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Controlling Chen's chaotic attractor using backstepping design based on parameters identification

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Abstract

In this Letter backstepping design is proposed for controlling Chen's chaotic attractor based on parameters identification. The observer is applied to the identification of the unknown parameters of Chen's chaotic system. And on this basis, an efficient backstepping design is developed for controlling Chen's chaotic system. Finally, numerical simulation are provided to show the effectiveness and feasibility of the developed controller design method.

1. Introduction

Dynamic chaos is a very interesting nonlinear effect which has been intensively studied during the last two decades. The effect is very common, it has been detected in a large number of dynamic systems of various physical nature. However, this effect is usually undesirable in practice, and it restricts the operating range of many electronic and mechanic devices. Recently, controlling this kind of complex dynamical systems has attracted a great deal of attention within the engineering society.

Chaos control, in a broader sense, can be divided into two categories [1]: one is to suppress the chaotic dynamical behavior and the other is to generate or enhance chaos in nonlinear systems (known as chaotincation or anti-control of chaos [2,3]). Nowadays, different techniques and methods [1–17] have been proposed to achieve chaos control. For instance, OGY method [4], differential geometric method [5], linear state space feedback [6], inverse optimal control [7] and output feedback control [8], among many others [1].

However, for some systems which have unknown parameters, the aforementioned many methods fail. An important problem in this field is how to achieve nonlinear control of complex dynamical systems with unknown parameters. This problem concerns the identification of the unknown parameters and the approach of controlling chaos. In this Letter, the observer is applied to the identification of the unknown parameters of Chen's system. Then, an efficient backstepping design is developed for controlling Chen's chaotic system. Then suggested tool enables stabilization of chaotic motion to a steady state as well as tracking of any desired trajectory to be achieved in a systematic way. And computer simulation is also given for the purpose of illustration and verification.

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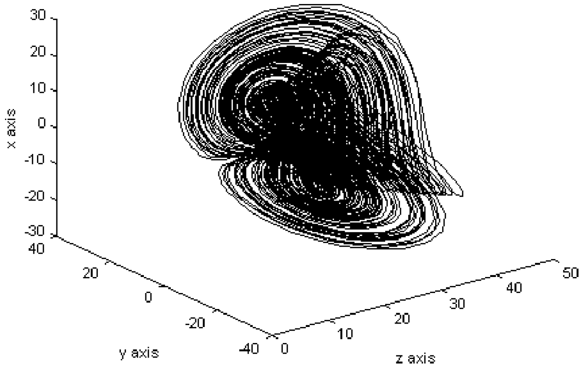

Fig. 1. Chen's chaotic attractor.

2. Chen's chaotic attractor

Recently, in the endeavor of anti-controlling chaos, Chen [2] developed a new chaotic system, the Chen's chaotic system, so called by other researchers, which was derived from the classical Lorenz system [3]. The nonlinear differential equations that describe Chen's attractor are

$$\begin{cases} \dot{x} = a(y-x), \\ \dot{y} = (c-a)x - xz + cy, \\ \dot{z} = xy - bz, \end{cases} \tag{1}$$

which has a chaotic attractor as shown in Fig. 1 when $a = 35$, $b = 3$, $c = 28$.

From a control engineering point of view, the chaotic Chen's attractor is relatively difficult to control as compared to the Loren's system and Chua's system due to the prominent three-dimensional and complex topological features of its attractor, especially its rapid change in velocity in the z-direction [8].

We are interested in developing control techniques able to drive a strange attractor with unknown parameters not only to a periodic orbit but also to a steady state. Since steady-state solutions represent the most practical operation mode in many chaotic systems such as electronic circuits [14]. It is assumed that only the parameter b is unknown. And we add a control input to the third state, so that the controlled system becomes

$$\begin{cases} \dot{x} = a(y-x), \\ \dot{y} = (c-a)x - xz + cy, \\ \dot{z} = xy - bz + u. \end{cases} \tag{2}$$

3. The identification of the unknown parameters

In this section, we will provide a observer that can identify the unknown parameter b of Chen's chaotic system.

Since the dynamic information about parameter b is hard to be known and b is a constant of system (1). We can assume that

$$\dot{b} = 0. \tag{3}$$

For unknown parameter b, it can act as state variable. Then system (1) can be augmented by above Eq. (3). And we imagine that all scalar variable x, y, z can be measured as a system output. In the following, we will design a observer which can identify the unknown parameter b.

