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Brief paper

Singularity-free adaptive control of discrete-time linear systems without prior knowledge of the high-frequency gain ☆

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ABSTRACT

This paper proposes a new output feedback model reference adaptive control (MRAC) method for discrete-time linear time-invariant systems with arbitrary relative degrees. The proposed method does not require any prior knowledge of the high-frequency gain represented by k_p, thereby completely eliminating the common design condition in the traditional discrete-time MRAC framework: the sign of k_p is known, as well as an upper bound on $|k_p|$. Specifically, an output feedback adaptive control law is developed, which incorporates a time-varying gain function to effectively address the singularity issue. The developed adaptive control law leads to the derivation of a linear estimation error equation for the closed-loop system. Consequently, a gradient algorithm based parameter update law is directly formulated by utilizing the estimation error and some other available signals without requiring any prior knowledge of k_p. In comparison to the traditional MRAC and Nussbaum function-based methods, it does not necessitate any additional design conditions or involve any transient performance issues, while still ensuring closed-loop stability and asymptotic output tracking for any given bounded reference signal. The simulation study showcases the design procedure and evaluates the efficacy of the proposed control method.

1. Introduction

The topic of adaptive control has been a prominent area of research in the control community, possessing both theoretical and practical significance due to its powerful capability in effectively handling parametric uncertainties within systems. In the past few decades, significant advancements have been made in adaptive control, encompassing both linear and nonlinear systems, leading to a plethora of remarkable achievements being published (see, for example, Annaswamy and Fradkov (2021), Astrom and Wittenmark (2013), Chen and Guo (1991), Chen and Zhang (1990), Ge, Hang, Lee, and Zhang (2001), Goodwin and Sin (1984), Krstic, Kanellakopoulos, and Kokotovic (1995), Landau, Lozano, M'Saad, and Karimi (2011), Liu, Wang, and Li (2024), Marino and Tomei (1995), Ordonez and Passino (1999), Ren, Zhao, and Gao (2023), Sastry and Bodson (1989), Tao (2014), Zhang, Zhang, Sun, Wang, and Zhang (2023)).

In the field of adaptive control, model reference adaptive control (MRAC) is a popular methodology with its origin dating back to the 1950s and continuing to be widely studied now. In Kalman (1958), the author endeavored to address the MRAC problem by devising an adaptive observer in response to the presence of unknown plant parameters. However, limited success was achieved due to the challenge of concurrently estimating both states and parameters. Fortunately, the direct controller parametrization introduced in Astrom and Wittenmark (1973), Monopoli (1974) represented a significant advancement, which emphasized the crucial necessity of estimating the controller parameters in order to achieve the objective of asymptotic output tracking in the MRAC framework. This prompted a majority of researchers to embrace this line of thinking and devote themselves to the development of appropriate parameter estimators. The readers are advised to consult several textbooks (Ioannou

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& Sun, 1996; Krstic et al., 1995; Narendra & Annaswamy, 1989; Sastry & Bodson, 1989; Tao, 2003) for further reference.

The design of a globally convergent model reference adaptive controller in traditional augmented error-based estimators can be achieved by incorporating prior sign information of the high-frequency gain Ioannou and Sun (1996), Tao (2003). We provide an illustrative example to elucidate this matter. Consider a general class of linear time-invariant (LTI) systems encompassing both continuous-time and discrete-time domains: $A(D)[y](t) = k_pB(D)[u](t)$, where k_p is the high-frequency gain, $y \in \mathbb{R}$ and $u \in \mathbb{R}$ are the system output and input, respectively, and $A(D)$ and $B(D)$ are monic pole and zero polynomials, respectively. Particularly, the symbol D, in the continuous-time case, is the Laplace transform variable or the time differentiation operator for all $t \in [0, \infty)$, as the case may be or, in the discrete-time case, is the z-transform variable or the time advance operator for all $t \in \{0, 1, 2, \dots\}$. In the traditional continuous-time MRAC framework, prior knowledge of the sign of k_p is required. For the discrete-time MRAC case, it requires not only the sign of k_p, but also an upper bound on $|k_p|$. Such information is crucial for the design of parameter update laws in order to prevent the occurrence of singularity problems in control laws. The readers are referred to Ioannou and Sun (1996), Tao (2003) for further details.

However, the constraints imposed by the prior knowledge of k_p largely restrict the application range of the MRAC technique. The relaxation of the sign information of k_p poses a long-term fundamental challenge in the field of adaptive control research. The researchers have exerted tremendous efforts in addressing this control problem. In Nussbaum (1983), the author proposed a controller by introducing a function, referred as Nussbaum gain function, to stabilize a class of first-order continuous-time systems without the sign information of k_p. Since then, the Nussbaum gain function-based adaptive control has been extensively investigated (see, for example, Chen (2019), Ge and Wang (2003), Lee and Narendra (1986), Wang and Liu (2022), Wang, Wen, and Guo (2020), Yang, Ge, and Lee (2009), Ye and Jiang (1998), Zhang, Zhao, Wang, Niu, and Xu (2023)). Besides, several other novel methods have also been reported to address the issue of unknown high-frequency gain in the adaptive control. For example, Kaloust and Qu (1995) designed a nonlinear robust control scheme to identify unknown control directions online by using bounding functions of unknown dynamics and achieved arbitrary small ultimate output tracking error. Lozano, Collado, and Mondie (1990) proposed a projection based adaptive control law for continuous-time LTI systems without requiring a prior knowledge of the high-frequency gain sign information but a lower bound on $|k_p|$ to ensure asymptotic tracking performance. Ortega, Gerasimov, Barabanov, and Nikiforov (2019) developed a dynamic regressor extension and mixing parameter estimation technique, and introduced a modified adaptive controller for multivariable continuous-time LTI systems. The proposed method in Ortega et al. (2019) eliminated the prior knowledge of the high-frequency gain matrix but requiring an interval excitation assumption. The authors of this paper have also made some efforts to address the singularity issue of the high-frequency gain matrix for multivariable nonlinear systems (Xu, Zhang, & Zhang, 2023; Zhang, Zhang, & Liu, 2022). The well-known backstepping technique, originally proposed in Krstic et al. (1995), is noteworthy due to its pivotal role in control design and stability analysis for high-order and nonlinear adaptive control systems.

Recently, Pin, Serrani, and Wang (2022) proposed a new MRAC method for a special class of continuous-time LTI systems with relative degree one. The proposed method did not need any information of the high-frequency gain and achieved asymptotic tracking performance based on a Lyapunov function based analysis. The investigation of systems with arbitrary relative degrees remains to be conducted, which may be resolved using a small-gain control design framework similar to that used in adaptive control for high-order relative degree cases. The readers are referred to Sastry and Bodson (1989), Tao (2003) for further information. Nevertheless, the new idea proposed in Pin et al. (2022) offers a novel solution to address the challenge of unknown high-frequency gain in adaptive control. Based on our previous experience in dealing with the control gain singularity problem (Xu et al., 2023; Zhang et al., 2022) and motivated by the new method in Pin et al. (2022) for continuous-time LTI systems with relative degree one, we provide a modified MRAC method for discrete-time LTI systems with arbitrary relative degrees in this paper. Note that the control method in Pin et al. (2022) addressing the continuous-time systems with relative degree one is not applicable to control the discrete-time systems with arbitrary relative degrees due to the essential differences between the stability characterizations of the continuous-time and discrete-time systems. Compared with the existing literature, this paper addresses the case for a general class of discrete-time LTI systems, and the proposed method does not require any prior knowledge of the high-frequency gain or encounter the transient performance issues. While, it still achieves asymptotic output tracking. In summary, the main contributions of this paper are as follows:

(i) This paper presents a unified solution to the singularity problem commonly encountered in the adaptive control gain issue for a general class of discrete-time LTI systems with arbitrary relative degrees.
(ii) The traditional discrete-time MRAC framework requires the prior knowledge of both the sign of k_p and an upper bound on $|k_p|$. The proposed control method presented in this paper completely eliminates the need for any prior knowledge of k_p, particularly without involving any transient performance issues.
(iii) An output feedback adaptive control law, which is singularity-free and incorporates a gradient algorithm based parameter update law, is developed. It does not necessitate any supplementary design conditions in comparison to the traditional MRAC framework, yet still guarantees closed-loop stability and asymptotic output tracking for any given bounded reference signal.

