaotic system

1. Introduction

Since its introduction by Pecora and Carrol in 1990 [1], chaos synchronization has received increasing attention due to its theoretical challenge and its great potential applications in secure communication, chemical reaction, biological systems and so on [2]. Basically, chaos synchronization problem can be formulated as follows. Given a chaotic system, which is considered as the master (or driving) system, and another identical system, which is considered as the slave (or response) system, the aim is to force the response of the slave system to synchronize the master system [3]. Great efforts have been devoted to achieving this goal in the last few years and a large variety of approaches has been proposed, such as sampled-data feedback synchronization method [4], impulsive control method [5], adaptive design method [6], and invariant manifold method [7], among many others (see [8–11] and references therein).

However, to our best knowledge, the aforementioned methods and many other existing synchronization methods mainly concern the synchronization of chaotic systems with low-dimensional attractor, char-

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acterized by one positive Lyapunov exponent. This feature limits the complexity of the chaotic dynamics. It is believed that the chaotic systems with higher-dimensional attractor have much wider application. In fact, the adoption of higher-dimensional chaotic systems has been proposed for secure communication and the presence of more than one positive Lyapunov exponent clearly improves security of the communication scheme by generating more complex dynamics. Moreover, most of the developed methods are valid only for the chaotic systems whose parameters are precisely known. But in practical situation, some system's parameters cannot be exactly known in priori, the effect of these uncertainties will destroy the synchronization and even break it. Therefore, synchronization of hyperchaotic systems in the presence of unknown parameters is essential.

In this Letter, a novel parameters identification and synchronization method is proposed for Rössler hyperchaotic system with all the system parameters unknown based upon adaptive control. By this method, one can achieve hyperchaotic synchronization and identify the unknown parameters simultaneously.

2. Problem formulation

Hyperchaotic systems are defined as having more than one positive Lyapunov exponent, that is, they are unstable in more than one direction. We consider the master hyperchaotic system in the form of

$$\dot{x}_d = f(x_d, \alpha), \tag{1}$$

where $x_d \in R^n$ is the state vector of the system, $\alpha \in R^m$ is the parameter vector. $f \in C^1(R^n \times R^m, R^n)$ is nonlinear function. The slave system is given by

$$\dot{x}_r = f(x_r, \alpha_1), \tag{2}$$

which has the same structure as the master system but the parameter vector $\alpha_1 \in R^m$ is completely unknown, or uncertain. In practical situation, the output signals of the master system (1) can be received by the slave system (2), but the parameter vector of the master system (1) may not be known a priori, even waits for identifying. Therefore, the goal of control is to design and implement an appropriate controller U for the slave system and a parameter adaptive estimation law of α_1, such that the controlled slave system

$$\dot{x}_r = f(x_r, \alpha_1) + U \tag{3}$$

could be synchronous with the master system (1), and the parameter α_1 approaches to α, i.e.,

$$\lim_{t\to+\infty} (x_r - x_d) = 0,$$
$$\lim_{t\to+\infty} (\alpha_1 - \alpha) = 0.$$

Since the description of the general case is rather messy and uneasy, at least in notations, to facilitate the description and discussion, we use Rössler hyperchaotic system as an example.

3. Identification and synchronization of uncertain Rössler hyperchaotic system

Rössler hyperchaotic system was provided by Rössler in describing dynamics of some hypothetical chemical reaction and is a first example of hyperchaotic system with two positive Lyapunov exponents. The nonlinear differential equations that describe Rössler hyperchaotic system are

$$\begin{cases} \dot{x} = -y - z, \\ \dot{y} = x + ay + w, \\ \dot{z} = b + xz, \\ \dot{w} = -cz + dw, \end{cases} \tag{4}$$

which has a hyperchaotic attractor when $a = 0.25$, $b = 3$, $c = 0.5$, $d = 0.05$ [12]. For convenience, we denoted the master Rössler hyperchaotic system as

$$\dot{\mathrm{x}}_m = R(\mathrm{x}_m), \tag{5}$$

which stands for

$$\begin{cases} \dot{x}_m = -y_m - z_m, \\ \dot{y}_m = x_m + ay_m + w_m, \\ \dot{z}_m = b + x_m z_m, \\ \dot{w}_m = -cz_m + dw_m. \end{cases}$$

