Robust Stabilization of LTI Systems via Indirect Adaptive Controllers

Ji Feng Zhang

Abstract—**This paper presents a robust indirect adaptive stabilization controller for systems with linear time-invariant (LTI) nominal parts, uncertain dynamic errors, and bounded disturbances. The *a priori* knowledge required for designing the adaptive controller is that the nominal part of the system is of known order and is stabilizeable.**

I. Introduction

Assume the single-input, single-output system is described by the following difference equation

$$A(z^{-1})y(t) = B(z^{-1})u(t) + \eta(t) + w(t) \tag{1.1}$$

where $y(t)$, $u(t)$, $\eta(t)$, and $w(t)$ are system output, input, uncertain dynamic error, and disturbance, respectively; $A(z^{-1})$ and $B(z^{-1})$ are polynomials in backward shift operator z^{-1}

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n},$$
$$B(z^{-1}) = b_1 z^{-1} + \cdots + b_n z^{-n}$$

with the unknown coefficient vector

$$\theta \triangleq [-a_1, \cdots, -a_n, \quad b_1, \cdots, b_n]^\tau.$$

The order n of the nominal part of the system is assumed to be known. The initial conditions $u(-1), \cdots, u(-n)$, $y(-1), \cdots, y(-n)$ for the system are arbitrary and may be unknown.

The adaptive control problems of system (1.1) have intensively been investigated in the past few decades, and there are a vast references, for instance, [1]–[10]. The conditions usually used on uncertain dynamic error $\eta(t)$ and disturbance $w(t)$ are as follows.

a) $\eta(t) \equiv w(t) \equiv 0$. In this case, the adaptive regulation problem is considered under the assumption that $A(z^{-1})$ and $B(z^{-1})$ are coprime (e.g., [5], [6]).
b) $\eta(t) \equiv 0$ and i) $\sup_{t\geq 0} |w(t)| < \infty$ or ii) $\{w(t)\}$ is a stochastic sequence possessing some stochastic properties, for example, $\{w(t), \mathcal{F}_t\}$ is a martingale difference sequence with $\sup_{t\geq 0} E(|w(t+1)|^\beta | \mathcal{F}_t) < \infty$ where $\beta > 2$ is a constant, and $\{\mathcal{F}_t\}$ is a family of nondecreasing σ-algebras; in this case, $\sup_{t\geq 0}(t+1)^{-1}\sum_{i=0}^t [w(i)]^2 < \infty$. For case i), adaptive stabilization algorithms are given in, for instance, [4] and [7]. While for case ii), adaptive LQG, tracking and stabilization controls are investigated in, for instance, [1]–[3] and [10].
c) $\eta(t)$ is small in some sense, and an upper bound for $\sup_{t\geq 0} |w(t)|$ is known. In this case, adaptive robust adaptive regulation algorithms are developed in, for instance, [8] and [9].
   In addition to that the order n is known and $A(z^{-1})$, $B(z^{-1})$ are coprime, more assumptions on the nominal part of system (1.1) are usually required. As examples, we list the following conditions:
d) $A(z^{-1})$ or $z^d B(z^{-1})$ is stable (e.g., [1]–[3], [10]).
e) A special area of the unknown parameter θ is available (e.g., [7], [10]).
f) A "partial state" form of system (1.1) is required (e.g., [6], [8], [9]).
g) On the parameter estimates, some conditions are imposed without verifying (e.g., [4]).

Recently, for the case where Conditions c) and f) mentioned above hold, Giri *et al.* [9] attacked this problem again to obtain a robust adaptive stabilization control via minimal *a priori* knowledge, i.e., only requiring that n is known and $A(z^{-1})$, $B(z^{-1})$ are coprime. There is, however, a circularity hidden in the argument as pointed out in [11].

The purpose of this paper, as in [9], is to give an adaptive controller, which stabilizes the closed-loop system with minimal *a priori* knowledge about the system structure, uncertain dynamic error, and disturbances. Precisely, throughout the paper, we assume only that

A.1) $A(z^{-1})$ and $B(z^{-1})$ are coprime.
A.2) $|\eta(t)| \leq \mu \sum_{i=0}^{t-1} \sigma^{t-i-1}(|y(i)| + |u(i)|)$, for some constant $\sigma \in [0, 1)$.
A.3) $\sup_{t\geq 0} \frac{1}{t+1}\sum_{i=0}^t [w(i)]^2 < \infty$.

It is worth emphasizing that no upper bound on $w(t)$ is required to be available, system (1.1) may have both uncertain dynamic errors and bounded disturbances, and none of Assumptions d)–g) (mentioned above) is needed.

The remainder of the paper is arranged as follows. Section II gives some preliminary results. Section III presents an adaptive controller which, under Assumptions A.1)–A.3), forces the input and output sequences of the closed-loop system to be bounded in the following averaging sense

$$\sup_{t\geq 0} \frac{1}{t+1}\sum_{k=0}^t (|y(k)|^2 + |u(k)|^2) < \infty. \tag{1.2}$$

Finally, Section IV gives some concluding remarks.

II. Preliminary Results

In the sequel, the norm $\|\cdot\|$ of a polynomial $X(z^{-1}) = \sum_{i=0}^\mu x_i z^{-i}$ is defined by $\|X(z^{-1})\| = (\sum_{i=0}^\mu |x_i|^2)^{1/2}$.

It is well known that under Assumption A.1) there exist two polynomials

$$G(z^{-1}) = 1 + \sum_{j=1}^{n-1} g_j z^{-j}, \quad H(z^{-1}) = \sum_{j=0}^{n-1} h_j z^{-j} \tag{2.1}$$

such that

$$A(z^{-1})G(z^{-1}) - B(z^{-1})H(z^{-1}) = 1. \tag{2.2}$$

From the fact that the coefficients of $G(z^{-1})$ and $H(z^{-1})$ are continuous functions of the coefficients of $A(z^{-1})$ and $B(z^{-1})$, it is not difficult to see the following.

Lemma 2.1: Under Assumption A.1), there exists a small positive number ν such that for any fixed $\theta' \in \Theta \triangleq \{\theta'' : \|\theta'' - \theta\| \leq \nu, \theta'' \in \mathbf{R}^{2n}\}$, the equation

$$A(\theta', z^{-1})G(\theta', z^{-1}) - B(\theta', z^{-1})H(\theta', z^{-1}) = 1$$

has a unique solution $(G(\theta', z^{-1}), H(\theta', z^{-1}))$ such that

$$\deg(G(\theta', z^{-1})) \leq n-1, \quad \deg(H(\theta', z^{-1})) \leq n-1,$$
$$\|G(\theta', z^{-1})\|^2 + \|H(\theta', z^{-1})\|^2 \leq 2(\|G(z^{-1})\|^2 + \|H(z^{-1})\|^2)$$

and

$$A(z^{-1})G(\theta', z^{-1}) - B(z^{-1})H(\theta', z^{-1}) \text{ is stable.}$$

Manuscript received September 29, 1994. This work was supported in part by Canadian NSERC grant A 1329 and the National Natural Science Foundation of China.