The remainder of this paper is organized as follows. The problem to be addressed is formulated in Section 2. The entire adaptive control design process, including the controller structure, the form of parameter update law, and the stability analysis, is presented in Section 3. The simulation results are illustrated in Section 4. Finally, concluding remarks are provided in Section 5.

Notations: In this work, we use $\mathbb{R}$ to denote the set of real numbers. We use z and z^{-1} to denote the time advance operator and time delay operator, respectively, i.e., $z[x](t) = x(t+1)$ and $z^{-1}[x](t) = x(t-1)$, where $t \in \{0, 1, 2, 3, \dots\}$, $x(t) \triangleq x(tT)$ for a sampling period $T > 0$, and $x(t)$ denotes any signal of any finite dimension. We also use L^∞ and L^2 to denote signal spaces defined as $L^\infty = \{x(t) : \|x(\cdot)\|_\infty < \infty\}$ and $L^2 = \{x(t) : \|x(\cdot)\|_2 < \infty\}$ with $\|x(\cdot)\|_\infty = \sup_{t\geq 0} \max_{1\leq i\leq n} |x_i(t)|$ and $\|x(\cdot)\|_2 = \sum_{t=0}^{\infty}(|x_1(t)|^2 + \cdots + |x_n(t)|^2)^{\frac{1}{2}}$, where $x(t) = [x_1(t), \dots, x_n(t)]^T$ denotes any signal on $\mathbb{R}^n$.

2. Problem statement

This section presents the formulation of the system model, control objective, and design conditions. Additionally, it addresses the technical issues to be resolved.

System model. Consider the following discrete-time LTI system model:

$$A(z)[y](t) = k_pB(z)[u](t), t \geq 0, \quad (1)$$

where $u(t), y(t) \in \mathbb{R}$ are the system input and output, respectively, $A(z)$ and $B(z)$ are monic polynomials of degrees n and m, respectively, with unknown constant coefficients, i.e.,

$$A(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0,$$
$$B(z) = z^m + b_{m-1}z^{m-1} + \cdots + b_1 z + b_0.$$

Without loss of generality, we assume that $A(z)$ is unstable. Note that $n - m > 0$ is the relative degree of the system (1), which reflects the input–output time delay (Tao, 2003). In this study, the constant high-frequency gain k_p is assumed to be completely unknown except that it must be non-zero to guarantee the controllability of system (1).

Control objective. The objective of this paper is to design an output feedback control input $u(t)$ for the system (1) with a_i, b_j and k_p being all unknown such that the closed-loop system are stable and the system output $y(t)$ tracks a given reference output $y^*(t) \in L^\infty$. The reference output $y^*(t)$ is usually generated by a reference model system

$$y^*(t) = \frac{1}{P_m(z)}[r](t), \tag{2}$$

where $P_m(z)$ is a monic and stable polynomial of degree $n-m$ and $r(t) \in \mathbb{R}$ is an external reference input signal such that $r(t) \in L^\infty$.

Assumptions. To achieve the control objective, the following assumptions are needed.

(A1) The polynomial $B(z)$ is stable.

(A2) The degree n of $A(z)$ is known.

(A3) The relative degree $n^* \triangleq n - m > 0$ is known.

Assumption (A1) means that all zeros of $B(z)$ are inside the unit circle of the complex z-plane, i.e., the system (1) is minimum phase. This is because MRAC will cancel the zeros of the system (1) and replace them with those of the reference model to achieve the tracking control objective. For the sake of stability, such cancellations should occur inside the unit circle of the complex z-plane. By the way, the polynomial $A(z)$ and $B(z)$ are not required to be coprime, i.e., we allow common zeros and poles in the transfer function of the system (1). Assumption (A2) is used to determine the dimension of the estimated parameter and can be relaxed as an upper bound on n is known. Assumption (A3) is devoted to choose the reference model. With assumption (A3), $P_m(z)$ in (2) can be chosen as z^{n^*}, which is a common choice in the discrete-time MRAC (Tao, 2003).

Remark 1. The traditional discrete-time MRAC (Tao, 2003) requires an additional assumption regarding the high-frequency gain k_p: an upper bound on $|k_p|$ is known, as well as the sign of k_p. In contrast, we assume that k_p is entirely unknown in this study. As a result, the control design is undertaken without any prior knowledge of k_p. Actually, k_p indicates the control direction and may be unknown for many control problems in engineering, for instance, uncalibrated visual servoing and autopilot design of uncertain ships (Psillakis, 2017).

Technical issues. To meet the control objective, we will solve the following technical issues in this work: (i) how to develop an adaptive control law that effectively addresses the singularity problem, while eliminating the need for any prior knowledge of k_p; (ii) how to ensure the implementability of the adaptive control law and the parameter update law in the presence of high-order time-delay caused by arbitrary relative degrees; and (iii) how to conduct an analysis on the closed-loop stability and output tracking, particularly focusing on the asymptotic convergence of the output tracking error. These concerns are all addressed in this work.

3. Adaptive control design

In this section, we first develop an adaptive control law incorporating a gradient algorithm based parameter update law. Following this, we present the principal outcome of this study and proceed to analyze stability and tracking performance of the system. By the way, the reading flow is optimized by presenting some long proofs in the Appendix.

3.1. Adaptive control law structure

To proceed, we present the following lemma that provides a fundamental design equation for subsequent control design of this section.

Lemma 1. *(Tao, 2003) There exist constant vectors $\theta_1^* \in \mathbb{R}^{n-1}$ and $\theta_2^* \in \mathbb{R}^n$ such that*

$$\theta_1^{*T}\omega_1(z)A(z) + k_p\theta_2^{*T}\omega_2(z)B(z) = A(z) - B(z)z^{n^*}, \tag{3}$$

where

$$\omega_1(z) = [z^{-n+1}, \dots, z^{-1}]^T,\ \ \omega_2(z) = [z^{-n+1}, \dots, z^{-1}, 1]^T. \tag{4}$$

The proof of this lemma can be seen in Tao (2003). Actually, Eq. (3) is the well-known matching equation for the output feedback model reference control of discrete-time LTI systems (Tao, 2003).