According to system (1), we have

$$bz = xy - \dot{z},$$

and the observer is

$$\dot{\hat{b}} = -G(z)z\hat{b} + G(z)(xy - \dot{z}), \tag{4}$$

where $G(z)$ is a gain function. Letting

$$e = b - \hat{b},$$

then

$$\dot{e}(t) = \dot{b} - \dot{\hat{b}} = -G(z)ze(t).$$

We can choose gain function $G(z)$ that makes the system

$$\dot{e}(t) + G(z)ze(t) = 0$$

exponential asymptotical stability for all z. That is, when $t \to \infty$, $\hat{b}(t)$ converges to $b(t)$ with exponent rate. Obviously, the gain function $G(z)$ can be chosen as k/z. Then we get

$$\dot{e}(t) + ke(t) = 0,$$

where $k > 0$, and it determines the convergence rate. But $\dot{z}$ is hard to be observed, and observer (4) is not applicable value. Hence, we introduce a auxiliary variable

$$\delta = \hat{b} + Q(z), \tag{5}$$

where $Q(z)$ is a design function and satisfies the following condition:

$$G(z) = \frac{dQ(z)}{dz}.$$

According to (5), we can get

$$\dot{\delta} = \dot{\hat{b}} + \frac{dQ(z)}{dz}z$$
$$= -G(z)z\big(\delta - Q(z)\big) + G(z)xy$$
$$= -G(z)z\delta + G(z)\big(zQ(z) + xy\big) \tag{6}$$

and

$$\hat{b} = \delta - Q(z). \tag{7}$$

So, if the design function $Q(z)$ make

$$\dot{e}(t) + \frac{dQ(z)}{dz}ze(t) = 0$$

exponential asymptotical stability, then, when $t \to \infty$, $\hat{b}(t)$ converges to $b(t)$ with exponent rate. And observer (6) and (7) is the identification observer of the unknown parameter b of system (1), where the $G(z)$ is a gain function and $Q(z)$ a design function, satisfying $dQ(z)/dz = G(z)$.

Note that observer (6) and (7) only rely on the third equation of system (1). That is, when the structure of the first and second equations of system (1) or the parameters a and b are altered, the results of identification are not influenced. So, the observer has strong robustness. Letting $Q(z) = k\ln z$, then the observer becomes

$$\begin{cases} \dot{\delta} = -k\delta - k^2\ln z + \frac{kxy}{z}, \\ \hat{b} = \delta - k\ln z. \end{cases} \tag{8}$$

4. Controlling Chen's system using backstepping design

Backstepping design is a systematic Lyapunov-based control technique, which can be applied to strict-feedback systems, pure-feedback systems and block-strict-feedback systems [14]. At first, we assume that the parameter b of controlled system (2) has been identified, and $b = \hat{b}$. Then the objective is to find a control law u for stabilizing the state of system (2) at the origin. Starting from the first equation, a stabilizing function $\alpha_1(x)$ has to be designed for the virtual control y in order to make the derivative of $V_1(x) = x^2/2$, that is,

$$\dot{V}_1 = -ax^2 + axy,$$

negative definite when $y = \alpha_1(x)$. By choosing $\alpha_1(x) = 0$ and defining the error variable $\bar{y}$ as

$$\bar{y} = y - \alpha_1(x), \tag{9}$$

the following $(x, \bar{y})$-subsystem is obtained:

$$\begin{cases} \dot{x} = -ax + a\bar{y}, \\ \dot{\bar{y}} = (c-a)x - xz + c\bar{y}, \end{cases}$$

for which a candidate Lyapunov function is $V_2(x, \bar{y}) = V_1(x) + (1/2)\bar{y}^2$. Since its time derivative

$$\dot{V}_2 = -ax^2 + \bar{y}(cx - xz + c\bar{y})$$

becomes negative definite by choosing the virtual control Z as

$$z = \alpha_2(x, \bar{y}) = c + (c+1)\frac{\bar{y}}{x},$$

the deviation of z from the stabilizing function α_2,

$$\bar{z} = z - \left[c + (c+1)\frac{\bar{y}}{x}\right], \tag{10}$$

gives the following system in the $(x, \bar{y}, \bar{z})$ coordinates:

$$\begin{cases} \dot{x} = -ax + a\bar{y}, \\ \dot{\bar{y}} = (c-a)x - x(z + \alpha_2) + c\bar{y}, \\ \dot{\bar{z}} = x\bar{y} - \hat{b}(z + \alpha_2) - \frac{c+1}{x^2}[(c-a)x^2 - x^2(z+\alpha_2) \\ \qquad + (c+a)x\bar{y} - \bar{a}^2] + u. \end{cases}$$

