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ADAPTIVE STABILIZATION FOR CONTINUOUS TIME SYSTEMS WITH DISTURBANCES

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SUMMARY

This paper concerns adaptive stabilization for single-input/single-output (SISO) continuous time systems with unknown coefficients and containing stochastic or deterministic disturbances. The conditions used here are possibly the weakest: neither the positive realness condition nor the availability of the upper bound of system disturbances is needed; the only condition imposed on the system structure is stabilizability, which is necessary for stabilizing a system even in the case where the system coefficients are known. The adaptive control given in the paper is switched at stopping times either on external excitations or on certainty equivalence controls defined by the pole assignment method at fixed times. It is shown that after a finite period of time the external excitation is no longer used and the system is stabilized in the long run average sense.

KEY WORDS Adaptive stabilization Stochastic system Continuous time Stopping time Pole placement

1. INTRODUCTION

For the last two decades much attention has been paid to adaptive stabilization of both discrete time (see e.g. References 1–10) and continuous time (see e.g. References 11–21) stochastic and deterministic systems. Authors of previous papers, in addition to the stabilizability assumption which is necessary for the problem in question, require various extra conditions. For example, input strict passitivity and *a priori* knowledge on the location of the parameters are required in Reference 19; a lower bound for the coprimeness degree is needed in References 5, 7, 8 and 19; a location restriction on the unstable zeros is used in Reference 11; the minimum phase condition and the strictly positive realness condition on the transfer function of the system noise are applied in Reference 17; some conditions on the system input or output are needed in References 18 and 20; and CB is assumed known in Reference 21, where B and C are matrix coefficients for the system input and output respectively.

The purpose of this paper is to remove all extra restrictions on the system structure, i.e. to adaptively stabilize a system under the stabilizability assumption only. Let us explain this more precisely.

Let S be the integral operator

$$Sy_t = \int_0^t y_s \, ds$$

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and let the SISO continuous time stochastic system be described by

$$A(S)y_t = y_0 + SB(S)u_t + C(S)w_t + S\eta_t \quad \forall t \geqslant 0 \tag{1}$$

where $A(S)$, $B(S)$ and $C(S)$ are polynomials in S with unknown coefficients but known orders:

$$A(S) = 1 + \sum_{i=1}^{p} a_i S^i, \qquad B(S) = \sum_{i=1}^{q} b_i S^{i-1}, \qquad C(S) = \sum_{i=0}^{l} c_i S^i \tag{2}$$

In (1), $\{w_t, \mathscr{F}_t\}$ is a standard Wiener process with respect to a non-decreasing and right-continuous σ-algebra $\{\mathscr{F}_t\}$ defined on a probability space $(\Omega, P, \mathscr{F})$, y_0 is the initial value, $\mathscr{F}_t$-adapted y_t and u_t (i.e. y_t and u_t are $\mathscr{F}_t$-measurable,) are the system output and input respectively and $\mathscr{F}_t$-adapted η_t is the system disturbance, which is different from that driven by the Wiener process and may be deterministic.

When $l = p$ and $c_p = ga_p$, system (1) has the state space representation

$$dx_t = Ax_t \, dt + Bu_t \, dt + C \, dw_t + D\eta_t \, dt \tag{3}$$

$$dy_t = D^T x_t \, dt + g \, dw_t \tag{4}$$

with

$$A = \begin{bmatrix} -a_1 & 1 & & \\ -a_2 & & \ddots & \\ \vdots & & & 1 \\ -a_m & & & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}, \qquad C = \begin{bmatrix} c_0 - ga_0 \\ \vdots \\ c_{m-1} - ga_{m-1} \end{bmatrix}, \qquad D \left.\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right\} m \tag{5}$$

where $m = \max\{p, q\}$, $a_0 = 1$, $a_i = 0$ for $i > p$, $b_j = 0$ for $j > q$, $c_k = 0$ for $k > l$ and X^T denotes the transpose of a vector or a matrix X.