With (3) to hand, we are ready to design the control law. First, we introduce $\rho^* = k_p$ and $\theta_3^* = \frac{1}{k_p}$. Define

$$\omega_1(t) = \omega_1(z)[u](t), \omega_2(t) = \omega_2(z)[y](t),$$
$$\omega(t) = \left[\omega_1^T(t), \omega_2^T(t), r(t)\right]^T \in \mathbb{R}^{2n},$$
$$\theta_p^* = \left[\theta_1^{*T}, \theta_2^{*T}, \theta_3^*\right]^T \in \mathbb{R}^{2n}, \delta_p^* = \rho^*\theta_p^* \in \mathbb{R}^{2n}.$$

Then, the control law in this paper is designed as

$$u(t) = \frac{1}{1 + \alpha(t)\rho(t)}\left(\theta_p^T(t)\omega(t) + \alpha(t)\delta_p^T(t)\omega(t)\right), \tag{5}$$

where $\theta_p(t), \delta_p(t), \rho(t)$ are the estimates of $\theta_p^*, \delta_p^*, \rho^*$, respectively, with $\theta_p(t) = \left[\theta_1^T(t), \theta_2^T(t), \theta_3(t)\right]^T$, and $\alpha(t) \in \mathbb{R}$ is a gain function which will be determined later to guarantee $1 + \alpha(t)\rho(t) \neq 0$.

3.2. Tracking error equation

Define the tracking error as $e(t) = y(t) - y^*(t)$. Then, we give the following lemma to specify a tracking error equation which plays a crucial role in the design of the parameter update law and subsequent stability analysis.

Lemma 2. *The adaptive control law (5), applied to the system (1), provides the tracking error equation as*

$$e(t + n^*) = \tilde{\theta}^T(t)\phi(t), \tag{6}$$

where $\tilde{\theta}(t) = \theta(t) - \theta^*$ with

$$\theta^* = \left[\theta_3^*, \theta_p^{*T}, \delta_p^{*T}, \rho^*\right]^T \in \mathbb{R}^{4n+2}, \tag{7}$$

$$\theta(t) = \left[\theta_3(t), \theta_p^T(t), \delta_p^T(t), \rho(t)\right]^T \in \mathbb{R}^{4n+2}, \tag{8}$$

and

$$\phi(t) = \left[\frac{e(t+n^*)}{\theta_3(t) + \alpha(t)}, \frac{\omega(t)}{\theta_3(t) + \alpha(t)}, \frac{\alpha(t)\omega(t)}{\theta_3(t) + \alpha(t)}, -\frac{\alpha(t)u(t)}{\theta_3(t) + \alpha(t)}\right]^T \in \mathbb{R}^{4n+2}. \tag{9}$$

Proof. The proof is given in the Appendix. □

From the structure of $\phi(t)$ in (9), it indicates that the signal $\theta_3(t)+\alpha(t)$ cannot be zero. The subsequent analysis will demonstrate the non-zero nature of $\theta_3(t)+\alpha(t)$ by designing the gain function $\alpha(t)$.

3.3. Parameter update law

To design a desired adaptive parameter update law, we first define an estimation error as

$$\epsilon(t) = e(t) + (\theta(t) - \theta(t-n^*))^T \phi(t-n^*). \tag{10}$$

From Assumption (A3) and the definitions of $\theta(t)$ in (8) and $\phi(t)$ in (9), we see that $\epsilon(t)$ is available at the present moment. With (6) and (10), we derive that

$$\epsilon(t) = \tilde{\theta}^T(t)\phi(t-n^*), \tag{11}$$

which is actually the estimation error equation.

Introducing the quadratic cost function $J = \frac{1}{2}\frac{\epsilon^2}{m^2}$, where $m = m(t)$ is a normalization signal to be defined, we derive its gradient with respect to $\theta(t)$ as

$$\frac{\partial J}{\partial \theta} = \frac{\epsilon(t)\phi(t-n^*)}{m^2(t)}.$$

Thus, we design a gradient algorithm based parameter update law for $\theta(t)$ as

$$\theta(t+1) = \theta(t) - \frac{\Gamma \epsilon(t)\phi(t-n^*)}{m^2(t)}, \tag{12}$$

where $0 < \Gamma = \Gamma^T < 2I_{4n+2}$ is the adaptive gain and

$$m(t) = \sqrt{1 + \phi^T(t-n^*)\phi(t-n^*)}. \tag{13}$$

Remark 2. The proposed method in our work deviates from the traditional MRAC framework which typically involves a bilinear regression form for the estimation error equation. The readers are referred to Ioannou and Sun (1996), Tao (2003) for further details. In our work, the control law (5) is designed in a manner that prevents the estimation error equation from taking on a bilinear form. Actually, the estimation error Eq. (11) in our work is of a linear regression form. As a result, we can directly formulate a gradient algorithm based parameter update law (12) utilizing the available signals $\epsilon(t)$ and $\phi(t-n^*)$. This eliminates the requirement for the prior knowledge of the high-frequency gain k_p that is necessary in the traditional MRAC framework. However, the cost of the proposed control method lies in the dimension of the estimated parameter vector $\theta(t)$ is relatively high compared with traditional MRAC design scheme. Despite this, the proposed algorithm is simple to implement and tune since the parameter update law (12) shares the same adaptation gain Γ.

The subsequent lemma elucidates that the parameter update law (12) possesses certain desirable properties.

Lemma 3. *The parameter update law (12) ensures that* $\theta(t) \in L^\infty$, $\frac{\epsilon(t)}{m(t)} \in L^2 \cap L^\infty$, *and* $\theta(t+i_0) - \theta(t) \in L^2$ *for any finite integer* $i_0 > 0$.

Proof. Consider the following positive definite function: $V(\tilde{\theta}) = \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$. Then, we have

$$\begin{aligned} & V(\tilde{\theta}(t+1)) - V(\tilde{\theta}(t)) \\ = & -\left(2 - \frac{\phi^T(t-n^*)\Gamma\phi(t-n^*)}{m^2(t)}\right)\frac{\epsilon^2(t)}{m^2(t)}. \end{aligned}$$

Since $0 < \Gamma = \Gamma^T < 2I_{4n+2}$, it follows from the definition of $m(t)$ in (13) that

$$V(\tilde{\theta}(t+1)) - V(\tilde{\theta}(t)) \leq -\beta\frac{\epsilon^2(t)}{m^2(t)}$$

for some constant $\beta > 0$. This implies that $\theta(t) \in L^\infty$ and $\frac{\epsilon(t)}{m(t)} \in L^2$. From (11), we get $\frac{\epsilon(t)}{m(t)} \in L^\infty$. From (12), we have $\theta(t+1) - \theta(t) \in L^2$. Finally, invoking the inequality

$$\|\theta(t+i_0) - \theta(t)\|_2 \leq \sum_{i=0}^{i_0-1} \|\theta(t+i+1) - \theta(t+i)\|_2,$$

we get $\theta(t+i_0) - \theta(t) \in L^2$ for any finite integer i_0. This completes the proof. □

3.4. Gain function design for $\alpha(t)$

To ensure that both the adaptive control law (5) and the regressor signal $\phi(t)$ are non-singular, the gain function $\alpha(t)$ is necessary to guarantee that

$$1 + \alpha(t)\rho(t) \neq 0, \tag{14}$$

$$\theta_3(t) + \alpha(t) \neq 0, \tag{15}$$

for any possible real value of $\rho(t)$ and $\theta_3(t)$. To achieve this objective, we design $\alpha(t)$ as

$$\alpha(t) = \begin{cases} -\left(|\theta_3(t)| + \underline{\alpha}\right), & \rho(t) < 0, \\ |\theta_3(t)| + \underline{\alpha}, & \rho(t) \geq 0, \end{cases} \tag{16}$$

where $\underline{\alpha} > 0$ can be arbitrary. With this choice of $\alpha(t)$, we give the following lemma.