The author is with the Institute of Systems Science, Chinese Academy of Sciences, P. R. China.

IEEE Log Number 9408804.

0018–9286/95$04.00 © 1995 IEEE

Here $(G(z^{-1}), H(z^{-1}))$ is the unique solution of (2.2) subject to (2.1), and

$$A(\theta', z^{-1}) = 1 + a_1' z^{-1} + \cdots + a_n' z^{-n},$$
$$B(\theta', z^{-1}) = b_1' z^{-1} + \cdots + b_n' z^{-n}$$

where a_j' and b_j', $j = 1, 2, \ldots, n$, are the components of θ', i.e., $\theta' = [a_1', \cdots, a_n', b_1', \cdots, b_n']^\tau$.

This lemma tells us that to get a stabilization controller, it suffices to require that the parameter estimate be located in a sufficiently small neighborhood of the true parameter θ.

In this paper, we will use the least-squares algorithm to generate the parameter estimates. To do this, we rewrite system (1.1) as

$$y(t) = \theta^\tau \varphi(t-1) + \eta(t) + w(t) \tag{2.3}$$

where

$$\varphi(t) = [y(t), \cdots, y(t-n+1), u(t), \cdots, u(t-n+1)]^\tau. \tag{2.4}$$

The least-squares (LS) algorithm is as follows

$$\theta(t) = \left[P(0) + \sum_{i=0}^{t-1} \varphi(i)\varphi^\tau(i)\right]^{-1} \times \sum_{i=0}^{t-1} \varphi(i) y(i+1), \quad \forall t \geq t_0 \tag{2.5}$$

or recursively

$$\theta(t+1) = \theta(t) + [1 + \varphi^\tau(t) P(t) \varphi(t)]^{-1} \times P(t)\varphi(t)[y(t+1) - \varphi^\tau(t)\theta(t)], \tag{2.6}$$

$$P(t+1) = P(t) - [1 + \varphi^\tau(t) P(t)\varphi(t)]^{-1} \times P(t)\varphi(t)\varphi^\tau(t) P(t) \tag{2.7}$$

with $P(t) = [P(0) + \sum_{i=0}^{t-1} \varphi(i)\varphi^\tau(i)]^{-1}$, where the initial value $P(0)$ is chosen such that

$$P(0) + \sum_{i=0}^{t_0-1} \varphi(i)\varphi^\tau(i) > 0, \quad \text{for some } t_0.$$

For the parameter estimates given by LS algorithm (2.6) and (2.7), the following lemma offers a general accuracy evaluation.

Lemma 2.2: Consider the plant (1.1) with Assumptions A.2) and A.3) and the least-squares estimator (2.6), (2.7). We have

$$\|\tilde{\theta}(t+1)\|^2 \leq \frac{C_1(t+1) + C_1 \mu^2 r(t)}{\lambda_{\min}^{(t)}}, \quad \forall t \geq 0 \tag{2.8}$$

where $\tilde{\theta}(t) = \theta - \theta(t)$, $r(t) = \alpha^t + \sum_{i=0}^t \|\varphi(i)\|^2$ with $\alpha > 1$ arbitrarily chosen, $\lambda_{\min}^{(t)}$ denotes the minimum eigenvalue of $P^{-1}(t+1)$ and C_1 is a constant independent of μ.

Proof: See Appendix A. □

From (2.8) it is easy to see that the less $(t+1)/\lambda_{\min}^{(t)}$ and $r(t)/\lambda_{\min}^{(t)}$ are, the more accurate the parameter estimate $\theta(t+1)$ is. Since both $r(t)$ and $\lambda_{\min}^{(t)}$ depend upon the system input sequence $\{u(t)\}$, and $r(t)/\lambda_{\min}^{(t)} \geq 1$, when one chooses $u(t)$ with the aim to get a satisfactory estimate for the unknown parameter θ, a compromise is required so as to insure that $r(t)$ does not diverge too fast and $\lambda_{\min}^{(t)}$ diverges fast enough. Lemma 2.3 below gives an example for choosing such kind of $u(t)$.

Lemma 2.3: Under Assumptions A.1)–A.3), if $r(t)$ is defined as in Lemma 2.2 and $u(t)$ is defined by

$$u(t) = \begin{cases} [r(t-1)]^{1/2}, & \text{if } t = 2nk + n, \quad k = 0, 1, 2, \cdots, \\ 0, & \text{otherwise} \end{cases} \tag{2.9}$$

then there exists a $\mu_0 > 0$ such that for any fixed μ: $0 \leq \mu \leq \mu_0$

$$\limsup_{t \to \infty} \|\tilde{\theta}(t)\| \leq C_2 \mu \tag{2.10}$$

where C_2 is a constant independent of μ.

Proof: See Appendix B. □

Corollary 2.1: Suppose that Assumption A.1) holds, $\eta(t) \equiv 0$ and $|w(t)| \leq \bar{w}$ for all $t \geq 0$. Let $u(t)$ be defined by (2.9), and $r(t) \equiv r(0) > 0$ with $r(0)$ arbitrary or $r(t)$ be given as in Lemma 2.2. If $\theta(t)$ is given by the LS algorithm (2.5) with $P(0) = 0$ and $t_0 = 5n$, then there exists a $w^* > 0$ such that for any $\bar{w} \in [0, w^*)$, $\|\tilde{\theta}(5n)\| \leq C_0 \bar{w}$, where C_0 is a constant independent of $\bar{w}$.

Proof: Following Step 2 of Appendix B, it is not difficult to see that there exists a constant $w^* > 0$ such that for any $\bar{w} \in [0, w^*]$

$$\lambda_{\min}\left(\sum_{i=0}^{5n-1} \varphi(i)\varphi^\tau(i)\right) \geq \lambda_{\min}\left(\sum_{i=2n}^{5n-1} \varphi(i)\varphi^\tau(i)\right) \geq C_0' r(3n-1) > 0 \tag{2.11}$$

where C_0' is a constant independent of μ.

