International Journal of Bifurcation and Chaos, Vol. 12, No. 4 (2002) 855–858

# THE COMPOUND STRUCTURE OF CHEN'S ATTRACTOR

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Received June 7, 2001; Revised June 28, 2001

This Letter reports the finding of the compound structure of Chen's attractor, which is obtained by merging together two simple attractors after performing a mirror operation. Also, the forming procedure of Chen's attractor is explored.

*Keywords*: Chaos; Chen's attractor; compound structure.

## 1. Introduction

Chaos has been extensively studied within the scientific, engineering and mathematical communities during the last three decades. Chaos has been found to be useful, or has great potential to be useful, in many disciplines, for example, information processing, collapse prevention of power systems, high-performance circuits and devices, thorough liquid mixing with low power consumption, and biomedical engineering applications in the research of human brain and heart [Chen, 2001; Chen & Dong, 1998]. All these strongly motivate the current research on the new task of generating chaos purposely, or enhancing existing chaos [Wang & Chen, 2000].

In 1963, Lorenz found the first chaotic attractor [Sparrow, 1982], which is merely a third-order autonomous system with only two multiplication terms. In 1999, Chen found another chaotic attractor, which is the *dual* to the Lorenz system and similarly has a simple structure but displays even more sophisticated dynamical behaviors [Chen & Ueta, 1999; Ueta & Chen, 2000]. Here, the duality is based on a classification condition formulated by Vaněček and Čelikovský [1996]: For the linear part

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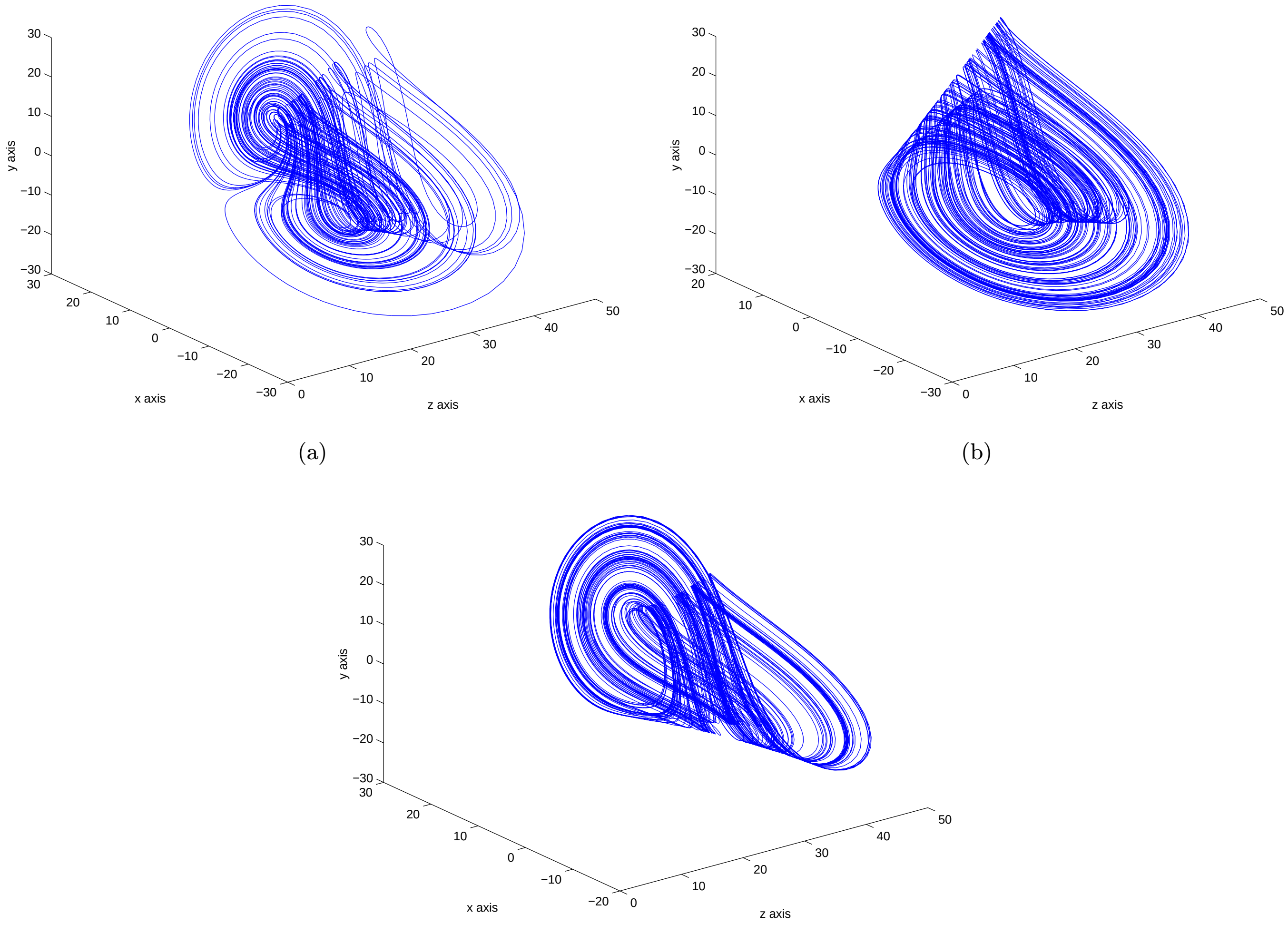
of the system, $A = [a_{ij}]_{3\times 3}$, the Lorenz system satisfies $a_{12}a_{21} > 0$ while the Chen system satisfies $a_{12}a_{21} < 0$. Very recently, Lü *et al.* found a new chaotic system [Lü & Chen, 2002; Lü *et al.*, 2002a], which satisfies the condition $a_{12}a_{21} = 0$ and represents the transition between the Lorenz and Chen attractors.