Lemma 4. *The gain function* $\alpha(t)$ *designed in (16) ensures that (14) and (15) always hold.*

Proof. Firstly, we consider the condition (15). When $\rho(t) < 0$, we have

$$\begin{aligned} \theta_3(t) + \alpha(t) &= \theta_3(t) - |\theta_3(t)| - \underline{\alpha} \\ &= \begin{cases} -\underline{\alpha}, & \theta_3(t) \geq 0, \\ 2\theta_3(t) - \underline{\alpha}, & \theta_3(t) < 0. \end{cases} \end{aligned}$$

Since $\underline{\alpha} > 0$, it yields $\theta_3(t) + \alpha(t) < 0$ when $\rho(t) < 0$. Similarly, when $\rho(t) \geq 0$, we have

$$\begin{aligned} \theta_3(t) + \alpha(t) &= \theta_3(t) + |\theta_3(t)| + \underline{\alpha} \\ &= \begin{cases} 2\theta_3(t) + \underline{\alpha}, & \theta_3(t) \geq 0, \\ \underline{\alpha}, & \theta_3(t) < 0. \end{cases} \end{aligned}$$

This implies that $\theta_3(t) + \alpha(t) > 0$ with arbitrary $\underline{\alpha} > 0$ when $\rho(t) \geq 0$. Thus, (15) holds.

Then, we consider the condition (14). When $\rho(t) < 0$, we have

$$\begin{aligned} 1 + \alpha(t)\rho(t) &= 1 - \left(|\theta_3(t)| + \underline{\alpha}\right)\rho(t) \\ &= 1 - |\theta_3(t)|\rho(t) - \underline{\alpha}\rho(t). \end{aligned}$$

Since $\rho(t) < 0$, we get $1 - |\theta_3(t)|\rho(t) - \underline{\alpha}\rho(t) > 0$ is always true with arbitrary $\underline{\alpha} > 0$. When $\rho(t) \geq 0$, we have

$$\begin{aligned} 1 + \alpha(t)\rho(t) &= 1 + \left(|\theta_3(t)| + \underline{\alpha}\right)\rho(t) \\ &= 1 + |\theta_3(t)|\rho(t) + \underline{\alpha}\rho(t). \end{aligned}$$

It follows from $\underline{\alpha} > 0$ that $1 + |\theta_3(t)|\rho(t) + \underline{\alpha}\rho(t) > 0$ is always true when $\rho(t) \geq 0$. Thus, (14) holds. This completes the proof. □

The controller (5) eliminates the need for prior knowledge of k_p in the parameter update law (12). However, the introduction of denominator terms $1+\alpha(t)\rho(t)$ and $\theta_3(t)+\alpha(t)$ may cause high gain problem during the adaptive process. To address this issue, we choose $\alpha(t)$ as defined in (16) to achieve a unified singularity-free design for both the control law (5) and the parameter update law (12). Lemma 4 demonstrates that $1+\alpha(t)\rho(t)$ and $\theta_3(t)+\alpha(t)$ are consistently non-zero with $\alpha(t)$ being designed in (16). As a result, the control law (5) and the parameter update law (12) are always implementable in the control process.

3.5. Stability analysis

The analysis for closed-loop system performance is now underway. The main result of this paper is given in the following. To optimize the reading flow, the proof of the main result is placed in the Appendix.

Theorem 1. *Under Assumptions (A1)-(A3), if the adaptive control law (5) with the parameter update law (12) is applied to the system (1), then the closed-loop system is stable and of asymptotic output tracking, i.e.,* $\lim_{t\to\infty}(y(t)-y^*(t))=0$.

Proof. The proof is given in the Appendix. □

So far, we have developed a modified MRAC method for discrete-time LTI systems with arbitrary relative degrees. Notably, the proposed method eliminates both the sign information of k_p and the upper bound on $|k_p|$, which still achieves asymptotic output tracking.

4. Simulation study

This section provides two examples to showcase the design process and substantiate the theoretical findings.

4.1. Simulation for systems with $n^*=1$

Simulation model. Consider the following system

$$A_s(z)[y](t)=k_{ps}B_s(z)[u](t), \tag{17}$$

where $k_{ps}=1$, and

$$A_s(z)=(z+1)(z-2)\left(z+\frac{1}{2}\right), \tag{18}$$

$$B_s(z)=z\left(z+\frac{1}{2}\right). \tag{19}$$

Form (18) and (19), we see that $A_s(z)$ is unstable and $B_s(z)$ is stable. Consequently, the simulation model (17) is minimum-phase. Moreover, $A_s(z)$ and $B_s(z)$ have a common factor $z+\frac{1}{2}$ and the relative degree is one, that is, $n^*=1$. The reference output signal is chosen as

$$y^*(t)=4\sin\frac{1}{2}t-\frac{1}{3}\cos\frac{3}{10}t. \tag{20}$$

Control law parameter setting. From (18) and (19), we calculate θ_1^* and θ_2^* as $\theta_1^*=\left[0,-\frac{1}{2}\right]^T$ and $\theta_2^*=\left[-1,-\frac{5}{2},-\frac{1}{2}\right]^T$. With $\rho^*=k_p$ and $\theta_3^*=\frac{1}{k_p}$, we have $\rho^*=1$ and $\theta_3^*=1$. With $\theta_p^*=[\theta_1^{*T},\theta_2^{*T},\theta_3^*]^T$ and $\delta_p^*=\rho^*\theta_p^*$, we derive $\theta_p^*=\delta_p^*=\left[0,-\frac{1}{2},-1,-\frac{5}{2},-\frac{1}{2},1\right]^T$. From (7), we obtain

$$\theta^*=\left[1,0,-\frac{1}{2},-1,-\frac{5}{2},-\frac{1}{2},1,0,-\frac{1}{2},-1,-\frac{5}{2},-\frac{1}{2},1,1\right]^T.$$

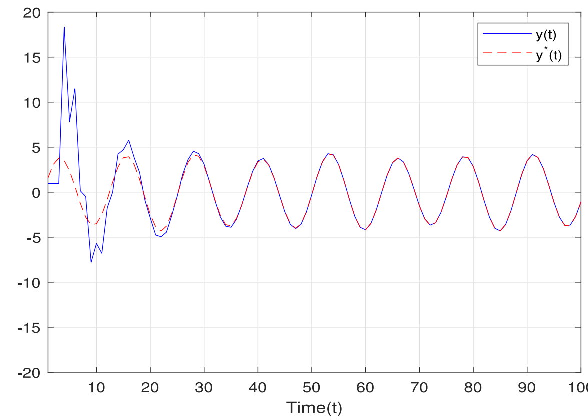

Fig. 1. Trajectories of the output $y(t)$ and the reference output $y^*(t)$ ($n^*=1$).