By iterating the previous steps, the derivative of $V_3(x, \bar{y}, \bar{z}) = V_2 + (1/2)\bar{z}^2$, that is,

$$\dot{V}_3 = -ax^2 - \bar{y}^2 + \bar{z}\bigg[-\hat{b}\bar{z} + u - (c+1)(c-a) - \hat{b}c$$
$$+ (c+1)z - (c+1)\big(a + c + \hat{b}\big)\frac{y}{x}$$
$$+ a(c+1)\frac{y^2}{x^2}\bigg],$$

becomes negative definite by choosing the input

$$u = \hat{b}c + (c+1)(c-a) + (c+1)\big(a + c + b\hat{b}\big)\frac{y}{x}$$
$$- a(c+1)\frac{y^2}{x^2} - (c+1)z, \tag{11}$$

which proves that the origin has been stabilized in the $(x, \bar{y}, \bar{z})$ coordinates. According to (9) and (10), the origin in the (x, y, z) coordinates has the same properties.

According to (8), we can get the control law of the controlled system (2) with unknown parameter b:

$$\begin{cases} u = \hat{b}c + (c+1)(c-a) + (c+1)(a+c+b\hat{b})\frac{y}{x} \\ \qquad - a(c+1)\frac{y^2}{x^2} - (c+1)z, \\ \dot{\delta} = -k\delta - k^2 \ln z + \frac{kxy}{z}, \\ \hat{b} = \delta - k \ln z, \end{cases} \tag{12}$$

where $k > 0$, and it is a control constant.

5. Tracking any desired trajectory

This section is to find a control law u so that a scalar output $y(t)$ tracks any desired trajectory $r(t)$, including stable or unstable limit cycles as well as chaotic trajectories.

Let $y(t) = y$ be the output and $\bar{y}$ be the deviation of y from the target, i.e., $\bar{y} = y - r(t)$. Given $V_2 = (1/2)\bar{y}^2$, its time derivative

$$\dot{V}_2 = \bar{y}\big[(c-a)x - xz + c\bar{y} + cr(t) - \dot{r}(t)\big]$$

becomes negative by choosing the virtual control z as

$$z = \alpha_2 = \frac{cr - \dot{r} + (c+1)\bar{y}}{x} + (c-a).$$

Again, given $V_3 = V_2 + (1/2)\bar{z}^2$, where $\bar{z} = z - \alpha_2$ is the deviation of the virtual control from the stabilizing function, the time derivative

$$\begin{aligned} \dot{V}_3 &= -\bar{y}^2 \\ &+ \bar{z}\bigg[-\hat{b}\bar{z} + u - (c-a)\big(c+\hat{b}+1\big) + rx + (c+1)z \\ &+ \frac{(a+\hat{b})r + (a+\hat{b}+1)\dot{r} + \ddot{r} - (c+1)(a+\hat{b}+c)y}{x} \\ &+ a\frac{(c+1)y^2 - (r+\dot{r}y)}{x^2}\bigg] \end{aligned}$$

is negative by choosing the input

$$\begin{aligned} u &= (c-a)\big(c+\hat{b}+1\big) - rx - (c+1)z \\ &- \frac{(a+\hat{b})r + (a+\hat{b}+1)\dot{r} + \ddot{r} - (c+1)(a+\hat{b}+c)y}{x} \\ &- a\frac{(c+1)y^2 - (r+\dot{r}y)}{x^2}, \end{aligned} \tag{13}$$

which assures that $y(t) = y$ tracks the reference signal $r(t)$. Similar results can be obtained by choosing x or z as output.

According to (8), the control law of the controlled system (2) with unknown parameter b are

$$\begin{cases} u = (c-a)(c+\hat{b}+1) - rx - (c+1)z \\ \qquad - \frac{(a+\hat{b})r + (a+\hat{b}+1)\dot{r} + \ddot{r} - (c+1)(a+\hat{b}+c)y}{x} \\ \qquad - a\frac{(c+1)y^2 - (r+\dot{r}y)}{x^2}, \\ \dot{\delta} = -k\delta - k^2 \ln z + k\frac{xy}{z}, \\ \hat{b} = \dot{\delta} - k \ln z. \end{cases} \tag{14}$$

Notice that, although the control law (14) could appear complex, they enable systems (2) to be controlled using a single input. Different approaches can be investigated if we assume that the system can be controlled by two scalar inputs.

