STOCHASTIC ADAPTIVE CONTROL FOR SYSTEMS WITH NOISE BEING AN ARMA PROCESS*

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Abstract: Stochastic adaptive control is considered for the discrete-time multi-input and multi-output system of multi-delay with noise expressed by an ARMA process. The CARIMA model is a special case of the system in question. The optimal adaptive control law is given and it is shown that a quadratic cost function is minimized and the closed-loop system is stable. Further, when the system is of minimum phase, the convergence rates of parameter estimates and of the cost-function are also derived.

Key Words: Stochastic system, adaptive control, quadratic cost, convergence analysis.

§ 1. Introduction

This paper considers the stochastic adaptive control problem for the system described by the following model

$$A(z)y_n = B(z)u_{n-d} + e_n, \quad y_i = 0, \ u_j = 0, \ i<0, \ j<0, \tag{1}$$

where y_n and u_n are the m-output and l-input respectively and e_n is the system noise modelled by an ARMA process:

$$D(z)e_n = C(z)w_n \tag{2}$$

driven by a martingale difference sequence $\{w_n, \mathscr{F}_n\}$ with respect to a family of non-decreasing σ-algebras $\{\mathscr{F}_n\}$ and with properties:

$$\sup_n E[\|w_{n+1}\|^2|\mathscr{F}_n] \leqslant \gamma < \infty, \text{ a.s.}, \ \forall n \geqslant 0, \tag{3}$$

$$\lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n} w_i w_i^{\tau} = R > 0, \text{ a.s.} \tag{4}$$

Here $d \geqslant 1$ is the system time delay, and $A(z)$, $B(z)$ and $C(z)$ are matrix polynomials in the shift-back operator z with unknown coefficients A_i, B_j, C_k, $i=1, \cdots, p$; $j=1, \cdots, q$; $k=1, \cdots, r$:

$$A(z) = I + A_1 z + \cdots + A_p z^p, \qquad p \geqslant 0, \tag{5}$$

$$B(z) = B_1 + B_2 z + \cdots + B_q z^{q-1}, \quad q \geqslant 1, \tag{6}$$

$$C(z) = I + C_1 z + \cdots + C_r z^r, \qquad r \geqslant 0, \tag{7}$$

while

$$D(z) = 1 + \alpha_1 z + \cdots + \alpha_s z^s \tag{8}$$

Received March 15, 1988.

* Project supported by the National Natural Science Foundation of China and the TWAS Research Grant No. 87—43.

is an arbitrary (maybe unstable) scalar polynomial with known parameters.

When $D(z)=1$, model (1) becomes the usual ARMAX model for which the adaptive control problem is the topic of many previous papers, e.g. [1—8]. In this case the system noise is an MA process which is of zero mean. However, in many application problems it may have a constant bias and using an ARMAX model for adaptive control often leads to an unsatisfactory result. For this reason a so-called CARIMA model (meaning that $D(z)=1-z$) is considered in [9—17] and simulations show that such a model often gives satisfactory results. But, in general, neither stability of the closed-loop adaptive control systems nor consistency of the parameter estimates are established in those papers.

Recently, for the system described by the CARIMA model, by using an implicit method Scattolini has given in [18] an adaptive control minimizing a long-run average of conditional expectations of quadratic loss function $v(t)$ and has shown stability of the closed-loop system. In the proof of the optimality of adaptive control given in [18] the assumption $E\|v(t)\|^2<\infty$ is necessary. However, satisfaction of this condition is not clear for the designed adaptive control.

Developments in this paper are devoted to the following directions:

i) $D(z)$ is not restricted to being $1-z$ but is an arbitrary polynomial.

ii) In the cost function the conditional expectation is no longer used.

iii) Stability of the closed-loop adaptive system and minimality of the cost function are simultaneously achieved by using an implicit method.

iv) An explicit method is applied to obtain rates of convergence of the cost function to its minimal value and of the parameter estimates to its true values when the open-loop system is of minimum phase.

The arrangement of the paper is as follows. In Section 2 the optimal control law for a quadratic cost-function is given for system (1) with known parameters. Section 3 derives the optimal adaptive control minimizing the closed-loop system when the open-loop system is not necessarily stable and possibly is of non-minimum phase. In Section 4 the convergence rates for both the cost function and the parameter estimates are demonstrated for the adaptive control system of minimum phase.

§ 2. Optimal Control for a Quadratic Cost

In this section we assume parameters in system (1) are known and solve the optimal control for a quadratic cost

$$\limsup_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n}\|A^0(z)y_i-B^0(z)y_i^*+Q(z)D(z)u_{i-d}\|^2, \tag{9}$$

where y_n^* is a specified $\mathscr{F}_{n-d}$-measurable bounded reference signal, $A^0(z)$, $B^0(z)$ and $Q(z)$ are given according to the design requirements and control u_n should be $\mathscr{F}_n$-measurable. In particular, setting $Q(z)\equiv 0$ leads to the usual model reference adaptive control problem. Further, if $A^0(z)\equiv B^0(z)\equiv I$ and $Q(z)\equiv 0$, then the cost (9) turns out to be the one for tracking.

In [8] it is shown that for the Diophantine equation

$$(\det C(z))I = F(z)(\mathrm{Adj}\, C(z))A(z)D(z) + z^d G(z), \tag{10}$$

the solution

$$F(z) = I + F_1 z + \cdots + F_{d-1} z^{d-1},\ G(z) = G_0 + G_1 z + \cdots + G_{p_1} z^{p_1}, \tag{11}$$

with $p_1 = \max(mr-d, (m-1)r+p+s-1)$ exists and is unique. The matrix coefficients F_i, $G_j (i=1,\cdots, d-1;\ j=0, \cdots, p_1)$ are easily expressed via coefficients of $\det C(z)$ and $(\mathrm{Adj}\, C(z))A(z)D(z)$.

