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Singularity-free adaptive control of MIMO discrete-time nonlinear systems with general vector relative degrees[☆]

Yuchun Xu[a,b], Yanjun Zhang[c,*], Ji-Feng Zhang[a,b]

[a] Key Laboratory of Systems and Control, Institute of Systems Science, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China
[b] School of Mathematics Sciences, University of Chinese Academy of Sciences, Bejing 100149, China
[c] School of Automation, Beijing Institute of Technology, Beijing 100081, China

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ABSTRACT

This paper develops a singularity-free adaptive tracking control scheme for a general class of multi-input and multi-output uncertain discrete-time nonlinear systems with non-canonical control gain matrices. The estimation of the control gain matrices, especially in some non-canonical forms, may be singular during parameter adaptation, which leads to the singularity problems of the adaptive control laws. This paper employs the matrix decomposition technique to solve the problem under a linearly parameterized adaptive control framework. The state and output feedback cases are addressed, respectively, to ensure closed-loop stability and asymptotic output tracking. Compared with the existing results, the features of the proposed adaptive control scheme include: (i) the proposed control laws do not involve the high-gain issue commonly encountered in robust control methods; (ii) two different filtered tracking error signals are introduced for the state and output feedback cases, respectively. These filters are crucial to avoid causality contradiction of the adaptive control laws commonly encountered in adaptive control of discrete-time systems; and (iii) a future time signal estimation-based adaptive control law is developed to ensure asymptotic output tracking for the output feedback case without requiring the high-gain observer. Finally, an illustrative example is given to verify the validity of the proposed control scheme.

1. Introduction

Regarding their significance in science and engineering, adaptive control systems have been extensively studied in the control community. Dealing with unknown control gains is an important technical issue and still attracts the interest of many researchers. In this regard, many works have been published to handle the singularity problems caused by the unknown control gains (Chang, 2000; De Mathelin & Bodson, 1995; Feng, Yu, & Han, 2013; Hou, Shi, Fang, & Duan, 2023; Hu, Fei, Ma, Wu, & Geng, 2016; Li, Liu, Li, & Xu, 2021; Li, Qiang, Zhuang, & Kaynak, 2004; Morse, 1993; Su, Xue, Liang, & Chen, 2022; Wang, Guo, Wen, Hu, & Qiao, 2019; Wang, Marconi, & Kellett, 2022; Weller & Goodwin, 1994; Wu, Yu, & Man, 1998; Yang, Zhang, & Fridman, 2022; Zhang, Xia, Shen, & Cui, 2021; Zheng, Pan, Guo, & Yu, 2019).

The control gains for single-input and single-output (SISO) uncertain systems are just scalars. Simply embedding some robust techniques (e.g., parameter projection, σ-modification) in the adaptive updating laws can ensure the estimates of the control gains are non-zero, and lead to well-defined adaptive control laws. In comparison, the control gains are matrices for multi-input and multi-output (MIMO) uncertain systems. Using parameter projection or σ-modification-based adaptive updating laws still relies on solid knowledge of the control gain matrices and lacks robustness to parameter variations (Tao, 2014). In other words, the design conditions are quite restrictive and largely limit the application range. In this respect, an arising question is whether a method can effectively handle the control gain matrix singularity problem under some relaxed design conditions. To this end, Morse (1993) introduced the matrix decomposition technique into the adaptive control systems, and successfully solved the control gain singularity problem for a class of uncertain linear time-invariant (LTI) systems. Since then, the matrix

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* Corresponding author.
E-mail addresses: xuyuchun@amss.ac.cn (Y. Xu), yanjun@bit.edu.cn (Y. Zhang), jif@iss.ac.cn (J.-F. Zhang).

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decomposition technique has been widely used in various control problems (Costa, Hsu, Imai, & Kokotovic, 2003; Hsu, Costa, & Lizarralde, 2007; Imai, Costa, Hsu, Tao, & Kokotovic, 2004; Li, Xie, & Chen, 2008; Mirkin & Gutman, 2005). The readers are referred to the monograph (Tao, 2003) that systematically synthesizes the matrix decomposition-based model reference adaptive control methods for MIMO LTI systems covering continuous-time and discrete-time cases.

Meanwhile, the application of the matrix decomposition technique in LTI systems motivates the researchers to consider how to employ matrix decomposition in nonlinear adaptive control systems. Based on this motivation, many works have been done mainly using robust control and approximation techniques. For instance, Chen, Behal, and Dawson (2008) designed a matrix decomposition-based singularity-free robust controller to achieve uniformly ultimately bounded tracking by using state feedback and high-gain observer-based output feedback, respectively. Wu, Chen, Wu, and Zhang (2018) dealt with the singularity problem for adaptive control of continuous-time nonlinear systems with time-varying parameterized uncertainties, and realized asymptotic output tracking by state feedback and uniformly ultimately bounded output tracking by high-gain observer-based output feedback, respectively. Wang, Isidori, and Su (2015) provided an extended observer-based method for asymptotic stabilization of nonlinear MIMO systems, given that the high-frequency gain matrix is bounded. The fuzzy/neural network approximation-based adaptive control methods were also proposed to overcome the singularity problems with the matrix decomposition technique (Arefi & Jahed-Motlagh, 2013; Boulkroune, Merazka, & Li, 2018; Li, Yang, Hu, & Yang, 2007; Zhang, Tao, Chen, Wen, & Zhang, 2019). Besides, the matrix decomposition technique has also been widely used in some application studies. For example, Ma, Jiang, Tao, and Cheng (2015) developed a fault-tolerant adaptive control scheme for attitude tracking of flexible spacecraft based on the gain matrix decomposition. Patterson, Sabelhaus, and Majidi (2022) addressed the state tracking problem for a soft robot manipulator by using a singular value decomposition compensator-based approach. It is noteworthy that the well-known backstepping technique developed in Krstic, Kanellakopoulos, and Kokotovic (1995) is the main tool for control design and stability analysis in nonlinear adaptive control systems.

Compared with the continuous-time case, dealing with the control gain singularity problems for the discrete-time case involves an additional technical issue: the designed control laws must have reasonable causality properties. In other words, the existing matrix decomposition-based methods for continuous-time systems are not suitable for the discrete-time counterparts. Based on this fact, some researchers turn to study the control gain matrix singularity problems of MIMO discrete-time nonlinear adaptive control systems. For instance, Qi, Tao, and Jiang (2014) proposed an indicator function-based adaptive control scheme for a class of MIMO time-varying systems by employing the parameter projection technique to solve the singularity problem under the condition that some knowledge of the control gain matrices is known. Fu, Chen, Wang, and Wu (2020) developed a supervised neural dynamic programming approach for a class of MIMO nonlinear systems given that the control gain matrix is bounded. In another study, Alanis, Sanchez, Loukianov, and Perez-Cisneros (2009) proposed an extended Kalman filter-based training algorithm to overcome the controller singularity problem. It is of note that some control systems considered in the literature are restricted to the control gains in some triangular structures that do not involve the singularity problem (Ge, Zhang, & Lee, 2004; Yang, Li, Ge, & Lee, 2010; Zhang, Ge, & Lee, 2005). Reviewing the literature, we found that rare studies have applied the matrix decomposition technique to solve the control gain singularity problems in MIMO discrete-time nonlinear systems, especially for the control gain matrices in non-canonical forms.

Therefore, despite many works devoted to addressing the control gain singularity problems in linear and nonlinear systems, some open issues are still yet to be solved, especially for the discrete-time nonlinear case. Recently, a matrix decomposition-based state feedback adaptive control scheme was proposed in Zhang, Zhang, and Liu (2022) for a class of MIMO discrete-time nonlinear systems, where the control gain is in non-canonical form, and the system vector relative degree is $[1, 1, \ldots, 1]$. The proposed method in Zhang et al. (2022) can be applied to a special class of nonlinear systems with specific vector relative degree $[1, 1, \ldots, 1]$. However, the following questions are still open. (i) How to design a singularity-free adaptive control scheme by using the matrix decomposition technique for MIMO discrete-time nonlinear systems with general vector relative degrees? (ii) Can the scheme still work for the general relative degrees case and achieve the desired system performance (i.e., closed-loop stability and asymptotic output tracking)? (iii) For the case when the state is unavailable, how to develop an output feedback adaptive control scheme without using the high-gain observer to ensure asymptotic output tracking? In this paper, we will systematically address the above questions. Specifically, a state feedback adaptive control law is proposed for a general class of MIMO discrete-time nonlinear systems with general vector relative degrees. Then, for the case when the system state is unavailable, a further time signal estimation-based adaptive output feedback control law is developed to ensure closed-loop stability and asymptotic output tracking. The proposed adaptive control laws do not involve high-gain or causality contradiction problems. Overall, the novelties and contributions of this paper are as follows.

(i) For a general class of MIMO discrete-time nonlinear systems with vector relative degree $[\rho_1, \rho_2, \ldots, \rho_M]$, a singularity-free state feedback adaptive control law is developed by using the matrix decomposition technique. This control law ensures closed-loop stability and asymptotic output tracking.
(ii) When the system state is unavailable, a singularity-free output feedback adaptive control law is developed under a future time signal estimation-based control framework. This control law ensures closed-loop stability and asymptotic output tracking and, particularly, does not involve the high-gain issue.
(iii) To design adaptive control laws with reasonable causality properties, two new filtered tracking error signals are introduced to construct the modified estimation error signals. Such modifications successfully prevent the causality contradiction issue in the adaptive control laws often encountered in discrete-time adaptive control systems.

The remainder of this paper is organized as follows. Section 2 formulates the system model and the control problems to be solved in this paper. Section 3 and Section 4, as the main parts of this paper, give the details of the matrix decomposition-based state feedback and output feedback adaptive control designs, respectively. Section 5 provides a simulation example. Finally, Section 6 gives the concluding remarks.

2. Problem statement

This section formulates the system model, the control objective, the design conditions, and the technical issues to be solved in this paper.

2.1. System model

Consider a class of uncertain MIMO discrete-time nonlinear systems of the form

$$\begin{aligned}\xi_{i,j}(t+1) &= \xi_{i,j+1}(t), j = 1, 2, \ldots, \rho_i - 1, \\ \xi_{i,\rho_i}(t+1) &= \theta_{f_i}^{*T}\phi_{f_i}(x(t)) + \sum_{j=1}^{M}\theta_{g_{ij}}^{*T}\phi_{g_{ij}}(x(t))u_j(t) + d_i(t), \\ y_i(t) &= \xi_{i,1}(t), i = 1, 2, \ldots, M, \end{aligned} \quad (1)$$

where $x(t) = [\xi_1^T(t), \xi_2^T(t), \ldots, \xi_M^T(t)]^T \in \mathbb{R}^{\sum_{i=1}^M \rho_i}$ with $\xi_i(t) = [\xi_{i,1}(t), \xi_{i,2}(t), \ldots, \xi_{i,\rho_i}(t)]^T \in \mathbb{R}^{\rho_i}$ is the state vector, $u(t) = [u_1(t), u_2(t), \ldots, u_M(t)]^T \in \mathbb{R}^M$ is the input vector, and $y(t) = [y_1(t), y_2(t), \ldots, y_M(t)]^T \in \mathbb{R}^M$ is the output vector. In addition, $\theta_{f_i}^* = [\theta_{f_{i_1}}^*, \ldots, \theta_{f_{ip_i}}^*]^T \in \mathbb{R}^{p_i}, \theta_{g_{ij}}^* = [\theta_{g_{ij_1}}^*, \ldots, \theta_{g_{ijq_{ij}}}^*]^T \in \mathbb{R}^{q_{ij}}$ are unknown constant parameters, and

$$\begin{aligned}\phi_{f_i}(x(t)) &= [f_{i_1}(x(t)), \ldots, f_{ip_i}(x(t))]^T \in \mathbb{R}^{p_i}, \\ \phi_{g_{ij}}(x(t)) &= [g_{ij_1}(x(t)), \ldots, g_{ijq_{ij}}(x(t))]^T \in \mathbb{R}^{q_{ij}}, \end{aligned} \quad (2)$$

with $f_{i_k} : \mathbb{R}^{\sum_{i=1}^M \rho_i} \to \mathbb{R}$ and $g_{ij_k} : \mathbb{R}^{\sum_{i=1}^M \rho_i} \to \mathbb{R}$ are known mappings. Moreover, the external disturbances $d_i(t), i = 1, 2, \ldots, M$, are bounded and modeled as

$$d_i(t) = \theta_{d_i}^{*T}\phi_{d_i}(t), i = 1, 2, \ldots, M, \quad (3)$$

where $\phi_{d_i}(t) = [d_{i_1}(t), \ldots, d_{is_i}(t)]^T \in \mathbb{R}^{s_i}$ with $d_{i_k} : \mathbb{R} \to \mathbb{R}$ is known mapping, and $\theta_{d_i}^* = [\theta_{d_{i_1}}, \ldots, \theta_{d_{is_i}}]^T \in \mathbb{R}^{s_i}$ is unknown constant parameter. Note that many practical systems can be modeled as (1), such as rigid robots and motors (Dawson, Carroll, & Schneider, 1994), ships (Tee & Ge, 2006), and jet engines and aircraft (Diao & Passino, 2001). Moreover, the disturbance model (3) can represent uncertain constant disturbances, sinusoidal disturbances with unknown magnitudes, and the combinations of them. For more details of (3), the readers are referred to Song and Tao (2018).

