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Leader-Following Adaptive Tracking Control for A Class of Uncertain Discrete-Time Nonlinear Multi-Agent Systems Under Event-Triggered Communication*

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Abstract This paper studies the leader-following adaptive tracking control problem for multi-agent systems comprising a leader agent and N follower agents with uncertain nonlinear dynamics. Specifically, a novel event-triggered communication based adaptive distributed observer is developed to enable each follower agent to estimate the leader's information. Then, new forms of adaptive control law and parameter update law are designed with the estimated leader's signals. The developed distributed adaptive control strategy has several characteristics: (i) With the introduced time-varying observer gain, the designed adaptive distributed observer eliminates the need for global graph information but ensures convergence of the estimates; (ii) By appropriately designing the event-triggered mechanism, the communication frequency among follower agents is reduced in the sense that the communication rate decays over time; (iii) The newly designed adaptive control law ensures a linear estimation error equation, facilitating the development of a stable parameter update law without requiring prior knowledge of uncertain system parameters. The stability of closed-loop system and leader-following asymptotical tracking are achieved. Simulation study demonstrates the theoretical results.

Keywords Adaptive control, event-triggered communication, heterogeneous nonlinear agent, leader-following tracking.

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1 Introduction

In recent decades, the cooperative control of multi-agent system (MAS) has emerged as a key area of interest within the control community, driven by its wide-ranging engineering demands such as mobile robotics and satellite coordination. Significant research efforts have been devoted to this domain, leading to remarkable advancements and a wealth of impactful studies[1–3].

The goal of cooperative control of MAS is to realize desired collective behaviors through distributed control strategies that utilize communication among neighboring agents. From the perspective of task objectives, this field encompasses a variety of specific research areas, including consensus, formation control, sensor network coverage, distributed estimation, distributed optimization, and so on. Among these, the consensus problem serves as a foundational aspect. The consensus problem of MAS primarily manifests in two paradigms: Leaderless consensus and leader-following tracking. Initial research on consensus problems primarily focused on convergence analysis under certain graph conditions[4–6] and later has been extended to consensus algorithm design from various perspectives, such as MAS with specific agent dynamics[7–9] and MAS consensus under constrained network communication[10–12].

In real applications, the control system may suffer from various uncertainties. Taking possible uncertainties of agent dynamics into account, adaptive control techniques are incorporated to the study of consensus control of MAS by many researchers to achieve desired performance[13–18]. For example, the leader-following consensus was considered in [13] for a class of first-order uncertain MAS with Lyapunov function based analysis. In [14], output consensus was achieved for a class of high-order nonlinear uncertain MAS by integrating adaptive distributed observer method and adaptive control technique. For strict-feedback nonlinear MAS with parameter uncertainty, [15] solved the adaptive consensus control problem by using the fusion least-square algorithm and the backstepping technique. Notably, the backstepping technique, which was initially developed in [19] for single-agent system, has become a powerful tool for the consensus algorithm design of uncertain high-order MAS. The aforementioned adaptive consensus control methods require the uncertainty associated with agent control gain is nonexist[13–16] or the sign of uncertain control gain is prior known[17, 18]. However, the sign of control gain in system dynamics, which is also referred as the control direction, maybe unknown in practical engineering. For single-agent systems, some classical approaches have been proposed to handle unknown control direction issue, including Nussbaum function methods[20] and multi-model switching methods[21]. Nevertheless, relatively few studies have considered the control direction issue of cooperative adaptive control in MAS. Towards this challenge, some achievements have been made in the recent decade[22–29]. For example, [22, 25, 26] developed multiple Nussbaum gain functions to combat unknown control directions by assuming control directions of all agents are identical. Afterwards, by constructing a compensator network, [27, 28] tackled output regulation problem under unknown control directions with Nussbaum gain based schemes, where the agents are not required to have identical control directions. It is worth mentioning that all these methods focus on MAS with continuous-time dynamics, where the design of Nussbuam gain functions and backstepping technique are critical for their effectiveness.

In addition to system uncertainties, consumption of communication resources is another problem worth considering since the design of consensus control law should be distributed, which relies on the communication between neighboring agents. As an effective tool to address the challenges associated with resource constraints, event-triggered control has emerged as a significant research area in MAS. In traditional time-triggered control, agents exchange information and update their control inputs at fixed time intervals, which can lead to unnecessary communication and computation. Event-triggered control, in contrast, allows agents to exchange information or update control actions only when specific events or conditions are met. This reduces communication overhead, making event-triggered control especially valuable in scenarios with limited bandwidth, energy constraints, or high communication costs. Specifically, seminal works[30, 31] established foundational event-triggered protocols for single-integrator MAS consensus attainment. The literature [32] systematically addressed both homogeneous and heterogeneous linear agent dynamics through adaptive event-based cooperative strategies. Recent advancements have been extended to nonlinear MAS domains, with [33, 34] developing stability-guaranteed triggering mechanisms for high-order and Euler-Lagrange nonlinearities, respectively. In [35], a distributed event-triggered observer architecture was introduced to resolve cooperative output regulation challenges in heterogeneous linear continuous-time MAS.

As mentioned above, the reported literature about cooperative adaptive control of MAS with unknown control directions focus on continuous-time system dynamics with discrete-time systems being omitted. And, most works relied on the Nussbaum gain function to combat control gain uncertainty. As indicated in [36], the Nussbaum gain method is easy to cause poor transient performance due to the oscillation nature of Nussbaum function. Thus, how to develop a distributed adaptive control strategy for discrete-time MAS possessing unknown control directions without relying Nussbaum gain method is still unclear. Recently, [37] and [38] developed novel model reference adaptive control methods for single-agent systems in continuous-time with relative degree one and in discrete-time with arbitrary relative degree, respectively, which remove the need on control direction information without resorting to Nussbaum gain method. Considering unknown control directions issue in cooperative adaptive control, the novel method in [37] and [38] inspires us to develop a distributed adaptive control scheme for discrete-time MAS to overcome the control gains uncertainty constraint. Motivated by above observations, this work establishes a distributed adaptive control scheme that resolves leader-following tracking problem in discrete-time MAS. Specifically, we consider the follower agents dynamics are nonlinear, heterogeneous, and with parameter uncertainties. Particularly, the control directions related parameters, i.e., the control gains of agents, are totally unknown except for non-zero conditions. In addition, considering communication resource consumption, an event-triggered mechanism is developed to reduce the frequency of communication among neighboring follower agents. In summary, this work makes the following contributions:

(i) We propose a novel distributed leader-following adaptive tracking control scheme for a class of MAS with heterogeneous nonlinear agent dynamics. Specifically, a new adaptive distributed observer is designed for each follower to estimate leader signal under event-triggered communication among neighboring agents. Then, new forms of control law and parameter update law are developed to achieve closed-loop stability and leader-following tracking control objective under system parameter uncertainties.

(ii) With time-varying observer gain, the proposed distributed adaptive observer does not need global graph information to achieve estimation convergence, which is distinct from some existing works (see [39]). Moreover, the well-designed event-triggered mechanism with time-varying threshold can reduce the communication frequency of neighboring agents with a decaying communication rate index, which implies that little communication is required when the MAS tends to a steady state.

(iii) Unlike some existing results (see [17, 18]), the proposed adaptive control law induces a linear estimation error equation, which motivates a parameter update law without needing any prior sign information or bound knowledge of unknown parameters covering uncertain control gains of agents. This is essentially different from some Nussbaum gain based methods (see [22, 25, 26]). Moreover, no singularity problem and casuality contradiction issue are involved by designing a time-varying gain and introducing some filter operators.

The rest of this paper is structured as follows. Section 2 presents the problem formulation and outlines the key technical challenges to be addressed. In Section 3, we detail the design of the proposed leader-following adaptive control strategy. A novel event-triggered communication based distributed adaptive observer is first developed for every follower agent, followed by the development of control law and parameter update mechanism. The stability of the system and the leader-following tracking performance are then analyzed. Section 4 provides simulation study to validate the theoretical results. Finally, Section 5 concludes the paper.

Notations In this work, $\mathbb{R}$ represents the set of real numbers. Symbols $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the sets of real vectors of dimension n and real matrices of size $m\times n$, respectively. The symbol z is used to denote the time shift operator. To be specific, for any signal $x(t) \triangleq x(tT)$ for a sampling period $T>0$, $z[x](t)=x(t+1)$ and $z^{-1}[x](t)=x(t-1)$. L^∞ and L^2 denote signal spaces defined as $L^\infty=\{x(t):\|x(\cdot)\|_\infty<\infty\}$ and $L^2=\{x(t):\|x(\cdot)\|_2<\infty\}$ with $\|x(\cdot)\|_\infty=\sup_{t\geq 0}\max_{1\leq i\leq n}|x_i(t)|$ and $\|x(\cdot)\|_2=(\sum_{t=0}^{\infty}|x_1(t)|^2+\cdots+|x_n(t)|^2)^{\frac{1}{2}}$, where $x(t)=[x_1(t),\cdots,x_n(t)]^{\mathrm{T}}$ denotes any signal on $\mathbb{R}^n$. We use c to denote a generic signal bound and $\tau(t)$ to denote a generic $L^2\cap L^\infty$ function which converge to zero as $t\in\infty$. $\|\cdot\|$ denotes the Euclidean norm for vectors or induced 2-norm for matrices. Define $\mathrm{vec}(Y)=[Y_1^{\mathrm{T}},\cdots,Y_m^{\mathrm{T}}]^{\mathrm{T}}$ as the vector from matrix $Y\in\mathbb{R}^{n\times m}$, where Y_i is the ith column of matrix Y. The Kronecker product is denoted by $\otimes$. The symbol I represents the general identity matrix. We use $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ to denote the minimal magnitude of eigenvalue and maximal magnitude of eigenvalue of matrix A, respectively. For a sequence of real-valued matrices or vector $\{a(t)\}$ and a real number sequence $\{b(t)\}$, the notation $a(t)=O(b(t))$ indicates that there exists a constant $c\geq 0$ such that $\lim_{t\to\infty}\|a(t)/b(t)\|\leq c$.

2 Problem Statement

In this section, we present the system model, the control objective, and design conditions. Additionally, it claims the technical issues to be resolved in this paper.

2.1 System Model

We consider an MAS including a leader agent and N follower agents. The dynamics of each follower agent $i,\ i = 1, \cdots, N$, is described as follows:

$$\begin{aligned} &x_{i,j}(t+1) = x_{i,j+1}(t), \quad j = 1, \cdots, r_i - 1, \\ &x_{i,r_i}(t+1) = \theta_i^{*\mathrm{T}} f_i(x_i(t)) + b_i^* u_i(t), \\ &y_i(t) = x_{i,1}(t), \end{aligned} \tag{1}$$

where $x_i(t) = [y_i(t), y_i(t+1), \cdots, y_i(t+r_i-1)]^{\mathrm{T}} \in \mathbb{R}^{r_i}, u_i(t) \in \mathbb{R}, y_i(t) \in \mathbb{R}$ are the state vector, control input and system output of agent i, respectively. The vector function $f_i(x_i(t))$ is a known smooth nonlinear Lipschitz mapping. The parameters θ_i^* and $b_i^* \neq 0$ are entirely unknown systems parameters, and $r_i \geq 1$ represents the known system relative degree and is allowed to be different among agents. Notably, there are some practical systems can be modeled as (1), such as rigid robots and motors, ships, and jet engines and aircraft. The state vector $x_i(t)$ is measurable by each agent $i,\ i = 1, \cdots, N$.

The leader agent follows the following dynamics

$$v_0(t+1) = S v_0(t), \quad y_0(t) = F v_0(t), \tag{2}$$

where $v_0(t) \in \mathbb{R}^n$ and $y_0(t) \in \mathbb{R}$ denote the leader's state variable and output variable, respectively. The matrices $S \in \mathbb{R}^{n \times n}$ and $F \in \mathbb{R}^{1 \times n}$ represent the leader's dynamics matrix and measure matrix, respectively, and the pair (S, F) is assumed to be detectable. In this paper, the dynamics matrix S and the state variable $v_0(t)$ are not acquired by all follower agents. Only a subset of agents directly connected to the leader have access to the actual values of S and $v_0(t)$. Moreover, the matrix F is considered known, as it specifies the reference output signal that each follower agent is required to track (see [14]).