Denote

$$A^0(z)F(z) = \overline{F}(z) + z^d \overline{N}(z), \tag{12}$$

where

$$\overline{F}(z) = \overline{F}_0 + \overline{F}_1 z + \cdots + \overline{F}_{d-1} z^{d-1}. \tag{13}$$

Proposition. If $A^0(0)B_1 + Q(0)$ is non-degenerate, and $\det C(z)$ is stable, then the optimal control u_{n-d} that is $\mathscr{F}_{n-d}$-measurable can uniquely be defined from

$$\begin{aligned}&(A^0(z)F(z)(\mathrm{adj}\, C(z))B(z) + (\det C(z))Q(z) - \overline{N}(z)(\mathrm{adj}\, C(z))B(z)z^d)\, D(z)u_{n-d}\\ &= (\det C(z))B^0(z)y_n^* - (D(z)\overline{N}(z)(\mathrm{adj}\, C(z))A(z) + A^0(z)G(z))y_{n-d}\end{aligned} \tag{14}$$

and the minimal value of the cost is

$$\begin{aligned}&\limsup_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n} \|A^0(z)y_i - B^0(z)y_i^* + Q(z)D(z)u_{i-d}\|^2\\ &= \mathrm{tr}\sum_{j=0}^{d-1} \overline{F}_j R \overline{F}_j^\tau.\end{aligned} \tag{15}$$

Proof. First of all, if $A^0(0)B_1 + Q(0)$ is non-singular, then (14) is obviously solvable with respect to u_{n-d} and the solution is $\mathscr{F}_{n-d}$-measurable. From (1) and (10) it is easy to see

$$\begin{aligned}&(\det C(z))(y_n - F(z)w_n)\\ &= G(z)y_{n-d} - (\det C(z))F(z)w_n + F(z)(\mathrm{adj}\, C(z))A(z)D(z)y_n\\ &= G(z)y_{n-d} + F(z)(\mathrm{adj}\, C(z))B(z)D(z)u_{n-d}.\end{aligned} \tag{16}$$

Using the expression for $\det C(z)y_n$ given by (16) and noticing (12) and (13) we have

$$\begin{aligned}&(\det C(z))(A^0(z)y_n - B^0(z)y_n^* + Q(z)D(z)u_{n-d})\\ &= A^0(z)((\det C(z))F(z)w_n + G(z)y_{n-d} + F(z)(\mathrm{adj}\, C(z))B(z)D(z)u_{n-d})\\ &\quad + (\det C(z))(Q(z)D(z)u_{n-d} - B^0(z)y_n^*)\\ &= (\det C(z))\overline{F}(z)w_n + (\det C(z))\overline{N}(z)w_{n-d} + A^0(z)(G(z)y_{n-d}\\ &\quad + F(z)(\mathrm{adj}\, C(z))B(z)D(z)u_{n-d}) + (\det C(z))(Q(z)D(z)u_{n-d} - B^0(z)y_n^*)\\ &= (\det C(z))\overline{F}(z)w_n + (\det C(z))\overline{N}(z)w_{n-d} - D(z)\overline{N}(z)(\mathrm{adj}\, C(z))A(z)y_{n-d}\\ &\quad + \overline{N}(z)(\mathrm{adj}\, C(z))B(z)z^d D(z)u_{n-d},\end{aligned} \tag{17}$$

where the last equality is obtained by applying (14). Finally, paying attention to (1) from (17) we find that the control satisfying (14) leads to

$$A^0(z)y_n - B^0(z)y_n^* + Q(z)D(z)u_{n-d} = \overline{F}(z)w_n. \tag{18}$$

From this and using Lemma 2 of [5] it is easy to conclude (15). ∎

§ 3. Adaptive Control by an Implicit Approach

From now on we assume that in system (1) matrix coefficients A_i, B_j, C_k, $i=1, \cdots, p$; $j=1, \cdots, q$ and $k=1, \cdots, r$, are unknown and that p, q and r as the upper bounds for system orders and d as the lower bound for the system time delay are available. The question is to design $\mathscr{F}_n$-measurable adaptive control u_n that minimizes cost (9), but we pay no attention to the parameter identification problem. The system considered here may be unstable and of non-minimum phase.

We first rewrite the relationship (14) that should be satisfied by the optimal control.

Set

$$D(z)\overline{N}(z)(\operatorname{adj} C(z))A(z)+A^0(z)G(z)$$

$$\triangleq G_0(z)\triangleq G_{00}+G_{01}z+\cdots+G_{0g_0}z^{g_0}, \tag{19}$$

$$-(\det C(z))B^0(z)\triangleq G_1(z)\triangleq G_{10}+G_{11}z+\cdots+G_{1g_1}z^{g_1}, \tag{20}$$

$$A^0(z)F(z)(\operatorname{adj} C(z))B(z)+(\det C(z))Q(z)-z^d\overline{N}(z)(\operatorname{adj} C(z))B(z)$$

$$\triangleq G_2(z)\triangleq G_{20}+G_{21}z+\cdots+G_{2g_2}z^{g_2}, \tag{21}$$

$$\det C(z)\triangleq 1+C_1z+\cdots+C_{mr}z^{mr}, \tag{22}$$

and

$$\bar{y}_n=A^0(z)y_n-B^0(z)y_n^*+Q(z)D(z)u_{n-d}. \tag{23}$$

Then (17) becomes

$$(\det C(z))(\bar{y}_n-\overline{F}(z)w_n)=G_0(z)y_{n-d}+G_1(z)y_n^*+G_2(z)D(z)u_{n-d} \tag{24}$$

since it follows from (1) that

$$\det C(z)w_n=D(z)\operatorname{adj} C(z)(A(z)y_n-B(z)u_{n-d}). \tag{25}$$

We apply an implicit approach to solve the adaptive control problem. To be specific, we directly estimate coefficients appearing in the optimal control rather than unknown coefficients in the system. For this we denote by θ^* the coefficients to be estimated

$$\theta^*=[G_{00}G_{01}\cdots G_{0g_0}G_{10}\cdots G_{1g_1}G_{20}G_{21}\cdots G_{2g_2}\ c_1I\cdots c_{mr}I]^\tau \tag{26}$$

and set

$$\varphi_n^{*\tau}=[y_n^\tau\cdots y_{n-g_0}^\tau y_{n+d}^{*\tau}\cdots y_{n+d-g_1}^{*\tau}D(z)u_n^\tau\cdots D(z)u_{n-g_2}^\tau-\varphi_{n-1}^{*\tau}\theta_{n-1}^*\cdots-\varphi_{n-mr}^{*\tau}\theta_{n-mr}^*]. \tag{27}$$