2.2. Control objective and design conditions

Control objective. For any given bounded reference output signal $y^*(t) = [y_1^*(t), y_2^*(t), \ldots, y_M^*(t)]^T$, the control objective of this paper is to develop analytical singularity-free state feedback and output feedback adaptive control laws, respectively, and ensure closed-loop stability and asymptotic output tracking $\lim_{t\to\infty}(y(t) - y^*(t)) = 0$ for the system (1).

Assumptions. To meet the control objective, we need the following design conditions:

(A1): $f_{i_k}(x) : \mathbb{R}^{\sum_{i=1}^M \rho_i} \to \mathbb{R},\ i = 1, \ldots, M,\ k = 1, \ldots, p_i$ are Lipschitz mappings on $\mathbb{R}^{\sum_{i=1}^M \rho_i}$.

(A2): The control gain matrix $\{\theta_{g_{ij}}^{*T}\phi_{g_{ij}}(x(t))\}, i, j = 1, \ldots, M$, can be expressed as $\Theta_g^*\Phi_g(x(t))$ and satisfies $|\det\{\Theta_g^*\Phi_g(x(t))\}| \geq \epsilon > 0,\ \forall x(t) \in \mathbb{R}^{\sum_{i=1}^M \rho_i}$, where ϵ is an unknown constant, Θ_g^* is an unknown constant square matrix, and $\Phi_g(x(t))$ is a known time-varying square matrix.

(A3): All leading principal minors of Θ_g^*, denoted as $\Delta_i, i = 1, \ldots, M$, are nonzero and their signs are known.

Assumption (A2) can be regarded as a modified relative degree condition indicating that the system (1) has vector relative degree $[\rho_1, \ldots, \rho_M]$. The relative degree information is actually the input–output time delay and is crucial for adaptive control design (Goodwin & Sin, 1984; Ioannou & Sun, 2012; Narendra & Annaswamy, 1989; Tao, 2003). Assumption (A3) and the decomposed form $\Theta_g^*\Phi_g(x)$ in Assumption (A2) are used to handle the singularity problem of the adaptive control law. In practice, the sign of Δ_i has certain physical meanings and can be verified via the inherent system characteristics, although the control gain matrices may be unknown. We illustrate a control gain matrix of a linearized aircraft system model in Liu, Tao, and Joshi (2010) to explain this issue. The Ref. Liu et al. (2010) shows that when the inputs are engine throttle, elevator, and rudder, and the outputs are forward velocity, pitch angle, and yaw angle, the control gain matrix (denoted as K_p) is of the form $K_p = [k_{p1}, k_{p2}, k_{p3}] \in \mathbb{R}^{3\times3}$ with $k_{p1} = [b_{011}, b_{031}, 0]^T$, $k_{p2} = [b_{012}, b_{032}, 0]^T$, $k_{p3} = [0, 0, b_{064}\cos(1/\theta_0)]^T$. As claimed in Liu et al. (2010), b_{0ij} all have certain physical meanings: b_{011} is the control gain from engine throttle to forward acceleration; b_{012} is the control gain from the elevator to forward acceleration; b_{031} is the control gain from engine throttle to pitch acceleration; b_{032} is the control gain from the elevator to pitch acceleration; and b_{064} is the control gain from rudder to yaw acceleration. Based on the above physical meanings of b_{0ij} and practical experience, one can deduce their signs as follows: $b_{011} > 0$, $b_{012} > 0$, $b_{031} > 0$, $b_{032} < 0$, and $b_{064} < 0$. Here, θ_0 is the value of the Euler pitch angle θ at the wings-level steady-state equilibrium point that ensures $\cos\theta_0 > 0$. Thus, based on the structure of K_p and the sign information, one can directly calculate that $\Delta_1 > 0$, $\Delta_2 < 0$, and $\Delta_3 > 0$, although the accurate values of b_{0ij} and θ_0 are unknown.

2.3. Technical issues

For simplicity of presentation, we rewrite the system model (1) into the following compact form

$$\begin{bmatrix} y_1(t+\rho_1) \\ y_2(t+\rho_2) \\ \vdots \\ y_M(t+\rho_M) \end{bmatrix} = \begin{bmatrix} \xi_{11}(t+\rho_1) \\ \xi_{21}(t+\rho_2) \\ \vdots \\ \xi_{M1}(t+\rho_M) \end{bmatrix}$$
$$= \Theta_f^*\phi_f(x(t)) + \Theta_g^*\Phi_g(x(t))u(t) + \Theta_d^*\phi_d(t), \quad (4)$$

where $\Theta_f^* = \mathrm{diag}\{\theta_{f_1}^{*T}, \ldots, \theta_{f_M}^{*T}\} \in \mathbb{R}^{M\times\sum_{i=1}^M p_i}$, $\phi_f(x(t)) = [\phi_{f_1}^T(x(t)), \ldots, \phi_{f_M}^T(x(t))]^T$, $\phi_d(t) = [\phi_{d_1}^T(t), \ldots, \phi_{d_M}^T(t)]^T$ and $\Theta_d^* = \mathrm{diag}\{\theta_{d_1}^{*T}, \ldots, \theta_{d_M}^{*T}\} \in \mathbb{R}^{M\times\sum_{i=1}^M s_i}$.

Clarification of the singularity problem. If the parameters in (4) were known, a simple nominal control law could be designed as

$$\begin{aligned}u(t) = (\Theta_g^*\Phi_g(x(t)))^{-1}(&-\Theta_f^*\phi_f(x(t)) - \Theta_d^*\phi_d(t) \\ &+ [y_1^*(t+\rho_1), \ldots, y_M^*(t+\rho_M)]^T). \end{aligned} \quad (5)$$

Then, the closed-loop system is $y_i(t+\rho_i) = y_i^*(t+\rho_i)$, $i = 1, 2, \ldots, M$. This implies that the exact output tracking is achieved. However, the nominal control law (5) is not available since Θ_f^*, Θ_g^*, and Θ_d^* are unknown. A usual adaptive version of the nominal control law is

$$\begin{aligned}u(t) = (\Theta_g(t)\Phi_g(x(t)))^{-1}(&-\Theta_f(t)\phi_f(x(t)) - \Theta_d(t)\phi_d(t) \\ &+ [y_1^*(t+\rho_1), \ldots, y_M^*(t+\rho_M)]^T), \end{aligned} \quad (6)$$

where $\Theta_f(t)$, $\Theta_g(t)$, and $\Theta_d(t)$ are estimates of Θ_f^*, Θ_g^*, and Θ_d^*, respectively, and updated by parameter updating law. It is noteworthy that without parameter constraint in the process of adaptation, the matrix $\Theta_g(t)$ may be singular which results in the singularity of the adaptive control law (6). The problem is how to ensure $\Theta_g(t)$ is always non-singular in the process of parameter adaptation. Apparently, the usual adaptive control law (6) cannot fulfill this requirement.

A matrix decomposition-based solution. Recently, the literature (Zhang et al., 2022) proposed a matrix decomposition-based state feedback adaptive control scheme for a class of nonlinear systems with vector relative degree $[1, 1, \ldots, 1]$. In this case, the output dynamics can be expressed as the form $y(t+1) = \Theta_{cf}^{*}\phi_f(x(t)) + \Theta_{cb}^{*}u(t)$ with Θ_{cf}^{*} and Θ_{cb}^{*} are unknown constant parameters. However, it is still open to study (i) how to design a matrix decomposition-based state feedback adaptive control scheme for system (1) with vector relative degree $[\rho_1, \rho_2, \ldots, \rho_M]$; and (ii) more importantly, when the system state is unavailable, how to develop an output feedback adaptive control law to still ensure closed-loop stability and asymptotic output tracking?

Note that using a high-gain observer-based method generally does not achieve asymptotic output tracking unless the reference output signal and its derivatives up to some certain orders all converge to zero. On the other hand, since $x(t) = [\xi_1^T(t), \xi_2^T(t), \ldots, \xi_M^T(t)]^T = [y_1(t), y_1(t+1), \ldots, y_1(t+\rho_1-1), \ldots, y_M(t), y_M(t+1), \ldots, y_M(t+\rho_M-1)]^T$ consists of future time output signals, another key issue is that how to solve the causality contradiction problem when using output feedback. Overall, we will solve the following technical issues:

(i) how to develop a matrix decomposition-based singularity-free state feedback adaptive control law to achieve closed-loop stability and asymptotic output tracking for system (1) with vector relative degree $[\rho_1, \rho_2, \ldots, \rho_M]$;

(ii) how to develop a singularity-free output feedback adaptive control law without involving the high-gain issue to achieve closed-loop stability and asymptotic output tracking for system (1); and

(iii) how to avoid causality contradiction in the output feedback adaptive control law caused by the general high-order relative degree (time delay) of system (1).

3. State feedback adaptive control

In this section, we assume the system state $x(t)$ is measurable, which implies that $\xi_i(t)$ can be used for adaptive control design. The design procedure contains five steps: (i) derivation of a parameterized model; (ii) construction of a state feedback adaptive control law; (iii) specification of a modified tracking error model; (iv) development of parameter updating law with a modified estimation error signal; and (v) analysis of system performance.

Step 1: Derivation of a parameterized model. Based on Assumption (A2), we decompose Θ_g^{*} as $\Theta_g^{*} = LD^{*}U$ uniquely. Here, L is a unit lower triangular matrix, U is a unit upper triangular matrix, and $D^{*} = \operatorname{diag}\{d_1^{*}, d_2^{*}, \cdots, d_M^{*}\} = \operatorname{diag}\{\Delta_1, \frac{\Delta_2}{\Delta_1}, \cdots, \frac{\Delta_M}{\Delta_{M-1}}\}$. Using the matrices L, D^{*}, U, we express Θ_g^{*} as $\Theta_g^{*} = S^{*}D_sU_s$, where $S^{*} = LD^{*}D_s^{-1}L^T$ is a positive definite matrix, $U_s = D_s^{-1}L^{-1^T}D_sU$ is a unit upper triangular matrix, and D_s is a diagonal matrix of the form

$$D_s = \operatorname{diag}\{\operatorname{sign}[d_1^{*}]\gamma_1, \operatorname{sign}[d_2^{*}]\gamma_2, \ldots, \operatorname{sign}[d_M^{*}]\gamma_M\} \tag{7}$$

with $\gamma_i,\ i = 1, 2, \ldots, M$, being arbitrary positive constants to be chosen. Based on above operation, the system model (4) can be expressed as

$$\begin{bmatrix} y_1(t+\rho_1) \\ y_2(t+\rho_2) \\ \vdots \\ y_M(t+\rho_M) \end{bmatrix} = \begin{bmatrix} \xi_{11}(t+\rho_1) \\ \xi_{21}(t+\rho_2) \\ \vdots \\ \xi_{M1}(t+\rho_M) \end{bmatrix}$$
$$= \Theta_f^{*}\phi_f(x(t)) + S^{*}D_sU_s\bar{u}(t) + \Theta_d^{*}\phi_d(t), \tag{8}$$

where $\bar{u}(t) = \Phi_g(x(t))u(t)$. Note that $\Phi_g(x(t))$ is always nonsingular with a bounded inverse for all $x(t) \in \mathbb{R}^{\sum_{i=1}^{M}\rho_i}$. Then, as long as $\bar{u}(t)$ is chosen, the control law $u(t)$ can be calculated as $u(t) = \Phi_g^{-1}(x(t))\bar{u}(t)$. From (8), we have

$$S^{*-1}\begin{bmatrix} y_1(t+\rho_1) \\ y_2(t+\rho_2) \\ \vdots \\ y_M(t+\rho_M) \end{bmatrix} = D_s\Theta_1^{*}\phi_f(x(t)) + D_s\Theta_2^{*}\bar{u}(t)$$
$$+D_s\bar{u}(t) + D_s\Theta_3^{*}\phi_d(t), \tag{9}$$

where

$$\Theta_1^{*} = D_s^{-1}S^{*-1}\Theta_f^{*},\ \ \Theta_3^{*} = D_s^{-1}S^{*-1}\Theta_d^{*}, \tag{10}$$

and $\Theta_2^{*} = U_s - I$ is of the form

$$\Theta_2^{*} = \begin{bmatrix} 0 & \theta_{212}^{*} & \theta_{213}^{*} & \cdots & \theta_{21M}^{*} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \theta_{2(M-1)M}^{*} \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}. \tag{11}$$

The signal U_s is divided into the sum of two parts: I and $U_s - I$. With this operation, we will show that the control law below is always well-defined.