Remark 2.1 The follower agents with the dynamics (1) are heterogeneous as their system properties are not necessarily uniform. Specifically, each follower i may have a distinct relative degree r_i and unique nonlinear dynamics $f_i(x_i(t))$, which can differ from those of other agents. Moreover, the control gains $b_i,\ i = 1, \cdots, N$, may have different unknown signs, i.e., the control directions are allowed to be non-identical, which is different from some existing works (see [22, 25, 26]).

2.2 Graph Theory

In this work, the agents' communication network is represented by a directed graph $\overline{\mathcal{G}} = (\overline{\mathcal{V}}, \overline{\mathcal{E}})$. Here, the set of notes is defined as $\overline{\mathcal{N}} = \{0, 1, \cdots, N\}$, and $\overline{\mathcal{E}} \subset \overline{\mathcal{N}} \times \overline{\mathcal{N}}$ denotes the edge set. An edge from node j to node i (with $j \neq i$) is denoted by (j, i), which indicates that node i is a child of node j. In this framework, node 0 corresponds to the leader agent while node i, $i = 1, \cdots, N$, represent the follower agents. The network's connectivity is further characterized by the weighted adjacency matrix of $\overline{A} = [a_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$, $i, j \in \overline{\mathcal{N}}$. For every node i, the diagonal entry is zero ($a_{ii} = 0$), and for $i \neq j$, a_{ij} is positive if and only if there exists an edge from j to i in $\overline{\mathcal{E}}$, that is, node i can receive the information from node j. Otherwise, $a_{ij} = 0$ for $i \neq j$. In particular, for $i = 1, \cdots, N$, a positive value of a_{i0} signifies that agent i has access to the leader agent. Meanwhile, a_{0i} is set as 0 by assuming that no follower transmit information to the leader. Additionally, one can define a subgraph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ of $\overline{\mathcal{G}}$ consisting solely of the follower nodes. The edge set $\mathcal{E}$ is formed by removing all connections that involve node 0 from the original set $\overline{\mathcal{E}}$. Within this subgraph, the neighbor set for any node i is given by $\mathcal{N}_i = \{j \in \mathcal{N} | (j, i) \in \mathcal{E}\}$. Finally, the graph matrix $H = [h_{ij}] \in \mathbb{R}^{N\times N}$, $i, j \in \mathcal{N}$ is introduced, where $h_{ii} = \sum_{j=0}^{N} a_{ij}$, and for $i \neq j$, $h_{ij} = -a_{ij}$.

2.3 Control Objective and Assumptions

Control objective For the MAS comprising of N follower agents with the dynamics (1) and a leader agent with the dynamics (2), interconnected through a directed communication graph $\overline{\mathcal{G}}$, the goal is to design a distributed adaptive control law $u_i(t)$ for each follower agent i that operates under event-triggered communication. This design should ensure the stability of closed-loop system and guarantee that the output of each follower $y_i(t)$ asymptotically tracks the leader's output $y_0(t)$, i.e., $\lim_{t\to\infty}(y_i(t) - y_0(t)) = 0$, $i = 1, \cdots, N$.

Assumption 2.2 To achieve the control objective, the following conditions are assumed.

(A1) The communication network $\overline{\mathcal{G}}$ possesses at least one directed spanning tree with the leader node 0 serving as the root.

(A2) The modulus of all eigenvalues of S are not exceeding to 1.

Remark 2.3 Assumption (A1) is the connectivity condition of the graph $\overline{\mathcal{G}}$. This condition ensures that there is a directed path from the leader to every follower and is a standard requirement for fixed directed graph (see [7, 8]). Assumption (A2) is widely adopted in the literature (see [39, 40]) and is satisfied by a broad range of signals (e.g., step, ramp, and sinusoidal functions). Under Assumption (A2), the leader's state $v_0(t)$ and output $y_0(t)$ remain bounded. In practical applications, Assumption (A2) is reasonable because its violation would imply that the leader produces exponentially growing signals, a scenario that is unlikely to occur.

2.4 Technical Issues

For the leader-following tracking control of the considered MAS, we will solve the following technical issues in this work: (i) How to design an adaptive distributed observer for agent i to estimate the information of the leader without relying on global graph information? (ii) How to develop an event-triggered mechanism such that the communication frequency can be reduced among follower agents? (iii) How to devise adaptive control law and parameter update law without any prior system parameter knowledge? These concerns will be resolved in the following.

3 Distributed Leader-Following Adaptive Tracking Control Design

In this section, we propose a distributed adaptive control scheme to ensure that each follower's output $y_i(t)$ converges to the leader's output $y_0(t)$. The strategy begins with the development of a novel event-triggered communication based adaptive distributed observer. This observer leverages local available information from neighboring agents to estimate the leader's signals. Next, we introduce a new adaptive controller for each follower agent. This controller induces a stable parameter update law without relying on prior knowledge of system parameters. Finally, we analyze the stability of the closed-loop system and assess the tracking performance based on the proposed controller and parameter update law.

3.1 Event-Triggered Communication Based on Adaptive Distributed Observer

Since agents in the MAS interact with each other by the communication graph $\overline{\mathcal{G}}$, not every follower could directly receive the leader's signals. To overcome this, we design an adaptive distributed observer for each follower agent that estimates both the leader's dynamics matrix S and state $v_0(t)$. An event-triggered mechanism is embedded in the observer to reduce communication frequency. Specifically, for every follower agent i, $i = 1, \cdots, N$, the event-triggered communication based adaptive distributed observer is designed as follows:

$$\begin{aligned} S_i(t+1) &= S_i(t) + \mu(t) \sum_{j=0}^{N} a_{ij}(\widehat{S}_j(t) - \widehat{S}_i(t)), \\ v_i(t+1) &= S_i(t)v_i(t) + \mu(t)S_i(t) \sum_{j=0}^{N} a_{ij}(\widehat{v}_j(t) - \widehat{v}_i(t)), \end{aligned} \tag{3}$$

where the observer gain is $\mu(t) = (t+t_0)^{-\mu^*}$ with $t_0 \geq 0$ and $\mu^* \in (0,1)$, $S_i(t)$ and $v_i(t)$ denote the estimates of the leader's dynamics matrix S and state $v_0(t)$, respectively. For $i, j = 1, \cdots, N$, the values $\widehat{S}_i(t)$ and $\widehat{S}_j(t)$ are the triggered versions of $S_i(t)$ and $S_j(t)$, and similarly, $\widehat{v}_i(t)$ and $\widehat{v}_j(t)$ are the triggered estimates of $v_i(t)$ and $v_j(t)$. Particularly, $\widehat{S}_0(t) = S$ and $\widehat{v}_0(t) = v_0(t)$. Specifically, the signals $\widehat{S}_j(t), \widehat{S}_i(t), \widehat{v}_i(t), \widehat{v}_i(t)$ are determined by the following equations:

$$\begin{aligned} &\widehat{S}_i(t) = S_i(t_k^i), \quad \widehat{v}_i(t) = S_i(t_k^i)^{(t-t_k^i)} v_i(t_k^i), \\ &\widehat{S}_j(t) = S_j(t_{k'}^j), \quad \widehat{v}_j(t) = S_j(t_{k'}^j)^{(t-t_{k'}^j)} v_j(t_{k'}^j), \end{aligned} \tag{4}$$

where t_k^i denotes the kth triggering instant of agent i, and $t_{k'}^j$ is the latest triggering instant of agent j before time instant t, which can be expressed as $t_{k'}^j = \max_{m \in \mathbb{N}} \{t_m^j | t_m^j < t\}$.

For the proposed event-triggered communication based adaptive distributed observer (3), we need to introduce a event-triggered mechanism to determine when to trigger communication for agent i. For $i = 1, \cdots, N$, we first define

$$\begin{aligned} &s_i(t) = \mathrm{vec}(S_i(t)), \quad s = \mathrm{vec}(S), \\ &\widetilde{s}_i(t) = s_i(t) - s, \quad \widetilde{s}(t) = [\widetilde{s}_1^{\mathrm{T}}(t), \cdots, \widetilde{s}_N^{\mathrm{T}}(t)]^{\mathrm{T}}, \end{aligned}$$

$$\begin{aligned}&\widetilde{v}_i(t)=v_i(t)-v_0(t),\quad \widetilde{v}(t)=[\widetilde{v}_1^{\mathrm{T}}(t),\cdots,\widetilde{v}_N^{\mathrm{T}}(t)]^{\mathrm{T}},\\&e_{s_i}(t)=s_i(t_k^i)-s_i(t),\quad e_{v_i}(t)=\widehat{v}_i(t)-v_i(t),\\&e_s(t)=[e_{s_1}^{\mathrm{T}}(t),\cdots,e_{s_N}^{\mathrm{T}}(t)]^{\mathrm{T}},\quad e_v(t)=[e_{v_1}^{\mathrm{T}}(t),\cdots,e_{v_N}^{\mathrm{T}}(t)]^{\mathrm{T}}.\end{aligned}\tag{5}$$

Then, the event-triggered mechanism for agent $i,\ i=1,\cdots,N$, is designed as

$$t_{k+1}^i=\inf\{t_k^i>t|\|e_{s_i}(t)\|+\|e_{v_i}(t)\|\geq g_i(t)\},\tag{6}$$

where $g_i(t)>0$ is a bounded triggering threshold signal to be chosen such that $g_i(t)=O(\mu^{d_i}(t))$ with $d_i>0$ satisfying $\mu^*(1+d_i)>1$. Since $\mu(t)=(t+t_0)^{-\mu^*}$ with $t_0\geq 0$ and $\mu^*\in(0,1)$, one could easily get $\lim_{t\to\infty}g_i(t)=0,\ i=1,\cdots,N$.

Remark 3.1 The event-triggered communication based adaptive distributed observer (3) is designed to simultaneously estimate the leader's system matrix S and state $v_0(t)$. In contrast to some existing work (see [39]), the observer gain here is time-varying, which eliminates the need for global graph information (i.e., spectral radius of H) to ensure convergence. It will be demonstrated that this time-varying observer gain guarantees that the estimates $S_i(t)$ and $v_i(t)$, $i=1,\cdots,N$, convergent to the actual leader's matrix S and state $v_0(t)$.

Remark 3.2 In the adaptive distributed observer (3), the estimates $S_i(t)$ and $v_i(t)$ are updated using $\widehat{S}_i(t)$ and $\widehat{v}_i(t)$, which are specified in (4). Here, $\widehat{S}_i(t)$ and $\widehat{S}_j(t)$ are selected as the latest triggering values, namely $S_i(t_k^i)$ and $S_j(t_{k'}^j)$, respectively. In contrast, $\widehat{v}_i(t)$ and $\widehat{v}_j(t)$ are refreshed constantly according to (4) at each time instant, which is reasonable by noting the leader dynamics (2).

Remark 3.3 It is both intuitive and effective for the event-triggered mechanism designed in (6). The signals $e_{s_i}(t)$ and $e_{v_i}(t)$ quantify the errors introduced by event-triggered communication. Thus, we denote the errors $e_{s_i}(t)$ and $e_{v_i}(t)$ as event-triggered errors. With the event-triggered mechanism (6), the event-triggered errors $e_{s_i}(t)$ and $e_{v_i}(t)$ are ensured to be not too large at all times. Indeed, since the triggering threshold $g_i(t)$ is positive and decays to zero, it could reduce the communication frequency and ensure the event-triggered errors decay to zero as time goes to infinity. Furthermore, to facilitate the convergence analysis, we design the triggering threshold $g_i(t)$ related to the time-varying observer gain $\mu(t)$. With this choice, we could derive the asymptotical convergence of the estimates $S_i(t)$ and $v_i(t)$ in the following.