The estimate

$$\theta_n^*=[G_{00n}\cdots G_{0g_0n}G_{10}\cdots G_{1g_1n}G_{20n}\cdots G_{2g_2n}c_{1n}I\cdots c_{mrn}I]^\tau \tag{28}$$

is recursively given by the following algorithm

$$\theta_n^*=\theta_{n-d}^*+\frac{\bar{a}}{r_{n-d}^*}\varphi_{n-d}^*(\bar{y}_n^\tau-\varphi_{n-d}^{*\tau}\theta_{n-d}^*) \tag{29}$$

with θ_0^* arbitrary but making G_{200} non-singular, where $\bar{a}$ is a positive constant and

$$r_n^*=1+\sum_{i=0}^n\|\varphi_i^*\|^2. \tag{30}$$

Equality (24) hints to us that we can take control u_n satisfying

$$\theta_n^{*\tau}\varphi_n^*=0.$$

However, its solvability is not guaranteed. To overcome this difficulty we suggest controlling a diminishing dither $\{v_n\}$ which is a sequence of mutually independent random vectors and is independent of $\{w_n\}$ and $\{y_n^*\}$. The components of v_n are also independent and with continuous distributions. Assume

$$Ev_n=0,\ Ev_nv_n^\tau=\frac{1}{n^\varepsilon}I,\ \|v_n\|^2\leqslant\sigma^2/n^\varepsilon,\ \text{for } n\geqslant 1 \text{ and } v_i=0 \text{ for } i\leqslant 0,$$

$\varepsilon\in(0,1/2)$, $\sigma^2>0$ is a constant.

From [4] it is known that $u_n^{(1)}$ can recursively be defined from

$$G_{20n}u_n^{(1)}=G_{20n}u_n-\theta_n^{*\tau}\varphi_n^*. \tag{31}$$

Define adaptive control

$$u_n=u_n^{(1)}+v_n. \tag{32}$$

Theorem 1. *Suppose that*

i) $A^0(z)$ *is stable and* $(\det C(z))-1/2\bar{a}$ *is strictly positive real and*

ii) $m=l$ *and* $(\det A^0(z))B(z)+A(z)(\operatorname{adj} A^0(z))Q(z)D(z)$ *is stable. Then adaptive control* (32) *stabilizes the closed-loop system and minimizes the cost*

$$\limsup_{n\to\infty}\frac{1}{n}\sum_{i=0}^{n}(\|y_i\|^2+\|D(z)u_i\|^2)<\infty,\ \text{a.s.}, \tag{33}$$

$$\limsup_{n\to\infty}\frac{1}{n}\sum_{i=0}^{n}\|A^0(z)y_i-B^0(z)y_i^*+Q(z)D(z)u_{i-d}\|^2$$

$$=\operatorname{tr}\sum_{i=0}^{d-1}\bar{F}_iR\bar{F}_i^\tau,\ \text{a.s.} \tag{34}$$

Proof. Set

$$z_{i-d}=\bar{y}_i-\bar{F}(z)w_i-\theta_{i-d}^{*\tau}\varphi_{i-d}^*,\quad \tilde{\theta}_n^*=\theta^*-\theta_n^*. \tag{35}$$

By (23)—(27) it is obvious that

$$(\det C(z))(\bar{y}_n-\bar{F}(z)w_n-\theta_{n-d}^{*\tau}\varphi_{n-d}^*)=\theta^{*\tau}\varphi_{n-d}^*-\theta_{n-d}^{*\tau}\varphi_{n-d}^*=\tilde{\theta}_{n-d}^{*\tau}\varphi_{n-d}^*. \tag{36}$$

By a treatment similar to Lemma 2 of [8] we have

$$\sum_{i=0}^{\infty}\|z_i\|^2/r_i^*<\infty,\ \text{a.s.} \tag{37}$$

and

$$\sup_n \operatorname{tr}\tilde{\theta}_n^{*\tau}\tilde{\theta}_n^*<\infty,\ \text{a.s.} \tag{38}$$

Since v_n is independent of $((D(z)-1)u_n+u_n^{(1)})$, by Lemma 2 of [5] we find that

$$\sum_{i=1}^{n}\|D(z)u_i\|^2\xrightarrow[n\to\infty]{}\infty$$

and hence $r_n^*\xrightarrow[n\to\infty]{}\infty$ a.s.

Then applying the Kronecker lemma to (37) we have

$$\sum_{i=0}^{n}\|z_i\|^2=o(r_n^*). \tag{39}$$

By (1) and (23) it is easy to see

$$\begin{aligned} &A(z)(\operatorname{adj} A^0(z))D(z)\bar{y}_n \\ &\quad = ((\det A^0(z))B(z) + A(z)(\operatorname{adj} A^0(z))Q(z)D(z))D(z)u_{n-d} \\ &\quad\quad + (\det A^0(z))C(z)w_n - A(z)(\operatorname{adj} A^0(z))B^0(z)D(z)y_n^*. \end{aligned} \tag{40}$$

From (31) and (32) it is known that $\theta_n^{*\tau}\varphi_n^* = G_{20n}v_n$. Then by Condition ii), (4) and (35), from (40) we assert that

$$\sum_{i=0}^{n} \|D(z)u_{i-d}\|^2 = O\left(\sum_{i=0}^{n} \|z_{i-d}\|^2 + n\right). \tag{41}$$

Further, from this and by (23) and stability of $A^0(z)$ we find

$$\sum_{i=0}^{n} \|y_i\|^2 = O\left(\sum_{i=0}^{n} \|z_{i-d}\|^2 + n\right), \tag{42}$$

and

$$r_n^* = O\left(\sum_{i=0}^{n} \|z_i\|^2 + n\right), \tag{43}$$

which together with (39) implies

$$\sum_{i=0}^{n} \|z_i\|^2 = o(n). \tag{44}$$

Then, (33) follows from (41), (42) and (44), while (34) is obtained by (35) and Lemma 2 of [5]. In fact, for any $\eta \in (1/2, 1)$ we have

$$\begin{aligned} \limsup_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n} \|\bar{y}_i\|^2 &= \limsup_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n} \|\bar{F}(z)w_i + z_{i-d} + G_{20i-d}v_{i-d}\|^2 \\ &= \limsup_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n} \|\bar{F}(z)w_i\|^2 + \limsup_{n\to\infty}\Bigg(\frac{1}{n} \sum_{i=0}^{n} \|z_{i-d} + G_{20i-d}v_{i-d}\|^2 \\ &\quad + O\left(\frac{1}{n}\Big(\sum_{i=0}^{n} \|z_{i-d} + G_{20i-d}v_{i-d}\|^2\Big)^{\eta}\right)\Bigg) \\ &= \operatorname{tr} \sum_{j=0}^{d-1} \bar{F}_j R \bar{F}_j^{\tau}. \end{aligned}$$