Step 2: Construction of an adaptive control law. For any given bounded reference output signal $y^{*}(t)$, the state feedback adaptive control law is designed as

$$u(t) = \Phi_g^{-1}(x(t))\bar{u}(t),$$
$$\bar{u}(t) = (I+\Theta_2(t))^{-1}\Bigg(-\Theta_1(t)\phi_f(x(t)) - \Theta_3(t)\phi_d(t)$$
$$-\Theta_4(t)\begin{bmatrix} \sum_{i=0}^{\rho_1-1} a_{1i}(y_1(t+i) - y_1^{*}(t+i)) \\ \vdots \\ \sum_{i=0}^{\rho_M-1} a_{Mi}(y_M(t+i) - y_M^{*}(t+i)) \end{bmatrix}$$
$$+\Theta_4(t)\begin{bmatrix} y_1^{*}(t+\rho_1) \\ \vdots \\ y_M^{*}(t+\rho_M) \end{bmatrix}\Bigg), \tag{12}$$

where $\Theta_i(t),\ i = 1, 2, 3, 4$, are estimates of $\Theta_1^{*}, \Theta_2^{*}, \Theta_3^{*}, (S^{*}D_s)^{-1}$, respectively. In particular, $a_{ji},\ i = 0, \ldots, \rho_j - 1$, are constant parameters to be chosen such that all zeros of $P_{mj}(z), j = 1, 2, \ldots, M$, are inside the unit circle of the complex z-plane with

$$P_{mj}(z) = z^{\rho_j} + a_{j,\rho_j-1}z^{\rho_j-1} + \cdots + a_{j1}z + a_{j0}. \tag{13}$$

Moreover, $\Theta_2(t)$ is of the form

$$\Theta_2(t) = \begin{bmatrix} 0 & \theta_{212}(t) & \theta_{213}(t) & \cdots & \theta_{21M}(t) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \theta_{2(M-1)M(t)} \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \tag{14}$$

where $\theta_{2ij}(t)$ is the estimate of θ_{2ij}^{*}. Here, $\det\{I+\Theta_2(t)\}$ is equal to one, which suggests that the adaptive control law (12) is always nonsingular and does not involve the high-gain issue. Also, $u(t)$ contains $y_i(t+j),\ i = 1, 2, \ldots, M,\ j = 1, \ldots, \rho_i - 1$, which are part of the system state and available at the current time instant.

Step 3: Specification of a modified tracking error model. Define the tracking error as $e(t) = y(t) - y^{*}(t) \in \mathbb{R}^M$. Together with (9) and (12), we have

$$S^{*-1}\begin{bmatrix} y_1(t+\rho_1) \\ y_2(t+\rho_2) \\ \vdots \\ y_M(t+\rho_M) \end{bmatrix} = D_s\Theta_4(t)\begin{bmatrix} y_1^{*}(t+\rho_1) \\ y_2^{*}(t+\rho_2) \\ \vdots \\ y_M^{*}(t+\rho_M) \end{bmatrix}$$
$$-D_s\tilde{\Theta}_1(t)\phi_f(x(t)) - D_s\tilde{\Theta}_2(t)\bar{u}(t) - D_s\tilde{\Theta}_3(t)\phi_d(t)$$
$$-D_s\Theta_4(t)\begin{bmatrix} \sum_{i=0}^{\rho_1-1} a_{1i}(y_1(t+i) - y_1^{*}(t+i)) \\ \vdots \\ \sum_{i=0}^{\rho_M-1} a_{Mi}(y_M(t+i) - y_M^{*}(t+i)) \end{bmatrix}. \tag{15}$$

Then, with some manipulations on (15), we obtain

$$S^{*-1}\begin{bmatrix} e_1(t+\rho_1)+\sum_{i=0}^{\rho_1-1}a_{1i}e_1(t+i) \\ \vdots \\ e_M(t+\rho_M)+\sum_{i=0}^{\rho_M-1}a_{Mi}e_M(t+i)\end{bmatrix} =$$
$$-D_s\tilde{\Theta}_1(t)\phi_f(x(t))-D_s\tilde{\Theta}_2(t)\bar{u}(t)-D_s\tilde{\Theta}_3(t)\phi_d(t)$$
$$-D_s\tilde{\Theta}_4(t)\begin{bmatrix} \sum_{i=0}^{\rho_1-1}a_{1i}e_1(t+i)-y_1^*(t+\rho_1) \\ \vdots \\ \sum_{i=0}^{\rho_M-1}a_{Mi}e_M(t+i)-y_M^*(t+\rho_M)\end{bmatrix}, \tag{16}$$

where $\tilde{\Theta}_i(t)=\Theta_i(t)-\Theta_i^*$. Define

$$\begin{aligned} P_m(z) &= \text{diag}\{P_{mj}(z)\},\ \Psi^*=\left[\Theta_1^*,\Theta_2^*,\Theta_3^*,(S^*D_s)^{-1}\right], \\ \Psi(t) &= [\Theta_1(t),\Theta_2(t),\Theta_3(t),\Theta_4(t)], \\ \tilde{\Psi}(t) &= \Psi(t)-\Psi^*=[\tilde{\Theta}_1(t),\tilde{\Theta}_2(t),\tilde{\Theta}_3(t),\tilde{\Theta}_4(t)], \\ \varphi(t) &= [-\phi_f^T(x(t)),-\bar{u}^T(t),-\phi_d^T(t), \\ &\quad -\sum_{i=0}^{\rho_1-1}a_{1i}e_1(t+i)+y_1^*(t+\rho_1),\cdots, \\ &\quad -\sum_{i=0}^{\rho_M-1}a_{Mi}e_M(t+i)+y_M^*(t+\rho_M)]^T. \end{aligned} \tag{17}$$

Then, (16) can be written as

$$P_m(z)[e](t)=S^*D_s\tilde{\Psi}(t)\varphi(t). \tag{18}$$

To implement $\bar{u}(t)$, we express $\tilde{\Psi}(t)\varphi(t)$ as $\tilde{\Psi}(t)\varphi(t)=[(\Psi_1(t)-\Psi_1^*)\varphi_1(t),\ldots,(\Psi_M(t)-\Psi_M^*)\varphi_M(t)]^T$, where

$$\begin{aligned} \Psi_1(t)-\Psi_1^* &= \left[\theta_{212}(t)-\theta_{212}^*,\cdots,\theta_{21M}(t)-\theta_{21M}^*,\right. \\ &\quad \left.\tilde{\Theta}_{11}(t),\tilde{\Theta}_{31}(t),\tilde{\Theta}_{41}(t)\right], \\ &\ \ \vdots \\ \Psi_{M-1}(t)-\Psi_{M-1}^* &= \left[\theta_{2(M-1)M}(t)-\theta_{2(M-1)M}^*,\right. \\ &\quad \left.\tilde{\Theta}_{1(M-1)}(t),\tilde{\Theta}_{3(M-1)}(t),\tilde{\Theta}_{4(M-1)}(t)\right], \\ \Psi_M(t)-\Psi_M^* &= \left[\tilde{\Theta}_{1M}(t),\tilde{\Theta}_{3M}(t),\tilde{\Theta}_{4M}(t)\right], \end{aligned} \tag{19}$$

and

$$\begin{aligned} \varphi_1(t) &= \Bigg[-\bar{u}_2(t),-\bar{u}_3(t),\ldots,-\bar{u}_M(t),-\phi_f^T(x(t)), \\ &\quad -\phi_d^T(t),-\sum_{i=0}^{\rho_1-1}a_{1i}e_i(t+i)+y_1^*(t+\rho_1), \\ &\quad \cdots,-\sum_{i=0}^{\rho_M-1}a_{mi}e_M(t+i)+y_M^*(t+\rho_M)\Bigg]^T, \\ &\ \ \vdots \\ \varphi_{M-1}(t) &= \Bigg[-\bar{u}_M(t),-\phi_f^T(x(t)),-\phi_d^T(t), \\ &\quad -\sum_{i=0}^{\rho_1-1}a_{1i}e_i(t+i)+y_1^*(t+\rho_1),\ldots, \\ &\quad -\sum_{i=0}^{\rho_M-1}a_{mi}e_M(t+i)+y_M^*(t+\rho_M)\Bigg]^T, \\ \varphi_M(t) &= \Bigg[-\phi_f^T(x(t)),-\phi_d^T(t), \\ &\quad -\sum_{i=0}^{\rho_1-1}a_{1i}e_i(t+i)+y_1^*(t+\rho_1),\ldots, \\ &\quad -\sum_{i=0}^{\rho_M-1}a_{mi}e_M(t+i)+y_M^*(t+\rho_M)\Bigg]^T. \end{aligned} \tag{20}$$

To avoid the causality contradiction problem, we introduce a stable and strictly proper filter

$$h(z)=\frac{1}{z-\alpha}, \tag{21}$$

where α is a constant chosen such that $0\le\alpha<1$. Then, the impulse response function of $h(z)$ is $\frac{1}{\alpha}(\alpha^t-\delta(t))\ge 0$ with $\delta(t)$ being the unit impulse response. A filtered tracking error is defined as

$$\bar{e}(t)=P_m(z)h(z)[e](t)=[\bar{e}_1(t),\bar{e}_2(t),\ldots,\bar{e}_M(t)]^T. \tag{22}$$

From (13), (17), and (21), we see that $P_m(z)h(z)[e](t)=\text{diag}\{P_{mi}(z)h(z)[e_i](t)\}$. To show $\bar{e}(t)$ is available at the current time instant, we divide $P_{mi}(z)$ into a product of $z-\alpha_{i0}$ and $P_{mi0}(z)$, where $P_{mi0}(z)=z^{\rho_i-1}+\sum_{j=0}^{\rho_i-2}b_{ij}z^j$. Thus, $\bar{e}(t)$ can be expressed as

$$\bar{e}(t)=\text{diag}\left\{\frac{z-\alpha_{i0}}{z-\alpha}P_{mi0}(z)[e_i](t)\right\} \tag{23}$$

with $P_{mi0}(z)[e_i](t)=e_i(t+\rho_i-1)+\sum_{j=0}^{\rho_i-2}b_{ij}e_i(t+j)$. Note that $e_i(t+j)=y_i(t+j)-y_i^*(t+j)$, $i=1,2,\ldots,M$, $j=1,\ldots,\rho_i-1$. Since $y_i(t+j)$, $i=1,2,\ldots,M$, $j=1,\ldots,\rho_i-1$, are part of the system state vector, the signals $P_{mi0}(z)[e_i](t)$, $i=1,\ldots,M$, are surely available at the current time instant. Therefore, as inferred from (23), $\bar{e}$ is available at the current time instant. If we do not introduce the filter $h(z)$, then $\bar{e}$ may be of the form $\bar{e}=P_m(z)[e]$. With $P_{mi}(z)[e_i]=e_i(t+\rho_i)+\sum_{j=0}^{\rho_i-1}a_{ij}e_i(t+j)$ and $y_i(t+\rho_i)$ is a future time signal, we see that $P_m(z)[e]$ is unavailable at the current time instant. Accordingly, using a combination of $h(z)$ and $P_m(z)$ to define $\bar{e}$ is necessary to avoid the causality contradiction problem in the state feedback case. The filtered error $\bar{e}(t)$ is a desired tracking error signal crucial for designing an estimation error and a parameter updating law. Moreover, the condition that the impulse response function of $h(z)$ is greater than or equal to zero is to guarantee the sign of the tracking error $e(t)$ cannot be changed before and after filtering by $h(z)$, and so is the output feedback case.

Step 4: Development of a parameter updating law. Before designing the parameter updating law, we first define an estimation error as

$$\varepsilon(t)=\bar{e}(t)+\Phi(t)\sigma(t), \tag{24}$$

where $\Phi(t)$ is an estimate of S^*D_s and $\sigma(t)=[\sigma_1(t),\sigma_2(t),\ldots,\sigma_M(t)]^T$ with

$$\begin{aligned} \sigma_j(t) &= \Psi_j(t)\delta_j(t)-h(z)[\Psi_j\varphi_j](t), \\ \delta_j(t) &= h(z)[\varphi_j](t). \end{aligned} \tag{25}$$

Note that $\bar{e}(t),\varepsilon(t),\sigma_j(t),\delta_j(t)$ are all available at the current time instant. To obtain the control signal $u(t)$, we need to update $\Theta_i(t)$, $i=1,2,3,4$, which is equivalent to update $\Psi(t)$. Moreover, $\varepsilon(t)$ depends on $\Phi(t)$ that also needs to be updated. Thus, we design the parameter updating law as

$$\Psi_j^T(t+1)=\Psi_j^T(t)-\frac{\text{sign}\{d_j^*\}\gamma_j\varepsilon_j(t)\delta_j(t)}{m^2(t)}, \tag{26}$$

$$\Phi(t+1)=\Phi(t)-\frac{\beta\varepsilon(t)\sigma^T(t)}{m^2(t)}, \tag{27}$$

where $j = 1, 2, \ldots, M$,

$$m(t) = \sqrt{1 + \sum_{j=1}^{M} \sigma_j^2(t) + \sum_{j=1}^{M} \delta_j^T(t)\delta_j(t)}, \quad (28)$$

and $\mathrm{sign}\{d_j^*\}$ and γ_j are specified in (7) such that $0 < \gamma_j^2 < 2\lambda_{\min}\{S^{*-1}\}$. Also, $\beta \in \mathbb{R}$ is a constant parameter such that $0 < \beta < \frac{2\lambda_{\min}\{S^{*-1}\}}{\lambda_{\max}\{S^{*-1}\}}$ with $\lambda_{\min}\{S^{*-1}\}$ and $\lambda_{\max}\{S^{*-1}\}$ are the minimum and maximum eigenvalues of S^{*-1}, respectively.