It is notable that Elwakil and Kennedy [2001] and Özoğuz *et al.* [2001] produced a modified Lorenz system, in which one multiplier is replaced by an absolute-valued function $f(x) = |x|$ while the other multiplier is replaced with a bipolar switching constant. In their design, an additional control parameter is used to verify the compound nature of the resulting butterfly-shaped attractor. That is, with the control gains, it is possible to confine the chaotic dynamics to one or another of the butterfly wings of the attractor, forming two simple attractors, which, when merged together, form the whole butterfly. We have given a rather detailed analysis of this modified Lorenz system in [Lü *et al.*, 2002b]. It is therefore interesting to ask if Chen's attractor also has a compound structure of two simple attractors. This Letter provides a positive answer to this question.

## 2. Compound Structure of Chen's Attractor

The nonlinear differential equations that describe Chen's attractor are:

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = (c - a)x - xz + cy \\ \dot{z} = xy - bz \end{cases} \tag{1}$$



(a)

(b)

(c)

Fig. 1. Chen's attractor and its forming attractors. (a) $m = 0$, (b) $m = -20$, (c) $m = 20$.

which has a chaotic attractor as shown in Fig. 1(a) when $a = 35$, $b = 3$, $c = 28$.

In order to investigate the compound structure of Chen's attractor, we add a constant gain to the second equation, that is,