According to (3) and (5), we have

$$\widetilde{s}_i(t+1)=\widetilde{s}_i(t)+\mu(t)\sum_{j=0}^N a_{ij}(s_j(t)-s_i(t))+\mu(t)\sum_{j=0}^N a_{ij}(e_{s_j}(t)-e_{s_i}(t)),\quad i=1,\cdots,N,$$

$$\begin{aligned}\widetilde{v}_i(t+1)&=S_i(t)v_i(t)-Sv_0(t)+\mu(t)S_i(t)\sum_{j=0}^N a_{ij}(\widehat{v}_i(t)-\widehat{v}_i(t))\\&=S\widetilde{v}_i(t)+\mu(t)S\sum_{j=0}^N a_{ij}(\widetilde{v}_j(t)-\widetilde{v}_i(t))+\widetilde{S}_i(t)v_0(t)+\widetilde{S}_i(t)\widetilde{v}_i(t)\end{aligned}$$

$$+\mu(t)\widetilde{S}_i(t)\sum_{j=0}^{N}a_{ij}(\widetilde{v}_j(t)-\widetilde{v}_i(t))+\mu(t)S_i(t)\sum_{j=0}^{N}a_{ij}(e_{v_j}(t)-e_{v_i}(t)),\quad i=1,\cdots,N,$$

where $s_0(t)=s$, $e_{s_0}(t)=0$, $\widetilde{v}_0(t)=0$, and $e_{v_0}(t)=0$. Note that the graph matrix $H=[h_{ij}]\in\mathbb{R}^{N\times N}$, $i,j\in\mathcal{N}$, with $h_{ii}=\sum_{j=0}^{N}a_{ij}$, and for $i\neq j$, $h_{ij}=-a_{ij}$. Then, it follows from definitions in (5) that the adaptive distributed observer (3) can be transformed into a compact form as follows:

$$\widetilde{s}(t+1)=(I_{Nn^2}-\mu(t)(H\otimes I_{n^2}))\widetilde{s}(t)-\mu(t)(H\otimes I_{n^2})e_s(t), \tag{7}$$

$$\begin{aligned}\widetilde{v}(t+1)=&((I_N\otimes S)-\mu(t)(H\otimes S))\widetilde{v}(t)-(\widetilde{S}_d(t)-\mu(t)A(t))\widetilde{v}(t)\\&+\widetilde{S}_d(t)(I_N\otimes v_0(t))-\mu(t)B(t)e_v(t),\end{aligned} \tag{8}$$

where $\widetilde{S}_d(t)=\text{block diag}\{\widetilde{S}_1(t),\cdots,\widetilde{S}_N(t)\}$ with $\widetilde{S}_i(t)\triangleq S_i(t)-S$, $i=1,\cdots,N$, and

$$A(t)=\begin{pmatrix}H_1\otimes\widetilde{S}_1(t)\\ \vdots\\ H_N\otimes\widetilde{S}_N(t)\end{pmatrix},\quad B(t)=\begin{pmatrix}H_1\otimes S_1(t)\\ \vdots\\ H_N\otimes S_N(t)\end{pmatrix}, \tag{9}$$

with H_i being the ith row of H. The compact form (7) and (8) is beneficial for the following convergence analysis of estimates $S_i(t)$ and $v_i(t)$, $i=1,\cdots,N$.

Next, we proceed to analyse the convergence of the proposed adaptive distributed observer (3). First, we introduce the following lemma, which describes the algebra property of the graph matrix H.

Lemma 3.4 *Under Assumption* (A1), *every eigenvalue of the matrix H has a positive real part.*

The proof of Lemma 3.4 can be seen in the literature (see [41]). Lemma 3.4 shows that the matrix H is nonsingular and the minimal magnitude of eigenvalue of H is positive, i.e., $\lambda_{\min}(H)>0$, which is crucial for the convergence analysis.

It follows from the form of adaptive distributed observer (3) that the convergence of estimate $v_i(t)$ relies on the convergence of estimate $S_i(t)$. Thus, we fist need to obtain the convergence of $S_i(t)$. To this end, the following lemma is needed.

Lemma 3.5 *For $\mu(t)=(t+t_0)^{-\mu^*}$ with $t_0\geq 0$, $\mu^*\in(0,1)$, and constants $b>0$ and $d>0$ such that $\mu^*(1+d)>1$, the following assertions hold as $t\to\infty$.*

$$\prod_{i=1}^{t}(1-b\mu(i))=O\left(\exp\left(\frac{b}{\mu^*-1}(t+t_0)^{1-\mu^*}\right)\right), \tag{10}$$

$$\sum_{i=1}^{t}\prod_{l=i+1}^{k}(1-b\mu(l))\,\mu^{d+1}(i)=O((t+t_0)^{-\mu^*(1+d)}). \tag{11}$$

The proof of this lemma can be seen in the literature (see [42]). This lemma demonstrates that the chosen time-varying observer gain $\mu(t)$ has some desirable properties. We now present the convergence result for the estimate $S_i(t)$ of each agent i, $i=1,\cdots,N$.

Lemma 3.6 *Under Assumption* (A1)*, the estimated leader dynamics matrix* $S_i(t)$, $i=1,\cdots,N$*, asymptotically converges to the true matrix* S *via the proposed adaptive distributed observer* (3)*, i.e.,* $\lim_{t\to\infty} S_i(t)=S$, $i=1,\cdots,N$.

Proof See the Appendix. ∎

Lemma 3.6 establishes the convergence result of $S_i(t)$. Building on this result, the convergence of estimate $v_i(t)$, $i=1,\cdots,N$, could be obtained, which is specified as follows.

Lemma 3.7 *Under Assumptions* (A1) *and* (A2)*, the observer states* $v_i(t)$, $i=1,\cdots,N$*, asymptotically converges to the leader's state* $v_0(t)$ *via the proposed adaptive distributed observer* (3)*, i.e.,* $\lim_{t\to\infty} v_i(t)=v_0(t)$, $i=1,\cdots,N$.

Proof See the Appendix. ∎

Remark 3.8 Lemma 3.6 and Lemma 3.7 collectively demonstrate that the developed adaptive distributed observer (3) is cable of simultaneously estimate the leader's dynamics matrix S and state $v_0(t)$ for every agent. Benefited from the properties of $\mu(t)$ in Lemma 3.5, the convergence results are derived without the need for global graph knowledge (i.e., spectral radius of H), which is different from some existing works in discrete-time MAS (see [39]). Moreover, distinct from the continuous-time event-triggered communication based adaptive distributed observer in [35], where a constant positive observer gain is sufficient for the convergence, a time-varying observer gain is utilized to ensure the validity of (3). Generally, an arbitrary positive constant observer gain is incapable of achieving the convergence of (3) unless the spectral radius of H is known to determine an upper bound for the observer gain (see [39]).

Based on these two lemmas, the subsequent result follows straightforwardly.

Lemma 3.9 *For any* $k\geq 0$*, we have*

$$\lim_{t\to\infty}(FS_i^k(t)v_i(t)-y_0(t+k))=0,\quad i=1,\cdots,N. \tag{12}$$

Proof See the Appendix. ∎

Lemma 3.9 demonstrates that asymptotical estimates for the leader signals $y_0(t+k)$, $k\geq 1$ can be achieved with estimated leader's dynamics matrix $S_i(t)$ and state $v_i(t)$, which is crucial for the following adaptive control design of each follower agent.

In continuous-time systems, event-triggered control must address Zeno behavior, i.e., the occurrence of infinitely many events in a finite time, to ensure a reduction in communication compared to non-triggered designs. In contrast, discrete-time systems inherently have a minimum inter-event interval (the sampling time), so Zeno behavior is not a concern. Instead, [43] introduced the communication rate as a metric to evaluate the effectiveness of the event-triggered mechanism in discrete-time systems. By proving a decaying communication rate, [43] showed that their event-triggered mechanism could significantly reduce the communication frequency. Then, to evaluate the validity of the proposed event-triggered mechanism (6), we analyse its communication rate. First, we give the definition of communication rate introduced in [43] as follows.

Definition 3.10 (see [43]) Let $K_i(t)$ be the number of triggering events for agent i, $i = 1, \cdots, N$, during the interval $[0, t]$. The communication rate is defined by $\lambda_c(t) = \frac{\sum_{i=1}^{N} K_i(t)|\mathcal{N}_i|}{t\sum_{i=1}^{N}|\mathcal{N}_i|}$, where $|\mathcal{N}_i|$ denotes the number of child neighbors of node i.

The above definition shows that the communication rate $\lambda_c(t)$ lies in region $[0, 1]$. Particularly, $\lambda_c(t) = 1$ means communication between follower agents occurs at every time instant up to t. Therefore, a decreasing $\lambda_c(t)$ implies a reduction in communication frequency. Based on this definition, we show the proposed event-triggered mechanism leads to a decaying $\lambda_c(t)$ under appropriate design parameters.

Lemma 3.11 *If the triggering threshold $g_i(t)$, $i = 1, \cdots, N$, satisfies*

$$\underline{\lim}_{t\to\infty}\frac{g_i(t+t^{\sigma})}{g_i(t)} > 0, \quad \underline{\lim}_{t\to\infty}\frac{g_i(t)}{r(t)t^{\sigma}} > 0, \tag{13}$$

where $r(t) \triangleq (t+t_0)^{-\mu^*(1+d)}$ and $\sigma \in (0, 1)$. Then, we get the communicate rate $\lambda_c(t)$ satisfies $\lim_{t\to\infty}\lambda_c(t) = 0$.

Proof See the Appendix. ∎

Remark 3.12 Condition (13) is easily met with the chosen $g_i(t) = O(\mu^{d_i}(t))$. For instance, one can validate (13) with $g_i(t)$ selected as $\mu^{d_i}(t)$ with $d_i > 0$. It is well recognized that there is a trade-off between the convergence rate and the communication resource utilization under the event-triggered mechanism. Specifically, for a given observer gain $\mu(t)$, a larger parameter d_i causes $g_i(t)$ to decay more rapidly, increasing the number of triggering events (and thus communication frequency), but also accelerating the convergence of the adaptive distributed observer. Therefore, selecting appropriate parameters in (6) is crucial for balancing these competing requirements.

3.2 Controller Structure

In this subsection, we develop a new form of adaptive controller for each follower agent i. By employing the estimated leader system matrix $S_i(t)$ and the state $v_i(t)$, we define the following auxiliary signals

$$p_{r_i}(t) = FS_i^{r_i}(t)v_i(t) - \sum_{k=0}^{r_i-1} a_{i,r_i-k}(y_i(t+k) - FS_i^{k}(t)v_i(t)), \quad i = 1, \cdots, N, \tag{14}$$

$$q_i(t) = y_i(t+r_i) - p_{r_i}(t), \quad i = 1, \cdots, N, \tag{15}$$

where $a_{i,1}, \cdots, a_{i,r_i}$ are some constant parameters to be chosen such that $P_i(z) = z^{r_i} + a_{i,1}z^{r_i-1} + \cdots + a_{i,r_i-1}z + a_{i,r_i}$ is a stable polynomial.

We define the leader-following tracking error by $e_i(t) = y_i(t) - y_0(t)$, $i = 1, \cdots, N$. The next lemma establishes the relationship between the leader-following tracking error and the observer estimation error.