Now, we give the following lemma to show the properties of the estimated parameters.

Lemma 1. *The parameter updating law (26)–(27) ensures*
(i) $\Psi_j(t) \in L^\infty$, $\Phi(t) \in L^\infty$, $j = 1, 2, \ldots, M$;
(ii) $\frac{\varepsilon(t)}{m(t)} \in L^2 \cap L^\infty$, $\Phi(t+1) - \Phi(t) \in L^2 \cap L^\infty$, and $\Psi_j(t+1) - \Psi_j(t) \in L^2 \cap L^\infty$, $j = 1, 2, \ldots, M$.

The proof of this lemma is similar to that of Lemma 2 in Zhang et al. (2022), and omitted here. This lemma shows that the parameter estimates have some important properties, which are essential for the stability analysis. Although $\Psi_j(t+1) - \Psi_j(t)$ and $\Phi(t+1) - \Phi(t)$ converge to zero asymptotically, it does not mean the convergence of $\Psi_j(t)$ and $\Phi(t)$.

Step 5: Analysis of system performance. Firstly, we introduce some useful notations: (i) c represents a signal bound; (ii) $\tau(t)$ denotes a generic $L^2 \cap L^\infty$ function that goes to zero as $t \to \infty$; and (iii) $L^{\infty e}$ denotes the set $L^{\infty e} = \{v(t) | \forall s < \infty, v_s(t) \in L^\infty\}$, for any function $v(t)$ with $v_s(t) = v(t), t \le s$, and $v_s(t) = 0, t > s$.

Before giving the main result, we first specify two constructive lemmas which are crucial for stability analysis.

Lemma 2. *For any given discrete-time signals $v_i(t) \in \mathbb{R}^{p_i}$, $i = 1, 2, 3$, with $p_1 = p_2$ such that $v_1(t) = q(z)[v_2](t)$, where $q(z)$ is a proper and stable rational function whose impulse response is larger than or equal to zero and $v_2(t) \in L^{\infty e}$, $v_3(t) \in L^{\infty e}$. If $\|v_2(t)\| \le \tau_1(t) \sup_{k \le t} \|v_3(k)\| + \tau_2(t)$, $\forall t \ge 0$, then*

$$\|v_1(t)\| \le \tau_3(t) \sup_{k \le t} \|v_3(k)\| + \tau_4(t), \forall t \ge 0, \quad (29)$$

where $\tau_i(t) > 0$, $i = 1, 2, 3, 4$, are all $L^2 \cap L^\infty$ functions.

Proof. Ignoring the exponentially decaying effect of the initial conditions, we obtain $v_1(t) = \sum_{k=0}^{t} q_z(t-k) v_2(k)$ from $v_1(t) = q(z)[v_2](t)$, where $q_z(t) \ge 0$ is the impulse response of $q(z)$. Next, we define $\mu_3(t) = \sum_{k=0}^{t} q_z(t-k)\tau_1(k)$ and $\mu_4(t) = \sum_{k=0}^{t} q_z(t-k)\tau_2(k)$, which can also be expressed as the forms $\mu_3(t) = q(z)[\tau_1](t)$ and $\mu_4(t) = q(z)[\tau_2](t)$. Note that $q(z)$ is stable and $\tau_1(t), \tau_2(t) \in L^2 \cap L^\infty$. Then, we can get $\mu_3(t), \mu_4(t) \in L^2 \cap L^\infty$. Setting $\tau_3(t) = \mu_3(t)$ and $\tau_4(t) = \mu_4(t)$, we get (29). □

Lemma 3. *(Tao, 2003) Let $c(zI - A)^{-1}b$ be a minimal realization of the stable and proper rational function $q(z)$. Then,*

$$\begin{aligned} &\kappa^T(t) q(z)[\gamma](t) - q(z)[\kappa^T \gamma](t) \\ &= q_c(z)[(q_b(z)z)[\gamma^T](z-1)[\kappa]](t), \end{aligned} \quad (30)$$

where $q_c(z) = c(zI - A)^{-1}$, $q_b(z) = (zI - A)^{-1}b$, and $\kappa(t)$ and $\gamma(t)$ are any signals of appropriate dimensions.

It is straightforward to obtain Lemma 3 from the discrete-time version of the well-known swapping lemma in Tao (2003). We omit the proof for space.

Based on above derivations, we give the first main result of this paper.

Theorem 4. *Under Assumptions (A1)-(A3), if the state feedback adaptive control law (12) with the parameter updating law (26)–(27) is applied to the system (1), then the closed-loop system is stable and of asymptotic output tracking $\lim_{t\to\infty}(y(t) - y^*(t)) = 0$.*

Proof. The proof contains four steps. First, we show

$$\sum_{j=1}^{M} |\sigma_j(t)| \le \tau \sup_{k \le t} \|e(k)\| + \tau. \quad (31)$$

Since $\|y^*(t)\| \le c$ and f_{ik} is Lipschitz mapping, we derive that $\|\varphi_j(t)\| \le c\|\xi(t)\| + c\|e(t)\| + c$. Then, it follows from $y_i(t) = \xi_{i,1}(t)$ that $\|\varphi_j(t)\| \le c\|e(t)\| + c$. With $\sigma_j(t) = \Psi_j(t)\delta_j(t) - h(z)[\Psi_j \varphi_j](t)$ and $\delta_j(t) = h(z)[\varphi_j](t)$, it is inferred from Lemma 3 that

$$\begin{aligned} \sigma_j(t) &= \Psi_j(t) h(z)[\varphi_j](t) - h(z)[\Psi_j \varphi_j](t) \\ &= h_c(z)[(h_b(z)z)[\varphi_j^T](z-1)[\Psi_j^T]](t), \end{aligned} \quad (32)$$

where $c(zI - A)^{-1}b$ denotes a minimal realization of $h(z)$ and $h_c(z) = c(zI - A)^{-1}$, $h_b(z) = (zI - A)^{-1}b$. Since $h_b(z)z$ is proper and stable, we have $\|h_b(z)z[\varphi_j^T](t)\| \le c + c \sup_{k \le t} \|e(k)\|$. With $|[(h_b(z)z)[\varphi_j^T](z-1)[\Psi_j^T]](t)| \le \|\Psi_j(t+1) - \Psi_j(t)\| \|h_b(z)z[\varphi_j](t)\|$, it is inferred from Lemma 1 that

$$|[(h_b(z)z)[\varphi_j^T](z-1)[\Psi_j^T]](t)| \le \tau \sup_{k \le t} \|e(k)\| + \tau. \quad (33)$$

Since the impulse response function of $h(z)$ is greater than or equal to zero, this together with (32), (33), and Lemma 2, results in $|\sigma_j(t)| \le \tau \sup_{k \le t} \|e(k)\| + \tau$, $\forall j = 1, 2, \ldots, M$. Thus, (31) holds.

Second, we prove that

$$m(t) \le c \sup_{k \le t} \|e(k)\| + c. \quad (34)$$

Starting from the definition of $\delta_j(t)$ and the fact that $h(z)$ is strictly proper and stable, we get $\|\delta_j(t)\| \le c \sup_{k \le t} \|\varphi_j(k)\| + c$. Then, with $\|\varphi_j(t)\| \le c\|e(t)\| + c$, we obtain

$$\|\delta_j(t)\| \le c \sup_{k \le t} \|e(k)\| + c. \quad (35)$$

From the definition of $m(t)$ in (28), we conclude that $m(t) \le 1 + \sum_{j=1}^{M} |\sigma_j(t)| + \sum_{j=1}^{M} \|\delta_j(t)\|$. Then, combining (31) and (35) implies (34) holds.

Third, we show that $e(t) \in L^\infty$. From (24), we have

$$\|\bar{e}(t)\| \le m(t) \left\| \frac{\varepsilon(t)}{m(t)} \right\| + \|\Phi(t)\sigma(t)\|. \quad (36)$$

Since $\frac{\varepsilon(t)}{m(t)} \in L^2 \cap L^\infty$, $\Phi(t) \in L^\infty$, using (31) gives $\|\bar{e}(t)\| \le \tau \sup_{k \le t} \|e(k)\| + \tau$. Then, $\sup_{k \le t} \|\bar{e}(k)\| \le \tau \sup_{k \le t} \|e(k)\| + \tau$. Let $P_m^{-1}(z)$ be the inverse of $P_m(z)$. Therefore, $P_m^{-1}(z)h^{-1}(z)$ is also proper and stable. From the definition of $\bar{e}(t)$ in (22), we derive that $\|e(t)\| \le c \sup_{k \le t} \|\bar{e}(t)\| + c \le \tau \sup_{k \le t} \|e(k)\| + c$, which demonstrates that $e(t) \in L^\infty$.

Finally, we prove closed-loop stability and $\lim_{t\to\infty}(y(t) - y^*(t)) = 0$. Since $\sup_{k \le t} \|\bar{e}(t)\| \le \tau \sup_{k \le t} \|e(k)\| + \tau$ and $e(t) \in L^\infty$, we have $\bar{e}(t) \in L^2 \cap L^\infty$. With $P_m^{-1}(z)h^{-1}(z)$ being proper and stable, we obtain $e(t) \in L^2 \cap L^\infty$. Then, it is concluded that $\lim_{t\to\infty} e(t) = 0$. In addition, with the boundedness of $e(t)$, we obtain the boundedness of $\varphi_j(t)$, $\sigma_j(t)$, $\delta_j(t)$ and $m(t)$, respectively. Moreover, from $y_j(t) = \xi_{j,1}(t)$, we derive the boundedness of $x(t)$. Finally, we can get the boundedness of $u(t)$. Thus, all signals in the closed-loop system are bounded. This completes the proof. □

So far, we have developed a singularity-free state feedback adaptive control law, where the determinant of the adaptive control gain matrix is always equal to one. Such a control law ensures asymptotic output tracking.

4. Output feedback adaptive control

This section considers the case when $x(t)$ is unavailable, and only the output is measurable. In this case, the state feedback adaptive control law will no longer be effective. As shown in the literature, the high-gain observer may be first employed to estimate the state $x(t)$, followed by designing an output feedback control law to achieve uniformly ultimately bounded tracking.

Instead of using the high-gain observer, this paper proposes a future time signal estimation-based adaptive control scheme. Compared with the existing results, the proposed output feedback adaptive control scheme ensures asymptotic output tracking and does not involve the high-gain issue. The basic idea is first to estimate the system parameters and use the derived parameter estimates to construct the estimates of some unknown further time signals. The future time signal estimation does not depend on any persistently exciting condition. Thus, the estimates cannot converge to their corresponding unknown signals. Nevertheless, we show that the developed output feedback adaptive control law ensures asymptotic output tracking.

The design procedure contains six steps: (i) estimation of the system parameters; (ii) estimation of future time signals; (iii) construction of an output feedback adaptive control law; (iv) specification of a modified tracking error model; (v) construction of controller parameter updating law; and (vi) analysis of the system performance. The details are as follows.