Noticing (2.3) and the fact that $\eta \equiv 0$, by (2.5) we have

$$\tilde{\theta}(5n) = \left[\sum_{i=0}^{5n-1} \varphi(i)\varphi^\tau(i)\right]^{-1} \sum_{i=0}^{5n-1} \varphi(i) w(i+1)$$

which, together with (2.11) and $|w(t)| \leq \bar{w}$, implies

$$\|\tilde{\theta}(5n)\| \leq \left[(C_0' \alpha^{3n-1})^{-1} \sum_{i=0}^{5n-1} \|\varphi(i)\|\right] \bar{w}.$$

Let $C_0 = (C_0' \alpha^{3n-1})^{-1} \sum_{i=0}^{5n-1} \|\varphi(i)\|$. Then following Step 1 of Appendix B we see that C_0 is a constant depending on w^*, but independent of $\bar{w}$. □

Remark 2.1: This corollary tells us that in the case where the system disturbances are small, one can use a small excitation to get a "precise enough" estimate for the system parameter.

III. Stabilization in the Averaging Sense

In this section, we will find a way to construct an adaptive control which stabilizes the closed-loop system under Assumptions A.1)–A.3).

For convenience, write $\theta(t)$ in the component form

$$\theta^\tau(t) = [-a_1(t) \cdots - a_n(t) \quad b_1(t) \cdots b_n(t)], \quad \forall t \geq 0$$

and let

$$A_t(z^{-1}) = 1 + a_1(t) z^{-1} + \cdots + a_n(t) z^{-n},$$
$$B_t(z^{-1}) = b_1(t) z^{-1} + \cdots + b_n(t) z^{-n}.$$

Definition 3.1: By the statement that the following Bezout equation

$$A_t(z^{-1}) G_t(z^{-1}) - B_t(z^{-1}) H_t(z^{-1}) = 1 \tag{3.1}$$

is solvable we mean that there exists a unique pair of $(G_t(z^{-1}), H_t(z^{-1}))$ such that (3.1) holds and

$$G_t(z^{-1}) = 1 + \sum_{k=1}^{n-1} g_k(t) z^{-k}, \quad H_t(z^{-1}) = \sum_{j=0}^{n-1} h_j(t) z^{-j}$$

for some real numbers $g_k(t)$, $k = 1, \cdots, n-1$ and $h_j(t)$, $j = 0, \cdots, n-1$.

From (2.10) of Lemma 2.3 we see that if the uncertain dynamic error is small enough, i.e., μ is small enough, then the estimate value $\theta(t)$ given by the LS algorithm (2.6) and (2.7) is indeed located in a small neighborhood of the true parameter value θ, provided that the input $u(t)$ defined by (2.9) is applied. This naturally leads us to design the adaptive stabilization control along the following lines:

A) Introduce an appropriate criterion to judge whether or not the parameter estimate is satisfactory;
B) Apply an excitation signal to the system, simultaneously estimate the unknown parameters via least-squares algorithm until a "satisfactory" estimate is obtained according to the criterion;
C) Construct a control law via the obtained "satisfactory" parameter estimates, use this control law to control the system until some "unsatisfactory" phenomenon appears and then turn to B); if there is no "unsatisfactory" phenomenon in step C), then the designed adaptive control law is used for ever.

More precisely, the adaptive control is given by

$$u(t) = \begin{cases} [r(t-1)]^{1/2}, \text{if } t \in [\tau_i, \sigma_{i+1}) \text{ and } t = \tau_i + 2nk + n \\ \quad \text{for some } i \geq 0 \text{ and } k \geq 0; \\ 0, \quad \text{if } t \in [\tau_i, \sigma_{i+1}) \text{ for some } i \geq 0, \\ \quad \text{but } t \neq \tau_i + 2nk + n \text{ for any } k \geq 0; \\ H_{\sigma_i}(z^{-1}) y(t) - (G_{\sigma_i}(z^{-1}) - 1) u(t), \\ \quad \text{if } t \in [\sigma_i, \tau_i] \text{ for some } i \geq 1 \end{cases} \tag{3.2}$$

where $\{\tau_i\}$ and $\{\sigma_i\}$ are the "switching time" sequences defined as follows: $\tau_0 = 0$, and for any $i \geq 1$

$$\sigma_i = \min\Big\{ t \geq \tau_{i-1} + 3n\text{: (3.1) is solvable, and}$$
$$\sum_{k=0}^{t} (y(k) - \varphi^\tau(k-1)\theta(t))^2 \leq [\gamma(t)]^2 s_t(\alpha^t)$$

where

$$\gamma(t) = \frac{1}{2n[\|G_t(z^{-1})\|_2^2 + \|H_t(z^{-1})\|_2^2]} \Big\} \tag{3.3}$$

$$\tau_i = \min\Big\{ t > \sigma_i : \sum_{k=0}^{t} (y(k) - \varphi^\tau(k-1)\theta(\sigma_i))^2 > [\gamma(\sigma_i)]^2 s_t(\alpha^{\sigma_i}) \Big\} \tag{3.4}$$

where $s_t(x)$ is given by $s_0(x) = 1$

$$s_t(x) = t \times \max\left\{ x, \frac{1}{k}\sum_{j=0}^{k-1}(y_j^2 + u_j^2),\ k = 1, \cdots, t \right\}, \quad \forall t \geq 1. \tag{3.5}$$

Remark 3.1: In the procedure (3.2)–(3.5) constructing adaptive control, a term α^t is introduced. The idea comes from the fact that to get a "precise enough" parameter estimate, a "large enough" signal/noise ratio is required. From the conditions of the paper, however, we do not know how strong the system disturbance is. So, an increasing term like α^t is needed to guarantee a proper signal/noise ratio for any bounded disturbances.

Remark 3.2: Unlike in [8] and [9], no normalized factor is needed in the procedure of estimating unknown parameter θ and of constructing adaptive control, so the number σ appearing in Assumption A.2) is not required to be available.

Theorem 3.1: Under Assumptions A.1)–A.3), there exists a $\mu^* > 0$ such that for any fixed μ: $0 \leq \mu \leq \mu^*$, the adaptive control law defined by (3.2)–(3.5) stabilizes the closed-loop system in the averaging sense of (1.2).

Proof: We divide the argument into two steps. □

Step 1: Show that there exists an integer $i_0 \geq 1$ such that $\sigma_{i_0} < \infty$ and $\tau_{i_0} = \infty$.