§ 4. Convergence Rates

As pointed out in [20] the problem of simultaneously identifying parameters and controlling the plant is important and difficult. It is the topic of the present section. When $D(z) = 1$, the problem has been discussed in [4—11], but for general $D(z)$ it is still left open. We now use an explicit approach to design adaptive control.

Assume $m = l$ and the system time delay d is known.

We consider the loss function (9) with $Q(z) = 0$ and set

$$\theta^{\tau} = [-A_1 \cdots -A_p B_1 \cdots B_q C_1 \cdots C_r]. \tag{45}$$

Given the initial value $\theta_0^{\tau} = [-A_{10} \cdots -A_{p0} B_{10} \cdots B_{q0} C_{10} \cdots C_{r0}]$ with $\det B_{10} \neq 0$ we recursively estimate θ by an ELS algorithm:

$$\theta_{n+1} = \theta_n + a_n P_n \varphi_n (D(z) y_{n+1}^{\tau} - \varphi_n^{\tau}\theta_n), \tag{46}$$

$$P_{n+1} = P_n - a_n P_n \varphi_n \varphi_n^{\tau} P_n, \quad a_n = (1 + \varphi_n^{\tau} P_n \varphi_n)^{-1}, \tag{47}$$

$$P_0 = \alpha I, \quad \alpha = m(p+r) + lq,$$

$$\varphi_n^\tau=[D(z)y_n^\tau\cdots D(z)y_{n-p+1}^\tau D(z)u_{n-d+1}^\tau\cdots D(z)u_{n-d-q+2}^\tau$$
$$D(z)y_n^\tau-\varphi_{n-1}^\tau\theta_n\cdots D(z)y_{n-r+1}^\tau-\varphi_{n-r}^\tau\theta_{n-r+1}]. \tag{48}$$

Set

$$A_n(z)=I+A_{1n}z+\cdots+A_{pn}z^p,\ B_n(z)=B_{1n}+B_{2n}z+\cdots B_{qn}z^{q-1},$$
$$C_n(z)=I+C_{1n}z+C_{rn}z^r,$$

where the coefficients are given by

$$\theta_n^\tau=[-A_{1n}\cdots-A_{pn}B_{1n}\cdots B_{qn}C_{1n}\cdots C_{rn}].$$

The control purpose is to make the left side of (17) as close to zero as possible. So motivated by the terms after the second equality of (17) we should define adaptive control $u_n^{(2)}$ as follows

$$B_{1n}D(z)u_n^{(2)}+(A^0(z)-I)B_{1n}D(z)u_n$$
$$=(\det C_n(z))(B^0(z)y_{n+d}^*-\overline{N}_n(z)(D(z)y_n-\theta_n^\tau\varphi_{n-1}))$$
$$-A^0(z)(G_n(z)y_n+(F_n(\operatorname{adj}C_n)B_n)(z)D(z)u_n-B_{1n}D(z)u_n), \tag{49}$$

where $F_n(z)=I+F_{1n}z+\cdots+F_{d-1n}z^{d-1}$ and $G_n(z)$ are the solution of the Diophantine equation

$$(\det C_n(z))I=(F_n(\operatorname{adj}C_n)A_n)(z)D(z)+G_n(z)z^d, \tag{50}$$

$\overline{N}_n(z)$ is a matrix polynomial satisfying

$$A^0(z)F_n(z)=\overline{F}_{0n}+\overline{F}_{1n}z+\cdots+\overline{F}_{d-1n}z^{d-1}+\overline{N}_n(z)z^d \tag{51}$$

and $(F_n(\operatorname{adj}C_n)B_n)(z)$ means the product of $F_n(z)$, $\operatorname{adj}C_n(z)$ and $B_n(z)$.

To guarantee strong consistency of parameter estimates the input and output of the system should not grow up too fast. So for some time intervals we need to take $u_n^{(1)}$ as defined by (31) or even as zero. To be specific, we define the desired control u_n^s as

$$u_n^s=\begin{cases}u_n^{(2)}, \text{ if } n \text{ belongs to some } [\tau_k,\sigma_k)\cap A\\ 0, \text{ if } n \text{ belongs to some } [\tau_k,\sigma_k)\cap A^c\\ u_n^{(1)}, \text{ if } n \text{ belongs to some } [\sigma_k,\tau_{k+1})\end{cases} \tag{52}$$

and the adaptive control as the disturbed version of u_n^s:

$$u_n=u_n^s+v_n, \tag{53}$$

where $\{v_n\}$ is defined in Section 3 with ε given by (56), A is the set of integers

$$A=\{j:\|D(z)u_j^{(2)}\|^2\leqslant j^{1+\delta}\} \tag{54}$$

and $\{\tau_k\}$ and $\{\sigma_k\}$ are stopping times defined as follows

$$0=\tau_1<\sigma_1<\tau_2<\sigma_2<\cdots,$$

$$\sigma_k=\sup\left\{t>\tau_k:\ \sum_{i=\tau_k}^{j-1}\|y_i\|^2\leqslant(j-1)^{1+\delta/2}+\|y_{\tau_k}\|^2, \forall j\in(\tau_k,t]\right\}, \tag{55}$$

$$\tau_{k+1}=\inf\left\{t>\sigma_k:\ \sum_{i=\sigma_k}^{t}\|y_i\|^2\leqslant\frac{t\log t}{2^k},\ \sum_{i=\tau_k}^{\sigma_k-1}\|y_i\|^2\leqslant\frac{t\log t}{2^k},\right.$$
$$\left.\sum_{i=\sigma_k}^{t}\|D(z)u_i\|^2\leqslant\frac{t\log t}{2^k},\ \sum_{i=\tau_k}^{\sigma_k-1}\|D(z)u_i\|^2\leqslant\frac{t\log t}{2^k}\right\}, \tag{56}$$

where

$$\delta\in\left[0, \frac{1-2\varepsilon(1+t)}{2t+3}\right),\quad \varepsilon\in\left(0, \frac{1}{2(t+1)}\right),\ t=mp+\max(p, q, r)\ (\geqslant 1).$$

Remark. Because $\det B_{10}\neq 0$ and the dither v_n is introduced in (53), by an argument similar to that used in [8] we can prove $\det B_{1n}\neq 0$ a.s. for any $n\geqslant 0$. Hence (49) is solvable with respect to $u_n^{(2)}$.

