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Adaptive Stabilization of Bilinear Systems

Xi Sun and Ji Feng Zhang

***Abstract*—For a class of deterministic and stochastic bilinear systems, "modified self-tuning regulators" are given by using recursive least-squares and modified recursive least-squares, respectively. The closed-loop systems are shown globally stable and asymptotically optimal in some sense without any restriction on location of poles or zeros of nominal linear part of the open-loop system considered. Simulations show that the methods used here work very well.**

I. Introduction

Since Åström and Wittenmark [1] introduced self-tuning controller for minimum-phase systems, much work has been done. Clarke [2] proposed a self-tuning controller for nonminimum-phase systems. Sin and Goodwin [3] analyzed the stability and the convergence of self-tuning controllers by using martingale type approach. Tsiligiannis and Svoronos [4] and Chai [5] analyzed stability and convergence of deterministic or stochastic Clarke–Gawthrop's self-tuning algorithm. It is worth noticing that all the works mentioned-above are focused only on linear systems.

However, actually, many systems are nonlinear, among which bilinear systems are of special importance, because bilinear systems not only widely exist in industrial processes such as chemical, biological or thermal processes (e.g., [6]), but also can be used to approximate a general nonlinear system in a more precise way. Svoronos *et al.* [7] introduced a self-tuning regulator for a bilinear system whose nominal linear part has stable zeros only. In order to guarantee stability of the self-tuning regulating system, they assumed that the output of the closed-loop system is ergodic, which seems to be restrictive for control problems. Cho and Marcus [8] developed a self-tuning algorithm and analyzed its stability for only first order bilinear systems. Sun *et al.* [9] deal with the case where $\sup_{t\geq 0}|y(t+1)-y(t)|<\infty$ and b_0, d_0 are known, where $y(t)$, b_0 and d_0 are given in (1) below.

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This note based on [9], gives global stabilization adaptive controls for general bilinear systems. The algorithm works well for bilinear systems whose open-loop nominal linear part may have stable or unstable poles or zeros. It is shown that the closed-loop system is globally stable and asymptotically optimal in some sense. Two simulation examples are given in Section VI.

II. Adaptive Control for Deterministic Systems

Consider a deterministic bilinear system (DBLS):

$$A(q^{-1})y(t)=q^{-d}B(q^{-1})u(t)+q^{-d}D(q^{-1})[y(t)u(t)]+\mu. \quad (1)$$

where $\{y(t)\}$, $\{u(t)\}$ denote system output and input, respectively; μ is constant disturbance; d is time-delay; $A(q^{-1})$, $B(q^{-1})$ and $D(q^{-1})$ are polynomials of unit shift-back operator q^{-1}:

$$A(q^{-1})=1+\sum_{i=1}^{n}a_iq^{-i}, \qquad B(q^{-1})=\sum_{i=0}^{m}b_iq^{-i},$$

$$D(q^{-1})=\sum_{i=0}^{l}d_iq^{-i}.$$

In the sequel, we assume that:

1. n, m, l as upper bounds of the polynomial orders and time-delay d are known;
2. $D(q^{-1})$ is stable, i.e., $d_0\neq 0$ and $D(q^{-1})\neq 0$ for any $|q|\geq 1$.

The purpose of a self-tuning regulator is to minimize the following cost function (e.g., [2], [10]):

$$J(u)=[P(q^{-1})y(t+d)-R(q^{-1})y^*(t+d)]^2,$$

where $P(q^{-1})\triangleq 1+\sum_{i=1}^{n_p}p_iq^{-i}$ and $R(q^{-1})\triangleq\sum_{i=0}^{n_r}r_iq^{-i}$ are known polynomials of q^{-1}, and $P(q^{-1})$ is stable, $\{y^*(t)\}$ as a reference signal is bounded (i.e., $\sup_{t\geq 0}|y^*(t)|<\infty$).