Moreover, $\omega_1(t)$ and $\omega_2(t)$ are specified as

$$\omega_1(t)=\omega_1(z)[u](t),\ \omega_1(z)=[z^{-2},z^{-1}]^T,$$
$$\omega_2(t)=\omega_2(z)[y](t),\ \omega_2(z)=[z^{-2},z^{-1},1]^T. \tag{21}$$

Let $\theta_3(t),\theta_p(t),\delta_p(t),\rho(t),\theta(t)$ be the estimates of $\theta_3^*,\theta_p^*,\delta_p^*,\rho^*,\theta^*$, respectively, with

$$\theta_p(t)=\left[\theta_{p1}(t),\theta_{p2}(t),\theta_{p3}(t),\theta_{p4}(t),\theta_{p5}(t),\theta_{p6}(t)\right],$$
$$\delta_p(t)=\left[\delta_{p1}(t),\delta_{p2}(t),\delta_{p3}(t),\delta_{p4}(t),\delta_{p5}(t),\delta_{p6}(t)\right].$$

Then, we get

$$\theta(t)=\left[\theta_3(t),\theta_p^T(t),\delta_p^T(t),\rho(t)\right]^T. \tag{22}$$

Choose $\theta(0)=[1.2,0.3,-0.6,-1.1,-2.2,-0.8,1.1,-0.1,-0.4,-1.1,-1.9,-0.4,1.1,1.2]^T$, $\Gamma=\text{diag}\{0.1,0.1,0.2,0.1,0.2,0.5,0.4,0.5,0.3,0.3,0.8,0.1,0.2,0.1\}$. Moreover, choose $y(0)=-1$ and $\underline{\alpha}=0.5$. Then, we can specify the control law and the parameter update law by (5) and (12), respectively.

Simulation figures. The system response for the case of $n^*=1$ is shown in Figs. 1–3. Fig. 1 presents the output response $y(t)$ versus the reference signal $y^*(t)$. This figure illustrates that $y(t)$ tracks $y^*(t)$ asymptotically. Fig. 2 shows the response of the control input $u(t)$. Fig. 3 shows the trajectories of part of parameters in $\theta(t)$. Notably, the proposed control method does not rely on any type of excitation conditions. Thus, parameter estimations may not converge to corresponding true values. Nevertheless, this does not affect the realization of the asymptotic output tracking control objective. In summary, the simulation results verify the validity of the proposed control method for systems with $n^*=1$.

4.2. Simulation for systems with $n^*=2$

Simulation model. Consider the same system model as (17) but with $A_s(z)=(z+1)(z-2)(z-1)$, $B_s(z)=z$. Moreover, we consider the same reference model as (20). In this case, the relative degree is two, that is, $n^*=2$.

Control law parameter setting. First, we calculate θ_1^* and θ_2^* as $\theta_1^*=[0,-2]^T$, $\theta_2^*=[4,0,-5]^T$. With $\theta_p^*=[\theta_1^{*T},\theta_2^{*T},\theta_3^*]^T$ and $\delta_p^*=\rho^*\theta_p^*$, we can get $\theta_p^*=\delta_p^*=[0,-2,4,0,5,1]^T$. Then, it follows from (7) that

$$\theta^*=[1,0,-2,4,0,-5,1,0,-2,4,0,-5,1,1]^T.$$

Moreover, $\omega_1(t)$ and $\omega_2(t)$ are the same as (21). $\theta(t)$, denoted as the estimate of θ^*, is the same as (22). Choose $\theta(0)$ =

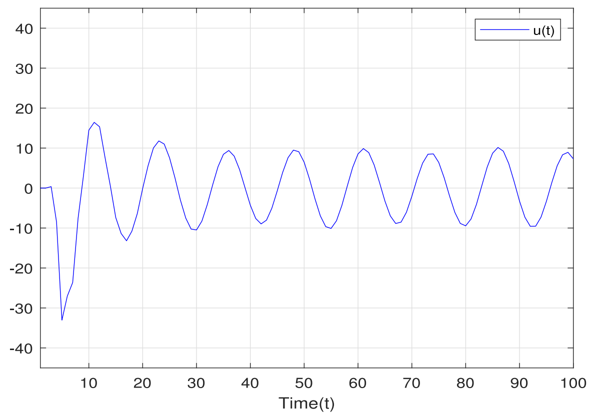

Fig. 2. Trajectory of the control law $u(t)$ ($n^* = 1$).

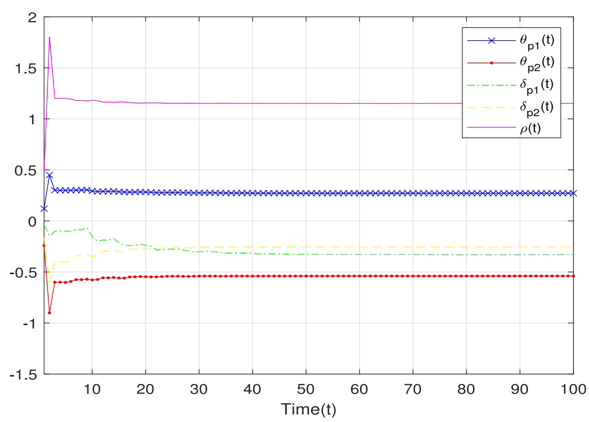

Fig. 3. Trajectories of part of parameters in $\theta(t)$ ($n^* = 1$).

$[1.2, 0.3, -1.6, 4.8, 1.5, -6, 1.4, 0.2, -2.1, 3.6, -0.2, -5.6, 1.5, 1.1]^T$, $\Gamma = \mathrm{diag}\,\{0.2, 0.1, 0.3, 0.4, 0.4, 0.6, 0.2, 0.2, 0.4, 0.4, 0.3, 0.5, 0.2, 0.1\}$. Moreover, choose $y(0) = -2$ and $\underline{\alpha} = 0.5$. Similarly, we also can determine the control law and the parameter update law by (5) and (12), respectively.

Simulation figures. The system response for the case of $n^* = 2$ is shown in Figs. 4–6. From these figures, we see that the proposed control method is also valid for systems with $n^* = 2$.

5. Concluding remarks

In this paper, we present a novel output feedback MRAC method for discrete-time LTI systems with arbitrary relative degrees. This method eliminates the need for the high-frequency gain assumption and achieves asymptotic output tracking. The proposed control method operates without the need for Nussbaum gain functions and eliminates the requirement of using projection operators to maintain non-singularity of the adaptive control gain. Furthermore, the convergence of tracking error does not necessitate any type of excitation conditions. The simulation study illustrates the control design process and validates the efficacy of the proposed control strategy. Several intriguing topics warrants further investigation. For instance, it is imperative to expand the proposed scheme to address multi-input and multi-output scenarios. In other words, the challenge lies in tackling the elimination of the assumption regarding the high-frequency gain matrix. Applying the proposed methodology to

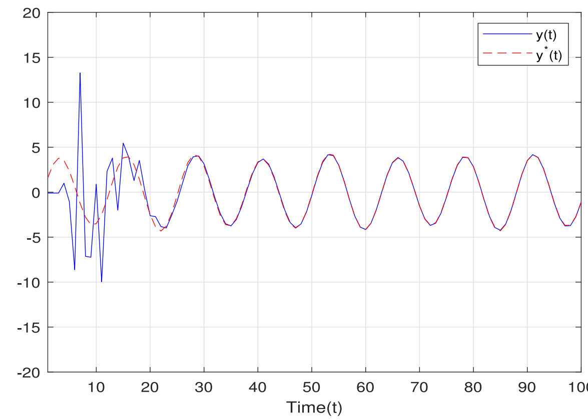

Fig. 4. Trajectories of the output $y(t)$ and the reference output $y^*(t)$ ($n^* = 2$).