We first prove that it is impossible that $\tau_{i_0'} < \infty$ and $\sigma_{i_0'+1} = \infty$ for some integer $i_0' \geq 0$. In fact, if there were an $i_0' \geq 0$ such that $\tau_{i_0'} < \infty$ and $\sigma_{i_0'+1} = \infty$, then by (3.2) we would have

$$u(t) = \begin{cases} [r(t-1)]^{1/2}, & \text{if } t = \tau_{i_0'} + 2nk + n \\ & \text{for } k = 0, 1, \cdots, \\ 0, & \text{if } t \geq \tau_{i_0'}, \text{ but } t \neq \tau_{i_0'} + 2nk + n \\ & \text{for any } k \geq 0. \end{cases} \tag{3.6}$$

In this case, by Lemma 2.3 we conclude that there exist constants $\mu_0 > 0$ and T_1, which depends on μ_0 only, such that for any fixed μ: $0 \leq \mu \leq \mu_0$

$$\|\tilde{\theta}(t)\| \leq 2C_2\mu, \quad \forall t \geq T_1 \tag{3.7}$$

where C_2 is a constant depending on μ_0, but independent of μ.

By Lemma 2.1 and Assumption A.1) it is easy to see that there exists $\mu_1 \in (0, \mu_0]$, which depends on C_2, n and $\|G(z^{-1})\|^2 + \|H(z^{-1})\|^2$ only, such that for any fixed μ: $0 \leq \mu \leq \mu_1$

$$\|\tilde{\theta}(t)\|^2 < \frac{1}{32n^3[\|G(z^{-1})\|^2 + \|H(z^{-1})\|^2]^2}, \quad \forall t \geq T_1 \tag{3.8}$$

and simultaneously, such that (3.1) is solvable and the solution $(G_t(z^{-1}), H_t(z^{-1}))$ satisfies

$$\|G_t(z^{-1})\|^2 + \|H_t(z^{-1})\|^2 \leq 2[\|G(z^{-1})\|^2 + \|H(z^{-1})\|^2]$$

where $(G(z^{-1}), H(z^{-1}))$ is the unique solution of (2.2) subject to (2.1). This leads to

$$\gamma(t) = \frac{1}{2n[\|G_t(z^{-1})\|^2 + \|H_t(z^{-1})\|^2]} \geq \frac{1}{4n[\|G(z^{-1})\|^2 + \|H(z^{-1})\|^2]} \triangleq \gamma. \tag{3.9}$$

By (2.3), Assumptions A.2)–A.3) and (3.8) we get that

$$\begin{aligned} &\sum_{k=0}^{t} (y(k) - \varphi^\tau(k-1)\theta(t))^2 \\ &\leq \|\tilde{\theta}(t)\|^2 \sum_{k=0}^{t-1} \|\varphi(k)\|^2 + C_3(t+1) + C_3\mu^2 \sum_{k=0}^{t-1} [y^2(k) + u^2(k)] \\ &\leq \|\tilde{\theta}(t)\|^2 n \sum_{k=0}^{t-1} [y^2(k) + u^2(k)] + C_3'(t+1) + C_3\mu^2 \sum_{k=0}^{t-1} [y^2(k) + u^2(k)] \\ &\leq \left(\frac{1}{2}\gamma^2 + C_3\mu^2\right) \sum_{k=0}^{t-1} [y^2(k) + u^2(k)] + C_3'(t+1), \quad \forall t \geq T_1 \end{aligned} \tag{3.10}$$

where C_3 and C_3' are constants independent of μ.

Recalling that $\alpha > 1$ implies that $\lim_{t\to\infty} \frac{\alpha^t}{t+1} = \infty$, by (3.5) and (3.9) we see that there exist $\mu_2 \in (0, \mu_1]$, which depends on γ and C_3 only, and T_2, which depends on γ^{-1}, C_3' and α only, such that for any fixed μ: $0 \le \mu \le \mu_2$

$$\sum_{k=0}^{T_2}(y(k) - \varphi^\tau(k-1)\theta(T_2))^2 \le \gamma^2 s_{T_2}(\alpha^{T_2}) \le [\gamma(T_2)]^2 s_{T_2}(\alpha^{T_2})$$

which contradicts the assumption that $\sigma_{i_0'+1} = \infty$.

We now prove that there exists an integer $i_0 \ge 1$ such that $\sigma_{i_0} < \infty$ and $\tau_{i_0} = \infty$.

In fact, if $\sigma_i < \infty$ and $\tau_i < \infty$ for all integer i, then similar to Step 1 in Appendix B we would obtain that

$$\sup_{i\ge 1}\ \sup_{t\in(\tau_{i-1},\sigma_i)} r(t)/r(t-1) < \infty.$$

In this case, by the argument used in Step 2 of Appendix B we can prove that there exists a constant C_4 (independent of μ) such that

$$\lambda_{\min}^{(\sigma_i-1)} \ge C_4 r(\sigma_i - 1)$$

which together with Lemma 2.2 and the definition of $r(t)$ yields

$$\|\tilde\theta(\sigma_i)\|^2 \le C_4'\sigma_i/\alpha^{\sigma_i} + C_4'\mu^2 \tag{3.11}$$

where C_4' is a constant independent of μ.

From this and by the argument similar to that for proving (3.10) one gets

$$\sum_{k=0}^{t}(y(k) - \varphi^\tau(k-1)\theta(\sigma_i))^2 \le n(C_4'\sigma_i/\alpha^{\sigma_i} + C_4'\mu^2)\sum_{k=0}^{t-1}[y^2(k) + u^2(k)] + C_3'(t+1). \tag{3.12}$$

Therefore, there exist i_0'' and $\mu_3 \in (0, \mu_2]$, both of which depend on γ, C_4', C_3', n and α only, such that for any $i \ge i_0''$ and any fixed μ: $0 \le \mu \le \mu_3$, one has $\gamma(\sigma_i) \ge \gamma$ and

$$\sum_{k=0}^{t}(y(k) - \varphi^\tau(k-1)\theta(\sigma_i))^2 \le \gamma^2 s_t(\alpha^{\sigma_i}) \le [\gamma(\sigma_i)]^2 s_t(\alpha^{\sigma_i}), \quad \forall t > \sigma_i.$$

This contradicts that $\tau_i < \infty$ for all integer i. Thus, there must be an integer i_0 such that $\sigma_{i_0} < \infty$ and $\tau_{i_0} = \infty$.