From (1) is is easy to see that

$$\begin{aligned}P(q^{-1})y(t+d)&=G(q^{-1})y(t)+F(q^{-1})B(q^{-1})u(t)\\&\quad+F(q^{-1})D(q^{-1})y(t)u(t)+F(1)\mu\triangleq\varphi^\tau(t)\theta. \quad (2)\end{aligned}$$

where $F(q^{-1})\triangleq 1+\sum_{i=1}^{d-1}f_iq^{-i}$ and $G(q^{-1})\triangleq\sum_{i=0}^{n_g}g_iq^{-i}$ (with $n_g\leq\max\{n_p-d,\ n+1\}$) are the unique solutions to equation

$$P(q^{-1})=F(q^{-1})A(q^{-1})+q^{-d}G(q^{-1})$$

and

$$\varphi^\tau(t)=[u(t),\cdots,u(t-m+d-1),\ y(t)u(t),\cdots,\ y(t-l+d-1)u(t-l+d-1),\ y(t),\cdots,y(t-n_g),\ 1],$$

$$\theta^\tau=[\alpha_0,\cdots,\alpha_{m+d-1},\ \beta_0,\cdots,\beta_{l+d-1},\ \gamma_0,\cdots,\gamma_{n_g},\ \gamma],\qquad \gamma=F(1)\mu. \quad (3)$$

Therefore, the optimal self-tuning control should satisfy the following equation:

$$\varphi^\tau(t)\theta=R(q^{-1})y^*(t+d).$$

Furthermore, by "certainty equivalence principle" we may define an optimal adaptive self-tuning control as follows

$$\varphi^\tau(t)\hat\theta(t)=R(q^{-1})y^*(t+d), \quad (4)$$

where

$$\hat\theta(t)=[\hat\alpha_0(t),\cdots,\hat\alpha_{m+d-1}(t),\quad \hat\beta_0(t),\cdots,\hat\beta_{l+d-1}(t),\quad \hat\gamma_0(t),\cdots,\hat\gamma_{n_g}(t),\quad \hat\gamma(t)]$$

is LS estimate of parameter θ at time t and is generated by

$$\hat{\theta}(t) = \hat{\theta}(t-d) + \frac{a(t)P(t-2d)\varphi(t-d)}{1 + a(t)\varphi^\tau(t-d)P(t-2d)\varphi(t-d)} \cdot [P(q^{-1})y(t) - \varphi^\tau(t-d)\hat{\theta}(t-d)], \quad (5)$$

$$P(t-d) = P(t-2d) - \frac{a(t)P(t-2d)\varphi(t-d)\varphi^\tau(t-d)P(t-2d)}{1 + a(t)\varphi^\tau(t-d)P(t-2d)\varphi(t-d)} \quad (6)$$

with $P(-d) = P(-d+1) = \cdots = P(-1) = I$ and $0 < \inf_{t\geq 0} a(t) \leq \sup_{t\geq 0} a(t) < \infty$.

However, it is easy to see that $u(t)$ in (4) is solvable if and only if $\hat{\alpha}_0(t) + \hat{\beta}_0(t)y(t) \neq 0$. In order to guarantee solvability and boundedness of control $u(t)$, we modify the standard self-tuning regulator (4) slightly:

$$\varphi^\tau(t)\hat{\theta}(t) = R(q^{-1})y^*(t+d) - \lambda\delta(t)u(t), \quad (7)$$

where $\lambda > 0$ is a constant, and

$$\delta(t) = \begin{cases} 1, & \text{if } \hat{\alpha}_0(t) + \hat{\beta}_0(t)y(t) \geq 0, \\ -1, & \text{if } \hat{\alpha}_0(t) + \hat{\beta}_0(t)y(t) < 0. \end{cases} \quad (8)$$

III. Stability Analysis for System (1), (7)

Lemma 3.1: The modified adaptive self-tuning regulator (5)–(7) has the following property:

$$\lim_{t\to\infty} \frac{|\psi(t)|}{[1 + a(t)\varphi^\tau(t-d)P(t-2d)\varphi(t-d)]^{1/2}} = 0,$$

where

$$\psi(t+d) = P(q^{-1})y(t+d) - R(q^{-1})y^*(t+d) + \lambda\delta(t)u(t). \quad (9)$$

Proof: See [10, Lemma 3.3.7].