Lemma 3.13 *For every follower agent i, the leader-following tracking error $e_i(t)$ satisfies*

$$P_i(z)[e_i](t) = \widetilde{q}_i(t), \quad i = 1, \cdots, N, \tag{16}$$

where $\widetilde{q}_i(t) = q_i(t) + e_{o_i}(t)$ with

$$e_{o_i}(t) = (FS_i^{r_i}(t)v_i(t) - y_0(t + r_i)) + \sum_{k=0}^{r_i-1} a_{i,r_i-k}(FS_i^k(t)v_i(t) - y_0(t + k)). \tag{17}$$

Proof For any interger $k \geq 0$, $y_i(t + k) - FS_i^k(t)v_i(t) = y_i(t + k) - y_0(t + k) + y_0(t + k) - FS_i^k(t)v_i(t) = e_i(t + k) + y_0(t + k) - FS_i^k(t)v_i(t),\ i = 1, \cdots, N$. Substituting this expression into the definition of $q_i(t)$ from (15) gives

$$\begin{aligned} q_i(t) &= y_i(t + r_i) - FS_i^{r_i}(t)v_i(t) + \sum_{k=0}^{r_i-1} a_{i,r_i-k}(y_i(t+k)-FS_i^k(t)v_i(t)) \\ &= e_i(t + r_i) + (y_0(t + r_i) - FS_i^{r_i}(t)v_i(t)) + a_{i,1}e_i(t + r_i - 1) + a_{i,1}(y_0(t + r_i - 1) \\ &\quad -FS_i^{r_i-1}(t)v_i(t)) + \cdots + a_{i,r_i}e_i(t) + a_{i,r_i}(y_0(t) - Fv_i(t)) \\ &= P_i(z)[e_i](t) - e_{o_i}(t), \quad i = 1, \cdots, N. \end{aligned}$$

Therefore, it is easy to drive the conclusion. This completes the proof. ∎

Remark 3.14 The decomposition $\widetilde{q}_i(t) = q_i(t) + e_{o_i}(t)$ shows that the leader-follower tracking error $e_i(t)$ is caused by two components. The term $e_{o_i}(t)$ characterizes the estimation error between true leader outputs and the estimated ones. Thus, its zero convergence is guaranteed by Lemma 3.9. Meanwhile, $q_i(t)$ captures the difference between agent's output and its estimate of the leader's output. Consequently, if the control law $u_i(t)$ is designed to drive $q_i(t)$ to zero, then the stability of $P_i(z)$ guarantees asymptotical output tracking by $\lim_{t\to\infty} P_i(z)[e_i](t) = 0$. In other words, by focusing on regulating $q_i(t)$, we indirectly control the tracking error. This is a certainly equivalence based design idea. Indeed, if $S_i(t) = S$ and $v_i(t) = v_0(t)$, then the equation (16) would become an equality with $q_i(t)$ becoming the leader-following tracking error. Notably, the establishments of auxiliary signals $p_{r_i}(t)$ and $q_i(t)$ in (15) only rely on local estimation of leader's information with the developed adaptive distributed observer (3). Thus, by converting the original leader-following tracking problem to a local tracking control problem for the signal $p_{r_i}(t)$ to be tracked and the error signal $q_i(t)$, we could overcome the difficulty caused by heterogeneous nonlinear dynamics of the follower agent.

Motivated by Lemma 3.13, we consider taking $q_i(t)$ as the error variable to be regulated for each agent i. We now design the control law for agent i with the system dynamics (1). First, we define $\rho_i^* = \frac{1}{b_i^*},\ i = 1, \cdots, N$, and $\delta_i^* = \rho_i^*\theta_i^*,\ i = 1, \cdots, N$. Then, for agent i, the adaptive controller $u_i(t)$ is given by

$$u_i(t) = (1+\alpha_i(t)b_i(t))^{-1}(-\delta_i^{\mathrm{T}}(t)f_i(x_i(t)) - \alpha_i(t)\theta_i^{\mathrm{T}}(t)f_i(x_i(t)) + (\rho_i(t)+\alpha_i(t))p_{r_i}(t)), \quad (18)$$

where $\delta_i(t), \theta_i(t), \rho_i(t)$, and $b_i(t)$ denote estimates of parameters $\delta_i^*, \theta_i^*, \rho_i^*$, and b_i^*, respectively. Additionally, the function $\alpha_i(t)$ is a time-varying gain to be designed later to guarantee the inverse operation $1+\alpha_i(t)b_i(t)$ remains nonzero.

Remark 3.15 In the controller (18), it is assumed that the nonlinear function $f_i(x_i(t))$ and the state $x_i(t)$ are available for every follower i. Moreover, the auxiliary signal $p_{r_i}(t)$ defined in (14) is constructed entirely from the agent's output measurements $y_i(t+k)$, $k = 0, 1, \cdots, r_i - 1$, and the estimates $S_i(t)$ and $v_i(t)$, all of which are accessible to each agent. Consequently, the control law (18) is implementable since no undetectable signals are involved. However, note that the inverse operation in (18) may lead to high gain issues. Hence, the time-varying gain $\alpha_i(t)$ needs to be designed appropriately to ensure $1+\alpha_i(t)b_i(t) \neq 0$, which will be performed in the subsequent discussion.

3.3 Parameter Update Law

In this subsection, we devise a parameter update law for the controller parameter estimates. For this purpose, we first derive the closed-loop error equation for each follower i based on the control law (18). For $i = 1, \cdots, N$, we rewrite (18) in the form

$$u_i(t) = -\delta_i^{\mathrm{T}}(t)f_i(x_i(t)) - \alpha_i(t)\theta_i^{\mathrm{T}}(t)f_i(x_i(t)) + (\alpha_i(t)+\rho_i(t))p_{r_i}(t) - \alpha_i(t)b_i(t)u_i(t). \quad (19)$$

According to the system dynamics (1), the output dynamics can be expressed compactly as $y_i(t+r_i) = \theta_i^{*\mathrm{T}} f_i(x_i(t)) + b_i^* u_i(t)$, $i = 1, \cdots, N$. Next, by adding and subtracting the term $\alpha_i(t)y_i(t+r_i)$ in (19), we obtain

$$\begin{aligned} u_i(t) &= -\delta_i^{\mathrm{T}}(t)f_i(x_i(t)) - \alpha_i(t)\theta_i^{\mathrm{T}}(t)f_i(x_i(t)) + (\rho_i(t)+\alpha_i(t))p_{r_i}(t) - \alpha_i(t)b_i(t)u_i(t) \\ &\quad +\alpha_i(t)y_i(t+r_i) - \alpha_i(t)y_i(t+r_i) \\ &= -\delta_i^{\mathrm{T}}(t)f_i(x_i(t)) - \alpha_i(t)\widetilde{\theta}_i^{\mathrm{T}}(t)f_i(x_i(t)) + (\rho_i(t)+\alpha_i(t))p_{r_i}(t) - \alpha_i(t)\widetilde{b}_i(t)u_i(t) \\ &\quad -\alpha_i(t)y_i(t+r_i), \quad i = 1, \cdots, N, \end{aligned} \quad (20)$$

where $\widetilde{\theta}_i(t) \triangleq \theta_i(t) - \theta_i^*$ and $\widetilde{b}_i(t) \triangleq b_i(t) - b_i^*$. Since $\rho_i^* = \frac{1}{b_i^*}$ and $\delta_i^* = \rho_i^*\theta_i^*$, it follows that

$$\rho_i^* y_i(t+r_i) = \delta_i^{*\mathrm{T}} f_i(x_i(t)) + u_i(t), \quad i = 1, \cdots, N. \quad (21)$$

Substituting (20) into (21) gives

$$\begin{aligned} \rho_i^* y_i(t+r_i) &= -\widetilde{\delta}_i^{\mathrm{T}} f_i(x_i(t)) - \alpha_i(t)\widetilde{\theta}_i^{\mathrm{T}}(t)f_i(x_i(t)) - \alpha_i(t)\widetilde{b}_i(t)u_i(t) + (\alpha_i(t)+\rho_i(t))p_{r_i}(t) \\ &\quad -\alpha_i(t)y_i(t+r_i), \quad i = 1, \cdots, N, \end{aligned} \quad (22)$$

where $\widetilde{\delta}_i(t) \triangleq \delta_i(t) - \delta_i^*$. Introducing the parameter error $\widetilde{\rho}_i(t) \triangleq \rho_i(t) - \rho_i^*$, we rearrange (22) to obtain

$$
\begin{aligned}
(\rho_i(t)+\alpha_i(t))y_i(t+r_i) = &-\widetilde{\delta}_i^{\mathrm{T}} f_i(x_i(t)) - \alpha_i(t)\widetilde{\theta}_i^{\mathrm{T}}(t) f_i(x_i(t)) - \alpha_i(t)\widetilde{b}_i(t)u_i(t) + \widetilde{\rho}_i(t)y_i(t+r_i) \\
&+(\rho_i(t)+\alpha_i(t))p_{ri}(t), \quad i=1,\cdots,N.
\end{aligned}
$$

Assuming $\rho_i(t)+\alpha_i(t) \neq 0$ for $i=1,\cdots,N$, the error variable $q_i(t)$ defined in (15) can be expressed as

$$
q_i(t) = -\widetilde{\delta}_i^{\mathrm{T}} \frac{f_i(x_i(t))}{\rho_i(t)+\alpha_i(t)} - \widetilde{\theta}_i^{\mathrm{T}}(t)\frac{\alpha_i(t)f_i(x_i(t))}{\rho_i(t)+\alpha_i(t)} - \widetilde{b}_i(t)\frac{\alpha_i(t)u_i(t)}{\rho_i(t)+\alpha_i(t)} + \widetilde{\rho}_i(t)\frac{y_i(t+r_i)}{\rho_i(t)+\alpha_i(t)}. \quad (23)
$$

For each $i=1,\cdots,N$, we define the following parameter vector.

$$
\beta_i^* = [\delta_i^{*\mathrm{T}}, \theta_i^{*\mathrm{T}}, b_i^*, \rho_i^*]^{\mathrm{T}}, \quad \beta_i(t) = [\delta_i^{\mathrm{T}}(t), \theta_i^{\mathrm{T}}(t), b_i(t), \rho_i(t)]^{\mathrm{T}}, \quad \widetilde{\beta}_i(t) = \beta_i(t) - \beta_i^*.
$$

Also, for $i=1,\cdots,N$, define the regressor

$$
\omega_i(t) = \left[-\frac{f_i^{\mathrm{T}}(x_i(t))}{\rho_i(t)+\alpha_i(t)}, -\frac{\alpha_i(t)f_i^{\mathrm{T}}(x_i(t))}{\rho_i(t)+\alpha_i(t)}, -\frac{\alpha_i(t)u_i(t)}{\rho_i(t)+\alpha_i(t)}, \frac{y_i(t+r_i)}{\rho_i(t)+\alpha_i(t)} \right]^{\mathrm{T}}.
$$

With these definitions, (23) becomes

$$
q_i(t) = \widetilde{\beta}_i^{\mathrm{T}}(t)\omega_i(t), \quad i=1,\cdots,N. \quad (24)
$$

Equation (24) characters the relationship between the parameter estimates and the error variable $q_i(t)$. For convenience of statement, we refer the equation (24) as the tracking error equation for the error variable $q_i(t)$. It is crucial to note that $\rho_i(t)+\alpha_i(t)$ must never vanish during adaptation to avoid singularities. The design of the time-varying gain $\alpha_i(t)$ will ensure this condition.

While the state $x_i(t)$ and the outputs $y_i(t+k)$, $k=0,1,\cdots,r_i-1$, are available at the current time, the signal $y_i(t+r_i)$ is not directly measurable. This makes the regressor $\omega_i(t)$ impractical for immediate use. To avoid this casuality contradiction issue in adaptive design, we introduce stable filter operators defined as

$$
h_i(z) = \frac{1}{z-\lambda_i}, \quad i=1,\cdots,N, \quad (25)
$$

where $0 \leq \lambda_i < 1$ are chosen constants. Define the filtered versions of $q_i(t)$ and $\omega_i(t)$ by $\overline{q}_i(z) = h_i(z)[q_i](t)$ and $\overline{\omega}_i(t) = h_i(z)[\omega_i](t)$. Operating on both sides of (24) with the filter (25) yields

$$
\overline{q}_i(t) = h_i(z)[\beta_i\omega_i](t) - \beta_i^{*\mathrm{T}}\overline{\omega}_i(t), \quad i=1,\cdots,N, \quad (26)
$$

which we refer as the filtered tracking error equation for error variable $q_i(t)$. The filtered signals $\overline{q}_i(t)$ and $\overline{\omega_i}(t)$ are now available at the current time, thereby facilitating the subsequent design of the parameter update law.