Theorem 2. *Assume that*

i) $\det C(z)-1/2\bar{a}$ *is strictly positive real and* $A^0(z)$ *is stable with* $A^0(0)=I$;

ii) $C^{-1}(z)-\frac{1}{2}I$ *is strictly positive real and* $B(z)$ *is stable;*

iii) $A(z)$, $B(z)$ *and* $C(z)$ *have no common left-factor and* A_p *is of full rank.*

Then adaptive control (53) *leads to that*

a) *the closed-loop system is stabilized,* i.e.

$$\limsup_{n\to\infty}\frac{1}{n}\sum_{i=0}^{n}(\|y_i\|^2+\|D(z)u_i\|^2)<\infty\quad \text{a.s.;}\tag{57}$$

b) *the cost is minimized,* i.e.

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=0}^{n}(A^0(z)y_i-B^0(z)y_i^*)(A^0(z)y_i-B^0(z)y_i^*)^\tau=\sum_{j=0}^{d-1}\bar{F}_jR\bar{F}_j^\tau\tag{58}$$

with the rate of convergence

$$\left\|\frac{1}{n}\sum_{i=0}^{n}(A^0(z)y_i-B^0(z)y_i^*)(A^0(z)y_i-B^0(z)y_i^*)^\tau-\frac{1}{n}\sum_{i=0}^{n}(\bar{F}(z)w_i)(\bar{F}(z)w_i)^\tau\right\| =O(n^{-1/2s});\tag{59}$$

c) *the parameter estimates are strongly consistent with the rate of convergence*

$$\|\theta-\theta_n\|^2=O\left(\frac{(\log n)(\log\log n)^c}{n^{1-(t+1)\varepsilon}}\right),\quad \forall c>1.\tag{60}$$

Proof. We complete the proof in three steps.

Step 1. We first show that θ_n is strongly consistent, so that for any sample θ_n is bounded.

By an argument similar to that used for Theorem 3 of [5] we can show

$$\|\theta-\theta_n\|^2=O\left(\frac{(\log n)(\log\log n)^c}{n^{1-(t+1)(\varepsilon+\delta)}}\right),\quad \forall c>1,\tag{61}$$

if we can verify

$$\frac{1}{n}\sum_{i=0}^{n}(\|y_i\|^2+\|D(z)u_i\|^2)=O(n^\delta).\tag{62}$$

It is clear that (61) implies the strong consistency of θ_n and that by the stability of $B(z)$ for (62) it suffices to show

$$\frac{1}{n}\sum_{i=0}^{n}\|y_i\|^2=O(n^\delta).\tag{63}$$

If $\tau_k<\infty$ and $\sigma_k=\infty$ for some k, then (63) is automatically satisfied because of (55). Also, if $\sigma_k<\infty$ and $\tau_{k+1}=\infty$ for some k, then (63) holds because of Theorem 1. Thus, we need only to show (63) for the case where $\tau_k<\infty$, $\sigma_k<\infty$ for all k. In this case, for sufficiently large k by (56) and then by (55) we have

$$\sup_{\tau_k\le n<\sigma_k}\frac{1}{n^{1+\delta}}\sum_{i=0}^{n}\|y_i\|^2$$

$$\leqslant \sup_{\tau_k\le n<\sigma_k}\frac{1}{n^{1+\delta}}\Big(\sum_{i=\tau_1}^{\sigma_1-1}\|y_i\|^2+\sum_{i=\sigma_1}^{\tau_2}\|y_i\|^2+\cdots+\sum_{i=\sigma_{k-1}}^{\tau_k}\|y_i\|^2+\sum_{i=\tau_k}^{n}\|y_i\|^2\Big)$$

$$\leqslant \sup_{\tau_k\le n<\sigma_k}\frac{1}{n^{1+\delta}}\Big(2\sum_{i=1}^{k-1}\frac{\tau_{i+1}\log\tau_{i+1}}{2^i}+\sum_{i=\tau_k}^{n}\|y_i\|^2\Big)$$

$$\leqslant 2+\sup_{\tau_k\le n<\sigma_k}\frac{1}{n^{1+\delta}}\sum_{i=\tau_k}^{n}\|y_i\|^2$$

$$\leqslant 2+\sup_{\tau_k\le n<\sigma_k}\frac{1}{n^{1+\delta}}(n^{1+\delta}+\|y_{\tau_k}\|^2)$$

$$\leqslant 3+\frac{\tau_k\log\tau_k}{\tau_k^{1+\delta}}<4. \tag{64}$$

Similarly, we can show

$$\sum_{i=0}^{\tau_k}\|D(z)u_i\|^2\leqslant 3\tau_k\log\tau_k,\quad \sum_{i=0}^{\tau_k}\|y_i\|^2\leqslant 3\tau_k\log\tau_k. \tag{65}$$

From (64) we see that for (63) it suffices to prove

$$\sup_{\sigma_k\le n<\tau_{k+1}}\frac{1}{n^{1+\delta}}\sum_{i=0}^{n}\|y_i\|^2=O(1). \tag{66}$$

We show (66) for $n\in[\sigma_k,\sigma_k+d-1]\cap[\sigma_k,\tau_{k+1})$ and $n\in[\sigma_k+d,\tau_{k+1})$ separately. If $n\in[\sigma_k,\sigma_k+d-1]\cap[\sigma_k,\tau_{k+1})$, then from (64) it follows that

$$\sum_{i=0}^{n}\|y_i\|^2=O(\tau_k^{1+\delta})+\sum_{i=\sigma_k}^{n}\|y_i\|^2=O(n^{1+\delta})+\sum_{i=\sigma_k}^{n}\|y_i\|^2. \tag{67}$$