Step 1: Estimation of the system parameters. For simplicity of presentation and without loss of generality, we assume $\rho_1 \le \rho_2 \le \cdots \le \rho_M$. Let $r = \sum_{i=1}^{M} p_i + M + \sum_{i=1}^{M} s_i$. From (8), we define

$$\begin{aligned}\phi(t) &= \phi(x(t), t) = [\phi_f(x(t)), \bar{u}(t), \phi_d(t)]^T \in \mathbb{R}^r,\\ \Lambda^* &= \{\Lambda^*_{ij}\} = [\Theta^*_f, \Theta^*_g, \Theta^*_d] \in \mathbb{R}^{M\times r},\end{aligned} \tag{37}$$

where $x(t) = [y_1(t), \ldots, y_1(t+\rho_1-1), \ldots, y_M(t), \ldots, y_M(t+\rho_M-1)]^T$, $\bar{u}(t) = [\bar{u}_1(t), \bar{u}_2(t), \ldots, \bar{u}_M(t)]^T$, and $d(t) = [d_1(t), d_2(t), \ldots, d_M(t)]^T$. Then, it follows from (4) that

$$[y_1(t+\rho_1), \ldots, y_M(t+\rho_M)]^T = \Lambda^*\phi(t). \tag{38}$$

Let $\Lambda(t) = [\Theta_f(t), \Theta_g(t), \Theta_d(t)]$ be the estimate of Λ^*. To reduce notation, we still use $\varepsilon(t)$ to denote the estimation error defined as

$$\varepsilon_i(t+\rho_i-\rho_M) = \Lambda_i(t)\phi(t-\rho_M) - y_i(t+\rho_i-\rho_M) \tag{39}$$

for $1 \le i \le M$ and $t \ge \rho_M$, where $\Lambda_i(t)$ and $\varepsilon_i(t)$ are the ith row of $\Lambda(t)$ and $\varepsilon(t)$, respectively. Using $\varepsilon(t)$, we design the parameter updating law for $\Lambda_i(t)$ as

$$\Lambda_i^T(t+1) = \Lambda_i^T(t) - \frac{\Gamma_i\varepsilon_i(t+\rho_i-\rho_M)\phi(t-\rho_M)}{m^2(t-\rho_M)} \tag{40}$$

for $1 \le i \le M$, where

$$m(t-\rho_M) = \sqrt{1+\phi(t-\rho_M)^T\phi(t-\rho_M)}. \tag{41}$$

and $\Gamma_i = \mathrm{diag}\{\gamma_{i_1}, \ldots, \gamma_{i_r}\}$ are the adaptation gains with $\gamma_{i_j} \in (0,2)$, $1 \le j \le r$. Since $\rho_i \le \rho_M$, the parameter updating law (40) only relies on known parameters and signals, which are certainly available at the current instant. Based on parameter updating law (40), the estimates $\Lambda_i(t)$, $i = 1, \ldots, M$, have the following important properties.

Lemma 5. *The parameter updating law (40) ensures that $\Lambda_i(t) \in L^\infty$, and $\frac{\varepsilon_i(t+\rho_i-\rho_M)}{m(t-\rho_M)}$, $\Lambda_i(t+i_0) - \Lambda_i(t) \in L^2$, $i = 1, \ldots, M$, where i_0 is an arbitrary positive integer.*

Proof. Consider the following positive definite functions $V_i(\tilde{\Lambda}_i) = \tilde{\Lambda}_i\Gamma_i^{-1}\tilde{\Lambda}_i^T$ for $i = 1, \ldots, M$, where $\tilde{\Lambda}_i = \Lambda_i(t) - \Lambda_i^*$ with Λ_i^* being the ith row of Λ^*. Then, we obtain

$$\begin{aligned}&V_i(\tilde{\Lambda}_i(t+1)) - V_i(\tilde{\Lambda}_i(t))\\ &= -\left(2 - \frac{\phi_i(t-\rho_M)^T\Gamma_i\phi_i(t-\rho_M)}{m^2(t-\rho_M)}\right)\frac{\varepsilon_i^2(t+\rho_i-\rho_M)}{m^2(t-\rho_M)}.\end{aligned}$$

Since Γ_i is a diagonal matrix with $\gamma_{i_j} \in (0,2)$ and $m(t-\rho_M) = \sqrt{1+\phi(t-\rho_M)^T\phi(t-\rho_M)}$, it yields that $V_i(\tilde{\Lambda}_i(t+1)) - V_i(\tilde{\Lambda}_i(t)) \le -\frac{\beta_i\varepsilon_i^2(t+\rho_i-\rho_M)}{m^2(t-\rho_M)}$, where $\beta_i = 2 - \max_{1\le j\le r}\{\gamma_{i_j}\} > 0$. It follows that $\Lambda_i(t) \in L^\infty$ and $V_i(\tilde{\Lambda}_i(t))$ is non-increasing. With the well-known monotone bounded theorem, we found that $V_i(\tilde{\Lambda}_i(t))$ is convergent. Accordingly, we have $\sum_{t=\rho_M}^{N}\frac{\beta_i\varepsilon_i^2(t+\rho_i-\rho_M)}{m^2(t-\rho_M)} \le V(\tilde{\Lambda}_i(\rho_M)) - V(\tilde{\Lambda}_i(N+1))$. It is found that $\sum_{t=\rho_M}^{\infty}\frac{\varepsilon_i^2(t+\rho_i-\rho_M)}{m^2(t-\rho_M)}$ is convergent, i.e., $\frac{\varepsilon_i(t+\rho_i-\rho_M)}{m(t-\rho_M)} \in L^2$. Furthermore, it is inferred from (40) that $\Lambda_i(t+1) - \Lambda_i(t) \in L^2$. □

Lemma 5 reveals some important properties about the estimated parameter $\Lambda(t)$ and the estimation error $\varepsilon(t)$, which are essential for the stability analysis.

Step 2: Estimation of future time signals. Using $\Lambda(t)$, we construct an estimation procedure to estimate some future time signals. Let $\widehat{y}(t+j)$ be an estimate of $y(t+j)$ at the current time instant with $\widehat{y}_j(t) = y_j(t)$. For $1 \le k_j \le \rho_j - \rho_{j-1}$ with $2 \le j \le M$ and $1 \le k_1 \le \rho_1$, we define

$$\begin{aligned}\widehat{\phi}(t-\rho_j+k_j) = [&\phi_f(\widehat{x}(t-\rho_j+k_j)), \bar{u}(t-\rho_j+k_j),\\ &\phi_d(t-\rho_j+k_j)]^T.\end{aligned} \tag{42}$$

where $\widehat{x}(t-\rho_j+k_j)$ is an estimate of $x(t-\rho_j+k_j)$. Recalling that

$$\begin{aligned}x(t) = [&y_1(t), y_1(t+1), \ldots, y_1(t+\rho_1-1), \ldots, y_M(t),\\ &y_M(t+1), \ldots, y_M(t+\rho_M-1)]^T,\end{aligned} \tag{43}$$

we have

$$\begin{aligned}&x(t-\rho_j+k_j)\\ &= [y_1(t-\rho_j+k_j), y_1(t+1-\rho_j+k_j), \ldots,\\ &\quad y_1(t+\rho_1-1-\rho_j+k_j), \ldots, y_j(t-\rho_j+k_j), \ldots,\\ &\quad y_j(t), y_j(t+1), \ldots\ldots, y_j(t+k_j-2), y_j(t+k_j-1), \ldots,\\ &\quad y_M(t-\rho_j+k_j), \ldots, y_M(t), y_M(t+1), \ldots,\\ &\quad y_M(t+\rho_M-1-\rho_j+k_j)]^T.\end{aligned} \tag{44}$$

Based on the structure of $x(t-\rho_j+k_j)$ in (44), we design an estimate of $x(t-\rho_j+k_j)$. Here, $\widehat{x}(t-\rho_j+k_j)$ is derived from $x(t-\rho_j+k_j)$ by replacing its unknown elements with their estimates and has the following form

$$\begin{aligned}&\widehat{x}(t-\rho_j+k_j)\\ &= \big[y_1(t-\rho_j+k_j), y_1(t+1-\rho_j+k_j), \ldots,\\ &\quad y_1(t+\rho_1-1-\rho_j+k_j), \ldots, y_j(t-\rho_j+k_j), \ldots,\\ &\quad \widehat{y}_j(t), \widehat{y}_j(t+1), \ldots, \widehat{y}_j(t+k_j-2), \widehat{y}_j(t+k_j-1), \ldots,\\ &\quad y_M(t-\rho_j+k_j), \ldots, \widehat{y}_M(t), \widehat{y}_M(t+1), \ldots,\\ &\quad \widehat{y}_M(t+\rho_M-1-\rho_j+k_j)\big]^T.\end{aligned} \tag{45}$$

In (45), the estimate $\widehat{x}(t-\rho_j+k_j)$ can be obtained from $x(t-\rho_j+k_j)$ by only replacing the unknown signals $y_j(t+i)$ with their estimates $\widehat{y}_j(t+i)$. So, the key issue is how to design the estimates $\widehat{y}_j(t+i)$. To ensure the estimation process with reasonable causality properties, we realize the future time signal estimation by the following procedure.

(i) Estimation of $y_M(t+i)$, $i = 1, \ldots, \rho_M - \rho_{M-1}$. According to the definition of $\phi(t)$ in (37), the signal $\phi(t-\rho_M+1)$ is available at the current time instant. Therefore, we derive an estimate of $y_M(t+1)$ as $\widehat{y}_M(t+1) = \Lambda_M(t)\phi(t-\rho_M+1)$. Using $\widehat{y}_M(t+1)$, we can directly derive an estimate $\widehat{\phi}(t-\rho_M+2)$ from $\phi(t-\rho_M+2)$

by replacing $y_M(t+1)$ with $\widehat{y}_M(t+1)$. Then, we further specify an estimate of $y_M(t+2)$ as $\widehat{y}_M(t+2) = \Lambda_M(t)\widehat{\phi}(t-\rho_M+2)$. Following this procedure, we can derive the estimates of $y_M(t+i)$ for $i = 1, 2, \ldots, \rho_M - \rho_{M-1}$ as follows:

$$\widehat{y}_M(t+i) = \Lambda_M(t)\widehat{\phi}(t-\rho_M+i), \tag{46}$$

with $\widehat{\phi}(t-\rho_M+1) = \phi(t-\rho_M+1)$.

(ii) Estimation of $y_M(t+j)$ and $y_{M-1}(t+k)$, $\rho_M - \rho_{M-1} < j \le \rho_M - \rho_{M-2}$, $1 \le k \le \rho_{M-1} - \rho_{M-2}$. By using (46), the estimate $\widehat{\phi}(t-\rho_{M-1}+1)$ can be obtained from $\phi(t-\rho_{M-1}+1)$ by replacing $y_M(t+i)$ with their estimates $\widehat{y}_M(t+i)$ for $i = 1, 2, \ldots, \rho_M - \rho_{M-1}$. In the estimation process, designing the estimates of $y_M(t+j)$, $\rho_M - \rho_{M-1} < j \le \rho_M - \rho_{M-2}$ needs the estimates of $y_{M-1}(t+j)$, $1 \le j \le \rho_{M-1} - \rho_{M-2}$. Therefore, we estimate them simultaneously in the following.

Using $\widehat{\phi}(t-\rho_{M-1}+1)$, we derive $\widehat{y}_{M-1}(t+1) = \Lambda_{M-1}(t)\widehat{\phi}(t-\rho_{M-1}+1)$ and $\widehat{y}_M(t+\rho_M-\rho_{M-1}+1) = \Lambda_M(t)\widehat{\phi}(t-\rho_{M-1}+1)$. Next, we get $\widehat{\phi}(t-\rho_{M-1}+2)$ from $\phi(t-\rho_{M-1}+2)$ by replacing $y_M(t+j)$, $1 \le j \le \rho_M - \rho_{M-1}+1$ and $y_{M-1}(t+1)$ with $\widehat{y}_M(t+j)$, $1 \le j \le \rho_M - \rho_{M-1} + 1$ and $\widehat{y}_{M-1}(t+1)$, respectively. Moreover, we design the estimates of $y_{M-1}(t+2)$ and $y_M(t+\rho_M-\rho_{M-1}+2)$ as $\widehat{y}_{M-1}(t+2) = \Lambda_{M-1}(t)\widehat{\phi}(t-\rho_{M-1}+2)$ and $\widehat{y}_M(t+\rho_M-\rho_{M-1}+2) = \Lambda_M(t)\widehat{\phi}(t-\rho_{M-1}+2)$, respectively. Following this procedure, we can sequentially derive the estimates of $y_M(t+\rho_M-\rho_{M-1}+i)$ and $y_{M-1}(t+i)$, $i = 1, \ldots, \rho_{M-1}-\rho_{M-2}$, as

$$\begin{aligned}\widehat{y}_M(t+\rho_M-\rho_{M-1}+i) &= \Lambda_M(t)\widehat{\phi}(t-\rho_{M-1}+i),\\ \widehat{y}_{M-1}(t+i) &= \Lambda_{M-1}(t)\widehat{\phi}(t-\rho_{M-1}+i).\end{aligned} \tag{47}$$

(iii) Estimation of other future time output signals. According to the estimation procedure shown in (i) and (ii), we can sequentially specify the estimate of the future time output signal $y_i(t+j)$. Overall, we first estimate $y_M(t+j), j = 1, \ldots, \rho_M - \rho_{M-1}$. Second, we estimate $y_M(t+j), j = \rho_M - \rho_{M-1}+1, \ldots, \rho_M - \rho_{M-2}$ and $y_{M-1}(t+k), k = 1, \ldots, \rho_{M-1}-\rho_{M-2}$. Third, we estimate $y_M(t+j), j = \rho_M - \rho_{M-2}+1, \ldots, \rho_M - \rho_{M-3}, y_{M-1}(t+k), k = \rho_{M-1}-\rho_{M-2}+1, \ldots, \rho_{M-1}-\rho_{M-3}$ and $y_{M-2}(t+i), i = 1, \ldots, \rho_{M-2}-\rho_{M-3}$. Finally, following this procedure provides the estimate of $y_i(t+j)$ as

$$\widehat{y}_i(t+j) = \Lambda_i(t)\widehat{\phi}(t-\rho_i+j), i = 1, \ldots, M, j = 1, \ldots, \rho_i - 1. \tag{48}$$

For the estimate $\widehat{y}_i(t+j)$ given in (48), the following lemma shows some important properties which are crucial for the output feedback control design and analysis.