Next, we proceed to develop a parameter update law for the estimated controller parameters using an estimation cost criterion (see [44]). With the filtered tracking error $\overline{q}_i(t)$, the estimation error $\varepsilon_i(t)$ is defined as

$$\varepsilon_i(t) = \overline{q}_i(t) - h_i(z)[\beta_i^{\mathrm{T}}\omega_i](t) + \beta_i^{\mathrm{T}}(t)h_i(z)[\omega_i](t), \quad i = 1, \cdots, N. \tag{27}$$

By combining this definition with the filtered tracking error equation (26), we derive the following estimation error equation as

$$\varepsilon_i(t) = \widetilde{\beta}_i^{\mathrm{T}}(t)\overline{\omega}_i(t), \quad i = 1, \cdots, N. \tag{28}$$

Using $\varepsilon_i(t), \quad i = 1, \cdots, N$, we then introduce a quadratic cost function $J_i = \frac{\varepsilon_i^2}{2m_i^2}$, where $m_i^2 = m_i^2(t) \triangleq 1 + \overline{\omega}_i^{\mathrm{T}}(t)\overline{\omega}_i(t)$ is a normalized signal. Based on the gradient of cost function J_i, the parameter $\beta_i(t)$ is updated by the following parameter update law

$$\beta_i(t+1) = \beta_i(t) - \frac{\mathit{\Gamma}_i\varepsilon_i(t)\overline{\omega}_i(t)}{m_i^2(t)}, \quad i = 1, \cdots, N, \tag{29}$$

with the adaptive gain $\mathit{\Gamma}_i$ satisfying $0 < \mathit{\Gamma}_i < 2I$.

Remark 3.16 It is notable that the signal $q_i(t)$ involves the unmeasurable output signal $y_i(t+r_i)$, which makes it unavailable at the current time. In this case, we use $\overline{q}_i(t)$ to define the estimation error $\varepsilon_i(t)$. Then, a linear regressive estimation error equation (28) can be derived as above. This formulation allows us to derive the gradient based parameter update law (29) for adjusting the controller parameters effectively.

Remark 3.17 Notably, the parameter update law (29) does not require any sign information for b_i, nor does the selection of the adaptive gain $\mathit{\Gamma}_i$ depend on knowing an upper bound for b_i. This advantage stems from the design of new adaptive control law (18), which leads to a linear tracking error equation (24) and, consequently, a linear regressive estimation error equation (28) rather than the usual bilinear form. Specifically, if we design a conventional adaptive control law of the form

$$u_i(t) = -\delta_i^{\mathrm{T}}(t)f_i^{\mathrm{T}}(x_i(t)) + \rho_i(t)p_{ri}(t), \quad i = 1, \cdots, N,$$

then the induced tracking error equation for the error variable $q_i(t)$, $i = 1, \cdots, N$, would be $q_i(t) = b_i^*\widetilde{\beta}_i^{\mathrm{T}}(t)\omega_i(t)$ with $\widetilde{\beta}_i(t) \triangleq \beta_i(t) - \beta_i^* = [\delta_i^{\mathrm{T}}(t), \rho_i(t)]^{\mathrm{T}} - [\delta_i^{*\mathrm{T}}, \rho_i^*]^{\mathrm{T}}$. After defining an estimation error $\varepsilon_i(t)$, one would typically arrive at a parameter update law to estimate $\beta_i^* = [\delta_i^{*\mathrm{T}}, \rho_i^*]^{\mathrm{T}}$ and b_i^* as follows

$$\beta_i(t+1) = \beta_i(t) - \frac{\mathrm{sign}[b_i^*]\mathit{\Gamma}_i\varepsilon_i(t)\zeta_i(t)}{m_i^2(t)}, \quad i = 1, \cdots, N,$$
$$b_i(t+1) = b_i(t) - \frac{\gamma_i\varepsilon_i(t)\xi_i(t)}{m_i^2(t)}, \quad i = 1, \cdots, N,$$

where $\zeta_i(t)$ and $\xi_i(t)$ are some regressors and the adaptive gain Γ_i satisfies $0 < \Gamma < 2I/b_i^0$ with b_i^0 being an upper bound of b_i. In contrast, the parameter update law (29) based on control law (18) avoids the need for both the sign information sign$[b_i^*]$ and an upper bound b_i^0. This simplification, afforded by the linear regressive estimation error equation in (28), highlights a key benefit of the new controller structure (18).

From the form of control law (18) and parameter update law (29), it is obvious that the control algorithm may become singular if either $1 + \alpha_i(t)b_i(t) = 0$ or $\alpha_i(t) + \rho_i(t) = 0$ occurs. To prevent such singularities, the following lemma gives the design of time-varying gain $\alpha_i(t)$.

Lemma 3.18 *If gain functions $\alpha_i(t),\ i = 1, \cdots, N$, is designed as*

$$\alpha_i(t) = \begin{cases} -\left(|\rho_i(t)| + \underline{\alpha}_i\right), & b_i(t) < 0, \\ |\rho_i(t)| + \underline{\alpha}_i, & b_i(t) \geq 0, \end{cases} \tag{30}$$

where $\underline{\alpha}_i$ is a arbitrary positive constant to be chosen, then the conditions $1 + \alpha_i(t)b_i(t) \neq 0,\ \rho_i(t) + \alpha_i(t) \neq 0,\ i = 1, \cdots, N$ would always hold for any $\rho_i(t) \in \mathbb{R}$ and $b_i(t) \in \mathbb{R}$.

The proof of Lemma 3.18 can be seen in the literature (see [38]). Lemma 3.18 ensures that no singularity problem would arise in the adaptive process with $\alpha_i(t),\ i = 1, \cdots, N$, defined as (30).

3.4 Stability Analysis

We now analyse the closed-loop control performance. We begin by establishing properties of the estimated parameters under the parameter update law (29).

Lemma 3.19 *Under the parameter update law* (29)*, for each agent i, the following properties hold: $\beta_i(t) \in L^\infty$, $\frac{\varepsilon_i(t)}{m_i(t)} \in L^2 \cap L^\infty$, and $\beta_i(t+1) - \beta_i(t) \in L^2$.*

Proof Choose a positive definite function as: $V_i(\widetilde{\beta}_i) = \widetilde{\beta}_i^{\mathrm{T}} \Gamma_i^{-1} \widetilde{\beta}_i$. Then, it yields

$$V_i(\widetilde{\beta}_i(t+1)) - V(\widetilde{\beta}_i(t)) = -\left(2 - \frac{\overline{\omega}_i^{\mathrm{T}}(t)\Gamma_i\overline{\omega}_i(t)}{m_i^2(t)}\right)\frac{\varepsilon_i^2(t)}{m_i^2(t)}.$$

Since $0 < \Gamma_i = \Gamma_i^{\mathrm{T}} < 2I$, it follows from the definition of $m_i(t)$ that $V_i(\widetilde{\beta}_i(t+1)) - V_i(\widetilde{\beta}_i(t)) \leq -k_i\frac{\varepsilon_i^2(t)}{m_i^2(t)}$ for some constant $k_i > 0$. This implies that $\beta_i(t) \in L^\infty$ and $\frac{\varepsilon_i(t)}{m_i(t)} \in L^2$. Then, we get $\frac{\varepsilon_i(t)}{m_i(t)} \in L^\infty$. Further, we have $\beta_i(t+1) - \beta_i(t) \in L^2$. This completes the proof. ∎

Based on the design process described, we now present the main theoretical result.

Theorem 3.20 *Under Assumptions* (A1) *and* (A2)*, for every follower agent i, the developed adaptive control law* (18) *and parameter update law* (29) *can achieve that the output $y_i(t)$ tracks the leader's output $y_0(t)$ asymptotically via the event-triggered communication based adaptive distributed observer* (3)*, i.e., $\lim_{t\to\infty}(y_i(t) - y_0(t)) = 0,\ i = 1, \cdots, N$.*

Proof First, we prove some boundedness property of the regressor $\overline{\omega}_i(t)$. By the definition of $\omega_i(t)$ and the boundedness of the parameter estimates, it yields $\|\omega_i(t)\| \leq c\max_{k\leq t}\|u_i(k)\| + c\max_{k\leq t+r_i}\|y_i(k)\| + c,\ i = 1, \cdots, N$, with $c > 0$ being a general constant bound. Lemma 3.6

and Lemma 3.7 imply the estimates $S_i(t)$ and $v_i(t)$ are bounded. Then, since $y_0(t)$ is bounded and $q_i(t)$ in (15) is defined based on measurable signals, we have $\|\omega_i(t)\| \leq c \max_{k\leq t} \|q_i(k)\| + c,\ i = 1, \cdots, N$. Since $h_i(z),\ i = 1, \cdots, N$, is stable filter and $\overline{\omega}_i(z) = h_i(z)[\omega_i](z)$, we have

$$\|\overline{\omega}_i(t)\| \leq c \max_{k\leq t} \|q_i(k)\| + c, \quad i = 1, \cdots, N. \tag{31}$$

Then, we prove the boundedness of closed-loop signals. It follows from the definition of normalization signal $m_i(t)$ and (31) that

$$\|m_i(t)\| \leq 1 + \|\overline{\omega}_i(t)\| \leq c \max_{k\leq t} \|q_i(k)\| + c, \quad i = 1, \cdots, N. \tag{32}$$

Without loss of generality, assuming that for $i = 1, \cdots, N$, the filter $h_i(z)$ is selected as $1/z$. From the definition of the estimation error $\varepsilon_i(t),\ i = 1, \cdots, N$, we obtain

$$\begin{aligned}\overline{q}_i(t) &= \varepsilon_i(t) + h_i(z)[\beta_i^{\mathrm{T}}\omega_i](t) - \beta_i^{\mathrm{T}}(t) h_i(z)[\omega_i](t)\\ &= \frac{\varepsilon_i(t)}{m_i(t)} m_i(t) - (\beta_i(t) - \beta_i(t-1))^{\mathrm{T}}\overline{\omega}_i(t).\end{aligned} \tag{33}$$

From Lemma 3.19, we have $\frac{\varepsilon_i(t)}{m_i(t)} \in L^2 \cap L^\infty$ and $\beta_i(t) - \beta_i(t-1) \in L^2 \cap L^\infty$. Hence, for $i = 1, \cdots, N$, it follows from (32) and (33) that

$$\|\overline{q}_i(t)\| \leq \left\|\frac{\varepsilon_i(t)}{m_i(t)}\right\| \|m_i(t)\| + \|\beta_i(t) - \beta_i(t-1)\| \|\overline{\omega}_i(t)\| \leq \tau(t) \max_{k\leq t} \|q_i(k)\| + c, \tag{34}$$

where $\tau(t)$ denotes a general $L^2 \cap L^\infty$ function which converges to zero as t goes to ∞. From the definition of $\overline{q}_i(t)$ and (34), it yields that $\overline{q}_i(t),\ i = 1, \cdots, N$, is bounded. Therefore, we get $q_i(t),\ i = 1, \cdots, N$, is bounded. Noting the definition of $q_i(t)$ in (15), we obtain $y_i(t),\ i = 1, \cdots, N$, is bounded. Then, it is easy to drive the control law $u_i(t),\ i = 1, \cdots, N$, designed as (18) are bounded with the boundness of $y_i(t)$ and the non-singularity of $1+\alpha_i(t)\rho_i(t)$ shown in Lemma 3.18. Further, all signals in the closed-loop signals can be proven as bounded.