By Condition i) det $C(z)$ is stable. Then from (16) for sufficiently large k we have

$$\sum_{i=\sigma_k}^{n}\|y_i\|^2=O(n)+O\Big(\sum_{i=0}^{n-d}\|y_i\|^2+\sum_{i=0}^{n-d}\|D(z)u_{i-d}\|^2\Big). \tag{68}$$

Combining (67) with (68) we see that for $n\in[\sigma_k,\sigma_k+d-1]\cap[\sigma_k,\tau_{k+1})$ (66) can be verified, if we can show

$$\sum_{i=0}^{n-d}\|D(z)u_i\|^2=O(n^{1+\delta}). \tag{69}$$

Notice in the present case

$$n-d\leqslant\sigma_k-1. \tag{70}$$

If $\tau_k\geqslant\sigma_k-d$, we have $n-d-\tau_k\leqslant\sigma_k-1-\tau_k\leqslant d-1$ and

$$\sum_{i=0}^{n-d}\|D(z)u_i\|^2=\sum_{i=0}^{\tau_k}\|D(z)u_i\|^2+\sum_{i=\tau_k+1}^{n-d}\|D(z)u_i\|^2\leqslant\sum_{i=0}^{\tau_k}\|D(z)u_i\|^2+dn^{1+\delta}=O(n^{1+\delta}),$$

where (54), (65) and (70) are implemented.

If $\tau_k<\sigma_k-d$, then by the stability of $B(z)$ and using (64), (65) and (70) we find

$$\sum_{i=0}^{n-d}\|D(z)u_i\|^2=\sum_{i=0}^{\sigma_k-d-1}\|D(z)u_i\|^2+\sum_{i=\sigma_k-d}^{n-d}\|D(z)u_i\|^2$$

$$=O\Big(\sum_{i=0}^{\sigma_k-1}\|y_i\|^2+\sigma_k\Big)+dn^{1+\delta}=O(n^{1+\delta}).$$

Thus (66) has been proved for $n\in[\sigma_k, \sigma_k+d-1]\cap[\sigma_k, \tau_{k+1})$. We now show it for $n\in[\sigma_k+d, \tau_{k+1})$. Write

$$\sum_{i=0}^{n}\|y_i\|^2=\sum_{i=0}^{\sigma_k+d-1}\|y_i\|^2+\sum_{i=\sigma_k+d}^{n}\|y_i\|^2$$

and notice that the first sum on its rigt-hand side is estimated by $O(n^{1+\delta})$. So for (66) we need only to show that the last term is also estimated by $O(n^{1+\delta})$. Noticing that for $n\in[\sigma_k, \tau_{k+1})$ from (52), (31) and (32) it follows that $\theta_n^{*\tau}\varphi_n^*=G_{20n}v_n$, from (23) and (35) we find

$$\sum_{i=\sigma_k+d}^{n}\|A^0(z)y_i\|^2=\sum_{i=\sigma_k+d}^{n}\|B^0(z)y_i^*+\bar{F}(z)w_i+G_{20i-d}v_{i-d}+z_{i-d}\|^2$$

which is of $O(n^{1+\delta})$, since by (38) G_{20i-d} is bounded and

$$\sum_{i=1}^{n}\|z_i\|^2$$

is estimated by (44). Then by the stability of $A^0(z)$ we have (66) and hence (63) which in turn implies (61).

Step 2. We now show that there is an integer k (possibly depending upon the sample) so that

$$\tau_k<\infty,\ \sigma_k=\infty\quad \text{a. s.} \tag{71}$$

If for some $k,\sigma_k<\infty$ and $\tau_{k+1}=\infty$, then Theorem 1 applies and (33) contradicts $\tau_{k+1}=\infty$. So for (71) we only need to show the impossibility that $\tau_k<\infty$, $\sigma_k<\infty$ for all $k\geqslant 0$.

Define

$$t_k=\sup\{n: j\in[\tau_k, \sigma_k)\cap\Lambda,\ \forall j\in[\tau_k, n]\}, \tag{72}$$

the existence of which for sufficiently large k is justified because of (54) and

$$\|D(z)u_{\tau_k}\|^2\leqslant\frac{\tau_k\log\tau_k}{2^{k-1}}$$

from (56).

By (16) and (65) we have

$$\sum_{i=0}^{\tau_k+d}\|y_i\|^2=O(\tau_k\log\tau_k).$$

Hence

$$\sum_{i=0}^{\tau_k+d}\|A^0(z)y_i\|^2=O(\tau_k\log\tau_k). \tag{73}$$

Set

$$\varphi_n^0=[D(z)y_n^\tau\cdots D(z)y_{n-p+1}^\tau D(z)u_{n-d+1}^\tau\cdots D(z)u_{n-d-q+2}^\tau w_n^\tau\cdots w_{n-r+1}^\tau]^\tau,$$

$$\xi_n=D(z)y_n-\theta_n^\tau\varphi_{n-1}-w_n,\quad r_n=1+\sum_{i=0}^{n}\|\varphi_i\|^2,\quad \tilde{\theta}_n=\theta-\theta_n.$$

From [5] and Theorem 3.1 we know that $C^{-1}(z)-1/2I$ implies

$$\sum_{i=0}^{n}\|\xi_i\|^2=O(\log r_n)(\log\log r_n)^c),\quad \forall c>1$$

from which and (62) it follows that

$$\sum_{i=0}^{n}\|\xi_i\|^2=O(\log^2 n). \tag{74}$$

In the sequel for any matrix polynomials $F_n(z)$ and $B_n(z)$ by $(F_nB_n)(z)$ we mean the product of $F_n(z)$ and $B_n(z)$, while

$$(F_nB)_n(z)=\sum_{i,j} B_{in}F_{jn-i}z^{i+j},$$

where $B_{in}(F_{in})$ is the matrix coefficient for z^i in $B_n(z)\,(F_n(z))$.