Step 2: Recalling the fact that $\sigma_{i_0} < \infty$ and $\tau_{i_0} = \infty$ for some integer $i_0 \ge 1$, by (3.1) and (3.3) we have

$$\begin{aligned} y(t) &= G_{\sigma_{i_0}}(z^{-1})[A_{\sigma_{i_0}}(z^{-1})y(t) - B_{\sigma_{i_0}}(z^{-1})u(t)] \\ &\quad + B_{\sigma_{i_0}}(z^{-1})[G_{\sigma_{i_0}}(z^{-1})u(t) - H_{\sigma_{i_0}}(z^{-1})y(t)], \\ u(t) &= H_{\sigma_{i_0}}(z^{-1})[A_{\sigma_{i_0}}(z^{-1})y(t) - B_{\sigma_{i_0}}(z^{-1})u(t)] \\ &\quad + A_{\sigma_{i_0}}(z^{-1})[G_{\sigma_{i_0}}(z^{-1})u(t) - H_{\sigma_{i_0}}(z^{-1})y(t)]. \end{aligned}$$

Hence, from (3.2) we get, for any $t \ge T_0 \triangleq \sigma_{i_0} + n$

$$y(t) = G_{\sigma_{i_0}}(z^{-1})[A_{\sigma_{i_0}}(z^{-1})y(t) - B_{\sigma_{i_0}}(z^{-1})u(t)], \tag{3.13}$$
$$u(t) = H_{\sigma_{i_0}}(z^{-1})[A_{\sigma_{i_0}}(z^{-1})y(t) - B_{\sigma_{i_0}}(z^{-1})u(t)]. \tag{3.14}$$

From (3.13), (3.14), (3.9), and (3.4) it follows that for any $t \ge T_0$

$$\begin{aligned} &\frac{1}{t}\sum_{j=0}^{t}(y^2(j) + u^2(j)) \\ &= \frac{1}{t}\sum_{j=T_0}^{t}(y^2(j) + u^2(j)) + \frac{1}{t}\sum_{j=0}^{T_0-1}(y^2(j) + u^2(j)) \\ &\le \frac{n}{t}(\|G_{\sigma_{i_0}}(z^{-1})\|^2 + \|H_{\sigma_{i_0}}(z^{-1})\|^2) \times \sum_{j=0}^{t}(y(j) - \varphi^\tau(j-1)\theta(\sigma_{i_0}))^2 + \frac{1}{t}\sum_{j=0}^{T_0-1}(y^2(j) + u^2(j)) \\ &= \frac{1}{\gamma(\sigma_{i_0})}\cdot\frac{1}{t}\sum_{j=0}^{t}(y(j) - \varphi^\tau(j-1)\theta(\sigma_{i_0}))^2 + C_5 \\ &\le \gamma(\sigma_{i_0})\frac{s_t(\alpha^{\sigma_{i_0}})}{t} + C_5 \end{aligned} \tag{3.15}$$

where $C_5 = \sum_{j=0}^{T_0-1}(y^2(j) + u^2(j))$.

Noticing that $t^{-1}s_t(\alpha^{\sigma_{i_0}})$ is nondecreasing, from (3.15) we get for any $t \ge T_0$ and any $l \in [T_0, t]$

$$\frac{1}{l}\sum_{j=0}^{l-1}(y^2(j) + u^2(j)) \le \gamma(\sigma_{i_0})\frac{s_l(\alpha^{\sigma_{i_0}})}{l} + C_5 \le \gamma(\sigma_{i_0})\frac{s_t(\alpha^{\sigma_{i_0}})}{t} + C_5$$

which together with (3.5) yields

$$\frac{s_t(\alpha^{\sigma_{i_0}})}{t} \le \max\left\{\alpha^{\sigma_{i_0}}; \frac{1}{l}\sum_{j=0}^{l-1}(y^2(j) + u^2(j)), l = 1, \cdots, T_0 - 1; \gamma(\sigma_{i_0})\frac{s_t(\alpha^{\sigma_{i_0}})}{t} + C_5\right\}. \tag{3.16}$$

Set

$$C_6 = \alpha^{\sigma_{i_0}} + C_5 + \max\left\{\frac{1}{l}\sum_{j=0}^{l-1}(y^2(j) + u^2(j)), \quad l = 1, \cdots, T_0 - 1\right\}.$$

Then (3.16) implies that for any $t \ge 1$

$$t^{-1}s_t(\alpha^{\sigma_{i_0}}) \le \gamma(\sigma_{i_0})t^{-1}s_t(\alpha^{\sigma_{i_0}}) + C_6$$

which means

$$t^{-1}s_t(\alpha^{\sigma_{i_0}}) \le (1 - \gamma(\sigma_{i_0}))^{-1}C_6$$

i.e., $t^{-1}s_t(\alpha^{\sigma_{i_0}})$ is bounded. Hence, the input and output sequences of the closed-loop system are bounded in the sense of (1.2).

IV. Concluding Remarks

For system (1.1), a robust indirect adaptive stabilization controller is designed. The construction of the controller is characterized by an excitation signal generated by the closed-loop system and an appropriate time splitting. The only *a priori* knowledge required for designing the adaptive controller is that the nominal part of the system is of known order and is stabilizeable. The system delay is not required to be available, i.e., b_1 may be zero. No matter what the feature of $w(t)$ is, deterministic or stochastic, the designed adaptive controller based on a unified algorithm makes the system input and output bounded in the averaging sense of (1.2). Similarly, we can design an adaptive controller such that the closed-loop system is uniformly bounded: $\sup_{t\geq 0}(\|y(t)\| + \|u(t)\|) < \infty$, if the system disturbance $\{w(t)\}$ is uniformly bounded: $\sup_{t\geq 0}\|w(t)\| < \infty$.

It is worth emphasizing that the excitation signal $r(t)$ used here is only an example, and it can be replaced by other ones. In fact, following the argument of Corollary 2.1, to get a "precise enough" parameter estimate, a "large enough" signal/noise ratio is needed. In other words, for a given system noise (including $w(t)$ and $\eta(t)$), a "precise enough" parameter estimate requires a "strong enough" excitation signal.

Obviously, Assumption A.1) is more restrictive than that $A(z^{-1})$ and $B(z^{-1})$ have no common unstable factor. Assumptions A.2) and A.3) are weak enough, however, to compensate for this deficiency. In fact, if $A(z^{-1})$ and $B(z^{-1})$ have a greatest common factor which is not one, but a stable polynomial, then there exist polynomials (of z^{-1}): $F(z^{-1})$, $A'(z^{-1})$, and $B'(z^{-1})$ such that i) $F(0) = 1$, $A'(0) = 1$; ii) $A'(z^{-1})$, $B'(z^{-1})$ are coprime; iii) $A(z^{-1}) = F(z^{-1})A'(z^{-1})$ and $B(z^{-1}) = F(z^{-1})B'(z^{-1})$. Let $\eta'(t) = [F(z^{-1})]^{-1}\eta(t)$ and $w'(t) = [F(z^{-1})]^{-1}w(t)$. Then one can see that all sequences $\{y(t)\}$ and $\{u(t)\}$ related by (1.1) satisfy

$$A'(z^{-1})y(t) = B'(z^{-1})u(t) + \eta'(t) + w'(t) \tag{4.1}$$

and *vice versa*. Furthermore, if the orders of $A'(z^{-1})$ and $B'(z^{-1})$ are known, then Assumptions A.1)–A.3) are true for model (4.1). Therefore, the conditions used in this paper are weaker than those in previous works.

