# IDENTIFICATION OF LINEAR SYSTEMS WITHOUT ASSUMING STABILITY AND MINIMUM PHASE*

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ABSTRACT

For both discrete- and continuous-time linear input-output systems, consistent estimates are given for unknown coefficients, system orders and time-delay. The proposed methods are characterized by the fact that there is no requirement for stability, minimum phase and any other behaviour of the system and by the fact that the designed experiment is diminishing. The latter fact is important when adaptive control problem is simultaneously solved.

**Keywords: identification, parameter estimation, continuous-time system**

## I. INTRODUCTION

We consider the identification problem for unknown orders, time-delay and matrix coefficients of the linear systems which are described as

$$A(z)y_n = B(z)u_n, \ n \geqslant 0 \text{ and } y_n = 0, \ u_n = 0 \text{ for } n < 0 \tag{1}$$

in the discrete-time case, and as

$$A(s)y_t = B(s)u_t, \ t \geqslant 0 \text{ and } y_t = 0, \ u_t = 0 \text{ for } t < 0 \tag{2}$$

in the continuous-time case, where $z$ and $s$ are the shift-back operator $(zy_n = y_{n-1})$ and the integral operator $\left( sy_t = \int_0^t y_\lambda d_\lambda \right)$ respectively, $y_n$ denotes the $m$-output and $u_n$ denotes the $l$-input and $A(z)$, $B(z)$ refer to the matrix polynomials

$$A(z) = I + A_1 z + \cdots + A_{p_0} z^{p_0}, \ p_0 \geqslant 0, \tag{3}$$

$$B(z) = B_{d_0} z^{d_0} + \cdots + B_{q_0} z^{q_0}, \ q_0 \geqslant d_0 \geqslant 1, \tag{4}$$

whose orders, time-delay and coefficients are unknown, i. e., the delay $d_0$, the system orders $(p_0, q_0)$ and the coefficients

$$\theta^\tau(p_0, d_0, q_0) = [-A_1 \cdots - A_{p_0} B_{d_0} \cdots B_{q_0}] \tag{5}$$

are to be estimated.

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This problem has been treated in various areas: In the time series analysis, this is the ARMA model with $\{u_n\}$, $n\in(-\infty, \infty)$ being an unavailable martingale difference sequence and in estimating its unknown parameters one usually assumes[1-3] that the system is stable or minimum-phase or even both. In the theory of linear systems when identification problem is concerned, the system input $\{u_n\}$ has to be designed (experiment design), the orders and the time-delay of the system are often supposed to be known[4-5] and one of the conditions such as stability condition, minimum-phase condition and the condition consisting in $\|y_n\|+\|u_n\|=O(n^\alpha)$, $\alpha>0$ is assumed to be satisfied[4-5]. In adaptive control theory, the input is designed with purpose not only for identifying the system but also for controlling the system and the assumptions made on the system are almost the same as those indicated above for theory of linear systems[5-11].

In this paper, not imposing on the systems (1) and (2) any conditions like stability, boundedness or growth rate condition for the system input-output, we design inputs for systems (1) and (2) respectively, and using the least squares algorithms, we obtain consistent estimates for the delay, orders and coefficients of the systems. The convergence rates of estimates for system coefficients are also established. The designed input is diminishing and this is important when adaptive control problem is simultaneously treated.

## II. Identification of Discrete-Time Systems

We now consider the systems (1), (3) and (4), where the true time-delay $d_0$ and the orders $(p_0, q_0)$ of the system are unknown, but assume that they belong to finite sets. To be specific, we need the following condition $H_1$:

$H_1$. There are known integers $p^*$, $q^*$ and $d^*\in[1, q^*]$ such that

$$(p_0, q_0)\in M\triangleq\{(p, q):\ 0\leqslant p\leqslant p^*,\ d^*\leqslant q\leqslant q^*\}, \tag{6}$$

$$d_0\in\overline{M}\triangleq\{d:\ d^*\leqslant d\leqslant q^*\}. \tag{7}$$

This condition means that the upper bounds for $p_0$ and $q_0$ are available.