When $C(S) = 0$ and η_t is deterministic, system (1) turns out to be a deterministic one:

$$\dot{x}_t = Ax_t + Bu_t + D\eta_t, \qquad y_t = y_0 + \int_0^t D^T x_s \, ds$$

Let us denote the collection of unknown coefficients of $A(S)$ and $B(S)$ by θ:

$$\theta = [-a_1, \ \ldots, \ -a_p, \ b_1, \ \ldots, \ b_q]^T \tag{6}$$

For θ we use the least squares (LS) estimate θ_t which is defined as (see e.g. References 17, 19, 20 and 22)

$$d\theta_t = R_t \varphi_t (dy_t - \varphi_t^T \theta_t \, dt) \quad \text{with} \quad R_t = \left(I + \int_0^t \varphi_s \varphi_s^T \, ds\right)^{-1} \tag{7}$$

and

$$\varphi_t^T = [y_t, \ \ldots, \ S^{p-1} y_t, \ u_t, \ \ldots, \ S^{q-1} u_t] \tag{8}$$

where $\theta_0 \in \mathscr{F}_0$ is arbitrarily chosen.

Based on θ_t, we want to design an adaptive control so that the system is stabilized under the following assumptions:

A1. $A(S)$ and $SB(S)$ are coprime, $b_q \neq 0$.

A2.
$$\sup_{t \geqslant 0} \frac{1}{t+1} \int_0^t \eta_s^2 \, ds < \infty \quad \text{a.s.}$$

Assumption A1 is not the weakest condition to stabilize a system for the case where θ is known, since in this case for stabilizability the necessary and sufficient condition is that the greatest common factor of $A(S)$ and $SB(S)$ be unity or a stable polynomial. However, in Lemma 6 in Appendix I it is shown that Assumptions A1 and A2 together are actually equivalent to the following.

A1′. The greatest common factor of $A(S)$ and $SB(S)$ is unity or a stable polynomial, $b_q \neq 0$, i.e. the system is stabilizable.

A2′.
$$\sup_{t \geq 0} \frac{1}{t+1} \int_0^t \eta_s^2 \, ds < \infty \quad \text{a.s.}$$

Therefore, without loss of generality, in the sequel we will use Assumptions A1 and A2 directly.

2. ADAPTIVE CONTROL

To define the adaptive control, we need the certainty equivalence control, the external excitation and the stopping times where the switches of control take place. This will be completed after several lemmas.

Lemma 1[17]

Let $k \geq 0$ be an integer and $E(S) = 1 + e_1 S + \cdots + e_k S^k$ with $e_k \neq 0$ be a stable polynomial, i.e. $E(z) \neq 0$ for any z with $\mathrm{Re}(z) \geq 0$, where $\mathrm{Re}(z)$ denotes the real part of a complex number z. Then there is a constant $\mu_e \geq 1$ (depending on $E(S)$ only) such that

$$\sum_{i=0}^{k} \int_0^t \left(\frac{S^i}{E(S)} x_\lambda \right)^2 d\lambda \leq \mu_e \int_0^t x_\lambda^2 \, d\lambda$$

for any square-integrable process $\{x_t\}$.

For the proof we refer to Reference 17.

If $A(S)$ and $SB(S)$ are coprime and $b_q \neq 0$, then for any polynomial

$$E(S) = 1 + e_1 S + \cdots + e_{p+q} S^{p+q} \quad \text{with} \quad e_{p+q} \neq 0 \tag{9}$$

there exists a unique pair of polynomials $(G(S), H(S))$ such that

$$A(S)G(S) - SB(S)H(S) = E(S) \quad \text{with} \quad \partial(G(S)) \leq q-1 \quad \text{and} \quad \partial(H(S)) = p \tag{10}$$

where here and hereafter $\partial(X(S))$ denotes the degree of polynomial $X(S)$ in S.