Lemma 3.2: Suppose that for some constants $0 \leq C_0 < \infty$ and $\epsilon \in [0, \lambda)$,

$$|P(q^{-1})y(t+d)| \leq C_0(1 + |y(t)|) + \epsilon \max\{|u(\tau)|: 0 \leq \tau \leq t\} + C_0 \max\{|\psi(\tau)|: 0 \leq \tau \leq t+d\}. \quad (10)$$

Then under Assumptions A.1) and A.2), the modified adaptive self-tuning regulator (5)–(7) leads to

a) $|u(t)| \leq C + C\max\{|\psi(\tau)|: 0 \leq \tau \leq t+d\}$, (11)

b) $|y(t)| \leq C + C\max\{|\psi(\tau)|: 0 \leq \tau \leq t+d\}$, (12)

c) $|y(t)u(t)| \leq C + C\max\{|\psi(\tau)|: 0 \leq \tau \leq t+d\}$, (13)

where $0 \leq C < \infty$ is constant.

Proof: For any $t \geq 0$ set

$$k(t) = \max\left\{k: |u(k)| = \max_{0\leq\tau\leq t} |u(\tau)|, \quad 0 \leq k \leq t\right\}. \quad (14)$$

From (1) and (9) we see that

$$\begin{aligned} y(t)u(t) = {} & [P(q^{-1})D(q^{-1})]^{-1}A(q^{-1})\psi(t+d) \\ & - D^{-1}(q^{-1})B(q^{-1})u(t) - D^{-1}(1)\mu \\ & + [P(q^{-1})D(q^{-1})]^{-1}A(q^{-1})R(q^{-1})y^*(t+d) \\ & - [P(q^{-1})D(q^{-1})]^{-1}A(q^{-1})[\lambda\delta(t)u(t)]. \end{aligned} \quad (15)$$

Set

$$\mathcal{M}_1 = \{t: |y(k(t))| \geq 2\xi\},$$

where $k(t)$ is given by (14),

$$\xi = \|D^{-1}(q^{-1})B(q^{-1})\| + \lambda \|[P(q^{-1})D(q^{-1})]^{-1}A(q^{-1})\|$$

with

$$\|M(q^{-1})\| = \sum_{i=0}^{\infty} |m_i|, \qquad \forall M(q^{-1}) = \sum_{i=0}^{\infty} m_i q^{-i}.$$

If $t \in \mathcal{M}_1$, then from (15) it is obvious that for some constant $C_1 < \infty$,

$$\begin{aligned} |u(k(t))| \leq {} & \frac{|[P(q^{-1})D(q^{-1})]^{-1}A(q^{-1})\psi(k(t)+d)|}{|y(k(t))|} \\ & + \frac{|D^{-1}(q^{-1})B(q^{-1})u(k(t))|}{|y(k(t))|} \\ & + \frac{|D^{-1}(1)\mu - [P(q^{-1})D(q^{-1})]^{-1}A(q^{-1})R(q^{-1})y^*(t+d)|}{|y(k(t))|} \\ & + \frac{|[P(q^{-1})D(q^{-1})]^{-1}A(q^{-1})[\lambda\delta(k(t))u(k(t))]|}{|y(k(t))|} \\ \leq {} & \frac{1}{2\lambda}\max\{|\psi(\tau)|: 0 \leq \tau \leq k(t)+d\} + \tfrac{1}{2}|u(k(t))| + C_1. \end{aligned}$$

From this we get the following:

$$|u(k(t))| \leq \frac{1}{\lambda}\max\{|\psi(\tau)|: 0 \leq \tau \leq k(t)+d\} + 2C_1. \quad (16)$$

If $t \notin \mathcal{M}_1$, then $|y(k(t))| < 2\xi$. Therefore, by (10) we get

$$|P(q^{-1})y(k(t)+d)| \leq C_0[1 + 2\xi] + \epsilon|u(k(t))| + C_0\max\{|\psi(\tau)|: 0 \leq \tau \leq k(t)+d\},$$

which together with (9) implies that

$$\begin{aligned} \lambda|u(k(t))| \leq {} & |\psi(k(t)+d)| + |P(q^{-1})y(k(t)+d)| + |R(q^{-1})y^*(t+d)| \\ \leq {} & (C_0+1)\max\{|\psi(\tau)|: 0 \leq \tau \leq k(t)+d\} + \epsilon|u(k(t))| + C_2, \end{aligned}$$

where $C_2 < \infty$ is constant. From this and (16) it is not difficult to see that

$$|u(k(t))|t \leq C\max\{|\psi(\tau)|: 0 \leq \tau \leq k(t)+d\} + C$$

for some constant $C < \infty$. Thus, from (14) we arrive at

$$\begin{aligned} |u(t)| \leq |u(k(t))| & \leq C\max\{|\psi(\tau)|: 0 \leq \tau \leq k(t)+d\} + C \\ & \leq C\max\{|\psi(\tau)|: 0 \leq \tau \leq t+d\} + C, \end{aligned}$$

i.e., (11) holds. (12) follows from (11) and (9), while (13) is implied by (11) and (15).