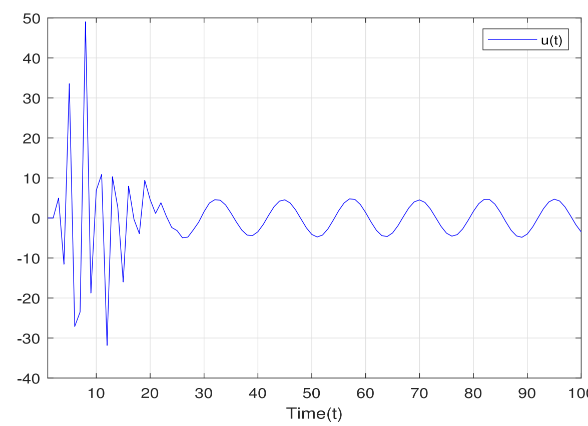

Fig. 5. Trajectory of the control law $u(t)$ ($n^* = 2$).

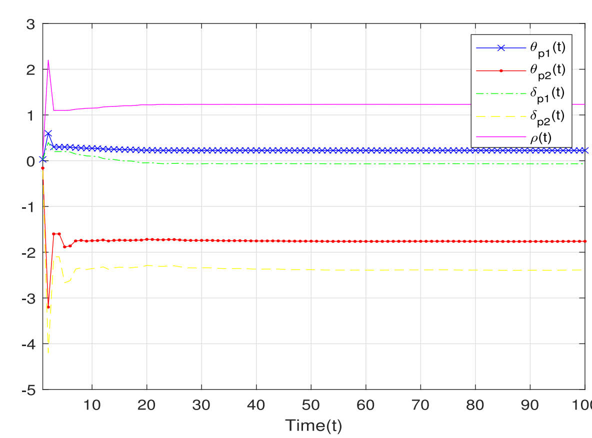

Fig. 6. Trajectories of part of parameters in $\theta(t)$ ($n^* = 2$).

indirect model reference adaptive control also merits further investigation. Moreover, assessing the robustness of the proposed control algorithm is crucial to expanding its applicability in practical settings.

Appendix

Proof of Lemma 2. First, we consider that $A(z)$ and $B(z)$ are coprime. From (3), we have

$$(\theta_1^{*T}\omega_1(z)-1)A(z)=-(\theta_2^{*T}\omega_2(z)k_p+z^{n^*})B(z).$$

It follows from $A(z)$ and $B(z)$ are coprime that there exists some polynomial $F(z)=-z^{-m}+f_{n-m-2}z^{-m-1}+\cdots+f_0z^{-n+1}$, which satisfies the equality

$$\theta_1^{*T}\omega_1(z)-1=F(z)B(z). \tag{A.1}$$

Using this result, we have

$$F(z)A(z)+k_p\theta_2^{*T}\omega_2(z)=-z^{n^*}. \tag{A.2}$$

Then, operating both sides of (A.2) on $y(t)$, we have $F(z)A(z)[y](t)+k_p\theta_2^{*T}\omega_2(z)[y](t)=-y(t+n^*)$. Combining the system model (1), we get

$$k_pF(z)B(z)[u](t)+k_p\theta_2^{*T}\omega_2(z)[y](t)=-y(t+n^*).$$

From (A.1), we have

$$k_p(\theta_1^{*T}\omega_1(z)-1)[u](t)+k_p\theta_2^{*T}\omega_2(t)=-y(t+n^*). \tag{A.3}$$

With the reference model, we have

$$\begin{aligned}&e(t+n^*)\\&=y(t+n^*)-y^*(t+n^*)\\&=k_p\left(-\theta_1^{*T}\omega_1(t)-\theta_2^{*T}\omega_2(t)+u(t)\right)-r(t)\\&=\rho^*\left(-\theta_1^{*T}\omega_1(t)-\theta_2^{*T}\omega_2(t)-\theta_3^*r(t)+u(t)\right)\\&=\rho^*\left(-\theta_p^{*T}\omega(t)+u(t)\right).\end{aligned} \tag{A.4}$$

With $\delta_p^*=\rho^*\theta_p^*$, it follows from Eq. (A.4) that

$$e(t+n^*)=-\delta_p^{*T}\omega(t)+\rho^*u(t). \tag{A.5}$$

Multiplying both sides of (A.5) by $\theta_3^*=\frac{1}{k_p}$, we obtain

$$\theta_3^*e(t+n^*)=-\theta_p^{*T}\omega(t)+u(t). \tag{A.6}$$

By rearranging the expression (5), one can get $u(t)=\theta_p^T(t)\omega(t)+\alpha(t)\delta_p^T(t)\omega(t)-\alpha(t)\rho(t)u(t)$. Together with (A.5), it yields

$$\begin{aligned}u(t)&=\theta_p^T(t)\omega(t)-\alpha(t)\left(-\delta_p^T(t)\omega(t)+\rho(t)u(t)\right)\\&\quad+\alpha(t)e(t+n^*)-\alpha(t)e(t+n^*)\\&=\theta_p^T(t)\omega(t)-\alpha(t)\left(-\delta_p^T(t)\omega(t)+\rho(t)u(t)\right)\\&\quad+\alpha(t)\left(-\delta_p^{*T}\omega(t)+\rho^*u(t)\right)-\alpha(t)e(t+n^*)\\&=\theta_p^T(t)\omega(t)-\alpha(t)\left(-\tilde{\delta}_p^T(t)\omega(t)+\tilde{\rho}(t)u(t)\right)\\&\quad-\alpha(t)e(t+n^*)\\&=\theta_p^T(t)\omega(t)+\alpha(t)\tilde{\delta}_p^T(t)\omega(t)-\alpha(t)\tilde{\rho}(t)u(t)\\&\quad-\alpha(t)e(t+n^*),\end{aligned} \tag{A.7}$$

where $\tilde{\delta}_p(t)\triangleq\delta_p(t)-\delta_p^*$ and $\tilde{\rho}(t)\triangleq\rho(t)-\rho^*$. Substituting (A.7) in (A.6), we have

$$\begin{aligned}&(\theta_3^*+\alpha(t))e(t+n^*)\\&=\tilde{\theta}_p^T(t)\omega(t)+\alpha(t)\tilde{\delta}_p^T(t)\omega(t)-\alpha(t)\tilde{\rho}(t)u(t),\end{aligned}$$

where $\tilde{\theta}_p(t)\triangleq\theta_p(t)-\theta_p^*$. Denote $\theta_3(t)$ as the estimate of θ_3^* and define $\tilde{\theta}_3(t)\triangleq\theta_3(t)-\theta_3^*$. Then, we obtain the following expression

$$\begin{aligned}&(\theta_3(t)+\alpha(t))e(t+n^*)\\&=\tilde{\theta}_3(t)e(t+n^*)+\tilde{\theta}_p^T(t)\omega(t)+\alpha(t)\tilde{\delta}_p^T(t)\omega(t)\\&\quad-\alpha(t)\tilde{\rho}(t)u(t).\end{aligned}$$

Given $\theta_3(t)+\alpha(t)\neq 0$ is always true, we get

$$\begin{aligned}e(t+n^*)&=\tilde{\theta}_3(t)\frac{e(t+n^*)}{\theta_3(t)+\alpha(t)}+\tilde{\theta}_p^T(t)\frac{\omega(t)}{\theta_3(t)+\alpha(t)}\\&\quad+\tilde{\delta}_p^T(t)\frac{\alpha(t)\omega(t)}{\theta_3(t)+\alpha(t)}-\tilde{\rho}(t)\frac{\alpha(t)u(t)}{\theta_3(t)+\alpha(t)}.\end{aligned}$$

From the definition of $\tilde{\theta}(t)$ and $\phi(t)$, we finally get $e(t+n^*)=\tilde{\theta}^T(t)\phi(t)$.