Finally, we prove the tracking performance. It follows from $\frac{\varepsilon_i(t)}{m_i(t)} \in L^2 \cap L^\infty$ and $\beta_i(t) - \beta_i(t-1) \in L^2 \cap L^\infty$ in Lemma 3.19 and (33) that $\overline{q}_i(t) \in L^2 \cap L^\infty$, i.e., $\lim_{t\to\infty} \overline{q}_i(t) = 0,\ i = 1, \cdots, N$. The stability of the filters $h_i(z)$, ensures that $\lim_{t\to\infty} q_i(t) = 0,\ i = 1, \cdots, N$. From Lemma 3.6 and Lemma 3.7, we obtain that $\lim_{t\to\infty} e_{o_i} = 0,\ i = 1, \cdots, N$, by noting its definition in (17). Then, combining $\lim_{t\to\infty} e_{o_i} = 0$ and $\lim_{t\to\infty} q_i(t) = 0$ gives $\lim_{t\to\infty} \widetilde{q}_i(t) = 0,\ i = 1, \cdots, N$. Further, invoking Lemma 3.13, we conclude that $\lim_{t\to\infty} P_i(z)[e_i](t) = 0,\ i = 1, \cdots, N$. Because $P_i(z)$ is a stable polynomial for $i = 1, \cdots, N$, it yields that $\lim_{t\to\infty} e_i(t) = 0,\ i = 1, \cdots, N$, which implies the asymptotical convergence of leader-following tracking errors $e_i(t),\ i = 1, \cdots, N$. This completes the proof. ∎

So far, a distributed leader-following adaptive tracking control strategy has been developed that enables each follower agent to asymptotically track the leader using only neighboring information. This strategy contains two main steps, i.e., estimating leader's dynamics matrix and state signal by an adaptive distributed observer and regulating the behavior of each follower by an adaptive controller. Particularly, by incorporating an event-triggered mechanism, the communication frequency among follower agents is reduced, as demonstrated by the decaying communication rate. Furthermore, the novel controller structure ensures that no prior system parameter information, such as sign$[b_i]$ and upper bound of b_i, is needed to achieve stable parameter update law design.

4 Simulation Study

This section presents a numerical example to illustrate the validity of the developed control method.

4.1 Simulaiton Model

Consider an MAS including a leader system with the dynamics (2) and four follower systems with the dynamics (1). Specifically, the four follower agents have the following system parameters and nonlinear dynamics.

$$
\begin{aligned}
&\theta_1^* = [1, -1], \quad b_1^* = 0.5, \quad f_1(x_1) = [\sin x_{11}, \sqrt{1 + \sin x_{12}}],\\
&\theta_2^* = [2, -1], \quad b_2^* = -1, \quad f_2(x_2) = [0.5 \sin \ln(x_{21})^2, \sqrt{(1 + \sin x_{22})^2}],\\
&\theta_3^* = [1, -1], \quad b_3^* = -1, \quad f_3(x_3) = [1 + \sin x_{33}, \sqrt{x_{31}^2 + x_{32}^2}],\\
&\theta_4^* = [1, -1], \quad b_4^* = -0.5, \quad f_4(x_4) = [\sin x_{41}, \sin x_{42}],
\end{aligned}
$$

where $x_1 = [x_{11}, x_{12}]$, $x_2 = [x_{21}, x_{22}]$, $x_3 = [x_{31}, x_{32}, x_{33}]$, $x_4 = [x_{41}, x_{42}]$ are the state variable of four follower agents. Thus, we derive that relative degrees of four follower agents are $r_1 = 2, r_2 = 2, r_3 = 3, r_4 = 2$. The leader system is with the initial value $v(0) = [0, 2]^{\mathrm{T}}$ and possesses the following system matrix

$$
S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{6} & \sin\frac{\pi}{6} \\ -\sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{bmatrix}, \quad F = [1, 0].
$$

The communication graph of the MAS is depicted as Figure 1, where $a_{ij} = 1$ if a directed edge from node j to node i exists.

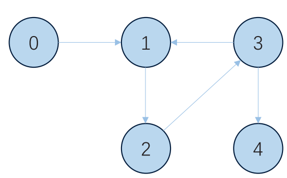

Figure 1 Communication graph

For the event-triggered communication based adaptive distributed observer (3), we choose the observer gain as $\mu(t) = t^{-0.4}$ and the triggering threshold $g_i(t) = t^{-0.7}$ for $i = 1, 2, 3, 4$, which satisfy $\mu^*(1 + d_i) > 1$. The initial estimates are set as $S_1(0) = 1.1S,\ S_2(0) = 0.8S,\ S_3(0) = 0.9S,\ S_4(0) = 1.2S$ and $v_1(0) = [-1, 3]^{\mathrm{T}},\ v_2(0) = [2, 2]^{\mathrm{T}},\ v_3(0) = [-2, 1]^{\mathrm{T}},\ v_4(0) = [3, 2.2]^{\mathrm{T}}$.

For the adaptive controller design of agent $i,\ i = 1, 2, 3, 4$, we choose the adaptive gain $\varGamma_i = I$ and $\underline{\alpha}_i = 0.5$ for $i = 1, 2, 3, 4$. Setting initial values as $\beta_1(0) = [1, -2.3, 1.1, -1.5, -0.5, 1.5]^{\mathrm{T}}$, $\beta_2(0) = [-2.1, 1.5, 2.2, -1.5, -1.5, -0.5]^{\mathrm{T}}$, $\beta_3(0) = [0.8, -2.5, -0.2, 2.5, -1.5, -1.2]^{\mathrm{T}}$, $\beta_4(0) = [-2.5, 1.1, 1.2, -0.8, -0.9, -3]^{\mathrm{T}}$ and $x_1(0) = [1, 5]^{\mathrm{T}}$, $x_2(0) = [1, 4]^{\mathrm{T}}$, $x_3(0) = [-2, 3, 2]^{\mathrm{T}}$, $x_4(0) = [-3, 3]^{\mathrm{T}}$. Moreover, we choose $a_{11} = -1, a_{12} = 1/4, a_{21} = -1, a_{22} = 1/4, a_{31} = -3/2, a_{32} = 3/4, a_{33} = -1/8, a_{41} = -1, a_{42} = 1/4$. Then, for every agent i, we could determine the control law $u_i(t)$ with (18) and the parameter update law with (29).

4.2 Simulation Results

The simulation results are presented in Figures 2–7. Figure 2 and Figure 3 display the trajectories of estimate errors $v_i(t) - v_0(t) = [v_{i1}(t) - v_{01}(t), v_{i2}(t) - v_{02}(t)]^{\mathrm{T}}$ and $S_i(t) - S = [S_{i11}(t) - S_{11}, S_{i12}(t) - S_{12}; S_{i21}(t) - S_{21}, S_{i22}(t) - S_{22}]$ for agent $i,\ i = 1, 2, 3, 4$, under the proposed adaptive distributed observer. Figures 2–3 confirms that the both the leader's dynamics matrix S and state $v_0(t)$ are asymptotically estimated, which is in accordance with the results of Lemma 3.6 and Lemma 3.7. Figure 4 illustrates the leader-following tracking performance and Figure 5 shows the trajectory of parameter estimates $b_i(t),\ i = 1, 2, 3, 4$, which demonstrates that the control strategy achieves both asymptotical tracking and closed-loop stability. To access the event-triggered mechanism, we simulate the trajectories of triggering instants and the communication rate, as shown in Figure 6 and Figure 7, respectively. These results indicate that the introduced event-triggered mechanism reduces the communication frequency, leading to a decaying communication rate. In a word, this simulation demonstrates the validity of the proposed distributed adaptive control method.

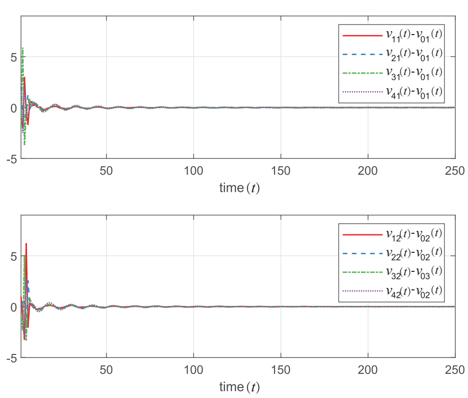

Figure 2 Trajectories of the leader's state estimate error

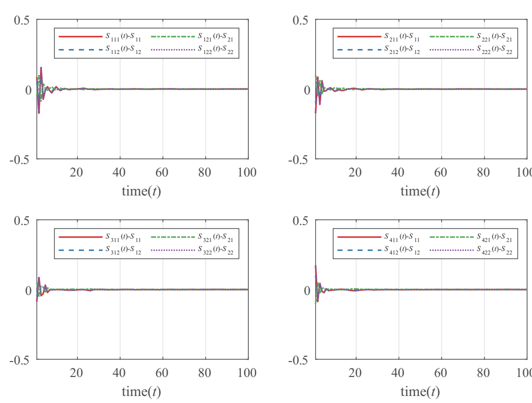

Figure 3 Trajectories of the leader's dynamics matrix estimate error

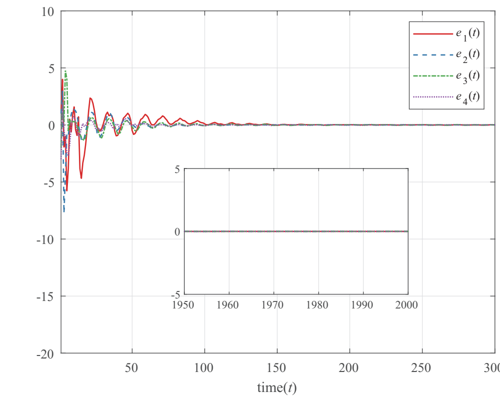

Figure 4 Trajectories of the leader-following tracking errors

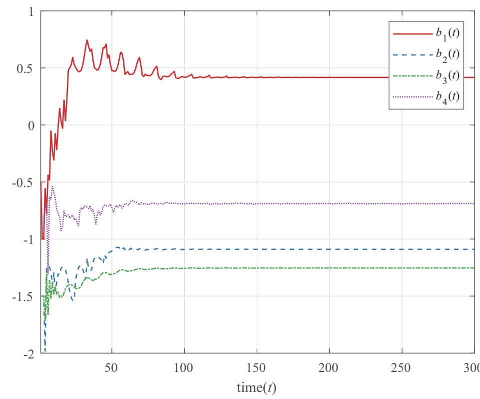

Figure 5 Trajectories of the control gain parameter estimates

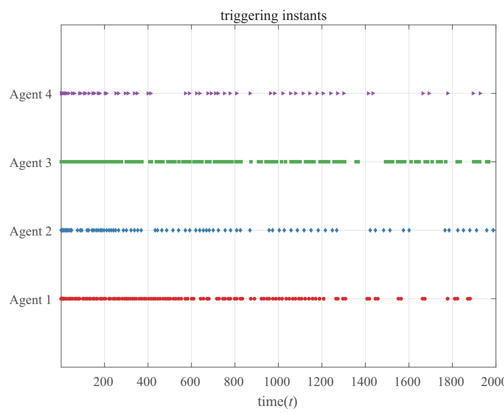

Figure 6 Trajectories of the triggering instants for four agents

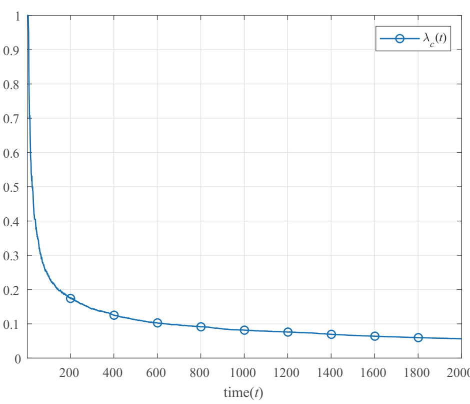

Figure 7 Trajectory of the communication rate

5 Concluding Remarks

This paper develops a distributed adaptive control approach to achieve leader-following tracking for an MAS with nonlinear uncertain follower agent dynamics. By designing an adaptive distributed observer, the follower agent could estimate the leader's information solely through local communication. Building on this, an adaptive control law along with a parameter update law is designed ensure that each follower's output converges to that of the leader despite parameter uncertainties. In particular, an event-triggered mechanism is integrated within the adaptive distributed observer to reduce communication frequency among the followers, as evidenced by a decaying communication rate. Notably, the proposed controller structure eliminates the need for any prior parameter knowledge when designing a stable adaptive update law. For future study, the switching topology of communication graph is worth investigating. In addition, a time-varying leader's dynamics matrix would be a bigger challenge to be solved, where the time-varying observer gain may fail for the observer design. Further, a more meaningful topic might be considering that the leader dynamics is a non-autonomous system. That is, the leader agent is driven by a external reference input. In this case, the common distributed observer technique may not be effective. This would be a future research focus.