Theorem 3.1: Under the assumptions of Lemma 3.2, the modified adaptive self-tuning regulator (5)–(7) leads to

a) $\lim_{t\to\infty}[P(q^{-1})y(t+d) - R(q^{-1})y^*(t+d) + \lambda\delta(t)u(t)]^2 = 0$,

b) $\sup_{t\geq 0}(|y(t)| + |u(t)|) < \infty$.

Proof: From (6) it is easy to see that $P(t) \leq I$, $\forall t \geq 0$, and hence, by Lemma 3.2, for some constant $C_3 < \infty$,

$$\varphi^\tau(t)P(t-d)\varphi(t) \leq |\varphi(t)|^2 \leq [C_3 + C_3\max\{|\psi(\tau)|: 0 \leq \tau \leq t\}]^2.$$

From this, Lemma 3.1 and the Key-Technical Lemma in [10] it follows that $\lim_{t\to\infty}\psi(t) = 0$, i.e., a) is true, while b) follows from a) and Lemma 3.2.

Remark: Assumption (10) is a technical constraint on the system output. In some case, for example, where $B(q^{-1}) = b_0$, $D(q^{-1}) = d_0$, $d = 1$, $P(q^{-1}) = 1$ and $A(q^{-1}) = 1 + a_1 q^{-1}$, Condition (10) does hold. In fact, noticing that the parameter estimates $\hat{\gamma}(t)$ and $\hat{\gamma}_0(t)$ given by (5) and (6) are bounded [10], by (7) we get

$$u(t) = [\lambda\delta(t) + \hat{\alpha}_0(t) + \hat{\beta}_0(t)y(t)]^{-1} \cdot[R(q^{-1})y^*(t+d) - \hat{\gamma}_0(t)y(t) - \hat{\gamma}(t)],$$

which implies that for some constant K

$$|u(t)| \le \frac{K}{\lambda}[1 + |y(t)|].$$

This and (9) yields (10).

It is worth noticing that $\epsilon \in [0, \lambda)$ does not mean that ϵ or λ must be very small.

IV. Adaptive Control for Stochastic Systems

Consider a stochastic bilinear system (SBLS):

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + q^{-d}D(q^{-1})[y(t)u(t)] + w(t) + \mu, \quad (17)$$

where $\{y(t)\}$, $\{u(t)\}$, $\{w(t)\}$ and μ are system output, input, stochastic noise and constant disturbance, respectively; d is time-delay; and $A(q^{-1})$, $B(q^{-1})$ and $D(q^{-1})$ have the same meanings as in Section II. In the sequel, we always assume that $\{w(t)\}$ is a martingale difference sequence with respect to a nondecreasing σ-algebra sequence $\{\mathcal{F}_t\}$, and that

$$E\{w(t)\,|\,\mathcal{F}_{t-1}\} = 0, \qquad E\{w^2(t)\,|\,\mathcal{F}_{t-1}\} = \sigma^2,$$

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} w^2(t) < \infty \quad \text{a.s.}$$

Similar to Section II, we take

$$J = \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T}[P(q^{-1})y(t+d) - R(q^{-1})y^*(t+d)]^2 \quad (18)$$

as a heuristic cost function, where $P(q^{-1}) \triangleq 1 + \sum_{i=1}^{n_p} p_i q^{-i}$ and $R(q^{-1}) \triangleq \sum_{i=0}^{n_r} r_i q^{-i}$ are known polynomials of q^{-1}, $P(q^{-1})$ is stable, $\{y^*(t+d)\}$ as a reference signal is bounded (i.e., $\sup_{T\ge 0}(1/T)\sum_{t=0}^{T}|y^*(t)|^2 < \infty$) and $y^*(t+d) \in \mathcal{F}_t$.

Similar to (2) we see

$$P(q^{-1})y(t+d) \triangleq \varphi^\tau(t)\theta + v(t+d), \quad (19)$$

where $v(t+d) = F(q^{-1})w(t+d)$; $F(q^{-1})$, $G(q^{-1})$, $\varphi(t)$ and θ are same as in Section II.