Appendix A

Proof of Lemma 2.2: From (2.3) and (2.6) it follows that

$$\tilde{\theta}(t+1) = \tilde{\theta}(t) - P(t)\varphi(t)(\tilde{\theta}^\tau(t+1)\varphi(t) + \eta(t+1) + w(t+1))$$

and by (2.7) and the matrix inverse formula we have $P^{-1}(t+1) = P^{-1}(t) + \varphi(t)\varphi^\tau(t)$. Consequently, we derive

$$\begin{aligned}
&\tilde{\theta}^\tau(t+1)P^{-1}(t+1)\tilde{\theta}(t+1)\\
&= [\tilde{\theta}^\tau(t+1)\varphi(t)]^2 + \tilde{\theta}^\tau(t)P^{-1}(t)\tilde{\theta}(t)\\
&\quad - 2(\tilde{\theta}^\tau(t+1)\varphi(t) + \eta(t+1) + w(t+1))\tilde{\theta}^\tau(t)\varphi(t)\\
&\quad + \varphi^\tau(t)P(t)\varphi(t)[\tilde{\theta}^\tau(t+1)\varphi(t) + \eta(t+1) + w(t+1)]^2\\
&= [\tilde{\theta}^\tau(t+1)\varphi(t)]^2 + \tilde{\theta}^\tau(t)P^{-1}(t)\tilde{\theta}(t)\\
&\quad + \varphi^\tau(t)P(t)\varphi(t)[\tilde{\theta}^\tau(t+1)\varphi(t) + \eta(t+1)\\
&\quad + w(t+1)]^2 - 2[\tilde{\theta}^\tau(t+1)\varphi(t)\\
&\quad + \eta(t+1) + w(t+1)][\tilde{\theta}^\tau(t+1) + (\tilde{\theta}^\tau(t+1)\varphi(t)\\
&\quad + \eta(t+1) + w(t+1))\varphi^\tau(t)P(t)]\varphi(t)\\
&\leq \tilde{\theta}^\tau(t)P^{-1}(t)\tilde{\theta}(t) - [\tilde{\theta}^\tau(t+1)\varphi(t)]^2 - 2[\eta(t+1)\\
&\quad + w(t+1)]\tilde{\theta}^\tau(t+1)\varphi(t).
\end{aligned}$$

Summing up both sides, noticing $2|ab| = 2(\frac{a}{\sqrt{2}})(\sqrt{2}b) \leq \frac{a^2}{2} + 2b^2$ and Assumptions A.2) and A.3) we have

$$\begin{aligned}
&\tilde{\theta}^\tau(t+1)P^{-1}(t+1)\tilde{\theta}(t+1)\\
&\leq \|\tilde{\theta}(0)\|^2 - \sum_{i=0}^{t}[\tilde{\theta}^\tau(i+1)\varphi(i)]^2\\
&\quad - 2\sum_{i=0}^{t}[\eta(i+1) + w(i+1)]\tilde{\theta}^\tau(i+1)\varphi(i)\\
&\leq \|\tilde{\theta}(0)\|^2 - \frac{1}{2}\sum_{i=0}^{t}\|\tilde{\theta}^\tau(i+1)\varphi(i)\|^2\\
&\quad + 2\sum_{i=0}^{t}[\eta(i+1) + w(i+1)]^2\\
&\leq C_1(t+1) + C_1\mu^2 r(t)
\end{aligned}$$

where C_1 is a constant independent of μ. Thus, (2.8) is true.

Appendix B

Proof of Lemma 2.3: We divide the argument into two steps.

Step 1: We first prove that there exists a constant $c > 1$ independent of μ such that

$$\sup_{t\geq 0} r(t+1)/r(t) \leq c, \quad \forall t \geq 0. \tag{B.1}$$

In fact, from (2.3) and (2.9) it is easy to see that

$$\begin{aligned}
\varphi(t) &=\\
&\begin{bmatrix} & \theta^\tau & & \\ I_{(n-1)\times(n-1)} & 0_{(n-1)\times 1} & 0_{(n-1)\times(n-1)} & 0_{(n-1)\times 1} \\ 0 & \cdots & \cdots & 0 \\ 0_{(n-1)\times(n-1)} & 0_{(n-1)\times 1} & I_{(n-1)\times(n-1)} & 0_{(n-1)\times 1}\end{bmatrix}\\
&\times \varphi(t-1)\\
&+ \begin{bmatrix} 0_{n\times 1} \\ \mathbb{1}_{\{t=2nk+n\}} \\ 0_{(n-1)\times 1}\end{bmatrix}[r(t-1)]^{1/2} + \begin{bmatrix} 1 \\ 0_{(2n-1)\times 1}\end{bmatrix}\\
&\times [\eta(t) + w(t)]
\end{aligned} \tag{B.2}$$

where $0_{x\times y}$ denotes an $x \times y$ zero matrix, and

$$\mathbb{1}_{\{t=2nk+n\}} = \begin{cases} 1, & \text{if } t = 2nk+n \text{ for some } k = 0,1,2,\cdots, \\ 0, & \text{otherwise.}\end{cases}$$

By (B.2) we see that there is a constant c_0 depending on θ and n only such that

$$\begin{aligned}
\|\varphi(t)\|^2 &\leq 3c_0\|\varphi(t-1)\|^2 + 3r(t-1) + 3[\eta(t) + w(t)]^2\\
&\leq 3(1+c_0)r(t-1) + 3[\eta(t) + w(t)]^2
\end{aligned}$$

which together with Assumptions A.2)–A.3) and the fact that

$$r(t) \triangleq \alpha^t + \sum_{i=0}^{t}\|\varphi(i)\|^2 \leq \alpha r(t-1) + \|\varphi(t)\|^2$$

gives the desired result (B.1).

Step 2: We now show that there exists a $\mu_0 > 0$ such that for any $\mu: 0 \leq \mu \leq \mu_0$

$$\lambda_{\min}\left(\sum_{i=2nk}^{2nk+3n-1}\varphi(i)\varphi^\tau(i)\right) \geq C_2' r(2nk+n-1) \tag{B.3}$$

where C_2' is a constant independent of μ.