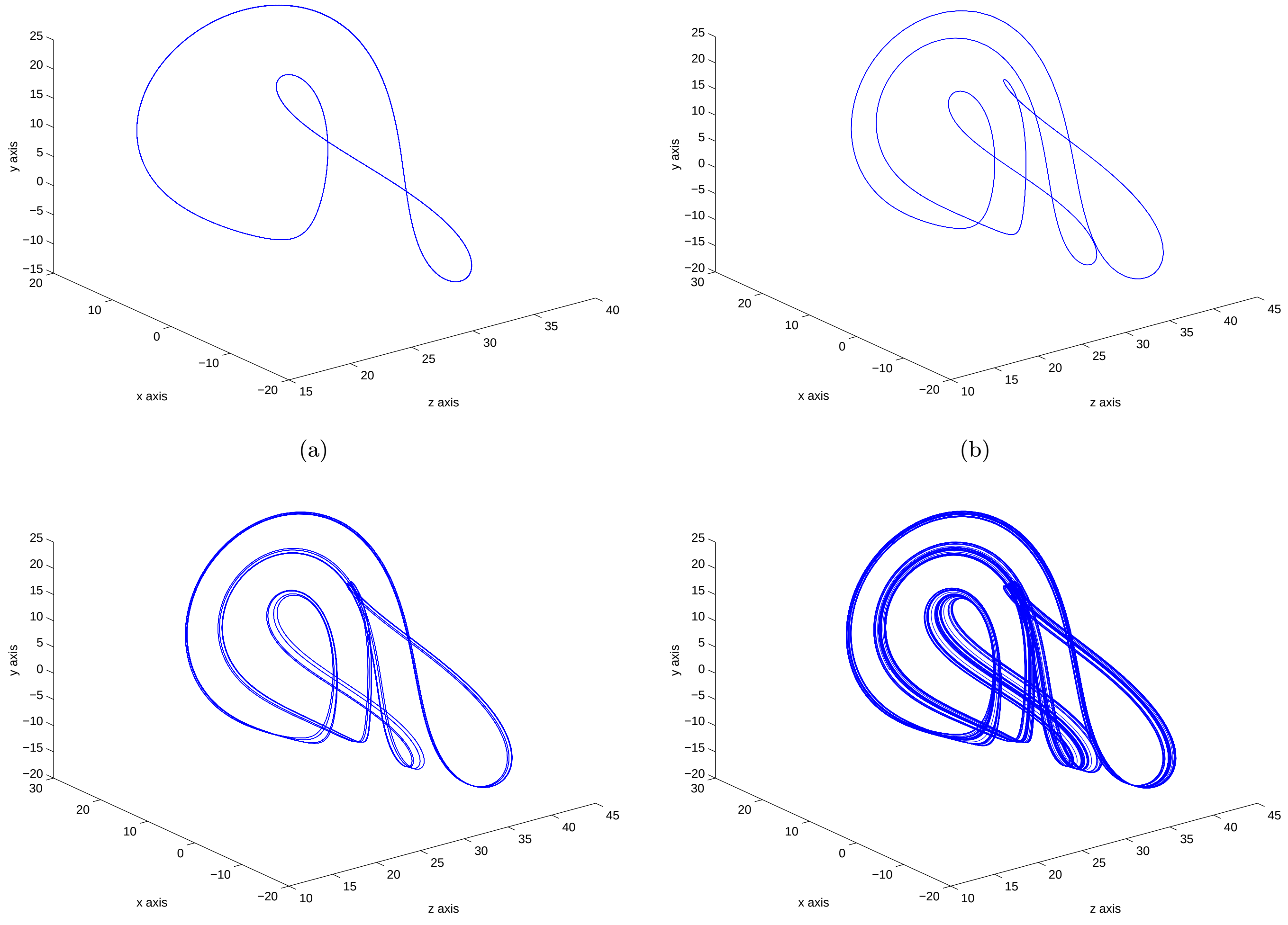
$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = (c - a)x - xz + cy + m \\ \dot{z} = xy - bz \end{cases} \tag{2}$$

when $m = -20$, we can confine Chen's attractor to the left-attractor shown in Fig. 1(b); while $m = 20$, we can get the mirror image of the left-attractor, i.e. the right-attractor shown in Fig. 1(c). This means that Chen's attractor is a compound structure obtained by merging together two simple attractors after performing one mirror operation.