By (19) and "certainty equivalence principle," similar to (7) we define modified adaptive self-tuning regulator in the following way:

$$\varphi^\tau(t)\hat{\theta}(t) = R(q^{-1})y^*(t+d) - \lambda\delta(t)u(t), \quad (20)$$

where $\lambda > 0$ is a constant, $\delta(t)$ is given by (8), and $\hat{\theta}(t)$ is given by

$$\hat{\theta}(t) = \hat{\theta}(t-d) + a(t-d)P(t-d)\varphi(t-d)e(t), \quad (21)$$

$$e(t) = P(q^{-1})y(t) - \varphi^\tau(t-d)\hat{\theta}(t-d), \quad (22)$$

$$r(t-d) = r(t-d-1) + |\varphi(t-d)|^2, \quad (23)$$

$$P(t-d) = \begin{cases} P'(t-d), & \text{if } r(t-d)\,\mathrm{tr}\,P'(t-d) \le K_1 \text{ and} \\ & \varphi^\tau(t-d)P'(t-d)\varphi(t-d) \le K_2, \\ \frac{r(t-2d)}{r(t-d)}P(t-2d), & \text{otherwise,} \end{cases} \quad (24)$$

$$a(t-d) = \begin{cases} 1, & \text{if } r(t-d)\,\mathrm{tr}\,P'(t-d) \le K_1 \text{ and} \\ & \varphi^\tau(t-d)P'(t-d)\varphi(t-d) \le K_2, \\ a'(t-d), & \text{otherwise,} \end{cases} \quad (25)$$

where K_1, K_2 are constants and $K_1 \ge \mathrm{tr}\,P(-1)$, $K_2 < 1$.

$$P'(t-d) = P(t-2d) - \frac{P(t-2d)\varphi(t-d)\varphi^\tau(t-d)P(t-2d)}{1 + \varphi^\tau(t-d)P(t-2d)\varphi(t-d)}, \quad (26)$$

$$a'(t-d) = \frac{1}{1 + \varphi^\tau(t-d)P(t-d)\varphi(t-d)}, \quad (27)$$

and $P(-d) = \cdots = P(-1) = I$, $r(-d) = \cdots = r(-1) = 1$, $\hat{\theta}(-d) = \cdots = \hat{\theta}(-1) = \hat{\theta}(0)$ are chosen arbitrarily.

V. Stability Analysis for System (17), (20)

From straightforward calculation and a similar argument as in [11] it is easy to get the following lemma.

Lemma 5.1: For system (17) the modified adaptive self-tuning regulator (20) leads to

$$1 - a(t-d)\varphi^\tau(t-d)P(t-d)\varphi(t-d) \ge K_3 > 0, \quad (28)$$

$$e(t) = \frac{\eta(t)}{1 - a(t-d)\varphi^\tau(t-d)P(t-d)\varphi(t-d)}, \quad (29)$$

$$\lim_{T\to\infty}\frac{1}{r(T)}\sum_{t=d}^{T} z^2(t) = 0 \quad \text{a.s.,} \quad (30)$$

where $K_3 = \min\{1 - K_2, 1/(1 + K_1)\}$ and

$$\eta(t) = P(q^{-1})y(t) - \varphi^\tau(t-d)\hat{\theta}(t), \qquad z(t) = \eta(t+d) - v(t+d). \quad (31)$$

Lemma 5.2: Suppose that A.1), A.2) hold, and that there are constants $c < \infty$ and $\epsilon \in [0, (1/4)\lambda)$ such that for any subset $\mathcal{N}_T$ of $\{0, 1, \cdots, T\}$,

$$\sum_{t\in\mathcal{N}_T}|P(q^{-1})y(t+d)|^2 \le c\left[T + \sum_{t\in\mathcal{N}_T}|y(t)|^2 + \sum_{t=0}^{T}\psi^2(t+d)\right] + \epsilon\sum_{t=0}^{T}u^2(t). \quad (32)$$

Then the modified adaptive self-tuning regulator (20)–(27) has the following properties:

a) $\frac{1}{T}\sum_{t=0}^{T}u^2(t) = O\left(\frac{1}{T}\sum_{t=0}^{T}\psi^2(t+d)\right) + O(1)$,

b) $\frac{1}{T}\sum_{t=0}^{T}y^2(t)u^2(t) = O\left(\frac{1}{T}\sum_{t=0}^{T}\psi^2(t+d)\right) + O(1)$,

c) $\frac{1}{T}\sum_{t=0}^{T}y^2(t) = O\left(\frac{1}{T}\sum_{t=0}^{T}\psi^2(t+d)\right) + O(1)$,

where $\psi(t+d)$ is given by (9).