From (10) and (1) it is clear that

$$\begin{aligned} E(S)y_t &= A(S)G(S)y_t - SB(S)H(S)y_t \\ &= G(s)[A(S)y_t - SB(S)u_t] + SB(S)[G(S)u_t - H(S)y_t] \\ &= G(S)[y_0 + C(S)w_t + S\eta_t] + SB(S)[G(S)u_t - H(S)y_t] \end{aligned} \tag{11}$$

and

$$\begin{aligned} E(S)u_t &= A(S)G(S)u_t - SB(S)H(S)u_t \\ &= H(s)[A(S)y_t - SB(S)u_t] + A(S)[G(S)u_t - H(S)y_t] \\ &= H(S)[y_0 + C(S)w_t + S\eta_t] + A(S)[G(S)u_t - H(S)y_t] \end{aligned} \tag{12}$$

Noticing that $\partial(G(S)) \leqslant q-1$, from (11) and Lemma 1 we see that in the case where θ is known, if $\partial(C(S)) \leqslant p$, $E(S)$ is stable and the control u_t is defined by

$$G(S)u_t - H(S)y_t = 0, \quad t \geqslant 0 \tag{13}$$

then under Assumptions A1 and A2 the system output is bounded in the average sense, i.e.

$$\sup_{t\geqslant 0} \frac{1}{t+1}\int_0^t y_s^2 \, ds < \infty \quad \text{a.s.} \tag{14}$$

Similarly, from (12), $\partial(H(S)) = p$ and Lemma 1 it is clear that under Assumptions A1 and A2 the control u_t defined by (13) is bounded in the average sense, i.e.

$$\sup_{t\geqslant 0} \frac{1}{t+1}\int_0^t u_s^2 \, ds < \infty \quad \text{a.s.} \tag{15}$$

if $\partial(C(S)) \leqslant q-1$ and $E(S)$ is stable.

Therefore, in the case where θ is known, if $\partial(C(S)) \leqslant \min\{p, q-1\}$ and Assumptions A1 and A2 hold, then for any stable $E(S)$ the control defined by (13) stabilizes the system, i.e.

$$\sup_{t\geqslant 0} \frac{1}{t+1}\int_0^t (y_s^2 + u_s^2) \, ds < \infty \quad \text{a.s.} \tag{16}$$

Replacing a_i, $i = 1, \ldots, p$, and b_j, $j = 1, \ldots, q$, in $A(S)$ and $SB(S)$ respectively by their estimates a_{it} and b_{jt} given by θ_t, we denote the results by $A_t(S)$ and $SB_t(S)$. In the case where $A_t(S)$ and $SB_t(S)$ are coprime, in a similar way to (10) we can obtain a pair of polynomials $(G_t(S), H_t(S))$.

If θ_{t_1} is an 'accurate' estimate for θ, then the certainty equivalence control given at time t_1 will hopefully work for $t \geqslant t_1$. However, in general we have no reason to expect that $\theta_t - \theta$ is small.

In order to obtain a 'good' estimate θ_t, we will use the excitation technique. For this we give two lemmas.

Lemma 2

Let

$$r_t = 1 + \int_0^t \| \varphi_s \|^2 \, ds$$

and let $\lambda_{\min}^{(t)}$ denote the smallest eigenvalue of the matrix R_t^{-1}, where φ_t and R_t are as given by (8) and (7) respectively. Then the parameter estimate θ_t given by (7) and (8) has the property

$$\| \theta_t - \theta \|^2 \leqslant \frac{\varkappa[(t+1)^{2l+1} + \log r_t]}{\lambda_{\min}^{(t)}} \quad \text{a.s.} \quad \forall t \geqslant 0$$

where $l = \partial(C(s))$ and $\varkappa$ is a random variable independent of time t.

The proof is given in Appendix II.

Lemma 3

Let α be an arbitrary positive constant. Define for $i = 1, 2, \ldots, p+q$

$$\beta_i = (-1)^{i+1}\alpha^i \frac{(p+q)!}{i! \times (p+q-i)!} \quad \text{with} \quad 0! \triangleq 1 \quad \text{and} \quad i! \triangleq 1 \times 2 \times \cdots \times i \tag{17}$$