Conflict of Interest

ZHANG Yanjun is a youth editorial board member for Journal of Systems Science & Complexity and was not involved in the editorial review or the decision to publish this article. All authors declare that there are no competing interests.

References

[1] Ren W and Beard R W, *Distributed Consensus in Multi-Vehicle Cooperative Control*, Springer, London, 2008.

[2] Lewis F, Zhang H, Hengster-Movric K, et al., *Cooperative Control of Multi-Agent Systems: Optimal and Adaptive Design Approaches*, Springer-Verlag, London, U.K., 2014.

[3] Yu W, Wen G, Chen G, et al., *Distributed Cooperative Control of Multi-Agent Systems*, John Wiley & Sons, Hoboken, 2017.

[4] Jadbabaie A, Lin J, and Morse A S, Coordination of groups of mobile autonomous agents using nearest neighbor rules, *IEEE Trans. Autom. Control*, 2003, **48**(6): 988–1001.

[5] Olfati-Saber R and Murray R M, Consensus problems in networks of agents with switching topology and time-delays, *IEEE Trans. Autom. Control*, 2004, **49**(9): 1520–1533.

[6] Moreau L, Stability of multiagent systems with time-dependent com munication links, *IEEE Trans. Autom. Control*, 2005, **50**(2): 169–182.

[7] Zhang H and Lewis F, Adaptive cooperative tracking control of high-order nonlinear systems with unknown dynamics, *Automatica*, 2012, **48**(7): 1432–1439.

[8] Huang J, Wang W, Wen C, et al., Distributed adaptive leader-follower and leaderless consensus control of a class of strict-feedback nonlinear systems: A unified approach, *Automatica*, 2020, **118**: 109021.

[9] Yang R and Zhou D, Distributed event-triggered tracking control of heterogeneous discrete-time multi-agent systems with unknown parameters, *Journal of Systems Science & Complexity*, 2022, **35**(4): 1330–1347.

[10] Zhang Q and Zhang J F, Quantized data-based distributed consensus under distributed time-varying communication topology, *SIAM J. Control Optim.*, 2013, **51**(1): 332–352.

[11] Hu M, Wang T, and Zhao Y, Consensus of switched multi-agent systems with binary-valued communications, *Sci. China Inf. Sci*, 2022, **65**: 162207.

[12] Yuan S, Yu C, and Sun J, Hybrid adaptive event-triggered consensus control with intermittent communication and control updating, *Journal of Systems Science & Complexity*, 2024, **37**(6): 2390–2405.

[13] Das A and Lewis F L, Distributed adaptive control for synchronization of unknown nonlinear networked systems, *Automatica*, 2010, **46**(12): 2014–2021.

[14] Huang J, Adaptive output synchronization for a class of uncertain nonlinear multi-agent systems over switching networks, *IEEE Trans. Autom. Control*, 2024, **69**(4): 2645–2651.

[15] Wang Y, Li W, and Zhang J F, Distributed adaptive nonlinear control with fusion least-squares, *SIAM J. Control Optim.*, 2022, **60**(5): 3148–3172.

[16] Xiao W, Cao L, Li H, et al., Observer-based adaptive consensus control for nonlinear multi-agent systems with time-delay, *Sci. China Inf. Sci.*, 2020, **63**(3): 132202.

[17] Baldi S, Yuan S, and Frasca P, Output synchronization of unknown heterogeneous agents via distributed model reference adaptation, *IEEE Trans. Control Net. Systems*, 2019, **6**(2): 515–525.

[18] Wang W, Wen C, and Huang J, Distributed adaptive asymptotically consensus tracking control of nonlinear multi-agent systems with unknown parameters and uncertain disturbances, *Automatica*, 2017, **77**: 133–142.

[19] Krstic M, Kanellakopoulos I, and Kokotovic P V, *Nonlinear and Adaptive Control Design*, Wiley, Hoboken, NJ, 1995.

[20] Wang C, Wen C, and Guo L, Multivariable adaptive control with unknown signs of the high-frequency gain matrix using novel Nussbaum functions, *Automatica*, 2020, **111**: 108618.

[21] Ma Y, Tao G, Jiang B, et al., Multi-model adaptive control for spacecraft under sign errors in actuator response, *J. Guid. Control Dynam.*, 2016, **39**(3): 628–641.

[22] Chen W, Li X, Ren W, et al., Adaptive consensus of multi agent systems with unknown identical control directions based on a novel Nussbaum-type function, *IEEE Trans. Autom. Control*, 2014, **59**(7): 1887–1892.

[23] Guo J, Zhang Y, and Zhang J, Distributed output feedback indirect MRAC of continuous-time multiagent linear systems, *SIAM J. Control Optim.*, 2025.

[24] Xu Y, Zhang Y, and Zhang J, Parameter estimation and tracking control of MIMO linear systems without prior knowledge of control signs and parameter bounds, *IEEE Trans. Autom. Control*, 2025, DOI: 10.1109/TAC.2024.3513039.

[25] Ding Z, Adaptive consenus output regulation of a class of nonlinear systems with unknown high-frequency gain, *Automatica*, 2015, **51**: 348–355.

[26] Liu L, Adaptive cooperative output regulation problem for a class of nonlinear multi-agent systems, *IEEE Trans. Autom. Control*, 2015, **60**(6): 1677–1682.

[27] Guo M, Xu D, and Liu L, Cooperative output regulation of heterogeneous nonlinear multi-agent systems with unknown control directions, *IEEE Trans. Autom. Control*, 2017, **62**(6): 3039–3045.

[28] Liu T and Huang J, Cooperative output regulation for a class of nonlinear multi-agent systems with unknown control directions subject to switching networks, *IEEE Trans. Autom. Control*, 2018, **63**(3): 783–790.

[29] Liu Z, Huang J, Wen C, et al., Distributed control of nonlinear systems with unknown time-varying control coefficients: A novel Nussbaum function approach, *IEEE Trans. Autom. Control*, 2023, **68**(7): 4191–4203.

[30] Dimarogonas D V, Frazzoli E, and Johansson K H, Distributed event triggered control for multi-agent systems, *IEEE Trans. Autom. Control*, 2013, **57**(5): 1291–1297.

[31] Fan Y, Liu L, Feng G, et al., Self-triggered consensus for multi-agent systems with Zeno-free triggers, *IEEE Trans. Autom. Control*, 2015, **60**(10): 2779–2784.

[32] Hu W, Liu L, and Feng G, Cooperative output regulation of linear multi-agent systems by intermittent communication: A unified framework of time- and event-triggering strategies, *IEEE Trans. Autom. Control*, 2018, **63**(2): 548–555.

[33] Meng H, Zhang H T, Wang Z, et al., Event-triggered control for semiglobal robust consensus of a class of nonlinear uncertain multiagent systems, *IEEE Trans. Autom. Control*, 2020, **65**(4): 1683–1690.

[34] Wei B, Xiao F, Fang F, et al., Velocity-free event-triggered control for multiple Euler-Lagrange systems with communication time-delays, *IEEE Trans. Autom. Control*, 2021, **66**(11): 5599–5605.

[35] Hao Y, Zhang J, and Liu L, Fully distributed event-triggered cooperative output regulation of multiagent systems under jointly connected digraphs, *IEEE Trans. Autom. Control*, 2023, **68**(7): 4241–4248.

[36] Oliveira T R, Peixoto A J, and Hsu L, Sliding mode control of uncertain multivariable nonlinear systems with unknown control direction via switching and monitoring function, *IEEE Trans. Autom. Control*, 2010, **55**(4): 1028–1034.

[37] Pin G, Serrani A, and Wang Y, Parameter-dependent input normalization: Direct-adaptive control with uncertain control direction, *Proc.* 61*th IEEE Conf. Decision Control*, 2022, 2674–2680.

[38] Xu Y, Zhang Y, and Zhang J F, Singularity-free adaptive control of discrete-time linear systems without prior knowledge of the high-frequency gain, *Automatica*, 2024, **165**: 111657.

[39] Huang J, The cooperative output regulation problem of discrete-time linear multi-agent systems by the adaptive distributed observer, *IEEE Trans. Autom. Control*, 2017, **62**(4): 1979–1983.

[40] Zhang K, Li Z Y, and Zhou B, Fully distributed output regulation of linear discrete-time multi-agent systems with time-varying topology and delays, *Automatica*, 2024, **167**: 111755.

[41] Godsil C and Royle G, *Algebraic Graph Theory*, Springer-Verlag, New York, 2001.

[42] Lu X, Wang T, and Zhao Y, Event-triggered control for consensus of multi-agent systems with binary-valued measurements, *IEEE Trans. Autom. Control*, 2024, **69**(7): 4630–4642.

[43] He X, Xing Y, Wu J, et al., Event-triggered distributed estimation with decaying communication rate, *SIAM J. Control Optim.*, 2022, **60**(2): 992–1017.

[44] Tao G, *Adaptive Control Design and Analysis*, John Wiley & Sons, Hoboken, NJ, 2003.

Appendix

Proof of Lemma 3.6 First, we denote $\Phi_1(t,s) = \prod_{k=s}^{t}(I - \mu(k)(H\otimes I))$. Then, it follows from (7) that

$$\widetilde{s}(t+1) = \Phi_1(t,t_0)\widetilde{s}(t_0) + \sum_{k=t_0}^{t} \Phi_1(t,k+1)\mu(k)(-H\otimes I_{n^2})e_s(k), \tag{A.1}$$

where $\Phi_1(t,t+1) \triangleq I$. Based on Assumption (A1) and Lemma 3.4, we obtain $\lambda_a \triangleq \lambda_{\min}(H) > 0$. Since $\mu(t)$ is non-increasing and satisfies $\lim_{t\to\infty}\mu(t) = 0$, there exists a finite time instant $t_1 > 0$ such that $\mu(t)H < I$. Then, for sufficiently large $t \geq t_1$, we have

$$\Phi_1(t,t_1) = \prod_{k=t_1}^{t}(I - \mu(k)(H\otimes I_{n^2})) \leq \prod_{k=t_1}^{t}(1-\lambda_a\mu(k))I. \tag{A.2}$$

Thus, for sufficiently large $t \geq t_1$, it follows from (A.1) and (A.2) that

$$\begin{aligned}\|\widetilde{s}(t+1)\| &\leq \|\Phi_1(t,t_1)\|\|\widetilde{s}(t_1)\| + \lambda_b \sum_{k=t_1}^{t} \|\Phi_1(t,k+1)\|\mu(k)\|e_s(k)\| \\ &\leq \prod_{k=t_1}^{t}(1-\lambda_a\mu(k))\|\widetilde{s}(t_1)\| + \lambda_b \sum_{k=t_1}^{t}\prod_{l=t_1+1}^{t}(1-\lambda_a\mu(l))\mu(k)\|e_s(k)\|, \qquad \text{(A.3)}\end{aligned}$$

where $0 < \lambda_b \triangleq \lambda_{\max}(H)$. From the event-triggered mechanism (6), it yields that $\|e_{s_i}(t)\| \leq g_i(t)$, $i = 1, \cdots, N$. Noting that the triggering threshold $g_i(t) = O(\mu^{d_i}(t))$ with d_i satisfying $\mu^*(1+d_i) > 1$, $i = 1, \cdots, N$, it follows from Lemma 3.5 and the definition of $e_s(t)$ in (5) that

$$\begin{aligned}&\prod_{k=t_1}^{t}(1-\lambda_a\mu(k)) \leq c\exp\left(\frac{\lambda_a}{\mu^*-1}(t+t_0)^{1-\mu^*}\right), \\ &\sum_{k=t_1}^{t}\prod_{l=t_1+1}^{t}(1-\lambda_a\mu(l))\mu(k)e_s(k) \leq c\,(t+t_0)^{-\mu^*(1+d)}, \qquad \text{(A.4)}\end{aligned}$$

where $c > 0$ denotes a general constant signal bound and $d \triangleq \min\{d_1, \cdots, d_N\}$. Then, for sufficiently large $t > t_1$, combining (A.3) and (A.4) gives

$$\|\widetilde{s}(t+1)\| \leq c\exp\left(\frac{\lambda_a}{\mu^*-1}(t+t_0)^{1-\mu^*}\right) + c\,(t+t_0)^{-\mu^*(1+d)} \leq c\,(t+t_0)^{-\mu^*(1+d)},$$

which indicates that $\lim_{t\to\infty}\|\widetilde{s}(t)\| = 0$. Thus, we have $\lim_{t\to\infty} s_i(t) = s$, i.e., $\lim_{t\to\infty} S_i(t) = S$, $i = 1, \cdots, N$. This completes the proof.

