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CONVERGENCE ANALYSIS FOR ADAPTIVE CONTROL SYSTEMS WITH UNKNOWN ORDERS, DELAY AND COEFFICIENTS**

This paper considers identification and adaptive control for linear discrete-time stochastic systems with unknown orders, time-delay and coefficients under uncorrelated noise. Under the assumption that a lower bound for the time-delay and upper bounds for system orders are known, 1) the consistent estimates for the time-delay, system orders and coefficients are recursively given; 2) the optimal adaptive controls are designed for both tracking and the quadratic loss function and 3) the rates of convergence both of the coefficient estimates to their true values and of the loss functions to their minimums are derived.

1. INTRODUCTION

Let the *a priori* information about the plant be merely that it is linear stochastic and bounds for its time-delay and orders are available. The question is how to design a control to minimize a tracking error or a quadratic loss function and simultaneously to get consistent estimates for time-delay, orders and coefficients of the system.

In time series analysis there is an extensive literature devoted to estimating orders and coefficients of a stationary ARMA process from a non-recursive point of view, see BOX and JENKINS [1], AKAIKE [2], [3], RISSANEN [4] and HANNAN and QUINN [5]. Recently, however, RISSANEN [6] established results concerning the recursive order estimation. But in the above works, some sort of stationarity and ergodicity of the stochastic processes involved are usually assumed. Therefore, the previously mentioned results cannot directly be applied to the ARMAX process when the exogenous input is a feedback control so that the process is neither ergodic nor stationary.

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To estimate the orders of a stochastic feedback control system the first step was made by CHEN and GUO [7], [8] who introduced a new information criterion CIC for both uncorrelated noise [7] and correlated noise cases [8]. Further effort in this direction was made by HEMERLY and DAVIS [9]. In their work, for systems with uncorrelated noise, by combining the PLS (Predictive Least Squares) criterion for order estimation with an adaptive control strategy minimizing a quadratic cost and applied to multidimensional ARX systems, it is shown that the combination enables us to estimate, recursively and in a strong consistent way, both the order and the coefficients of the controlled system, while achieving asymptotically optimal cost. However, all these papers not only need some strong assumptions because of the technical problem, but also need a great deal of computation since they require a set of parallel algorithms (one for each of the possible orders of the system) for estimating system coefficients and system states appearing in construction of an optimal quadratic adaptive control.

This paper is devoted to reducing the computing quantity and the assumptions required in [7]–[9]. Parameters we want to estimate are not only system orders but also the system time-delay which is not estimated in previous works. The knowledge about time-delay is unnecessary in some cases where adaptive tracking [10] or adaptive control with quadratic cost [11] are dealt with without paying attention to parameter estimation, but it is crucial for some control problems, for example, the minimum variance control is sensitive to time-delay [12]. The recursion is also given for criteria as in [9], but the number of system coefficients we need to estimate here is much less than that estimated in [7]–[9], since we have modified the criterion CIC used in [8] and use only one algorithm for estimating system coefficients and system states appearing in the LQ adaptive control problem. In addition, conditions used in this paper have essentially been weakened in comparison with those in [7]–[9]. As main results of the paper, for stochastic systems of possible non-minimum-phase with unknown orders, time-delay and coefficients, optimal adaptive controls are derived for tracking and quadratic index, respectively; rates of convergence both of the performance index to its minimum and of the parameter estimates to their true values are also established.

For clarity of the description, this paper deals with single-input and single-output systems only. The corresponding results for multidimensional systems can be obtained similarly. The arrangement of this paper is as follows. Section 2 presents methods and criteria for estimating system orders, time-delay and coefficients. Section 3 discusses sufficient conditions guaranteeing consistency of the estimates. Section 4 designs an optimal adaptive tracking control which makes the estimated parameters strongly consistent, while Section 5 gives an optimal quadratic adaptive control which guarantees the strong consistency of the estimated parameters and the asymptotical minimality of the loss function. The convergence rates both of the coefficient estimates to their true values and of the loss functions to their minimums are also derived in Sections 4 and 5. Finally, we conclude this paper in Section 6.

2. ESTIMATION METHODS FOR TIME-DELAY, ORDERS AND COEFFICIENTS

In this section we present methods estimating the unknown time-delay, orders and coefficients of a stochastic system with uncorrelated noise.

Let the single-input and single-output systems be described by a linear stochastic difference equation

$$A(z)y_n = B(z)u_n + w_n,\ n > 0; \quad y_n = u_n = w_n = 0,\ n \leqslant 0, \tag{2.1}$$

where y_n, u_n and w_n are the output, input and noise, respectively; $A(z)$ and $B(z)$ are polynomials in shift-back operator z:

$$A(z) = 1 + a_1 z + \ldots + a_{p_0} z^{p_0}, \quad p_0 \geqslant 0, \tag{2.2}$$

$$B(z) = b_{d_0} z^{d_0} + \ldots + b_{q_0} z^{q_0},\ q_0 \geqslant d_0 \geqslant 1. \tag{2.3}$$

The coefficients a_i $(i = 1, \ldots, p_0)$, b_j $(j = d_0, \ldots, q_0)$, the time-delay d_0 and the orders (p_0, q_0) are unknown but it is assumed that a lower bound for d_0 and upper bounds for p_0, q_0 are available, i.e., integers p^*, q^* and $q^* \geqslant d^* \geqslant 1$ are given such that

$$(p_0, q_0) \in M_0 \triangleq \{(p, q)\colon 0 \leqslant p \leqslant p^*,\ d^* \leqslant q \leqslant q^*\}, \tag{2.4}$$

$$d_0 \in M_d \triangleq \{d\colon d^* \leqslant d \leqslant q^*\}. \tag{2.5}$$

We now write down methods for estimating d_0, (p_0, q_0) and a_i $(i = 1, \ldots, p_0)$, b_j $(j = d_0, \ldots, q_0)$.

Corresponding to the largest possible orders and the smallest possible time-delay we take the stochastic regressor

$$\varphi_n^* = [y_n \ldots y_{n-p^*+1}\, u_{n-d^*+1} \ldots u_{n-q^*+1}]^\tau \tag{2.6}$$

and denote unknown coefficients by

$$\theta^* = [-a_1 \ldots -a_{p^*}\, b_{d^*} \ldots b_{q^*}]^\tau, \tag{2.7}$$

where $a_i = 0$ for $i > p_0$ and $b_j = 0$ for $j < d_0$ or $j > q_0$ by definition.

Given any initial value θ_0^*, the estimate

$$\theta_n^* = [-a_{1n} \ldots -a_{p^*n}\, b_{d^*n} \ldots b_{q^*n}]^\tau \tag{2.8}$$

for θ^* is given by the least-squares method:

$$\theta_n^* = \Big(\sum_{i=0}^{n-1} \varphi_i^* \varphi_i^{*\tau} + I\Big)^{-1} \sum_{i=0}^{n-1} \varphi_i^* y_{i+1} \tag{2.9}$$

or recursively given by:

$$\theta_{n+1}^* = \theta_n^* + b_n^* P_n^* \varphi_n^* (y_{n+1} - \varphi_n^{*\tau} \theta_n^*), \tag{2.10}$$