Set $\Phi(t) = A(z^{-1})\varphi(t)$ and $D = [D_1, D_2]^\tau$, where

$$D_1^\tau = \overbrace{\begin{pmatrix} 0 & b_1 & \cdots & \cdots & \cdots & \cdots & b_n & 0 & \cdots & 0 \\ 0 & 0 & \ddots & & & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & & & & & \ddots & 0 \\ 0 & \cdots & 0 & 0 & b_1 & \cdots & \cdots & \cdots & \cdots & b_n \end{pmatrix}}^{2n} \Bigg\} n$$

and

$$D_2^\tau = \overbrace{\begin{pmatrix} 1 & a_1 & \cdots & \cdots & \cdots & \cdots & a_n & 0 & \cdots & 0 \\ 0 & 1 & \ddots & & & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & & & & & \ddots & 0 \\ 0 & \cdots & 0 & 1 & a_1 & \cdots & \cdots & \cdots & \cdots & a_n \end{pmatrix}}^{2n} \Bigg\} n.$$

From (1.1) it is obvious that

$$\Phi(t) = DU(t) + W(t) \tag{B.4}$$

where

$$U(t) = [u(t), \cdots, u(t-2n+1)]^\tau,$$
$$W(t) = [\eta(t) + w(t), \cdots, \eta(t-n+1) + w(t-n+1), \underbrace{0, \cdots, 0}_{n}]^\tau.$$

Set $t_k = 2nk$. Then from (B.4) it is not difficult to see that for any $x \in \mathbf{R}^{2n}$ with $\|x\| = 1$

$$\sum_{i=t_k+n}^{t_k+3n-1} [x^\tau \Phi(i)]^2 \geq \frac{1}{2} \sum_{i=t_k+n}^{t_k+3n-1} [x^\tau DU(i)]^2 - \sum_{i=t_k+n}^{t_k+3n-1} [x^\tau W(i)]^2$$

which together with the fact $\lambda_{\min}(DD^\tau) \triangleq \varepsilon > 0$ implies that

$$\begin{aligned} \lambda_{\min}\left(\sum_{i=t_k+n}^{t_k+3n-1} \Phi(i)\Phi^\tau(i)\right) &\geq \frac{1}{2}\lambda_{\min}\left(\sum_{i=t_k+n}^{t_k+3n-1} DU(i)U^\tau(i)D^\tau\right) - \lambda_{\max}\left(\sum_{i=t_k+n}^{t_k+3n-1} W(i)W^\tau(i)\right) \\ &\geq \frac{1}{2}\lambda_{\min}(DD^\tau)\lambda_{\min}\left(\sum_{i=t_k+n}^{t_k+3n-1} U(i)U^\tau(i)\right) - n\sum_{i=t_k}^{t_k+3n-1} [\eta(i) + w(i)]^2 \\ &\geq \frac{1}{2}\varepsilon\lambda_{\min}\left(\sum_{i=t_k+n}^{t_k+3n-1} U(i)U^\tau(i)\right) - 2n(t_k+3n)W - 2n\sum_{i=t_k}^{t_k+3n-1} [\eta(i)]^2 \end{aligned} \tag{B.5}$$

where $W \triangleq \sup_{t\geq 0}(t+1)^{-1}\sum_{i=0}^{t}[w(i)]^2 < \infty$.

On the other hand, we have

$$\begin{aligned} \lambda_{\min}\left(\sum_{i=t_k+n}^{t_k+3n-1} \Phi(i)\Phi^\tau(i)\right) &= \inf_{\|x\|=1} \sum_{i=t_k+n}^{t_k+3n-1} (x^\tau\Phi(i))^2 \\ &\leq \lambda_{\min}\left(\sum_{i=t_k}^{t_k+3n-1} \varphi(i)\varphi^\tau(i)\right) \cdot \left[(n+1)\left(1 + \sum_{j=1}^{n} a_j^2\right)\right] \end{aligned}$$

which together with (B.5) yields

$$\begin{aligned} \lambda_{\min}\left(\sum_{i=t_k}^{t_k+3n-1} \varphi(i)\varphi^\tau(i)\right) &\geq \frac{\varepsilon}{2c_1}\lambda_{\min}\left(\sum_{i=t_k+n}^{t_k+3n-1} U(i)U^\tau(i)\right) \\ &\quad - 2nc_1^{-1}(t_k+3n)W - 2c_1^{-1}n\sum_{i=t_k}^{t_k+3n-1} [\eta(i)]^2 \end{aligned} \tag{B.6}$$

where $c_1 = (n+1)(1 + \sum_{j=1}^{n} a_j^2)$.

From (2.9) it is easy to get that

$$\sum_{i=t_k+n}^{t_k+3n-1} U(i)U^\tau(i) = r(t_k+n-1)I_{2n\times 2n}. \tag{B.7}$$

Noticing that $\lim_{k\to\infty}(t_k+3n)/r(t_k+n-1) = 0$ and that (B.1) implies

$$\sum_{i=t_k}^{t_k+3n-1} [\eta(i)]^2 \leq \sum_{i=0}^{t_k+3n-1} [\eta(i)]^2 \leq \mu^2 c_2 r(t_k+3n-2) \leq \mu^2 c_2 c^{2n-1} r(t_k+n-1)$$

where c_2 is a constant independent of μ, by (B.6) and (B.7) we obtain (B.3).

We now prove (2.10). To this end, for any fixed t let k_t be the largest integer such that $2nk_t + 3n - 1 \leq t$. Then it is obvious that $0 \leq t - (2nk_t + 3n - 1) < 2n$, i.e. $t < 2nk_t + 5n - 1$. This implies that $t - (2nk_t + n - 1) < 4n$. Hence, by (B.1) we have

$$r(t) \leq c^{t-(2nk_t+n-1)} r(2nk_t + n - 1) \leq c^{4n} r(2nk_t + n - 1).$$

From this and (B.3) we get

$$\lambda_{\min}^{(t)} \geq \lambda_{\min}\left(\sum_{i=0}^{2nk_t+3n-1} \varphi(i)\varphi^\tau(i)\right) \geq C_2' r(2nk_t+n-1) \geq C_2' c^{-4n} r(t)$$

which together with Lemma 2.1 and $\alpha > 1$ yields the desired result (2.10).

ACKNOWLEDGMENT

The author would like to express his gratitude to the reviewers for the constructive comments.

REFERENCES

[1] P. R. Kumar and P. Varaiya, *Stochastic Systems: Estimation, Identification, and Adaptive Control.* Englewood Cliffs, NJ: Prentice-Hall, 1986.
[2] P. E. Caines, *Linear Stochastic Systems.* New York: Wiley, 1988.
[3] H. F. Chen and L. Guo, *Identification and Stochastic Adaptive Control.* Boston, MA: Birkhäuser, 1991.