Lemma 6. *The estimates $\widehat{y}_i(t+j)$, $1 \le i \le M$, $1 \le j \le \rho_i - 1$, given in (48) are available at the current time instant and have the following properties*

$$\begin{aligned}&|\widehat{y}_i(t+j) - y_i(t+j)|\\ &\le \tau(t)\max_{k=0,1,\ldots,\rho_M-1}\{\|\phi(t-k)\|\} + \tau(t),\end{aligned} \tag{49}$$

where $\tau(t)$ denotes a generic L^2 function that goes to zero as t goes to infinity.

Proof. Based on the estimation procedure shown above, it is inferred that $\widehat{y}_i(t+j)$ is available at the current time instant. Now, we show that (49) holds. With the estimate of $y_M(t+1)$, it yields $\widehat{y}_M(t+1) - y_M(t+1) = \Lambda_M(t)\widehat{\phi}(t-\rho_M+1) - y_M(t+1) = (\Lambda_M(t) - \Lambda_M(t+1))\widehat{\phi}(t-\rho_M+1) + \frac{\varepsilon_M(t+1)}{m(t-\rho_M+1)}m(t-\rho_M+1)$. Lemma 5 gives $\Lambda_M(t) - \Lambda_M(t+1) \in L^2$ and $\frac{\varepsilon_M(t+1)}{m(t-\rho_M+1)} \in L^2$. Also, from (41), we get $m(t-\rho_M+1) \le \|\phi(t-\rho_M+1)\| + 1 = \|\widehat{\phi}(t-\rho_M+1)\| + 1$. Accordingly,

$$|\widehat{y}_M(t+1) - y_M(t+1)| \le \tau(t)\|\phi(t-\rho_M+1)\| + \tau(t). \tag{50}$$

Note that $\widehat{y}_M(t+2) - y_M(t+2) = \Lambda_M(t)\widehat{\phi}(t-\rho_M+2) - y_M(t+2) = \Lambda_M(t)\widehat{\phi}(t-\rho_M+2) - \Lambda_M(t)\widehat{\phi}(t-\rho_M+1) + (\Lambda_M(t) - \Lambda_M(t+2))\widehat{\phi}(t-\rho_M+1) + \frac{\varepsilon_M(t+2)}{m(t-\rho_M+2)}m(t-\rho_M+2)$. Since f_{i_k} is Lipschitz mapping, we have $\|\widehat{\phi}(t-\rho_M+2) - \widehat{\phi}(t-\rho_M+1)\| \le c|y_M(t+1) - \widehat{y}_M(t+1)|$. From (41), we have $m(t-\rho_M+2) \le \|\phi(t-\rho_M+2)\| + 1$. From Lemma 5, we have $\Lambda_M(t) - \Lambda_M(t+2) \in L^2$ and $\frac{\varepsilon_M(t+1)}{m(t-\rho_M+1)} \in L^2$, which together with (50) implies $|\widehat{y}_M(t+2) - y_M(t+2)| \le \tau(t)\max_{k=\rho_M-2,\rho_M-1}\{\|\phi(t-k)\|\} + \tau(t)$. For $i = 1, \ldots, \rho_M - \rho_{M-1} + 1$, we can sequentially derive from Lemma 5 and the Lipschitz property of f_{i_k} that

$$\begin{aligned}&|\widehat{y}_M(t+i) - y_M(t+i)|\\ &\le \tau(t)\max_{k=0,1,\ldots,\rho_M-1}\{\|\phi(t-k)\|\} + \tau(t).\end{aligned} \tag{51}$$

Moreover, $\widehat{y}_{M-1}(t+1) - y_{M-1}(t+1) = -y_{M-1}(t+1) + \Lambda_{M-1}(t)\widehat{\phi}(t-\rho_{M-1}+1) = \Lambda_{M-1}(t)(\widehat{\phi}(t-\rho_{M-1}+1) - \phi(t-\rho_{M-1}+1)) + (\Lambda_{M-1}(t) - \Lambda_{M-1}(t+\Lambda_{M-1}))\phi(t-\rho_{M-1}+1) + \frac{\varepsilon_{M-1}(t+1)}{m(t-\rho_{M-1}+1)}m(t-\rho_{M-1}+1)$. Since f_{i_k} is Lipschitz mapping, we get $|\widehat{\phi}(t-\rho_{M-1}+1) - \phi(t-\rho_{M-1}+1)| \le c\max_{k=1,\ldots,\rho_M-\rho_{M-1}}|y_M(t+k) - \widehat{y}_M(t+k)|$. Then, it follows from Lemma 5 and (51) that

$$\begin{aligned}&|\widehat{y}_{M-1}(t+1) - y_{M-1}(t+1)|\\ &\le \tau(t)\max_{k=0,1,\ldots,\rho_M-1}\{\|\phi(t-k)\|\} + \tau(t).\end{aligned} \tag{52}$$

Following the above recursive procedure, we can derive that $\widehat{y}_i(t+j)$ satisfies (49) for $i = M-1, M-2, \ldots, 1$ and $1 \le j \le \rho_i$ sequentially. This completes the proof. □

Lemma 6 presents a key property of the estimate $\widehat{y}_i(t+j)$. With this property, the output feedback control law can be designed to ensure the closed-loop signals are all bounded. Then, it follows from (49) that the error $\widehat{y}_i(t+j) - y_i(t+j)$ converges to zero.

Step 3: Construction of adaptive control law. For the output feedback case, we design the adaptive control law as

$$\begin{aligned}u(t) &= \Phi_g^{-1}(\widehat{x}(t))\bar{u}(t),\\ \bar{u}(t) &= (I + \Theta_2(t))^{-1}\Bigg(-\Theta_1(t)\phi_f(\widehat{x}(t)) - \Theta_3(t)\phi_d(t)\\ &\quad -\Theta_4(t)\begin{bmatrix}\sum_{i=0}^{\rho_1-1} a_{1i}(\widehat{y}_1(t+i) - y_1^*(t+i))\\ \vdots\\ \sum_{i=0}^{\rho_M-1} a_{Mi}(\widehat{y}_M(t+i) - y_M^*(t+i))\end{bmatrix}\\ &\quad +\Theta_4(t)\begin{bmatrix}y_1^*(t+\rho_1)\\ y_2^*(t+\rho_2)\\ \vdots\\ y_M^*(t+\rho_M)\end{bmatrix}\Bigg),\end{aligned} \tag{53}$$

where $\Theta_i(t), i = 1, 2, 3, 4$ are estimates of Θ_1^*, Θ_2^*, Θ_3^*, $(S^*D_s)^{-1}$, respectively, $\widehat{y}_i(t+j)$ is given in (48), and

$$\begin{aligned}\widehat{x}(t) = [&y_1(t), \widehat{y}_1(t+1), \ldots, \widehat{y}_1(t+\rho_1-1), \ldots,\\ &y_M(t), \widehat{y}_M(t+1), \ldots, \widehat{y}_M(t+\rho_M-1)]^T.\end{aligned} \tag{54}$$

Step 4: Specification of a modified tracking error model. Define an output estimation-based tracking error as $\widehat{e}_i(t+j) = \widehat{y}_i(t+j) - y_i^*(t+j)$ for $1 \le i \le M$ and $0 \le j \le \rho_i - 1$ with $\widehat{e}_i(t) = e_i(t)$. Together with (9) and (53), we have

$$\begin{aligned}P_m(z)[e](t) &= S^*D_s\tilde{\Psi}(t)\varphi(t) +\\ &\quad \begin{bmatrix}\sum_{i=0}^{\rho_1-1} a_{1i}[y_1(t+i) - \widehat{y}_1(t+i)]\\ \vdots\\ \sum_{i=0}^{\rho_M-1} a_{Mi}[y_M(t+i) - \widehat{y}_M(t+i)]\end{bmatrix}\\ &\quad + S^*D_s\Theta_1^*(\phi_f(x(t)) - \phi_f(\widehat{x}(t))).\end{aligned} \tag{55}$$

where $P_m(z)$ and $\tilde{\Psi}(t)$ have the same expressions as $P_m(z)$ and $\tilde{\Psi}(t)$ in (17) of the state feedback case, and

$$\varphi(t) = [-\phi_f^T(\widehat{x}(t)), -\bar{u}^T(t), -\phi_d^T(t), \\ -\sum_{i=0}^{\rho_1-1} a_{1i}\widehat{e}_1(t+i) + y_1^*(t+\rho_1), \cdots, \\ -\sum_{i=0}^{\rho_M-1} a_{Mi}\widehat{e}_M(t+i) + y_M^*(t+\rho_M)]^T. \tag{56}$$

Here, we introduce stable and strictly proper filters

$$H_i(z) = \frac{1}{z^{\rho_i} + \alpha'_{i,\rho_i-1}z^{\rho_i-1} + \cdots + \alpha'_{i1}z + \alpha'_{i0}} \tag{57}$$

for $i = 1, \ldots, M$, subject to their impulse response functions must be greater than or equal to zero. Then, we see that the simplest choice of $H_i(z)$ is $\frac{1}{z^{\rho_i}}$. We define $H(z) = \text{diag}\{H_i(z)\}$, followed by defining a filtered tracking error as

$$\bar{e}(t) = P_m(z)H(z)[e](t). \tag{58}$$

Note that $P_m(z)H(z)[e](t) = \text{diag}\{P_{mi}(z)H_i(z)[e_i](t)\}$. With the form of $P_{mi}(z)$ and $H_i(z)$, we have

$$P_{mi}(z)H_i(z)[e](t) \\ = \left(1 + \frac{(a_{i,\rho_i-1} - \alpha'_{i,\rho_i-1})z^{\rho_i-1} + \cdots + (a_{i0} - \alpha'_{i0})}{z^{\rho_i} + \alpha'_{i,\rho_i-1}z^{\rho_i-1} + \cdots + \alpha'_{i1}z + \alpha'_{i0}}\right)[e_i](t) \\ = e_i(t) + \frac{(a_{i,\rho_i-1} - \alpha'_{i,\rho_i-1})z^{\rho_i-1} + \cdots + (a_{i0} - \alpha'_{i0})}{z^{\rho_i} + \alpha'_{i,\rho_i-1}z^{\rho_i-1} + \cdots + \alpha'_{i1}z + \alpha'_{i0}}[e_i](t).$$

Since $\frac{(a_{i,\rho_i-1} - \alpha'_{i,\rho_i-1})z^{\rho_i-1} + \cdots + (a_{i0} - \alpha'_{i0})}{z^{\rho_i} + \alpha'_{i,\rho_i-1}z^{\rho_i-1} + \cdots + \alpha'_{i1}z + \alpha'_{i0}}$ is stable and strictly proper, $P_{mi}(z)H_i(z)[e](t)$ can be acquired from the current known output signals and the reference signals. Thus, the filtered tracking error $\bar{e}(t)$ is available at the current time instant. Therefore, using filtered tracking error (58) to define an estimation error and parameter updating law does not involve the causality contradiction problem.

Step 5: Construction of controller parameter updating law. To reduce notation, we still use $\varepsilon(t)$ to denote an estimation error as

$$\varepsilon(t) = \bar{e}(t) + \Phi(t)\sigma(t), \tag{59}$$

where $\Phi(t)$ is the estimate of S^*D_s and $\sigma(t) = [\sigma_1(t), \sigma_2(t), \ldots, \sigma_M(t)]^T$ with

$$\sigma_j(t) = \Psi_j(t)\delta_j(t) - H_j(z)[\Psi_j\varphi_j](t), \\ \delta_j(t) = H_j(z)[\varphi_j](t). \tag{60}$$

In particular, $\Psi_j(t), j = 1, \ldots, M$, have the same expressions as $\Psi_j(t)$ in (19), and $\varphi_j(t), j = 1, \ldots, M$, are derived from (20) by replacing $x(t)$ and $e_i(t+j)$ with their estimates $\widehat{x}(t)$ and $\widehat{e}_i(t+j)$. It is noteworthy that $\bar{e}(t), \varepsilon(t), \sigma_j(t), \delta_j(t)$ are all available at the current time instant. Then, we design the parameter updating law as follows:

$$\Psi_j^T(t+1) = \Psi_j^T(t) - \frac{\text{sign}\{d_j^*\}\gamma_j\varepsilon_j(t)\delta_j(t)}{m^2(t)}, \tag{61}$$

$$\Phi(t+1) = \Phi(t) - \frac{\beta\varepsilon(t)\sigma^T(t)}{m^2(t)}, \tag{62}$$

where $j = 1, 2, \ldots, M$, $m(t)$ has the same form as $m(t)$ in (28) and γ_j and β have the same meanings as γ_j and β clarified below (28), respectively.

