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A NEW CHAOTIC ATTRACTOR COINED

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This letter reports the finding of a new chaotic attractor in a simple three-dimensional autonomous system, which connects the Lorenz attractor and Chen's attractor and represents the transition from one to the other.

Keywords: Chaos; Chen's attractor; Lorenz attractor.

1. Introduction

In 1963, Lorenz found the first chaotic attractor in a three-dimensional autonomous system [Sparrow, 1982]:

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = cx - xz - y \\ \dot{z} = xy - bz\,, \end{cases} \tag{1}$$

which is chaotic when $a = 10$, $b = 8/3$, $c = 28$ (see Fig. 1).

In 1999, Chen found another chaotic attractor, also in a simple three-dimensional autonomous system, which nevertheless is not topologically equivalent to the Lorenz's [Chen & Ueta, 1999; Ueta & Chen, 2000]:

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = (c-a)x - xz + cy \\ \dot{z} = xy - bz\,, \end{cases} \tag{2}$$

which is chaotic when $a = 35$, $b = 3$, $c = 28$ (see Fig. 2).

It is notable that Vanecek and Celikovský [1996] classified a generalized Lorenz system family

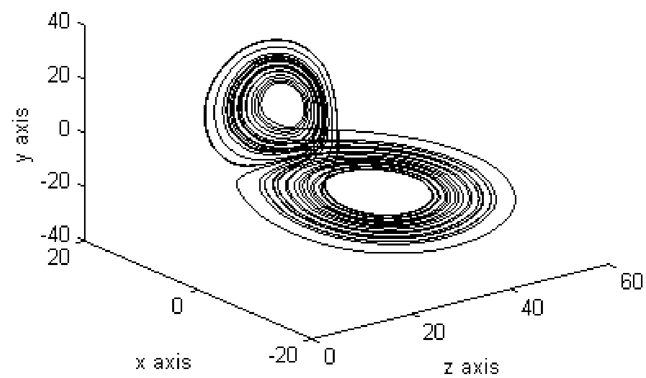


Fig. 1. Lorenz chaotic attractor. $a = 10$, $b = 8/3$, $c = 28$.

by a condition on its linear part $A = [a_{ij}]$:

$$a_{12}a_{21} > 0\,,$$

which includes the familiar Lorenz system as a special case (see their book for more details), while Chen's system satisfies

$$a_{12}a_{21} < 0\,.$$

Hence, Chen's system does not belong to this generalized Lorenz system family. In fact, Chen's

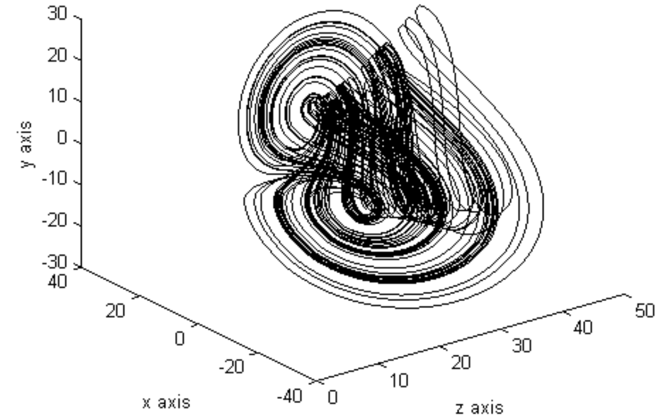


Fig. 2. Chen's chaotic attractor. $a = 35$, $b = 3$, $c = 28$.

system belongs to another canonical family of chaotic systems [Celikovský & Chen, 2002]. It is therefore interesting to ask if there is a chaotic system in between the Lorenz system and Chen's system that satisfies the condition

$$a_{12}a_{21} = 0\,.$$

This Letter provides a positive answer to this question.

2. The New Chaotic Attractor

The new chaotic attractor is generated by the following simple three-dimensional autonomous system:

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = -xz + cy \\ \dot{z} = xy - bz\,. \end{cases} \tag{3}$$

When the parameters $a = 36$ and $b = 3$ are fixed while parameter c varies, one can observe that the attractor generated by this system is similar to the Lorenz attractor for $12.7 < c < 17.0$,