From (1) we have

$$D(z)y_{n+d}=\theta^{\tau}\varphi^0_{n+d-1}+w_{n+d}=\tilde{\theta}^{\tau}_n\varphi^0_{n+d-1}+w_{n+d}+\theta^{\tau}_n\varphi^0_{n+d-1}$$

or

$$\tilde{\theta}^{\tau}_n\varphi^0_{n+d-1}+C_n(z)w_{n+d}=A_n(z)D(z)y_{n+d}-B_n(z)D(z)u_n. \tag{75}$$

Using (75), (50) and the following equality for $n\in[\tau_k,\ t_k]$

$$\begin{aligned}&A^0(z)\left(G_n(z)y_n+(F_n(\operatorname{adj}C_n)B_n)(z)D(z)u_n\right)\\&=B_{1n}D(z)v_n+(\det C_n(z)^*)\left(B^0(z)y^*_{n+d}-\overline{N}_n(z)\xi_n-\overline{N}_n(z)w_n\right)\end{aligned}$$

which follows from (49), we find that

$$\begin{aligned}
&A^0(z)\left(F_n(\operatorname{adj}C_n)\right)(z)\left(\tilde{\theta}^{\tau}_n\varphi^0_{n+d-1}+C_n(z)w_{n+d}\right)\\
&=A^0(z)\left(\left(F_n(\operatorname{adj}C_n)\right)A\right)_n(z)D(z)y_{n+d}-A^0(z)\left(\left(F_n(\operatorname{adj})C_n\right)B\right)_n(z)D(z)u_n\\
&=A^0(z)\left((\det C_n(z))I-G_n(z)z^d\right)y_{n+d}\\
&\quad+A^0(z)\left(\left(\left(F_n(\operatorname{adj}C_n)\right)A\right)_n(z)-\left(F_n(\operatorname{adj}C_n)A_n\right)(z)\right)D(z)y_{n+d}\\
&\quad-A^0(z)\left(\left(F_n(\operatorname{adj}C_n)\right)B\right)_n(z)D(z)u_n\\
&=A^0(z)(\det C_n(z))y_{n+d}-A^0(z)G_n(z)y_n\\
&\quad+A^0(z)\left(\left(\left(F_n(\operatorname{adj}C_n)\right)A\right)_n(z)-\left(F_n(\operatorname{adj}C_n)A_n\right)(z)\right)D(z)y_{n+d}\\
&\quad-A^0(z)\left(F_n(\operatorname{adj}C_n)B_n\right)(z)D(z)u_n\\
&\quad+A^0(z)\left(\left(F_n(\operatorname{adj}C_n)B_n\right)(z)-\left(\left(F_n(\operatorname{adj}C_n)\right)B\right)_n(z)\right)D(z)u_n\\
&=A^0(z)(\det C_n(z))y_{n+d}-B_{1n}D(z)v_n\\
&\quad-(\det C_n(z)^*)\left(B^0(z)y^*_{n+d}-\overline{N}_n(z)\xi_n-\overline{N}_n(z)w_n\right)\\
&\quad+A^0(z)\left(\left(\left(F_n(\operatorname{adj}C_n)\right)A\right)_n(z)-\left(F_n(\operatorname{adj}C_n)A_n\right)(z)\right)D(z)y_{n+d}\\
&\quad+A^0(z)\left(\left(F_n(\operatorname{adj}C_n)B_n\right)(z)-\left(\left(F_n(\operatorname{adj}C_n)\right)B\right)_n(z)\right)D(z)u_n\\
&=(\det C(z))\left(A^0(z)y_{n+d}-B^0(z)y^*_{n+d}\right)\\
&\quad+A^0(z)\left(\det C_n(z)-\det C(z)\right)y_{n+d}-B_{1n}D(z)v_n\\
&\quad+\left(\det C(z)-\det C_n(z)\right)B^0(z)y^*_{n+d}+(\det C_n(z))\left(\overline{N}_n(z)\xi_n\right)\\
&\quad+(\det C_n(z))\left(\overline{N}_n(z)w_n\right)\\
&\quad+A^0(z)\left(\left(\left(F_n(\operatorname{adj}C_n)\right)A\right)_n(z)-\left(F_n(\operatorname{adj}C_n)A_n\right)(z)\right)D(z)y_{n+d}\\
&\quad+A^0(z)\left(\left(F_n(\operatorname{adj}C_n)B_n\right)(z)-\left(\left(F_n(\operatorname{adj}C_n)\right)B\right)_n(z)\right)D(z)u_n.
\end{aligned}\tag{76}$$

Noticing that (12) implies

$$\begin{aligned}&A^0(z)\left(\left(F_n(\operatorname{adj}C_n)\right)C\right)_n(z)w_{n+d}\\&=(\det C(z))\overline{F}(z)w_{n+d}+\det C(z)N(z)w_n\\&\quad+A^0(z)\left(\left(\left(F_n(\operatorname{adj}C_n)\right)C\right)_n(z)-F(z)(\operatorname{adj}C(z))C(z)\right)w_{n+d},\end{aligned}$$

we then obtain from (76)

$$
\begin{aligned}
&(\det C(z))(A^0(z)y_{n+d}-B^0(z)y^*_{n+d})\\
&=(\det C(z))\bar{F}(z)w_{n+d}+A^0(z)(F_n \operatorname{adj} C_n)(z)\tilde{\theta}_n^\tau \varphi^0_{n+d-1})\\
&\quad+B_{1n}D(z)v_n+((\det C(z))\bar{N}(z)-((\det C_n)\bar{N})_n(z))w_n-(\det C_n(z))(\bar{N}_n(z)\xi_n)\\
&\quad-A^0(z)(\det C_n(z)-\det C(z))y_{n+d}-(\det C(z)-\det C_n(z))B^0(z)y^*_{r+d}\\
&\quad-A^0(z)(((F_n(\operatorname{adj} C_n))A)_n(z)-(F_n(\operatorname{adj} C_n)A_n)(z))D(z)y_{n+d}\\
&\quad-A^0(z)((F_n(\operatorname{adj} C_n)B_n)(z)-((F_n(\operatorname{adj} C_n))B)_n(z))D(z)u_n\\
&\quad+A^0(z)(((F_n(\operatorname{adj} C_n))C)_n(z)-F(z)(\operatorname{adj} C(z))C(z))w_{n+d}\\
&=(\det C(z))\bar{F}(z)w_{n+d}+\varepsilon_n,
\end{aligned}
\tag{77}
$$

where ε_n denotes the rest terms after the first equality of (77). Paying attention to (61) and (74) and by $\|v_n\|^2\leqslant\sigma^2/n^\varepsilon$ we have

$$\sum_{i=0}^{n}\|\varepsilon_i\|^2=O(n^{1-\varepsilon})+O\Big(\sum_{i=1}^{n+d}\frac{\log^2 i}{i^\beta}(\|\varphi_i^0\|^2+\|w_i\|^2+\|y_i\|^2+1)\Big),\tag{78}$$

where $\beta=1-(t+1)(\varepsilon+\delta)$.