Performing a proof similar to that of Lemma 1, one can see that the given parameter updating law (61)–(62) ensures the estimates have the following properties: (i) $\Psi_j(t) \in L^\infty$, $\Phi(t) \in L^\infty$, $j = 1, 2, \ldots, M$; and (ii) $\frac{\varepsilon(t)}{m(t)} \in L^2 \cap L^\infty$, $\Phi(t+1) - \Phi(t) \in L^2 \cap L^\infty$, and $\Psi_j(t+1) - \Psi_j(t) \in L^2 \cap L^\infty$, $j = 1, 2, \ldots, M$.

Step 6: Analysis of the system performance. Based on above derivations, we give the second main result of this paper as follows:

Theorem 7. *Under Assumptions (A1)-(A3), if the output feedback adaptive control law (53) with the parameter updating law (61)–(62) is applied to the system (1), then the closed-loop system is stable and of asymptotic output tracking* $\lim_{t\to\infty}(y(t) - y^*(t)) = 0$.

Proof. The proof contains four steps. First, we show

$$\sum_{j=1}^{M} |\sigma_j(t)| \le \tau \sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1} |\widehat{e}_i(t+j)| + \tau \sup_{k\le t}\|e(k)\| + \tau. \tag{63}$$

Since $\|y^*(t)\| \le c$ and f_{ik} is Lipschitz mapping, it yields $\|\varphi_j(t)\| \le c\|\widehat{x}(t)\| + c\|e(t)\| + c\sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1}|\widehat{e}_i(t+j)| + c$. Then, with the boundedness of $y^*(t)$, we get $\|\varphi_j(t)\| \le c\|\widehat{x}(t) - x^*(t) + x^*(t)\| + c\|e(t)\| + c\sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1}|\widehat{e}_i(t+j)| + c \le c\|\widehat{x}(t) - x^*(t)\| + c\|x^*(t)\| + c\|e(t)\| + c\sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1}|\widehat{e}_i(t+j)| + c \le c\|e(t)\| + c\sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1}|\widehat{e}_i(t+j)| + c$, where $x^*(t) \triangleq [y_1^*(t), \ldots, y_1^*(t+\rho_1-1), \ldots, y_M^*(t+\rho_M-1)]^T$. With $\sigma_j(t) = \Psi_j(t)\delta_j(t) - H_j(z)[\Psi_j\varphi_j](t)$, $\delta_j(t) = H_j(z)[\varphi_j](t)$ and using Lemma 3, we get

$$\sigma_j(t) = \Psi_j(t)H_j(z)[\varphi_j](t) - H_j(z)[\Psi_j\varphi_j](t) \\ = H_{C_j}(z)[(H_{B_j}(z)z)[\varphi_j^T](z-1)[\Psi_j^T]](t), \tag{64}$$

where $C_j(zI - A_j)^{-1}B_j$ denotes a minimal realization of $H_j(z)$, $H_{C_j}(z) = C_j(zI - A_j)^{-1}$, and $H_{B_j}(z) = (zI - A_j)^{-1}B_j$. Since $H_{B_j}(z)z$ is proper and stable, we have $\|H_{B_j}(z)z[\varphi_j^T](t)\| \le c + c\sup_{k\le t}\|e(k)\| + c\sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1}|\widehat{e}_i(t+j)|$. With $|[(H_{B_j}(z)z)[\varphi_j^T](z-1)[\Psi_j^T]](t)| \le \|\Psi_j(t+1) - \Psi_j(t)\|\|H_{B_j}(z)z[\varphi_j](t)\|$ and $\Psi_j(t+1) - \Psi_j(t) \in L^2 \cap L^\infty$, we have

$$|[(H_{B_j}(z)z)[\varphi_j^T](z-1)[\Psi_j^T]](t)| \\ \le \tau \sum_{i=1}^{M}\sum_{j_i=1}^{\rho_i-1} |\widehat{e}_i(t+j)| + \tau \sup_{k\le t}\|e(k)\| + \tau. \tag{65}$$

From (64), (65), and the fact that the impulse response of $H_j(z)$ is greater than or equal to zero, we obtain $|\sigma_j(t)| \le \tau\sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1}|\widehat{e}_i(t+j)| + \tau\sup_{k\le t}\|e(k)\| + \tau$, $\forall j = 1, 2, \ldots, M$. Thus, (63) holds.

Second, we demonstrate that

$$m(t) \le c\sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1}|\widehat{e}_i(t+j)| + c\sup_{k\le t}\|e(k)\| + c. \tag{66}$$

From the definition of $\delta_j(t)$ and the condition that $H_j(z)$ is strictly proper and stable, we get $\|\delta_j(t)\| \le c\sup_{k\le t}\|\varphi_j(k)\| + c$. Next, it follows from $\|\varphi_j(t)\| \le c\sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1}|\widehat{e}_i(t+j)| + c\|e(t)\| + c$ that

$$\|\delta_j(t)\| \le c\sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1}|\widehat{e}_i(t+j)| + c\sup_{k\le t}\|e(k)\| + c. \tag{67}$$

From the definition of $m(t)$, we conclude that $m(t) \le 1 + \sum_{j=1}^{M}|\sigma_j(t)| + \sum_{j=1}^{M}\|\delta_j(t)\|$. Then, it follows from (63) and (67) that (66) holds.

Third, we prove $e(t) \in L^\infty$. From (59), we have $\|\bar{e}(t)\| \le \|\varepsilon(t)\| + \|\Phi(t)\sigma(t)\| \le m(t)\left\|\frac{\varepsilon(t)}{m(t)}\right\| + \|\Phi(t)\sigma(t)\|$. Since $\frac{\varepsilon(t)}{m(t)} \in L^2 \cap L^\infty$ and $\Phi(t) \in L^\infty$, from $\sum_{j=1}^{M}|\sigma_j(t)| \le \tau\sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1}|\widehat{e}_i(t+$

$j)\| + \tau \sup_{k\le t}\|e(k)\| + \tau$, we get

$$\|\bar{e}(t)\| \le \tau \sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1} |\widehat{e}_i(t+j)| + \tau \sup_{k\le t}\|e(k)\| + \tau. \quad (68)$$

Then, we have

$$\sup_{k\le t}\|\bar{e}(k)\| \le \tau \sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1} |\widehat{e}_i(t+j)| + \tau \sup_{k\le t}\|e(k)\| + \tau. \quad (69)$$

From Lemma 6, we get

$$\begin{aligned} &|\widehat{e}_i(t+j)| = |\widehat{y}_i(t+j) - y_i^*(t+j)| \\ &\le |\widehat{y}_i(t+j) - y_i(t+j)| + |y_i(t+j) - y_i^*(t+j)| \\ &\le \tau(t) \max_{k=0,1,\ldots,\rho_M-1}\{\|\phi(t-k)\|\} + \tau(t) \\ &\quad + |e_i(t+j)|. \end{aligned} \quad (70)$$

With the definition of $\|\phi(t)\|$ and the Lipschitz property of f_{i_k}, we have

$$\begin{aligned} &\|\phi(t-k)\| = \|\phi(x(t-k), t-k)\| \\ &\le \|\phi(x(t-k), t-k) - \phi(x^*(t-k), t-k)\| \\ &\quad + \|\phi(x^*(t-k), t-k)\| \\ &\le \|\phi_f(x(t-k)) - \phi_f(x^*(t-k))\| \\ &\quad + \|\phi(x^*(t-k), t-k)\| \\ &\le c \sum_{i_k} |f_{i_k}(x(t-k)) - f_{i_k}(x^*(t-k))| + c \\ &\le c \sup_{k\le t+\rho_M-1}\|e(k)\| + c \end{aligned} \quad (71)$$

for $k = 0, 1, \ldots, \rho_M - 1$. Thus, we get

$$\max_{k=0,1,\ldots,\rho_M-1}\|\phi(t-k)\| \le c \sup_{k\le t+\rho_M-1}\|e(k)\| + c. \quad (72)$$

Then, it follows from (69) and (70) that

$$\begin{aligned} \sup_{k\le t}\|\bar{e}(k)\| &\le \tau \sum_{i=1}^{M}\sum_{j=1}^{\rho_i-1} |e_i(t+j)| + \tau \sup_{k\le t}\|e(k)\| \\ &\quad + \tau \max_{k=0,1,\ldots,\rho_M-1}\{\|\phi(t-k)\|\} + \tau \\ &\le \tau \sup_{k\le t+\rho_M-1}\|e(k)\| + \tau. \end{aligned} \quad (73)$$

With $P_m^{-1}(z)$ and $H^{-1}(z)$ being the inverse of $P_m(z)$ and $H(z)$, respectively, we get $P_m^{-1}H^{-1}(z)$ is proper and stable. From the definition of $\bar{e}(t)$ in (58), we derive that $\|e(t)\| \le c\sup_{k\le t}\|\bar{e}(t)\| + c \le \tau \sup_{k\le t+\rho_M-1}\|e(k)\| + c$, which implies that $e(t) \in L^\infty$.

Finally, we prove closed-loop stability and $\lim_{t\to\infty}(y(t) - y^*(t)) = 0$. Since $e(t) \in L^\infty$ and $\sup_{k\le t}\|\bar{e}(k)\| \le \tau \sup_{k\le t+\rho_M-1}\|e(k)\| + \tau$, we have $\bar{e}(t) \in L^2 \cap L^\infty$. With $P_m^{-1}H^{-1}(z)$ being proper and stable, we obtain $e(t) \in L^2 \cap L^\infty$. Then, we conclude that $\lim_{t\to\infty} e(t) = 0$. Further, the boundedness of all closed-loop signals can be obtained from the boundedness of $e(t)$. This completes the proof. □

So far, this section has developed a future time signal estimation-based output feedback adaptive control scheme, where the adaptive control law is always non-singular. Particularly, essentially different from the high-gain observer based adaptive control methods, the proposed adaptive control law does not involve the high-gain issue and ensures asymptotic output tracking.

5. Simulation example

This section gives a numerical example to show the design procedure and verify the validity of the proposed control scheme.

System model. Consider the following system model

$$\begin{aligned} \xi_{11}(t+1) &= \theta_{f_1}^{*T}\phi_{f_1}(x(t)) + \sum_{j=1}^{2}\theta_{g_{1j}}^{*T}\phi_{g_{1j}}u_j(t) + d_1(t), \\ \xi_{21}(t+1) &= \xi_{22}(t), \\ \xi_{22}(t+1) &= \theta_{f_2}^{*T}\phi_{f_2}(x(t)) + \sum_{j=1}^{2}\theta_{g_{2j}}^{*T}\phi_{g_{2j}}u_j(t) + d_2(t), \\ y_1(t) &= \xi_{11}(t), y_2(t) = \xi_{21}(t), \end{aligned} \quad (74)$$

where $x(t) = [\xi_{11}, \xi_{21}, \xi_{22}]^T$ is the state vector, and $u(t) = [u_1, u_2]^T$ and $y(t) = [\xi_{11}, \xi_{21}]^T$ are the system input and output vectors, respectively. The vector relative degree of the system model (74) is $\rho = [1, 2]$. Moreover, $\theta_{f_1}^{*T}\phi_{f_1} = 2.1\sqrt{1+\xi_{11}^2} + 1.7\xi_{22}\sin\xi_{21}$, $\theta_{f_2}^{*T}\phi_{f_2} = 0.6\xi_{21}\sin\xi_{22} + 0.8\xi_{11}$, $\theta_{g_{11}}^{*T}\phi_{g_{11}} = -21 - \sin^2\xi_{11} + \sin^2\xi_{21}$, $\theta_{g_{12}}^{*T}\phi_{g_{12}} = -11 - \cos^2\xi_{21} + \sin^2\xi_{22}$, $\theta_{g_{21}}^{*T}\phi_{g_{21}} = -3 + 3\sin^2\xi_{21}$, and $\theta_{g_{22}}^{*T}\phi_{g_{22}} = -30 + 3\sin^2\xi_{22}$. Additionally, $d_1(t) = \theta_{d_1}^{*T}\phi_{d_1}(t) = 0.02\sin(t/8)$ and $d_2(t) = \theta_{d_2}^{*T}\phi_{d_2}(t) = 0.05\cos(t/6)$. In this simulation, we assume that the system parameters are all unknown.