Proof: From (17) and (9) it follows that

$$\begin{aligned} A(q^{-1})\psi(t+d) &= P(q^{-1})B(q^{-1})u(t) + A(q^{-1})[\lambda\delta(t)u(t)] + P(1)\mu \\ &\quad + P(q^{-1})w(t+d) + P(q^{-1})D(q^{-1})[y(t)u(t)] \\ &\quad - A(q^{-1})R(q^{-1})y^*(t+d). \end{aligned} \quad (33)$$

Thus, because of the stability of $P(q^{-1})$ and $D(q^{-1})$ there exists a constant $c_1 < \infty$ such that

$$\sum_{t=0}^{T}y^2(t)u^2(t) \le c_1\sum_{t=0}^{T}\psi^2(t+d) + c_1\sum_{t=0}^{T}u^2(t) + c_1T. \quad (34)$$

Set

$$\mathcal{M}_2 = \{t\colon |y(t)| \ge \sqrt{2c_1},\, t \ge 0\}.$$

Then from (34) we see

$$\sum_{\substack{t\in\mathcal{M}_2\\0\le t\le T}} u^2(t) \le \frac{1}{2c_1}\sum_{\substack{t\in\mathcal{M}_2\\0\le t\le T}} y^2(t)u^2(t) \le \frac{1}{2}\sum_{t=0}^{T}\psi^2(t+d) + \frac{1}{2}\sum_{t=0}^{T}u^2(t) + \frac{1}{2}T. \quad (35)$$

Since $t \notin \mathcal{M}_2$ implies $|y(t)| < \sqrt{2c_1}$, from (9) and (32) it follows that

$$\sum_{\substack{t\notin\mathcal{M}_2\\0\le t\le T}} u^2(t) \le 2\lambda^{-1}\sum_{\substack{t\notin\mathcal{M}_2\\0\le t\le T}} [P(q^{-1})y(t+d)]^2 + 2\lambda^{-1}\sum_{\substack{t\notin\mathcal{M}_2\\0\le t\le T}} [\psi(t+d) + R(q^{-1})y^*(t+d)]^2$$
$$\le O(T) + 2\epsilon\lambda^{-1}\sum_{t=0}^{T}u^2(t) + 2\lambda^{-1}(c+2)\sum_{t=0}^{T}\psi^2(t+d). \quad (36)$$

By (35) and (36) we see that for some constant $c_2 < \infty$,

$$\sum_{t=0}^{T}u^2(t) \le c_2\sum_{t=0}^{T}\psi^2(t+d) + (\tfrac{1}{2} + 2\epsilon\lambda^{-1})\textstyle\sum_{t=0}^{T}u^2(t) + c_2T,$$

which together with $\epsilon \in [0,\ (1/4)\lambda)$ implies a). Result b) follows from (34) and a), while result c) comes from stability of $P(q^{-1})$, (9) and a).

Theorem 5.1: Under the conditions of Lemma 5.2, the modified adaptive self-tuning regulator (20)–(27), closed-loop system (17) and (20) are globally stable and asymptotically optimal:

i) $\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T}[y^2(t)u^2(t) + u^2(t) + y^2(t)] < \infty$ a.s.,

ii) $\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T}[P(q^{-1})y(t+d) - R(q^{-1})y^*(t+d) + \lambda\delta(t)u(t)]^2 = \Gamma$ a.s.

Proof: From (9), (22) and (20) it is easy to see that $\psi(t) = e(t)$. Noticing (28), (29) and (31) we get

$$e^2(t) \le K_3^{-2}[z(t-d) + v(t)]^2 \le 2K_3^{-2}[z^2(t-d) + v^2(t)],$$

and hence,

$$\frac{1}{T}\sum_{t=0}^{T}\psi^2(t+d) \le \frac{2K_3^{-2}}{T}\sum_{t=0}^{T}z^2(t) + K_5.$$

From this, (30) and Lemmas 5.2 it is not difficult to get the desired conclusions i) and ii) by a similar argument in [5].