Proof of Lemma 3.7 First, we denote $F_1(t) = (I \otimes S) - \mu(t)(H \otimes S)$, $F_2(t) = -(\widetilde{S}_d(t) - \mu(t)A(t))$, $F_3(t) = \widetilde{S}_d(t)(I \otimes v_0(t)) - \mu(t)B(t)e_v(t)$. Then, it follows from (8) that $\widetilde{v}(t+1) = (F_1(t) + F_2(t))\widetilde{v}(t) + F_3(t)$. Based on the convergence of $S_i(t)$ in Lemma 3.6, we have $\|F_2(t)\| \leq c\,(t+t_0)^{-\mu^*(1+d)}$ for sufficiently large $t > t_1$. Denote $\Phi_2(t,s) = \prod_{k=s}^{t}(F_1(k) + F_2(k))$. Thus, we get

$$\|\widetilde{v}(t+1)\| \leq \|\Phi_2(t,t_1)\|\|\widetilde{v}(t_1)\| + \sum_{k=t_1}^{t}\|\Phi_2(t,k+1)\|\|F_3(k)\|. \qquad \text{(A.5)}$$

Based on Assumption (A2), for sufficiently large $t > t_1$, we have

$$\left\|\prod_{k=s}^{t} F_1(k)\right\| = \left\|\prod_{k=s}^{t}(I-\mu(k)H)\otimes S^{t-s+1}\right\| \leq \left\|\prod_{k=s}^{t}(I-\mu(k)H)\right\| \leq \prod_{k=s}^{t}(1-\lambda_a\mu(k)). \qquad \text{(A.6)}$$

Thus, it follows from Lemma 3.5 and (A.6) that

$$\|\Phi_2(t,t_1)\|\|\widetilde{v}(t_1)\| \leq c\left\|\prod_{k=s}^{t} F_1(k)\right\| + c(t+t_0)^{-\mu^*(1+d)}$$

$$
\begin{aligned}
&\leq c\prod_{k=t_1}^{t}(1-\lambda_a\mu(k))+c(t+t_0)^{-\mu^*(1+d)}\\
&\leq c\exp\left(\frac{\lambda_a}{\mu^*-1}(t+t_0)^{1-\mu^*}\right)+c(t+t_0)^{-\mu^*(1+d)}\\
&\leq c(t+t_0)^{-\mu^*(1+d)}. \qquad \text{(A.7)}
\end{aligned}
$$

In addition, from Assumption (A2), the state variable $v_0(t)$ of the leader system (2) is bounded. Then, for sufficiently large $t > t_1$, it follows from Lemma 3.6 that $\|\widetilde{S}_d(t)(I\otimes v_0(t))\| \leq c(t+t_0)^{-\mu^*(1+d)}$. From the event-triggered mechanism in (6), we have $\|e_{v_i}(t)\| \leq g_i(t) \leq c\mu^{d_i}(t),\ i=1,\cdots,N$. From Lemma 3.6, the estimates $S_i(t),\ i=1,\cdots,N$, are bounded. Thus, the signal $B(t)$ is bounded. Then, for sufficiently large $t > t_1$, we have $\|\mu(t)B(t)e_v(t)\| = c\mu^{d_i+1}(t) \leq c(t+t_0)^{-\mu^*(1+d)}$. Then, we have $\|F_3(t)\| \leq c(t+t_0)^{-\mu^*(1+d)}$ for sufficiently large $t > t_1$. Further, it follows from Lemma 3.5 that

$$
\begin{aligned}
\sum_{k=t_1}^{t}\|\Phi_2(t,k+1)\|\|F_3(k)\| &\leq \sum_{k=t_1}^{t}\left\|\prod_{l=k+1}^{t}F_1(k)\right\|\|F_3(k)\|+c(t+t_0)^{-\mu^*(1+d)}\\
&\leq c\sum_{k=t_1}^{t}\prod_{l=s+1}^{t}(1-\lambda_a\mu(l))\mu^{1+d}(k)+c(t+t_0)^{-\mu^*(1+d)}\\
&\leq c(t+t_0)^{-\mu^*(1+d)}. \qquad \text{(A.8)}
\end{aligned}
$$

Finally, it follows from (A.5), (A.7) and (A.8) that $\|\widetilde{v}(t+1)\| \leq c(t+t_0)^{-\mu^*(1+d)}$, which implies $\lim_{t\to\infty}\widetilde{v}(t)=0$, i.e., $\lim_{t\to\infty}v_i(t)=v_0(t),\ i=1,\cdots,N$. This completes the proof.

Proof of Lemma 3.9 With the leader dynamics (2), we have $FS_i(t)v_i(t)-y_0(t+1) = F(S_i(t)v_i(t)-Sv_0(t))$. Since $S_i(t)v_i(t)-Sv_0(t) = S_i(t)(v_i(t)-v_0(t))+(S_i(t)-S)v_0(t)$, it follows from Lemma 3.6 and Lemma 3.7 that (12) holds for $k=1$. For $k>1$, we have

$$
S_i^k(t)-S^k = S_i^k(t)(S_i(t)-S)+S_i^{k-2}(t)(S_i(t)-S)S+\cdots+(S_i(t)-S)S^{k-1}.
$$

From Lemma 3.6, we obtain $S_i(t)$ is bounded and $\lim_{t\to\infty}(S_i(t)-S)=0$, which implies that $\lim_{t\to\infty}(S_i^k(t)-S^k)=0$. Moreover, we have

$$
\begin{aligned}
S_i^k(t)v_i(t)-S^kv_0(t) &= S_i^k(t)v_i(t)-S_i^k(t)v_0(t)+S_i^kv_0(t)-S^kv_0(t)\\
&= S_i^k(v_i(t)-v_0(t))+(S_i^k(t)-S^k)v_0(t).
\end{aligned}
$$

Since $v_0(t)$ and $S_i(t)$ are bounded and noting $y_0(t+k)=FS^kv_0(t)$ and $\lim_{t\to\infty}(v_i(t)-v_0(t))=0$, we obtain (12) holds for $k>1$. This completes the proof.

Proof of Lemma 3.11 First, it follows from (3) that $\|e_{s_i}(t+1)\| = \|\widehat{s}_i(t+1)-s_i(t+1)\| \leq \|\widehat{s}_i(t)-s_i(t)\|+\mu(t)\|\sum_{j=0}^{N}a_{ij}(s_j(t)-s_i(t))\|+\mu(t)\|\sum_{j=0}^{N}a_{ij}(e_{s_j}(t)-e_{s_i}(t))\|,\ i=1,\cdots,N$. Denote $d_0=\max_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}a_{ij}$. From Lemma 3.6, we have $\|s_i(t)-s\| \leq cr(t),\ i=1,\cdots,N$. Then, for sufficiently large $t\in[t_k^i,t_{k+1}^i)$, we have

$$
\begin{aligned}
\|e_{s_i}(t+1)\| &\le \|\widehat{s}_i(t)-s_i(t)\| + d_0\mu(t)\max_{i\in\mathcal{V}}\|s_i(t)-s\| + d_0\mu(t)g_i(t)\\
&\le \|\widehat{s}_i(t)-s_i(t)\| + c\mu(t)r(t) + c\mu(t)g_i(t)\\
&\le \|\widehat{s}_i(t)-s_i(t)\| + cr(t)\\
&= \|e_{s_i}(t)\| + cr(t), \quad i=1,\cdots,N.
\end{aligned} \tag{A.9}
$$

Moreover, it follows from the event-triggered mechanism (6) that $\|e_{v_i}(t)\| = \|\widehat{v}_i(t)-v_i(t)\| \le g_i(t)$ and $\|e_{s_i}(t)\| = \|\widehat{s}_i(t)-s_i(t)\| \le g_i(t)$ for sufficiently large $t\in[t_k^i, t_{k+1}^i)$. Besides, from Assumption (A2), we have $\|S\|\le 1$. Then, for sufficiently large $t\in[t_k^i,t_{k+1}^i)$, we have

$$
\begin{aligned}
\|e_{v_i}(t+1)\| \le{}& \|S\|\|\widehat{v}_i(t)-v_i(t)\| + \|\widehat{S}_i(t)-S\|\|\widehat{v}_i(t)-v_i(t)\| + \|(S_i(t)-\widehat{S}_i(t))v_i(t)\|\\
&+\left\|\mu(t)S_i(t)\sum_{j=0}^{N}a_{ij}(v_j(t)-v_i(t))\right\| + \left\|\mu(t)S_i(t)\sum_{j=0}^{N}a_{ij}(e_{s_j}(t)-e_{s_i}(t))\right\|\\
\le{}& \|\widehat{v}_i(t)-v_i(t)\| + cr(t)g_i(t) + c\mu(t)r(t) + c\mu(t)g_i(t)\\
\le{}& \|e_{v_i}(t)\| + cr(t), \quad i=1,\cdots,N.
\end{aligned} \tag{A.10}
$$

Define L_k^i as the time interval between the kth trigger time instant and the $(k+1)$th trigger time instant for agent i, $i=1,\cdots,N$. Then, from (A.9), (A.10), and the monotonicity of $r(t)$, we obtain

$$
\begin{aligned}
\|e_{s_i}(t_{k+1}^i)\| + \|e_{v_i}(t_{k+1}^i)\| &\le \|e_{s_i}(t_{k+1}^i-1)\| + \|e_{v_i}(t_{k+1}^i-1)\| + cr(t_{k+1}^i-1)\\
&\le \|e_{s_i}(t_k^i-1)\| + \|e_{v_i}(t_k^i-1)\| + c\sum_{k=t_k^i}^{t_{k+1}^i-1} r(k)\\
&\le cL_k^i r(t_k^i), \quad i=1,\cdots,N.
\end{aligned} \tag{A.11}
$$

Noting the event-triggered mechanism (6), we observer that to ensure the event is triggered for agent i at the time instant t_k^i, a necessary condition is $cL_k^i r(t_k^i) > g_i(t_{k+1}^i)$, $i=1,\cdots,N$, which is equivalent to $L_k^i > cr^{-1}(t_k^i)g_i(t_k^i+L_k^i)$, $i=1,\cdots,N$. From the conditions on the threshold $g_i(t)$, we have the following inequality holds $L_k^i \ge c(t_k^i)^\sigma$, $i=1,\cdots,N$. Then, for $k,s\in\mathbb{N}^+$ such that $k>s$, the following inequality holds $t_k^i \ge t_s^i + c\sum_{j=s}^{k-1}(t_j^i)^\sigma$, $i=1,\cdots,N$. Note that $t_j^i\ge j$. Then, the following inequality holds $t_k^i \ge t_s^i + c\sum_{j=s}^{k-1} j^\sigma \ge c(k-1)^{\sigma+1}$, $i=1,\cdots,N$. For $t_k^i\le t< t_{k+1}^i$, it yields $k<(\frac{t-t_s^i}{c})^{\frac{1}{\sigma+1}}+1$, $i=1,\cdots,N$, which means $K_i(t)\le(\frac{t-t_s^i}{c})^{\frac{1}{\sigma+1}}+1$, $i=1,\cdots,N$. Thus, for $i=1,\cdots,N$, it follows from $\sigma\in(0,1)$ that

$$
\lim_{t\to\infty}\frac{K_i(t)}{t} < \lim_{t\to\infty}\frac{1}{t}\left(\left(\frac{t-t_s^i}{c}\right)^{\frac{1}{\sigma+1}}+1\right) = 0,
$$

which combines the definition of $\lambda_c(t)$ imply $\lim_{t\to\infty}\lambda_c(t)=0$. This completes the proof.