We now show that the second term of (78) is also of order $O(n^{1-\varepsilon})$.

Set

$$S_i=\sum_{j=1}^{i}(\|\varphi_j^0\|^2+\|w_j\|^2+\|y_j\|^2+1),\quad \text{for } i\geqslant 1;$$

$$S_i=0,\ \text{for } i<1.$$

By (4) and (62) it is clear that $S_i=O(i^{1+\delta})$. Then, noticing

$$
\begin{aligned}
&(\log^2 i)(1+i)^\beta-i^\beta\log^2(1+i)\\
&=i^\beta(\log^2 i)\left(\left(1+\frac{1}{i}\right)^\beta-\left(1+\frac{\log\left(1+\frac{1}{i}\right)}{\log i}\right)^2\right)\\
&=i^\beta(\log^2 i)\left(\frac{\beta}{i}+o\left(\frac{1}{i}\right)\right)
\end{aligned}
$$

we find that

$$
\begin{aligned}
&\sum_{i=1}^{n}\frac{\log^2 i}{i^\beta}(\|\varphi_i^0\|^2+\|w_i\|^2+\|y_i\|^2+1)\\
&=\frac{\log^2 n}{n^\beta}S_n+\sum_{i=1}^{n-1}\left(\frac{\log^2 i}{i^\beta}-\frac{\log^2(i+1)}{(i+1)^\beta}\right)S_i\\
&=O\left(\frac{\log^2 n}{n^\beta}n^{1+\delta}\right)+O\left(\sum_{i=1}^{n-1}\frac{\log^2 i}{i(i+1)^\beta}i^{1+\delta}\right)=O(n^{1-\varepsilon})
\end{aligned}
$$

and

$$\sum_{i=1}^{n}\|\varepsilon_i\|^2=O(n^{1-\varepsilon}).\tag{79}$$

Since $\det C(z)$ is exponentially stable, from (77) and (79) we easily conclude that

$$\sum_{i=\tau_k}^{t_k}\|A^0(z)y_{i+d}\|^2=O(t_k).\tag{80}$$

Combining (73) and (80) by the stability of $A^0(z)$ we have

$$\sum_{i=0}^{t_k}\|y_{i+d}\|^2=O(t_k\log t_k)\tag{81}$$

and then from (49) we have

$$\|D(z)u_{t_k+1}^{(2)}\|^2=O\Big(t_k+\sum_{i=0}^{t_k+1}\|y_i\|^2+\sum_{i=0}^{t_k}\|D(z)u_i\|^2\Big)$$

$$=O\Big(t_k+\sum_{i=0}^{t_k+d}\|y_i\|^2\Big)=O(t_k\log t_k). \tag{82}$$

By the definition of t_k, from (82) we know that

$$t_k=\sigma_k-1. \tag{83}$$

Hence from (81)

$$\sum_{i=0}^{\sigma_k}\|y_i\|^2=O(\sigma_k\log\sigma_k).$$

This means that for all sufficiently large k

$$\sum_{i=\tau_k}^{\sigma_k}\|y_i\|^2\leqslant\sigma^{1+\delta}$$

which contradicts definition (55) for σ_k. Hence we are convinced that it is impossible that $\tau_k<\infty$, $\sigma_k<\infty$ for all k.

Step 3. By (83) we see that $\Lambda\supset[\tau_k,\sigma_k)$ for all k. Hence (71) implies that Λ^c is a finite set.

Now let k be large enough so that $\tau_k<\infty$, $\sigma_k=\infty$ and $t_k=\infty$ a.s. Then (77) is valid for all $n\geqslant\tau_k$. Hence

$$A^0(z)y_{n+d}-B^0(z)y^*_{n+d}=\bar F(z)w_{n+d}+HC^{n-\tau_k}x_{\tau_k+d}+H\sum_{j=\tau_k+d}^{n}C^{n-j}H^{\tau}s_j, \tag{84}$$

where

$$C=\underbrace{\begin{bmatrix}-c_1I & I & 0\cdots0 & 0\\ \vdots & 0 & I\ 0 & 0\\ \vdots & \vdots & \ddots & \\ \vdots & \vdots & 0 & I\\ -c_{mr}I & 0\cdots\cdots & 0 & 0\end{bmatrix}}_{m^2r},\quad H=\underbrace{[I0\cdots0]}_{m^2r},\ I=I_{m\times m},$$

and x_{τ_k} is defined by $(y_i, y_i^*, w_i, i\leqslant\tau_k)$. Notice that C^n exponentially tends to zero because det $C(z)$ is stable. Then by (79) and Lemma 2 of [5] from (84) we find that (59) is true, while (58) follows from (59), and (57) follows from (58) by using the stability of $A^0(z)$ and $B(z)$. Hence (62) holds for $\delta=0$. Finally, setting $\delta=0$ in (61) leads to (60).

§ 5. Conclusion

We have discussed the stochastic adaptive control problem for systems with noise being an ARMA process. When identification of system parameters is not concerned, the adaptive control problem is relatively simply solved in Section 3 by applying an implicit approach. The problem of simultaneously identifying system parameters and controlling the plant is much more complicated. To solve this problem by means of an explicit approach we have designed in Section 4 an optimal adaptive control with flexible structure. The adaptive control is essentially based on the ELS identification which gives a good convergence property, but it may be switched to the one based on the stochastic gradient algorithm for a finite number of times in order to guarantee the stability of the closed-loop system. Developing an adaptive control that is based on one algorithm only with convergence property as good as the ELS one belongs to further research.

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