The slave Rössler hyperchaotic system has the same structure as the master system but the system parameters are unknown, we denote it as

$$\dot{\mathrm{x}}_s = \hat{R}(\mathrm{x}_s), \tag{6}$$

which stands for

$$\begin{cases} \dot{x}_s = -y_s - z_s, \\ \dot{y}_s = x_s + \hat{a} y_s + w_s, \\ \dot{z}_s = \hat{b} + x_s z_s, \\ \dot{w}_s = -\hat{c} z_s + \hat{d} w_s. \end{cases}$$

In this section, a systematic design process of synchronization and identification of uncertain Rössler hyperchaotic system is provided.

3.1. Nonadaptive design based upon nonlinear observer

In the case that the state signals and system parameters are available, there are many synchronization method. We use the method based upon nonlinear observer to synchronize Rössler hyperchaotic system. Informally, an observer is a dynamical system designed to be driven by the output of another dynamical system. More precisely, for a given dynamical system

$$\dot{x}_d = f(x_d) \tag{7}$$

with output $o = s(x_d) \in R^k$, the dynamical system

$$\dot{x}_r = f(x_r) + g\big(o - s(x_r)\big) \tag{8}$$

is said to be a nonlinear observer of system (7), if x_r converges to x_d as $t \to \infty$, where $g : R^k \to R^n$ is a suitable chosen function. Moreover, system (8) is said to be a global observer of system (7) if x_r converges to x_d as $t \to \infty$ for any initial conditions $x_r(0), x_d(0)$ [13,14]. Obviously, system (8) is a (global) observer of system (7) if the error system

$$\begin{aligned} \dot{e} &= f(x_r) - f(x_d) + g\big(s(x_d) - s(x_r)\big) \\ &= f(x_d + e) - f(x_d) + g\big(s(x_d) - s(x_d + e)\big) \\ &= h(e, t) \end{aligned} \tag{9}$$

has a (global) asymptotically stable equilibrium point $e = 0$, where $e = x_r - x_d$. It is known that control theory offers us no general method to choose a output $o = s(x_d) \in R^k$ and a function $g : R^k \to R^n$ such that the nonlinear and nonautonomous system (9) has a (global) asymptotically stable equilibrium point $e = 0$. In addition, it is worth noting that, the output signal $o = s(x_d)$ is an artificial output of the system which can be properly designed to feed the nonlinear map $g : R^k \to R^n$. Since the adoption of a scalar signal is a suitable feature for practical applications, it is better to assume that $o = s(x_d) \in R$. Owing to the simple structure of the Rössler hyperchaotic system, we can design the observer with a scalar output $o = s(x_d) \in R$.

Theorem 1. *For Rössler hyperchaotic system* (5) *with system parameters satisfying the condition* $-d^2c + dca + dc^2a - c - 2c^2 - c^3 \neq 0$, *if* $o = s(x_m) = k_1x_m + k_2y_m + k_3z_m + k_4w_m - x_mz_m$ *and* $g(o - s(\mathrm{x}_s)) = \alpha[s(\mathrm{x}_m) - s(\mathrm{x}_s)]$, *then the dynamical system*

$$\dot{\mathrm{x}}_s = R(\mathrm{x}_s) + g\big(o - s(\mathrm{x}_s)\big) \tag{10}$$

is a global observer of Rössler hyperchaotic system (5). *Where* $\alpha = (0, 0, -1, 0)^\top$ *and* $\mathrm{k} = (k_1, k_2, k_3, k_4)$ *is the feedback gain vector properly selected.*

Proof. With the selection of output $o = s(\mathrm{x}_m)$ and the map g, the error system of (10) and (5) becomes linear and time-invariant, and can be expressed as

$$\dot{\mathrm{e}} = A\mathrm{e} - \alpha \mathrm{k}\mathrm{e}, \tag{11}$$

where $\mathrm{e} = (e_1, e_2, e_3, e_4) = (x_s - x_m, y_s - y_m, z_s - z_m, w_s - w_m)^\top$ and

$$A = \begin{pmatrix} 0 & -1 & -1 & 0 \\ 1 & a & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -c & d \end{pmatrix}.$$