VI. Simulation Experiments

Example 1: Consider a DBLS system:

$$(1 - 0.4q^{-1})y(t) = q^{-1}(1 + 1.6q^{-1})u(t) + q^{-1}(0.4 + 0.1q^{-1})[y(t)u(t)] + 1.$$

Let $y^*(t) \equiv 2$ for $t \ge 0$, $\lambda = 0.1$, $P(q^{-1}) = R(q^{-1}) = 1$. Then with modified adaptive self-tuning regulator (5)–(7), the output response curve of closed-loop system (1) and (7) is shown in Fig. 1.

We now give a simulation for the dissolved oxygen concentration model used in [12].

Example 2:

$$y(t+T) = y(t) - \frac{T}{V}Q(t)[(1+r)y(t) - C_i] - K_1Tu(t)[y(t) - C_s] - K_2T + \epsilon(t+T), \quad (37)$$

where $y(t)$ is dissolved oxygen concentration, $u(t)$ air flow rate, $Q(t)$ influent flow rate, V reactor volume, r ratio of recycled flow to

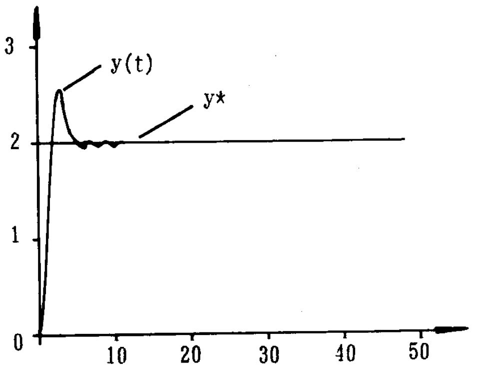

Fig. 1. Output response curve of Example 1.

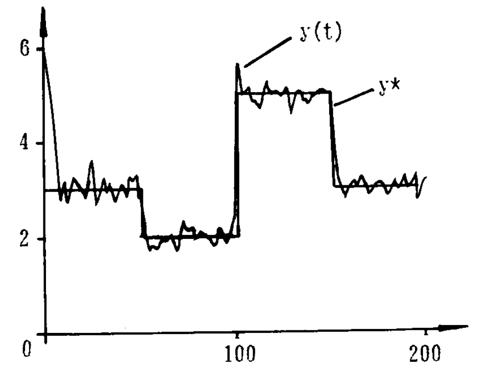

Fig. 2. Output response curve of Example 2.

influent flow, C_i and C_s the influent and saturation dissolved oxygen concentration respectively, T is sampling period, K_1 mass transfer coefficient, K_2 oxygen uptake rate, $\epsilon(t)$ disturbance.

Let

$$\varphi^\tau(t) = [u(t),\quad y(t)u(t),\quad y(t),\quad 1],$$

$$\theta^\tau = \left[K_1C_sT,\ -K_1T,\ \left(1 - \frac{T}{V}(1+r)Q(t)\right),\ -K_2T + \frac{TC_i}{V}Q(t)\right]$$

Then (37) can be rewritten as follows:

$$y(t+1) = \varphi^\tau(t)\theta + \epsilon(t+1).$$

The following parameter values are used for the simulation:

$V = 1000m^3$,	$Q = 2.0 \sim 2.8m^3\,min^{-1}$,
$u = 0 \sim 2m^3\,min^{-1}$,	$r = 2$,
$t = 0.5\,min$,	$C_i = 0.1mg\,l^{-1}$,
$C_s = 9.17mg\,l^{-1}$,	$K_1 = 0.65m^{-3}$,
$K_2 = 1.04mg\,l^{-1}\,min^{-1}$,	

$w(t)$ is uncorrelated noise with Gaussian distribution, $E\{w(t)\} = 0$ and $E\{w(t)^2\} = 0.15$, which is generated by a software random generator; λ, P, R are chosen as: $\lambda = 0.001$, $P(q^{-1}) = R(q^{-1}) = 1$. The initial parameter value $\hat{\theta}(0) = [1,\ 0,\ 0,\ 0]$.

Then with modified adaptive self-tuning regulator (20)–(27), the output response curve of closed-loop system (17) and (20) is indicated in Fig. 2.