When $A(z)$ and $B(z)$ are non-coprime, we rewrite $B(z)$ as $B(z)=B_1(z)B_2(z)$ such that $B_1(z)$ is of degree n_1, and $B_2(z)$ and $A(z)$ are coprime. Then, there exist unique $\bar{\theta}_1^*\in\mathbb{R}^{n-1-n_1}$ and $\theta_2^*\in\mathbb{R}^n$ such that

$$\begin{aligned}&z^{n-1-n_1}\bar{\theta}_1^{*T}\bar{\omega}_1(z)A(z)+k_p\theta_2^{*T}z^{n-1}\omega_2(z)B_2(z)\\&=z^{n-1-n_1}A(z)-z^{n-1}B_2(z)z^{n^*}\end{aligned}$$

with $\bar{\omega}_1(z)=\left[z^{-n+n_1+1},\ldots,z^{-1}\right]^T$. With some manipulations, we have

$$\begin{aligned}&z^{-n_1}\bar{\theta}_1^{*T}\bar{\omega}_1(z)A(z)+k_p\theta_2^{*T}\omega_2(z)B_2(z)\\&=z^{-n_1}A(z)-B_2(z)z^{n^*}.\end{aligned}$$

Therefore, there exists some polynomial $\bar{F}(z)$ of the form $\bar{F}(z)=-z^{-m+n_1}+\bar{f}_{n^*+n_1-2}z^{-m+n_1-1}+\cdots+\bar{f}_0z^{-n+n_1+1}$ such that

$$\bar{F}(z)B_2(z)=\bar{\theta}_1^{*T}\bar{\omega}_1(z)-1.$$

Let $F(z)=z^{-n_1}\bar{F}(z)$. Then, we also obtain (A.2). Following the same derivation as above, we also get the tracking error Eq. (6). This completes the proof. □

Proof of Theorem 1. For $\omega_1(t)$ and $\omega_2(t)$ in (4), we define

$$\bar{\phi}(t)=[\omega_1^T(t),\omega_2^T(t)]^T. \tag{A.8}$$

From (A.3) and $\theta_3^*=\frac{1}{k_p}$, we get $\theta_1^{*T}\omega_1(t)+\theta_2^{*T}\omega_2(t)=u(t)-\theta_3^*y(t+n^*)$. Combining (1) and (A.8), it yields

$$\bar{\phi}(t+1)=A^*\bar{\phi}(t)+b^*y(t+n^*), \tag{A.9}$$

where

$$A^*=\begin{bmatrix}E_{n-2} & 0_{(n-2)\times n}\\ \theta_1^{*T} & \theta_2^{*T}\\ 0_{(n-1)\times(n-1)} & E_{n-1}\\ A_1^{*T} & A_2^{*T}\end{bmatrix}\in\mathbb{R}^{(2n-1)\times(2n-1)},$$

$$b^*=\begin{bmatrix}b_1^*\\ b_2^*\end{bmatrix}\in\mathbb{R}^{2n-1},\ b_1^*=\begin{bmatrix}0\\ \vdots\\ 0\\ \theta_3^*\end{bmatrix}\in\mathbb{R}^{n-1},$$

$$E_i=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\ \vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&1\end{bmatrix}\in\mathbb{R}^{i\times(i+1)},$$

$$\begin{aligned}&\{A_1^{*T}\in\mathbb{R}^{n-1},A_2^{*T}\in\mathbb{R}^n,b_2^*\in\mathbb{R}^n\}\\&=\begin{cases}\{[z^{*T},k_p,0,\ldots,0],-p^{*T},[0,\cdots,0]\},\text{ if } n^*>1,\\ \{z^{*T}+k_p\theta_1^{*T},-p^{*T}+k_p\theta_2^{*T},[0,\cdots,0,1]\},\text{ if } n^*=1\end{cases}\end{aligned}$$

with $p^*=[a_0,\ldots,a_{n-1}]^T\in\mathbb{R}^n$ and $z^*=[z_0,\ldots,z_{m-1}]^T\in\mathbb{R}^m$. Consider the dynamic system (A.9). Let the first variable of $\bar{\phi}(t)$ be the output of (A.9). Then, we derive an input–output expression for the system (A.9) with the virtual input $y^*(t+n^*)$ and the virtual output $z^{-n+1}u(t)$ as $z^{n-1}[u](t) = c^*(zI_{2n-1}-A^*)^{-1}b^*[y](t+n^*)$, where $c^* = [1, 0, \dots, 0] \in \mathbb{R}^{2n-1}$. From the system model (1), it yields $A(z)z^{n-1}[u](t) = k_p c^*(zI_{2n-1} - A^*)^{-1}b^* z^{n^*}B(z)[u](t)$. This implies that $\det\{zI_{2n-1} - A^*\} = z^{n+n^*-1}B(z)$, which together with the Assumption (A1) shows that the eigenvalues of A^* are all inside the unit circle of the z-complex plane, i.e., A^* is stable. Thus, there exists a nonsingular matrix $P^* \in \mathbb{C}^{(2n-1)\times(2n-1)}$ such that $\|P^*A^*P^{*-1}\|_2 < 1$, where $\|\cdot\|_2$ denotes the induced Euclidean matrix norm. With this matrix P^*, the following vector norm $\|\cdot\|$ in $\mathbb{R}^{2n-1}$ is defined as $\|x\| = \|P^*x\|_2$. Then, it follows from (10) and (A.9) that

$$\bar{\phi}(t+1) = A^*\bar{\phi}(t) + b^*(y^*(t+n^*) + \epsilon(t+n^*) - (\theta(t+n^*) - \theta(t))^T\phi(t)). \tag{A.10}$$

Introducing the auxiliary signal

$$s(t) = \left|\frac{\epsilon(t+n^*)}{m(t+n^*)}\right| + \|\theta(t+n^*) - \theta(t)\|_2. \tag{A.11}$$

It follows from Lemma 3 that $s(t) \in L^2$. From (A.10) and (A.11), we obtain

$$\|\bar{\phi}(t+1)\| \leq (a_0 + c_1 s(t))\|\bar{\phi}(t)\| + c_2 \tag{A.12}$$

for some constants $a_0 \in (0, 1)$, $c_1 > 0$, and $c_2 > 0$. Since $s(t) \in L^2$, we get

$$\sum_{t=t_0}^{t_0+j} s(t) \leq (j+1)^{\frac{1}{2}} \left(\sum_{t=t_0}^{t_0+j} s^2(t)\right)^{\frac{1}{2}} \leq c_3(j+1)^{\frac{1}{2}}$$

for any $j \geq 1$ and some constant $c_3 > 0$. With this property, we have

$$\begin{aligned}\prod_{t=t_0}^{t_0+j}(a_0 + c_1 s(t)) &\leq \left(a_0 + \frac{c_1}{j+1}\sum_{t=t_0}^{t_0+j} s(t)\right)^{j+1} \\ &\leq \left(a_0 + \frac{c_1 c_3}{\sqrt{j+1}}\right)^{j+1} \\ &\leq a_0^{j+1}\left(1 + \frac{c_1 c_3}{a_0\sqrt{j+1}}\right)^{j+1} \\ &\leq a_0^{j+1} e^{\frac{c_1 c_3}{a_0}\sqrt{j+1}},\end{aligned}$$

which implies that $\lim_{t\to\infty} \sum_{j=1}^{k} \prod_{t=t_0}^{t_0+j}(a_0 + c_1 s(t)) < \infty$. From (A.12), we see that $\|\bar{\phi}(t)\|$ is bounded. Thus, $\|\omega(t)\|$ is bounded. Then, it yields $\|\phi(t)\|$ is bounded, which indicates that all closed-loop signals are bounded. With $\epsilon(t) = \frac{\epsilon(t)}{m(t)}m(t)$ and $m(t) \in L^\infty$, we have $\epsilon(t) \in L^2$. Thus, we conclude $\lim_{t\to\infty}\epsilon(t) = 0$. From Lemma 3, we have $\theta(t) - \theta(t-n^*) \in L^2$, which shows that $\lim_{t\to\infty}(\theta(t) - \theta(t-n^*)) = 0$. It follows from (10) that $\lim_{t\to\infty} e(t) = 0$. This completes the proof. □

References

Annaswamy, A. M., & Fradkov, A. L. (2021). A historical perspective of adaptive control and learning. *Annual Reviews in Control*, *52*, 18–41.