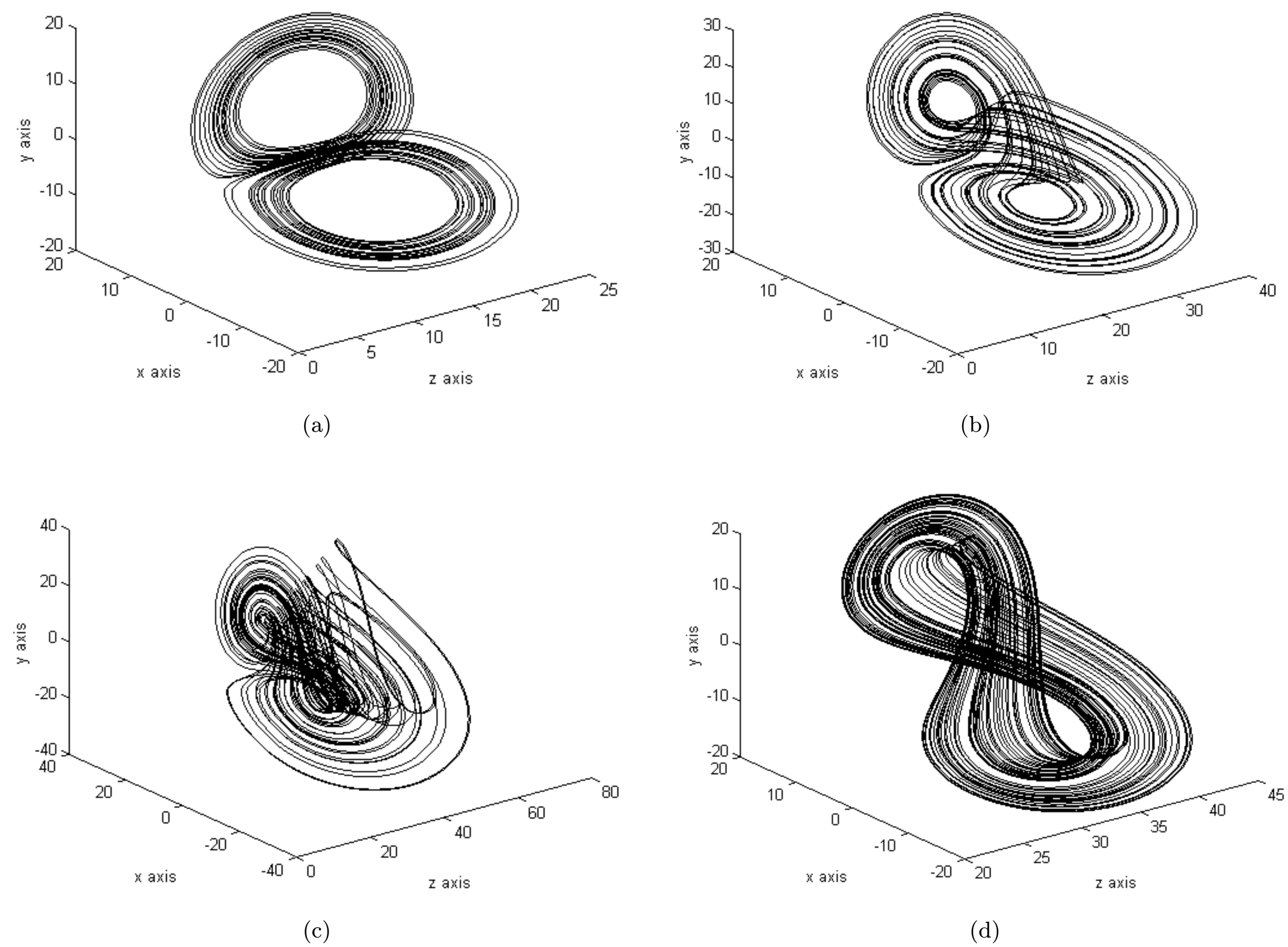
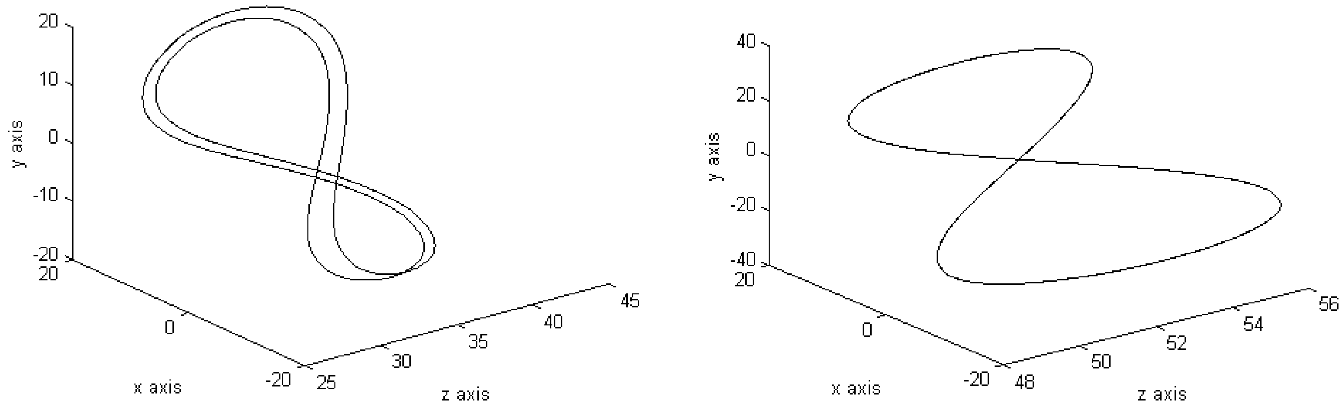


Fig. 3. The new chaotic attractor. (a) $c = 13.0$, (b) $c = 20.0$, (c) $c = 28.0$, (d) $c = 28.7$.



(a)

(b)

Fig. 4. Periodic orbits of the new chaotic attractor. (a) $c = 29.2$, (b) $c = 34.0$.

has a transitory shape when $18.0 < c < 22.0$, and then becomes similar to Chen's attractor when $23.0 < c < 28.5$, as depicted in Figs. 3(a)–3(c), respectively.

It is noticeable that this system has a rather wide range of parameter values in which the system displaces a chaotic attractor of different shapes. Numerical experience shows that observable chaos exists in the following ranges at the least:

$$\begin{gathered} 12.7 < c < 17.0\,, \quad 18.0 < c < 22.0\,, \\ 23.0 < c < 28.5\,, \quad 28.6 < c < 29.0\,, \\ 29.334 < c < 29.345\,. \end{gathered}$$

When $c < 12.6$, the system converges to a fixed point. And when $29.1 < c < 29.334$ and $29.345 < c < 35.0$, there is at least one periodic orbit in the system, as shown in Fig. 4. In fact, these two intervals correspond to the three periodic solution intervals of the Lorenz system [Sparrow, 1982].

3. Concluding Remarks

Obviously, the new system is not diffeomorphic with the Lorenz and Chen's systems since the eigenvalue structures of their corresponding equilibrium points are not equivalent. It is straightforward but somewhat tedious to verify that there is no nonsingular coordinate transforms that can convert one system to the other. Therefore, they are all not topologically equivalent. More detailed analysis on the new chaotic system will be reported in a forthcoming paper.

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