6. Simulation results

In order to verify the effectiveness of observer (8) and the control applicability of the proposed control laws (12) and (14), we assume $b = 2.655$ and initial conditions $x(0) = 2.0$, $y(0) = -1.5$, $z(0) = 0.5$. The fourth-order Runge–Kutta method is used to solve the systems of different equations, such as (12) and (14), with time step size equal 0.001 in all numerical simulations.

Letting $k = 0.8$, Fig. 2 shows the effectiveness of observer (8). We investigate the effectiveness of observer under the condition of without outer force and outer force $f = 3$.

With the control law (12) applied, the chaotic orbit of the system is quickly driven to its originally unstable zero equilibrium point as expected. Fig. 3 shows the time waveform of the control u switched on at $t = 50$, and the stabilization of the state variables x, y, z.

With the control law (14) applied, the scalar output $y(t)$ tracks the desired trajectory $r(t) = \sin t$ as ex-

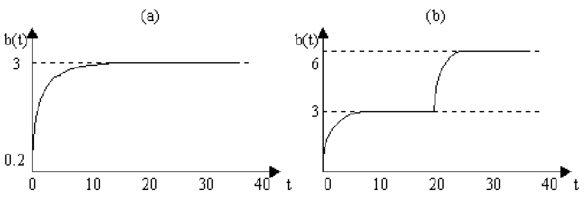

Fig. 2. The identification results of Chen's system. (a) Without outside force. (b) Under the work of outside force $f = 3$.

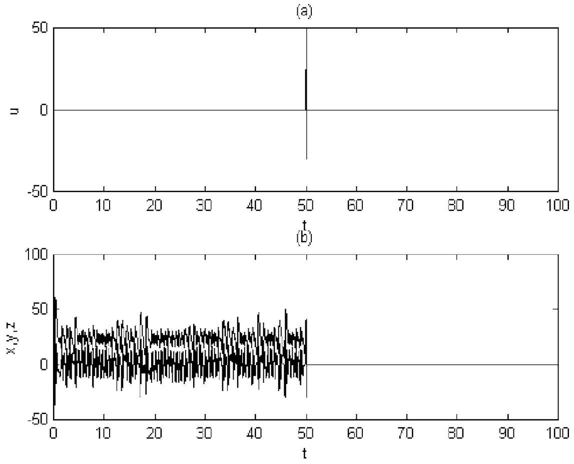

Fig. 3. Controlling Chen's system using backstepping design. (a) Time waveform of the control input u switched on at $t = 50$; (b) stabilization of the state variable x, y, z.

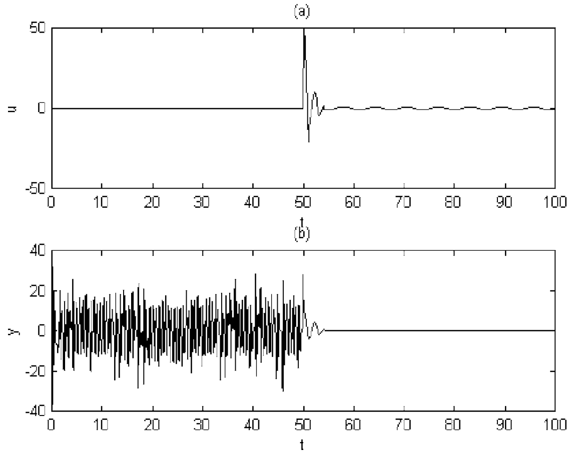

Fig. 4. Tracking $r(t) = \sin t$ using backstepping design. (a) Time waveform of the control input u switched on at $t = 50$; (b) time waveform of the output y of Chen's system.

pected. Fig. 4 shows the time waveform of the control switched on at $t = 50$, and the time waveform of the output y of the Chen's system.

7. Conclusion

In this Letter a Lyapunov-based approach, called backstepping design, has been proposed for controlling Chen's chaotic system with unknown parameter. This new and effect control law can drive a strange attractor not only to a periodic orbit but also to a steady state. And it is a systematic procedure for controlling chaotic or hyperchaotic dynamics. Especially, this method combine the identification of unknown parameters with backstepping design to control chaotic system with unknown parameters.

It should be noted that although this Letter is focused on the control of Chen's chaotic attractor with unknown parameter, we believe that the higher nonlinearity and complexity of the chaotic Chen's attractor justifies the practical application of the proposed method to some other complex dynamical systems with unknown parameters as well.

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