System (11) could be written in the form

$$\dot{\mathrm{e}} = A\mathrm{e} + \alpha u, \tag{12}$$

where $u = -\mathrm{ke}$ plays the role of a state feedback. On the other hand, the matrix

$$\big[\alpha, A\alpha, A^2\alpha, A^3\alpha\big] = \begin{pmatrix} 0 & 1 & 0 & -1-c \\ 0 & 0 & 1+c & a(1+c)+cd \\ -1 & 0 & 0 & 0 \\ 0 & c & cd & cd^2 \end{pmatrix}$$

is full rank because $\det[\alpha, A\alpha, A^2\alpha, A^3\alpha] = -d^2c + dca + dc^2a - c - 2c^2 - c^3 \neq 0$, so the single-input dynamic system (12) is controllable, i.e., all the eigenvalues are controllable. Thus, we can select an appropriate feedback gain vector k such that system (11) is globally asymptotically stable at zero, that is dynamical system (10) is a global observer of Rössler hyperchaotic system (5). We prove the theorem as desired. □

For instance, if the system parameters $a = 0.25$, $b = 3$, $c = 0.5$, $d = 0.05$, with the selection of feedback gain vector $\mathrm{k} = (-3.3712, -0.9561, 4.300,$

-5.8126), the eigenvalues of system (11) are $-1.0647\pm0.0657\mathrm{i}, -0.9353\pm0.0636\mathrm{i}$. So, from Theorem 1, we choose $U = g(o - s(\mathrm{x}_s)$ as above, the controlled Rössler hyperchaotic system

$$\dot{\mathrm{x}}_s = R(\mathrm{x}_s) + U$$

is synchronous with Rössler hyperchaotic system (5).

3.2. Adaptive synchronization and identification

The controller designed in Section 3.1 requires the knowledge of the system parameters. However, in many practical situations, it is difficult to exactly determine the values of the system parameters in advance. The effects of these uncertainties will destroy the synchronization and even break it. Therefore, adaptive synchronization of the chaotic system in the presence of unknown parameter is essential. To this end, we have the following theorem.

Theorem 2. *With the controller* $U = (u_1, u_2, u_3, u_4) = -o + s(\mathrm{x}_s)$, *where* $o = s(\mathrm{x}_m) = (-x_m, -(\hat{a}+1)y_m, x_m - z_m + \hat{c}w_m - x_m z_m, -y_m - (\hat{d}+1)w_m)$, *and the parameters adaptive laws of* $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$ *as below*

$$\begin{cases} \dot{\hat{a}} = -y_m e_2, \\ \dot{\hat{b}} = -e_3, \\ \dot{\hat{c}} = z_m e_4, \\ \dot{\hat{d}} = -w_m e_4 \end{cases} \tag{13}$$

then the controlled uncertain Rössler hyperchaotic system

$$\begin{cases} \dot{x}_s = -y_s - z_s + u_1, \\ \dot{y}_s = x_s + \hat{a}y_s + w_s + u_2, \\ \dot{z}_s = \hat{b} + x_s z_s + u_3, \\ \dot{w}_s = -\hat{c}z_s + \hat{d}w_s + u_4 \end{cases} \tag{14}$$

is synchronous with the master system (5) *and satisfies*

$$\lim_{t\to+\infty}(\hat{a}-a) = \lim_{t\to+\infty}(\hat{b}-b) = \lim_{t\to+\infty}(\hat{c}-c) = \lim_{t\to+\infty}(\hat{d}-d) = 0.$$

Proof. According to the drive system (5) and the controlled response system (14), we get the error dynamical system

$$\begin{cases} \dot{e}_1 = -e_1 - e_2 - e_3, \\ \dot{e}_2 = e_1 - e_2 + e_4 + (\hat{a}-a)y_m, \\ \dot{e}_3 = e_1 - e_3 + \hat{c}e_4 + \hat{b} - b, \\ \dot{e}_4 = -e_2 - \hat{c}e_3 - e_4 - (\hat{c}-c)z_m + (\hat{d}-d)w_m. \end{cases} \tag{15}$$