It is worth noticing that, unlike in reference [12], what we need is only to measure $y(t)$ and $u(t)$.

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On Statistical Properties of a Test for Model Structure Selection Using the Extended Instrumental Variable Approach

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***Abstract*—A procedure for structural estimation using the extended instrumental variable technique is presented. The statistical distribution of the test quantity is obtained in the white noise case. A criterion is defined by introducing a penalty term on the decreasing rate (with the number of parameters) of the proposed test quantity. The consistency of the estimate is proven. Monte Carlo simulations are included.**

I. INTRODUCTION

Consider an SISO system represented by the following representation:

$$y(t) = \sum_{k=1}^{n} a_k y(t-k) + \sum_{k=1}^{n} b_k u(t-k) + w(t) \tag{1.1}$$

where $u(t)$, $y(t)$, and $w(t)$ are the input, output, and noise, respectively, and n is the order of the system.

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The estimation of the order n is a crucial problem in system identification. This can be done by minimizing some criteria [5]–[7]. These criteria have the form:

$$CV(\hat{n}) \triangleq V(\hat{n}) + S(N, \hat{n}) \tag{1.2}$$

where $\hat{n}$ is the model order, $V(\hat{n}) \triangleq (1/N)\sum_{t=1}^{N} \hat{e}(t)^2$ is the loss function, N is the number of data, $\hat{e}(t)$ is the prediction error, $S(N, \hat{n})$ is the penalty term increasing with the number of parameters $(2\hat{n})$ and decreasing with N.

In general, the computation of $V(\hat{n})$ requires two passes through the data: parameter estimation and prediction error computation. For real time operation, a recursive algorithm for computing $V(\hat{n})$ is given in [1]. Another approach is to use the Rissanen's predictive least squares criterion [4] which doesn't require $\hat{e}(t)$ to be computed by the final model. However, the computational cost of both approaches is still high due to the parallel implementation of many identification algorithms (one for each model structure).

For reducing the computational cost, several nonparametric methods may be used. One of the most well-known approaches is the rank test using the instrumental variable (IV) technique [5], [7], [9]–[12]. To be more precise, define the following matrices:

$$R \triangleq [\tilde{y}(-1)\tilde{u}(-1) \vdots \tilde{y}(-2)\tilde{u}(-2) \vdots \cdots \vdots \tilde{y}(-\hat{n})\tilde{u}(-\hat{n})]$$

and

$$Z \triangleq [\tilde{z}_1 \tilde{z}_2 \cdots \tilde{z}_L] \tag{1.3}$$

where

$$\tilde{u}(k) \triangleq [u(k)u(k+1)\cdots u(k+N-1)]^T,$$

$$\tilde{z}_i \triangleq [z_i(0)z_i(1)\cdots z_i(N-1)]^T \tag{1.4}$$

$\tilde{y}(k)$ is defined similarly to $\tilde{u}(k)$ and $z_i(t)$ is an instrumental variable. If $\dim\{Z\} = L > \dim\{R\} = 2\hat{n}$, we have the extended instrumental variable (EIV) method [5], [7].

The principle of the method is as follows: Suppose that Z is appropriately chosen and $u(t)$ is persistently exciting of sufficiently high order, then

$$\lim_{N\to\infty} \mathrm{E}\left\{\frac{1}{N} Z^T[R, \tilde{y}(0)]\right\} \text{ has full rank if and only if } \hat{n} < n \tag{1.5}$$

where E{} is the expectation operator. The order n is estimated by the smallest value of $\hat{n}$ such that $(1/N)Z^T[R, \tilde{y}(0)]$ is not of full rank. The main problem when using this method is the determination of the zero threshold for the test quantity (the scalar which is null if and only if $(1/N)Z^T[R, \tilde{y}(0)]$ is not of full rank). The reason is that the statistical distribution of the test quantity is often unknown [3], [7].

In this note, using the EIV approach, a new test procedure is proposed. The limiting distribution of the test quantity is derived in the white noise case. By introducing a penalty term on the decreasing rate (with the number of parameters) of the test quantity, a criterion for consistent estimate is defined. The layout of this note is as follows: the test procedure is described in Section II; the statistical distribution in the white noise case is derived in Section III; a criterion for consistent estimate is presented in Section IV; the extension to MIMO system is shown in Section V; simulations are given in Section VI; Section VII contains the conclusion.