## 3. Forming Procedure of Chen's Attractor

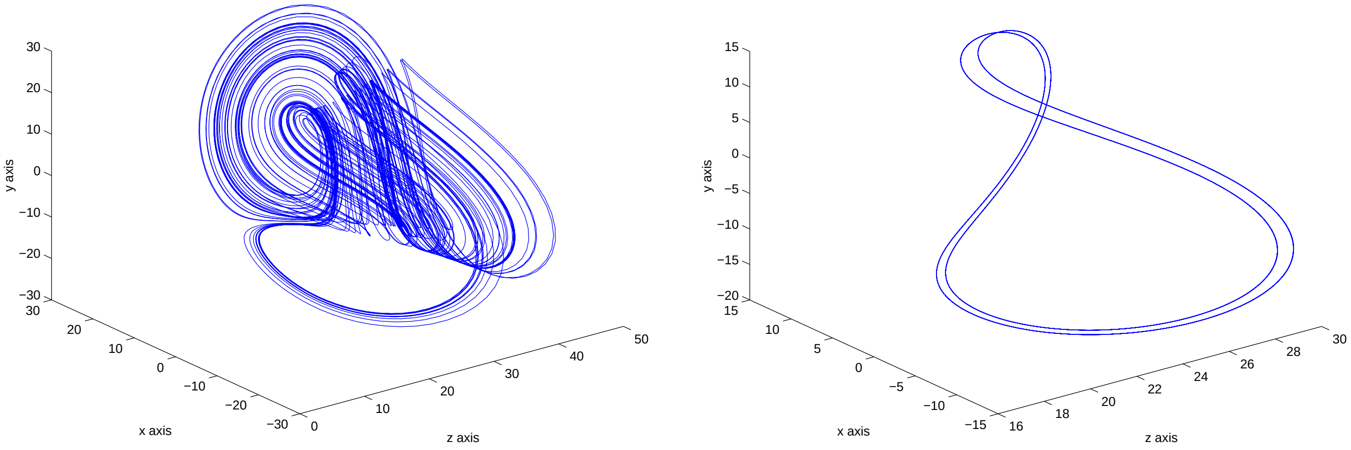
In this section, we investigate the dynamical behaviors of the controlled Chen's system (2) with the variations of parameter $m$ by both theoretical analysis and numerical computation. Also, we further explain how the two simple attractors merge together to form Chen's attractor. We only list the domains of parameter $m$ in the following.

When $|m| \geq 828$, the system (2) converges to a point; when $25.4 \leq |m| \leq 827$, the system (2) has a limit cycle [see Fig. 2(a)]; when $23 < |m| \leq 25.3$, there are period-doubling bifurcations [see Figs. 2(b)–2(d)]; when $19.3 \leq |m| \leq 23$, the attractor is a left-attractor (or right-attractor) [see Figs. 1(b) and 1(c)]; when $11 \leq |m| < 19.2$, the



(a) (b) (c) (d)

Fig. 2. Phase portraits of system (2). (a) $m = 30$, (b) $m = 24$, (c) $m = 23.3$, (d) $m = 23.1$, (e) $m = 19$, (f) $m = 3$.



(e) (f)

Fig. 2. (*Continued*)

system (2) has a partial attractor, which is bounded [see Fig. 2(e)]; when $|m| \leq 10$, the system (2) has a complete attractor; when $2 \leq |m| \leq 3.879$, there is a periodic window [see Fig. 2(f)].

According to Figs. 2(a)–2(f), we can see that the Chen attractor is a compound structure of two simple attractors and each of the simple attractors is derived from some simple limit cycle [see Fig. 2(a)].

## Acknowledgments

This work was supported by the National Natural Science Foundation of People's Republic of China (No. 10171099) and Hong Kong RGC (CERG 9040579 and SRG 7001328).

## 4. Conclusions

We have reported the finding of the compound structure of Chen's attractor, and investigated the forming of a dynamical procedure. There are abundant and complex dynamical behaviors still unknown about this interesting compound structure, which may contribute to a better understanding of the essence of Chen's system and its generalization. More detailed analysis is in order.

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