Consider a Lyapunov function as

$$V_1(\mathrm{e}, \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}) = V(\mathrm{e}) + \frac{1}{2}\left(\tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2\right),$$

where $V(\mathrm{e}) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2)$ and $\tilde{a} = \hat{a} - a$, $\tilde{b} = \hat{b} - b$, $\tilde{c} = \hat{c} - c$, $\tilde{d} = \hat{d} - d$. Taking the time derivative of $V_1(\mathrm{e}, \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d})$ along the trajectories of the error dynamical system (15) leads to

$$\begin{aligned} \frac{dV_1}{dt} &= -2V(\mathrm{e}) + \tilde{a}(y_m e_2 + 4\dot{\hat{a}}) + \tilde{b}(e_3 + 4\dot{\hat{b}}) \\ &\quad + \tilde{c}(-z_m e_4 + 4\dot{\hat{c}}) + \tilde{d}(w_m e_4 + 4\dot{\hat{d}}) \\ &= -2V(\mathrm{e}). \end{aligned} \tag{16}$$

Since $V_1(\mathrm{e}, \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d})$ is a positive definite function and $\frac{dV_1}{dt}$ is a negative semi-definite function, it follows that the equilibrium points $e_i = 0$ $(i = 1,2,3,4)$, $\hat{a} = a$, $\hat{b} = b$, $\hat{c} = c$, $\hat{d} = d$ of the system (13) and (15) are uniformly stable, i.e., $e_i(t) \in L_\infty$ $(i = 1,2,3,4)$, $\hat{a} \in L_\infty$, $\hat{b} \in L_\infty$, $\hat{c} \in L_\infty$, $\hat{d} \in L_\infty$. From (16) we can easily show that the square of $e_i(t)$ $(i = 1,2,3,4)$ is integrable with respect to time, i.e., $e_i(t) \in L_2$ $(i = 1,2,3,4)$. In addition, system (15) implies $\dot{e}_i(t) \in L_\infty$ $(i = 1,2,3,4)$ for any initial conditions, which in turn implies $e_i(t) \to 0$ $(i = 1,2,3,4)$ as $t \to +\infty$ [15]. Furthermore, from the error system, one can easily see that $\ddot{e}_i(t)$ is bounded $(i = 1,2,3,4)$, thus by basic calculus method one can arrive at the result $\lim_{t\to+\infty} e_i(t) = 0$ $(i = 1,2,3,4)$, which together with the above result implies the conclusion of Theorem 2. The proof is complete. □

It should be noted that unlike most existing adaptive synchronization methods, our adaptive synchronization method cannot only achieve synchronization but also identify the system parameters as well.

4. Numerical simulation

In order to verify the effectiveness of the proposed method, let the master signals are from Rössler hy-

