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Global stability of systems with amplitude and rate saturation compensation

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SUMMARY

Rate and amplitude saturation of the actuator is common in practical control systems. When the actuator is rate and amplitude saturated, the control performance can deteriorate rapidly, and in the worse case, the closed-loop system can become unstable. It is therefore important that both types of saturations are properly compensated. Following the approach for compensating amplitude constraints, a scheme for compensating systems with both rate and amplitude saturation is proposed in this paper. The conditions for the compensated system to be globally stable are derived, and from this result, a procedure for designing the rate and amplitude saturation compensators is devised. As it is difficult to design both the rate and the amplitude saturation compensators simultaneously, a two-step approach is adopted. In the proposed compensator design procedure, the amplitude saturation compensator is designed first, followed by the rate saturation compensator. As the compensators designed using the proposed procedure satisfy the conditions for global stability, the compensated system is therefore globally stable. It is also shown that these compensators can be designed using the LMI technique. The implementation of the design procedure is demonstrated by a simulation example. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS: global stability; amplitude and rate saturation; saturation compensation; strictly positive real control

1. INTRODUCTION

Rate and amplitude constraints are common in practical control systems. The amplitude constraint often arises from the physical size of the actuator, whilst the inertia of the driving mechanism in the actuator often leads to rate constraint. It is well known that if the actuator is amplitude saturated, the control performance can deteriorate rapidly, and the closed-loop system can even be unstable in the worst case. Clearly, the addition of rate saturation will in general make the situation worse, not better. It is, therefore, important to devise suitable compensation methods to ensure the closed-loop system remain stable when amplitude and/or rate saturation occur. It is shown in Reference [1] that a linear system subject to amplitude

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saturation of the actuator can be globally asymptotically stabilized, if and only if it is asymptotically null controllable with bounded controls. However, this result does not in general apply to linear feedback systems subject to amplitude constraint [2–4]. Several feedback control laws that ensure these nonlinear systems are globally asymptotically stable are proposed in References [5, 6]. In References [7–9], feedback control laws utilizing the low and high gain technique are derived, such that these nonlinear systems are semi-globally stable. Based on the singular perturbation approach, dynamical feedback control laws that tracks the set point subject to the maximum allowable control rate are proposed in Reference [10]. Similar to systems with only amplitude saturation, there are two approaches to compensate for systems subject to both amplitude and rate constraints. In the first approach, the controllers are designed taking into account both constraints [11–18]. A simple approach to tackle the saturation problem is to introduce a supervisory loop in the control system for adjusting the gain of the controller such that no rate and amplitude saturation of actuator occur [11–14]. A drawback of this approach is that the closed-loop response is likely to be sluggish, as the control tends to be very conservative. It is shown in Reference [15] that low gain feedback control laws can be designed, such that linear systems subject to both amplitude and rate saturation closed-loop system are semi-global stabilized. A continuous-time predictive control via state feedback for single-input single-output linear systems with actuator amplitude and rate saturation was derived in Reference [16], and the local stability of the closed-loop system is analysed using the Lyapunov function. By applying the separation principle and using a deadbeat observer for state estimation, it is shown in Reference [17] that global asymptotic stability and offset-free tracking of admissible set-point references are achieved for discrete-time systems without violating the constraints. An approach to design LQG-type of fixed-structure optimal control law that guarantees the domain of attraction for linear systems with independent amplitude and rate saturations is proposed in Reference [18]. In the second approach, the rate and the amplitude saturation compensators are designed separately [19–21] similar to that for compensating amplitude saturation [4], after the controller is designed assuming no saturation. The main advantage of this approach is that the closed-loop system can achieve the designed performance when there is no actuator saturation, as saturation compensators are activated only after saturation has occurred. Further, only linear control systems design techniques are required to design the controller and the saturation compensators [20]. The sequence of saturation in systems with both rate and amplitude constraints is important, as it leads to different analysis of the performance and stability of the closed-loop system. For physical systems, rate saturation is more likely to occur before amplitude saturation. This is because amplitude saturation only occurs after the actuator has reaches its limits. Before reaching this limit, the actuator is likely to be rate saturated first, especially when the change in control is large. Therefore, following the discussions in References [19–22], it is assumed here that rate constraint precedes amplitude constraint. By extending the results on the compensation for amplitude saturation, a compensation scheme for both rate and amplitude constraints is proposed in References [19–22]. Though some design guidelines for the rate and amplitude saturation compensators are proposed in these papers, only few stability results of the compensated system are presented there, and little results can be found elsewhere in the literature. For this reason, the main aim of this paper is to derive the global stability conditions for systems with both rate and amplitude compensation. The organization of the paper is as follows. In Section 2, the system with rate and amplitude constraints is presented. The model of the actuator with both constraints is then discussed, followed by the derivation of the compensators for these constraints. The conditions for global stability of the systems with rate and amplitude compensation are derived in Section 3. The design of both the rate and amplitude compensators that satisfy the global stability conditions are devised using the LMI in Section 4. The results presented in this paper are illustrated by a simulation example presented in Section 5.

2. COMPENSATION OF RATE AND AMPLITUDE CONSTRAINTS

Denote $R^{m\times n}$ the set of all real matrices with m rows by n columns, R^n the set of all real n dimension vector, and A' the transpose of matrix A.

Consider a linear multi-input multi-output (MIMO) system P described by the following minimal state-space realization.

$$\begin{aligned} \dot{x}_p(t) &= Ax_p(t) + Bu(t) \\ y(t) &= Cx_p(t) \end{aligned} \tag{1}$$

where $x_p(t) \in R^n$ is the system state, $u(t) \in R^m$ is the actuator output, $y(t) \in R^p$ is the system output. $A \in R^{n\times n}, B \in R^{n\times m}$ and $C \in R^{p\times n}$ are constant matrices. From (1), the transfer function matrix of the system P is $T_p(s) = C(sI - A)^{-1}B$.

Assume that the following linear controller K has been designed such that the closed-loop system without rate and amplitude saturation is asymptotically stable.

$$\begin{aligned} \dot{x}_k(t) &= Fx_k(t) + Ge(t) \\ v(t) &= Hx_k(t) \\ e(t) &= w(t) - y(t) \end{aligned} \tag{2}$$

where $w(t) \in R^P$ is the reference input, $v(t) \in R^m$ is the controller output, $x_k(t) \in R^q$ is the controller state. $F \in R^{q\times q}, G \in R^{q\times p}$ and $H \in R^{m\times q}$ are constant matrices, and the transfer function matrix of the controller is $K(s) = H(sI - F)^{-1}G$.

The rate and amplitude constraints for the actuator are given by

$$u(t) = \mathrm{sat}[v(t)] = (\mathrm{sat}[v_1(t)] \;\; \mathrm{sat}[v_2(t)] \;\; \cdots \;\; \mathrm{sat}[v_m(t)])'$$

and

$$\dot{u}(t) = \mathrm{sat}[\dot{v}(t)] = (\mathrm{sat}[\dot{v}_1(t)] \;\; \mathrm{sat}[\dot{v}_2(t)] \;\; \cdots \;\; \mathrm{sat}[\dot{v}_m(t)])'$$

where

$$\mathrm{sat}[v_i(t)] = \begin{cases} u_i^+, & v_i > u_i^+ \\ v_i(t), & u_i^- \leqslant v_i \leqslant u_i^+ \\ u_i^- & v_i < u_i^- \end{cases}$$