We also need the following identifiability condition $H_2$:

$H_2$. $A(z)$ and $B(z)$ have no common left factor and $A_{p_0}$ and $B_{q_0}$ are of row full rank.

Let $\{v_n, \mathfrak{F}_n\}$ be an arbitrary $l$-dimensional martingale difference sequence with properties

$$Ev_nv_n^\tau=\frac{1}{n^\varepsilon}I,\quad \|v_n\|^2\leqslant\sigma^2/n^\varepsilon,\quad \varepsilon\in\left[0,\ \frac{1}{2(t_0+1)}\right),$$

where $t_0\triangleq(m+1)p^*+q^*$, $\sigma^2>0$ is a constant and $\mathfrak{F}_n\triangleq\sigma\{v_i,\ i\leqslant n\}$. $\{v_n\}$ serves as the excitation source for the system and makes its identification possible.

Further, let $u_n^0$ be the $l$-dimensional input, which is $\sigma\{y_i,\ 0\leqslant i\leqslant n\}$ measurable and is designed for control purpose. For pure identification we may set $u_n^0\equiv0$, while in the case of adaptive control we allow $u_n^0$ to grow up but not faster than

$$\sum_{j=0}^{n} \|u_j^0\|^2 = O(n^{1+\delta}), \quad \delta \in \left[0, \frac{1-2\varepsilon(t_0+1)}{2t_0+3}\right]. \tag{8}$$

Finally, for identifying $d_0$ and $(p_0, q_0)$, we set the system input

$$u_n = u_n^0 + v_n. \tag{9}$$

We now describe the estimation algorithms: For any $(p, q) \in M$ and $d \in \overline{M}$, set

$$\theta^\tau(p, d, q) = [-A_1 \cdots - A_p B_d \cdots B_q], \tag{10}$$

$$\varphi_n^\tau(p, d, q) = [y_n^\tau \cdots y_{n-p+1}^\tau u_{n-d+1}^\tau \cdots u_{n-q+1}^\tau] \tag{11}$$

with agreement $A_i = 0$ for $i > p_0$ and $B_j = 0$ for $j < d_0$ and $j > q_0$.

Given initial value $\theta_0(p, d, q)$, the least squares estimate $\theta_n(p, d, q)$ for $\theta(p, d, q)$ is defined by

$$\theta_n(p, d, q) = \left(\sum_{i=0}^{n-1} \varphi_i(p, d, q)\varphi_i^\tau(p, d, q) + I\right)^{-1} \sum_{i=0}^{n-1} \varphi_i(p, d, q) y_{i+1}^\tau \tag{12}$$

or equivalently by the recursive algorithm

$$\begin{aligned} \theta_{n+1}(p, d, q) = \theta_n(p, d, q) + b_n(p, d, q)P_n(p, d, q)\varphi_n(p, d, q) \\ \cdot (y_{n+1} - \varphi_n^\tau(p, d, q)\theta_n(p, d, q)), \end{aligned} \tag{13}$$

$$\begin{aligned} P_{n+1}(p, d, q) = P_n(p, d, q) - b_n(p, d, q)P_n(p, d, q)\varphi_n(p, d, q) \\ \cdot \varphi_n^\tau(p, d, q)P_n(p, d, q), \quad P_0 = I, \end{aligned} \tag{14}$$

$$b_n(p, d, q) = (1 + \varphi_n^\tau(p, d, q)P_n(p, d, q)\varphi_n(p, d, q))^{-1}. \tag{15}$$

Let $\{a_n\}$ be any sequence of real numbers satisfying

$$a_n \xrightarrow[n\to\infty]{} \infty \text{ and } a_n / n^{1-(1+t_0)(\varepsilon+\delta)} \xrightarrow[n\to\infty]{} 0. \tag{16}$$

It is worth noting that $1-(1+t_0)(\varepsilon+\delta) \geqslant (1-\varepsilon)/2 > 0$, and hence the existence of $\{a_n\}$ is undoubted.