[4] B. Egardt and C. Samson, "Stable adaptive control of nonminimum phase systems," *Syst. Contr. Lett.*, vol. 2, no. 3, pp. 137–144, 1982.
[5] L. Praly, "Toward a globally stable direct adaptive control scheme for not necessarily minimum phase systems," *IEEE Trans. Automat. Contr.*, vol. 29, no. 10, pp. 946–949, 1984.
[6] G. Kreisselmeier, "An indirect adaptive controller with a self-excitation capability," *IEEE Trans. Automat. Contr.*, vol. 34, no. 5, pp. 524–528, 1989.
[7] R. Lozano-Leal, "Robust adaptive regulation without persistent excitation," *IEEE Trans. Automat. Contr.*, vol. 34, pp. 1260–1267, 1989.
[8] F. Giri, M. M'Saad, J. M. Dion, and L. Dugard, "On the robustness of discrete-time indirect adaptive (linear) controllers," *Automatica*, vol. 27, no. 1, pp. 153–159, 1991.
[9] F. Giri, M. M'Saad, L. Dugard, and J. M. Dion, "Robust adaptive regulation with minimal priori knowledge," in *Proc. 29th IEEE Conf. Decis. Contr.*, vol. 2, Honolulu, HI, 1990 pp. 985–990, see also *IEEE Trans. Automat. Contr.*, vol. 37, no. 3, pp. 305–315, 1992.
[10] M. Radenkovic and G. Leininger, "Robustness of the Åström-Wittenmark's Self-Tuning Controller," in *Proc. 31st Conf. Decis. Contr.*, AZ, 1992.
[11] J. F. Zhang, "Comments on 'Robust adaptive regulation with minimal prior knowledge,'" *IEEE Trans. Automat. Contr.*, vol. 39, no. 3, p. 605, 1994.

Decidability and Closure Properties of Weak Petri Net Languages in Supervisory Control

Alessandro Giua and Frank DiCesare

***Abstract*—We extend the class of control problems that can be modeled by Petri nets considering the notion of weak terminal behavior. Deterministic weak languages represent closed-loop terminal behaviors that may be enforced by nonblocking Petri net supervisors if controllable. The class of deterministic weak PN languages is not closed under the supremal controllable sublanguage operator.**

I. Introduction

In this note we present a notion of terminal behaviors for Petri nets (PN) called weak behaviors and study their use in supervisory control theory (SCT) [10]. Weak behaviors overcomes some of the problems due to the use of Petri net marked behaviors [2], [15], [13].

Since the weak behavior is specifically defined only for Petri nets, we will introduce it with an example.

In Fig. 1, we have shown a simple communication process where a sender S sends messages to a receiver R through an infinite-capacity channel C. The initial state of the system is shown in Fig. 1. A token in S_on (S_off) means that the sender is active (has disconnected). A token in R_on (R_off) means that the receiver is active (has disconnected). The tokens in C represent messages sent on the channel but not yet received.

We consider as final marking M_f, with $M_f(S\_off) = M_f(R\_off) = 1$ and $M_f(S\_on) = M_f(C) = M_f(R\_on) = 0$. We may consider as terminal behavior of the net its marked language, i.e., the set of all firing sequences that reach the final marking M_f. Since $M_f(C) = 0$, this means that in a terminal state there may not be tokens in C, i.e., all messages sent by S have been received

Manuscript received February 21, 1994; revised June 14, 1994.
A. Giua is with the Istituto di Elettrotecnica, Università di Cagliari, Italy.
F. DiCesare is with the Department of Electrical and Computer Science Engineering, Rensselaer Polytechnic Institute, Troy, NY USA.
IEEE Log Number 9408805.

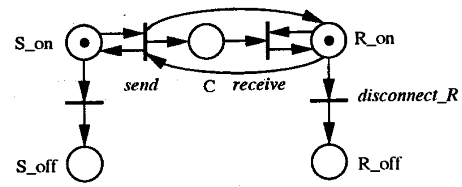

Fig. 1. A sender-receiver process.

by R. We may consider as terminal behavior of the net its weak language, i.e., the set of all firing sequences that reach a marking greater or equal to M_f. This means that we consider as terminal all those states in which both sender and receiver have disconnected, i.e., $M_f(S\_off) = M_f(R\_off) = 1$, regardless of the number of tokens contained in C.

The choice between weak and marked language as terminal behavior depends on the physical problem. For the example we considered here, accepting the weak language as terminal behavior means that we are not interested in ensuring that all messages sent are received or that we are not capable of enforcing this constraint. It may be interesting to note that a Petri net is, in effect, a weak counter, in the sense that if a transition firing may occur at a marking M then it may also occur at any marking $M' > M$ (see Lemma 2.1). Thus it also make sense to assume that if a marking M is final then any marking $M' > M$ is also final.

This note discusses the use of weak languages in supervisory control. In particular, since deterministic weak PN languages have not been studied before, we also devoted some time to the study of their properties. Our main results can be summarized as follows. First, the classes of weak and marked languages generated by deterministic nets are incomparable. Thus, taking also into account the weak behavior of deterministic nets (in addition to the marked behavior) we extend the class of control problems that can be modeled by PN. Second, deterministic weak PN languages are DP-closed, i.e., they represent closed-loop terminal behaviors that may be enforced by Petri net supervisors. This is an important result that does not hold for the class of deterministic marked PN languages. Third, the main properties of interest in supervisory control, such as controllability and L-closure, are decidable when this class of languages is considered. It is also decidable whether a system is weakly blocking. Finally, the class of deterministic weak PN languages is not closed under the supremal controllable sublanguage operator.

II. Background

A. Petri Nets

A Place/Transition net (P/T net) is a structure $(P, T, Pre, Post)$ where: P is a set of places; T is a set of transitions; *Pre*: $P \times T \rightarrow \mathbb{N}$ specifies the arcs directed from places to transitions; *Post*: $P \times T \rightarrow \mathbb{N}$ specifies the arcs directed from transitions to places. Here $\mathbb{N} = \{0, 1, 2, \cdots\}$. See [7], [11] for a more complete definition of Petri nets.

A marking is a vector $M: P \rightarrow \mathbb{N}$. $\mathbb{N}^{|P|}$ will denote the set of all possible markings that may be defined on the net. A P/T system or net system $\langle N, M_0 \rangle$ is a net N with an initial marking M_0. We will expand sometimes the definition of marking to a function: $M: P \rightarrow \mathbb{N}_\omega$, with $\mathbb{N}_\omega = \mathbb{N} \cup \{\omega\}$. ω is a new element such that for all $n \in \mathbb{N} : n < \omega$, and for all $n \in \mathbb{N}_\omega : \omega + n = n + \omega = \omega - n = \omega$.