Astrom, K. J., & Wittenmark, B. (1973). On self tuning regulators. *Automatica*, *9*(2), 185–199.

Astrom, K. J., & Wittenmark, B. (2013). *Adaptive control*. Courier Corporation.

Chen, Z. (2019). Nussbaum functions in adaptive control with time-varying unknown control coefficients. *Automatica*, *102*, 72–79.

Chen, H. F., & Guo, L. (1991). *Identification and stochastic adaptive control*. Springer Science & Business Media.

Chen, H. F., & Zhang, J. F. (1990). Identification and adaptive control for systems with unknown orders, time-delay and coefficients. *IEEE Transactions on Automatic Control*, *35*(8), 866–877.

Ge, S. S., Hang, C. C., Lee, T. H., & Zhang, T. (2001). *Stable adaptive neural network control*. Boston, MA: Kluwer Academic.

Ge, S. S., & Wang, J. (2003). Robust adaptive tracking for time-varying uncertain nonlinear systems with unknown control coefficients. *IEEE Transactions on Automatic Control*, *48*(8), 1463–1469.

Goodwin, G. C., & Sin, K. S. (1984). *Adaptive filtering prediction and control*. Englewood Cliffs, NJ: Prentice-Hall.

Ioannou, P., & Sun, J. (1996). *Robust adaptive control*. New Jersey: Prentice-Hall.

Kalman, R. (1958). Design of a self-optimizing control system. *Journal of Fluids Engineering*.

Kaloust, J., & Qu, Z. (1995). Continuous robust control design for nonlinear uncertain systems without a priori knowledge of control direction. *IEEE Transactions on Automatic Control*, *40*(2), 276–282.

Krstic, M., Kanellakopoulos, I., & Kokotovic, P. V. (1995). *Nonlinear and adaptive control design*. New York: Wiley and Sons.

Landau, Y. D., Lozano, R., M'Saad, M., & Karimi, A. (2011). *Adaptive control: algorithms, analysis and applications* (2nd ed.). London: Springer.

Lee, T. H., & Narendra, K. S. (1986). Stable discrete adaptive control with unknown high-frequency gain. *IEEE Transactions on Automatic Control*, *31*(5), 477–478.

Liu, R., Wang, H., & Li, W. (2024). Prescribed-time stabilization and inverse optimal control of stochastic high-order nonlinear systems. *Science China: Information Sciences*, *67*, Article 122202.

Lozano, R., Collado, J., & Mondie, S. (1990). Model reference robust adaptive control without a priori knowledge of the high freqency gain. *IEEE Transactions on Automatica Control*, *35*(1), 71–78.

Marino, R., & Tomei, P. (1995). *Nonlinear control design: geometric, adaptive and robust*. Hemel Hempstead, UK: Prentice-Hall.

Monopoli, R. V. (1974). Model reference adaptive control with an augmented error. *IEEE Transactions on Automatic Control*, *19*(4), 474–484.

Narendra, K., & Annaswamy, A. (1989). *Stable adaptive systems*. New Jersey: Prentice Hall.

Nussbaum, R. (1983). Some remarks on a conjecture in parameter adaptive control. *Systems & Control Letters*, *3*(5), 243–246.

Ordonez, R., & Passino, K. M. (1999). Stable multiple-input multiple-output adaptive fuzzy/neural control. *IEEE Transactions on Fuzzy Systems*, *7*(3), 345–353.

Ortega, R., Gerasimov, D. N., Barabanov, N. E., & Nikiforov, V. O. (2019). Adaptive control of linear multivariable systems using dynamic regressor extension and mixing estimators: Removing the high-frquency gain assumptions. *Automatica*, *110*, Article 108589.

Pin, G., Serrani, A., & Wang, Y. (2022). Parameter-dependent input normalization: direct-adaptive control with uncertain control direction. In *IEEE conference on decision and control* (pp. 2674–2680).

Psillakis, H. E. (2017). Consensus in networks of agents with unknown high-frequency gain signs and switching topology. *IEEE Transactions on Automatic Control*, *62*(8), 3993–3998.

Ren, X., Zhao, W., & Gao, J. (2023). Adaptive regulation for hammerstein and wiener systems with event-triggered observations. *Journal of Systems Science & Complexity*, *36*, 1878–1904.

Sastry, S. S., & Bodson, M. (1989). *Adaptive control: stability, convergence and robustness*. New Jersey: Prentice-Hall.

Tao, G. (2003). *Adaptive control design and analysis*. John Wiley & Sons.

Tao, G. (2014). Multivariable adaptive control: A survey. *Automatica*, *50*(11), 2737–2764.

Wang, Y., & Liu, Y. (2022). Adaptive output-feedback tracking for nonlinear systems with unknown control direction and generic inverse dynamics. *Science China: Information Sciences*, *65*, Article 182204.

Wang, C., Wen, C., & Guo, L. (2020). Multivariable adaptive control with unknown signs of the high-frequency gain matrix using novel Nussbaum functions. *Automatica*, *111*, Article 108618.

Xu, Y., Zhang, Y., & Zhang, J. F. (2023). Singularity-free adaptive control of MIMO discrete-time nonlinear systems with general vector relative degrees. *Automatica*, *153*, Article 111054.

Yang, C., Ge, S. S., & Lee, T. H. (2009). Output feedback adaptive control of a class of nonlinear discrete-time systmes with unknown control directions. *Automatica*, *45*(1), 270–276.

Ye, X., & Jiang, J. (1998). Adaptive nonlinear design without a priori knowledge of control directions. *IEEE Transactions on Automatic Control*, *43*(11), 1617–1621.

Zhang, Y., Zhang, J. F., & Liu, X. K. (2022). Matrix decomposition-based adaptive control of noncanonical form MIMO DT nonlinear systems. *IEEE Transactions on Automatic Control*, *67*(8), 4330–4337.

Zhang, Y., Zhang, Z., Sun, J., Wang, L., & Zhang, K. (2023). Higher-order properties and extensions for indirect MRAC and APPC of linear systems. *Science China: Information Sciences*, http://dx.doi.org/10.1007/s11432-023-3874-6.

Zhang, H., Zhao, X., Wang, H., Niu, B., & Xu, N. (2023). Adaptive tracking control for output-constrained switched MIMO pure-feedback nonlinear systems with input saturation. *Journal of Systems Science & Complexity*, *36*, 960–984.

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