Set

$$\sigma_n(p, d, q) = \sum_{i=0}^{n-1} \|y_{i+1} - \theta_n^\tau(p, d, q)\varphi_i(p, d, q)\|^2 \tag{17}$$

and

$$CIC_1(p, q)_n = \sigma_n(p, d^*, q) + (p+q)a_n. \tag{18}$$

The estimate $(p_n, q_n)$ for $(p_0, q_0)$ is defined by minimizing $CIC_1(p, q)_n$:

$$(p_n, q_n) = \arg\min_{(p,q)\in M} CIC_1(p, q)_n, \quad n \geqslant 1 \tag{19}$$

and the estimate $d_n$ for $d_0$ is given by

$$d_n = \arg\min_{d\in\overline{M}} CIC_2(d)_n, \quad n \geqslant 1, \tag{20}$$

where

$$CIC_2(d)_n = \sigma_n(p_n, d, q_n) - d \cdot a_n \tag{21}$$

with $p_n$, $q_n$ obtained from (19).

**Theorem 1.** *Assume that Conditions* $H_1$ *and* $H_2$ *hold and* the *control* $\{u_n\}$ *is defined by* (9) *with* (8) *satisfied. Then*

$$(p_n, d_n, q_n) \xrightarrow[n\to\infty]{} (p_0, d_0, q_0), \text{ a. s.} \tag{22}$$

*and*

$$\|\theta_n(p_n \vee p_0, d_n \wedge d_0, q_n \vee q_0) - \theta(p_n \vee p_0, d_n \wedge d_0, q_n \vee q_0)\| = O\left(\frac{1}{n^{1-(1+t_0)(\varepsilon+\delta)}}\right), \text{ a. s.,} \tag{23}$$

*where* $a \wedge b$ *and* $a \vee b$ *mean* $\min(a, b)$ *and* $\max(a, b)$ *respectively.*

This theorem says that $(p_n, q_n)$ and $d_n$ given by (19) and (20) are consistent estimates for system orders and time-delay respectively, while (23) indicates the convergence rates of estimates for system coefficients:

## III. Proof of Theorem 1

For every $(p, q) \in M$, denote by $\lambda_{\min}^{(p,q)}(n)$ the smallest eigenvalue of

$$\sum_{j=0}^{n-1} \varphi_i(p, d^*, q)\varphi_i^\tau(p, d^*, q).$$

We have[8]

**Lemma 1.** *If the positive real numbers* $\{a_n\}$ *in* (18) *and the system input* $\{u_n\}$ *(not necessarily given by* (9)*) satisfy the following conditions*

$$a_n \xrightarrow[n\to\infty]{} \infty \text{ and } a_n/\lambda_{\min}^{(p,q)}(n) \xrightarrow[n\to\infty]{} 0, \text{ a. s.} \tag{24}$$

*for* $(p, q) = (p_0, q^*)$ *and* $(p, q) = (p^*, q_0)$, *then* $(p_n, q_n)$ *and* $d_n$ *given by* (19) *and* (20) *respectively are consistent, i. e.*

$$(p_n, d_n, q_n) \xrightarrow[n\to\infty]{} (p_0, d_0, q_0), \text{ a. s.} \tag{25}$$

*Proof of Theorem* 1.

By Lemma 1 for proving (22), we only need to show that $\{a_n\}$ satisfying (16), satisfies (24) as well. For this it suffices to prove that

$$\liminf_{n\to\infty} n^{-1+(1+t_0)(\varepsilon+\delta)}\lambda_{\min}^{(p,q)}(n) \neq 0, \text{ a. s.} \tag{26}$$

for $(p, q) = (p_0, q^*)$ and $(p, q) = (p^*, q_0)$.

If (26) were not true, then there would exist a vector[7]

$$\eta = [\alpha^{(0)\tau} \cdots \alpha^{(p-1)\tau} \beta^{(d^*)\tau} \cdots \beta^{(q-1)\tau}]^\tau, \ \|\eta\| = 1$$

satisfying