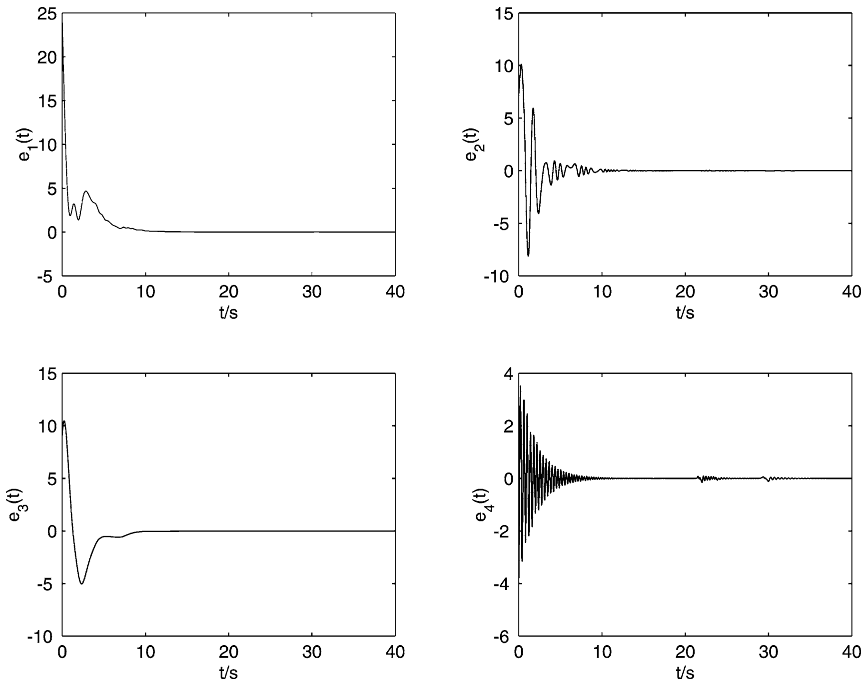

Fig. 1. Graphs of synchronization errors varying with time. $e_1(t) = x_s(t) - x_m(t)$, $e_2(t) = y_s(t) - y_m(t)$, $e_3(t) = z_s(t) - z_m(t)$, $e_4(t) = w_s(t) - w_m(t)$.

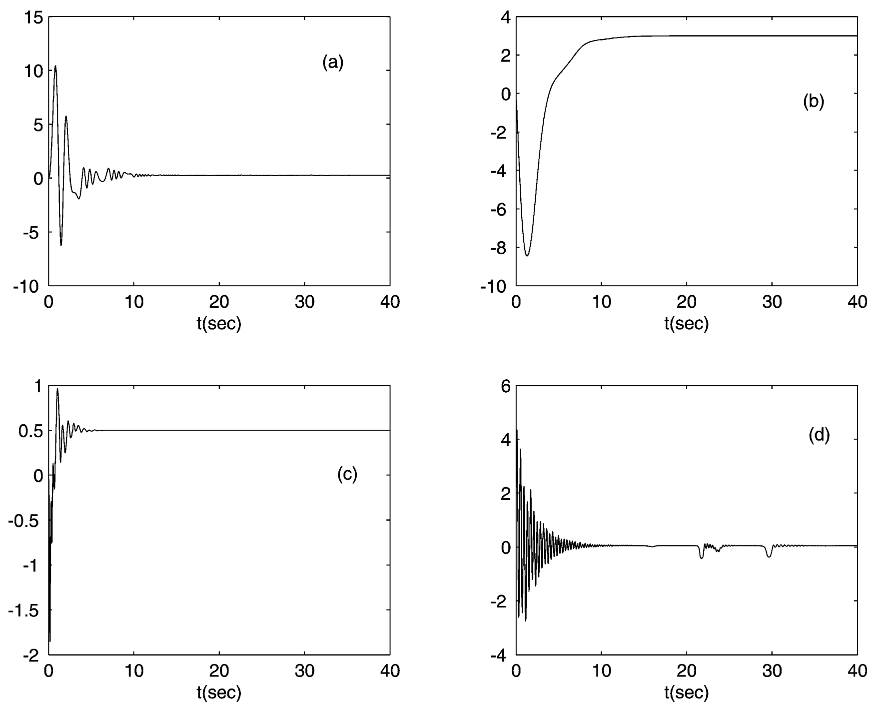

Fig. 2. Graphs of parameters identification results. (a): $\hat{a}(t)$, (b): $\hat{b}(t)$, (c): $\hat{c}(t)$, (d): $\hat{d}(t)$.

perchaotic system (5) with system parameters $a = 0.25$, $b = 3$, $c = 0.5$, $d = 0.05$ and initial condition $(-20, 0, 0, 15)$. Suppose initial condition of the controlled Rössler hyperchaotic system (14) is $(5.0, 7.0, 9.0, 11.0)$ and the unknown parameters have zero initial conditions. Numerical simulation shows that parameters identification and adaptive synchronization are achieved successfully. Figs. 1 and 2 display the results.

5. Conclusion

In this Letter, we introduce an adaptive synchronization and parameters identification method for Rössler hyperchaotic system with all the system parameters unknown. With this method one can achieve synchronization and parameters identification simultaneously. Lyapunov direct method is used to prove the stability of the method. Numerical experiment shows the effectiveness of the proposed method.

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