Parameterized model. From (74), we have

$$\Phi_g(x) = \begin{bmatrix} 20+\sin^2\xi_{11} & 1+\cos^2\xi_{21} \\ 1-\sin^2\xi_{21} & 10-\sin^2\xi_{22} \end{bmatrix},$$

and decompose Θ_g^* as

$$\Theta_g^* = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$

where S^*, D_s, U_s are three matrices of the right side, respectively. Then, we have

$$\begin{bmatrix} y_1(t+1) \\ y_2(t+2) \end{bmatrix} = \Theta_f^*\phi_f(x(t)) + S^*D_sU_s\bar{u}(t) + \Theta_d^*\phi_d(t),$$

where

$$\Theta_f^* = \begin{bmatrix} 2.1 & 1.7 & 0 & 0 \\ 0 & 0 & 0.6 & 0.8 \end{bmatrix}, \quad \Theta_d^* = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.05 \end{bmatrix},$$

and $\phi_f(x(t)) = [\sqrt{1+\xi_{11}^2}, \xi_{22}\sin\xi_{21}, \xi_{21}\sin\xi_{22}, \xi_{11}]^T$, $\phi_d(t) = [\sin\frac{t}{8}, \cos\frac{t}{6}]^T$ and $\bar{u}(t) = \Phi_g(x(t))u(t)$. Then, we have

$$\begin{aligned} S^{*-1}\begin{bmatrix} y_1(t+1) \\ y_2(t+2) \end{bmatrix} &= D_s\Theta_1^*\phi_f(x(t)) + D_s\Theta_2^*\bar{u}(t) \\ &\quad + D_s\bar{u}(t) + D_s\Theta_3^*\phi_d(t), \end{aligned} \quad (75)$$

where

$$S^{*-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \Theta_2^* = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$\Theta_1^* = D_s^{-1}S^{*-1}\Theta_f^* = \begin{bmatrix} -2.1 & -1.7 & 0 & 0 \\ 0 & 0 & -0.2 & -0.2667 \end{bmatrix},$$

$$\Theta_3^* = D_s^{-1}S^{*-1}\Theta_d^* = \begin{bmatrix} -2 & 0 \\ 0 & -0.5 \end{bmatrix}.$$

Simulation for the state feedback. When the system states are available, the state feedback adaptive control law is designed as

$$u(t) = \Phi_g^{-1}(x(t))\bar{u}(t),$$

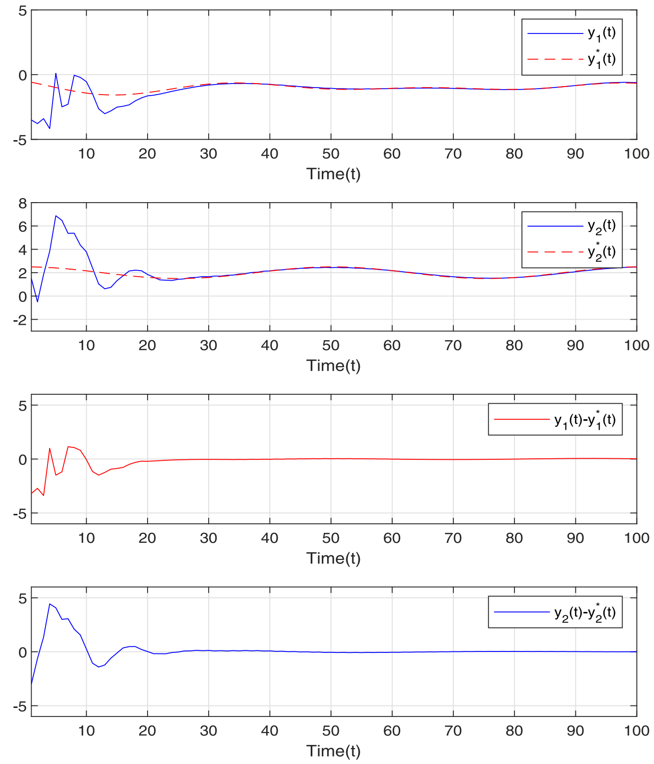

Fig. 1. Response of $y(t)$ v.s. $y^*(t)$ (state feedback case).

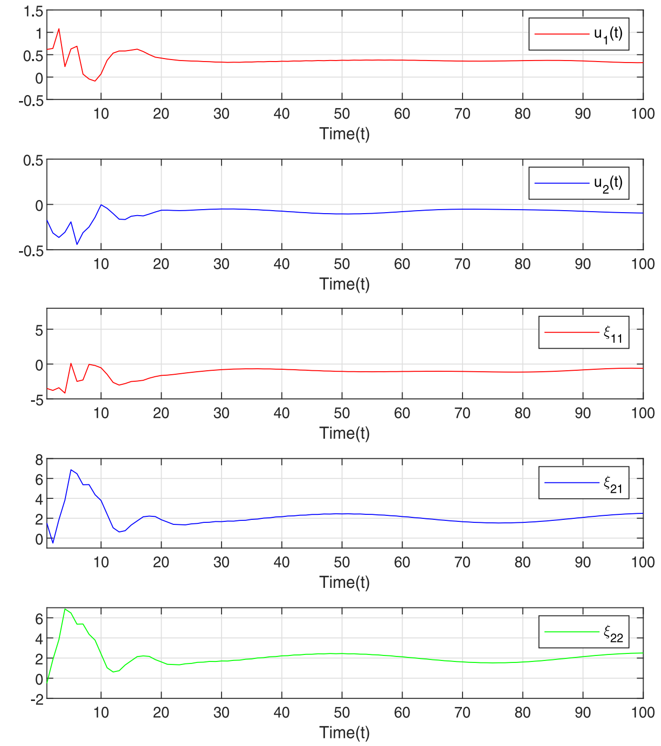

Fig. 2. Response of control input and system state variables (state feedback case).

$$\bar{u}(t) = (I + \Theta_2(t))^{-1} \Big(-\Theta_1(t)\phi_f(x(t)) - \Theta_3(t)\phi_d(t) - \Theta_4(t) \begin{bmatrix} a_{10}(y_1(t) - y_1^*(t)) \\ \sum_{i=0}^{1} a_{2i}(y_2(t+i) - y_2^*(t+i)) \end{bmatrix} + \Theta_4(t) \begin{bmatrix} y_1^*(t+1) \\ y_2^*(t+2) \end{bmatrix} \Big), \tag{76}$$

where $y^*(t) = [-1 - 0.5\sin\frac{t}{6} + 0.5\cos\frac{t}{7}, 2 + 0.5\cos\frac{t}{8}]^T$, and $\Theta_i(t)$, $1 \le i \le 4$, are the estimates of Θ_1^*, Θ_2^*, Θ_3^*, $(S^*D_s)^{-1}$, respectively. We choose $P_{m1}(z) = z - 0.2$ and $P_{m2}(z) = z^2 - 0.6z + 0.08$. Then, we get $a_{10} = -0.2$, $a_{20} = 0.08$ and $a_{21} = -0.6$. Choose $h(z) = \frac{1}{z-0.2}$. Then, the parameter updating law can be specified from (26)–(27). To preserve a reasonable paper length, we omit further details. The simulation results of the state feedback adaptive control law are given in Figs. 1–2.

Fig. 1 presents the output response $y(t)$ of the system (74) versus the reference signal $y^*(t)$. The figure illustrates that $y(t)$ tracks $y^*(t)$ asymptotically. Fig. 2 shows the response of the control input $u(t)$ and the system state variables, which indicates that the control input and the state variables are all bounded. In summary, the simulation results verify the validity of the proposed state feedback adaptive control law.

Simulation for the output feedback. Considering only the system output can be acquired, we need to design an output feedback adaptive control law.

In this case, the state $\xi_{22}(t) = y_2(t+1)$ cannot be used for adaptive control design. Therefore, we first estimate $y_2(t+1)$. Choose $\Gamma_2 = I_8$. Then, we can get $\Lambda_2(t)$ based on (39). Next, the estimate $\widehat{y}_2(t+1)$ can be acquired from $\widehat{y}_2(t+1) = \Lambda_2(t)\phi(t-1)$. Choose $H_1(z) = \frac{1}{z}$ and $H_2(z) = \frac{1}{z^2}$. The reference signal $y^*(t)$ is also chosen as $y^*(t) = [-1 - 0.5\sin\frac{t}{6} + 0.5\cos\frac{t}{7}, 2 + 0.5\cos\frac{t}{8}]^T$. Afterward, we can design the output feedback adaptive control law based on (53) and the parameter updating law based on (61)–(62) with $\widehat{y}_2(t+1)$. To increase the paper's readability, we ignore the specific design details and give the simulation results in Figs. 3–4.

Fig. 3 presents the output response $y(t)$ of the system (74) versus the reference signal $y^*(t)$. The figure shows that $y(t)$ also tracks $y^*(t)$ asymptotically. Fig. 4 depicts the response of the control input $u(t)$ and the response of system state variables, which also illustrates that the control input and the state variables are all bounded. It follows from Figs. 3–4 that the proposed output feedback adaptive control scheme is also valid.

6. Concluding remarks

This paper developed a matrix decomposition-based linearly parameterized adaptive control scheme for a class of MIMO discrete-time nonlinear systems with general vector relative degrees. The state and output feedback cases are addressed, which achieve desired system performance. Notably, we proposed a future time signal estimation-based output feedback adaptive control method which ensures that the control law is always nonsingular without involving the high-gain and causality contradiction issues. Simulation results have demonstrated the control design procedure and verified the effectiveness of the proposed adaptive control scheme. There are many important issues deserving further study such as how to deal with the case where the control gain matrix cannot be decomposed into a product of two square matrices, how to relax the Lipschitz condition and how to use quantized sensor measurements to ensure desired system performance.

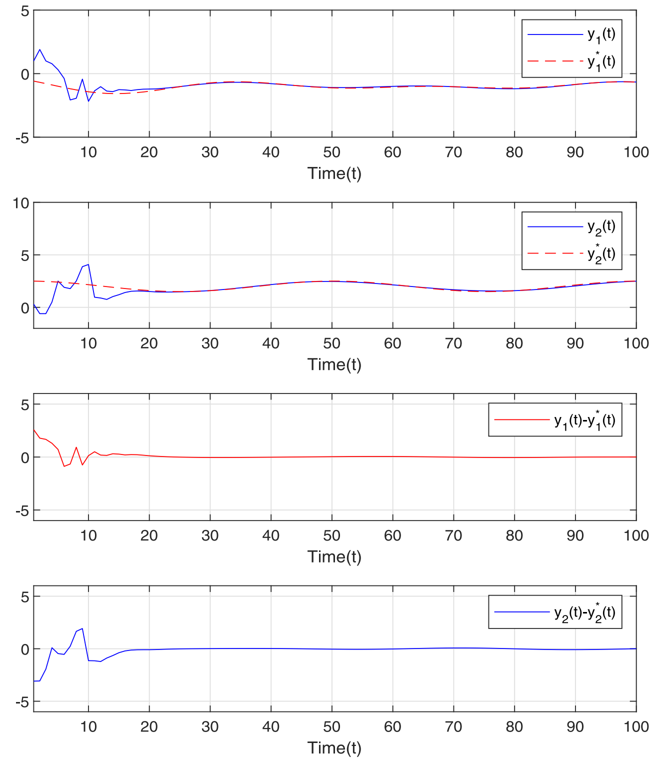

Fig. 3. Response of $y(t)$ versus $y^*(t)$ (output feedback case).

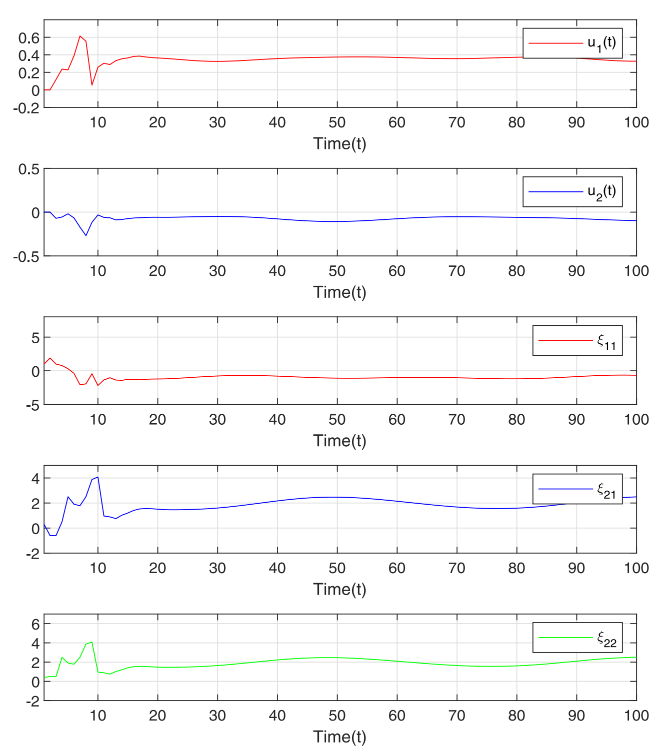

Fig. 4. Response of control input and system state variables (output feedback case).

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Yuchun Xu received the B.S. degree in mathematics from Shandong University, Jinan, China, in 2021. He is currently working toward the Ph.D. degree majoring in systems analysis and integration at Academy of Mathematics and Systems Science, Chinese Academy of Science, Beijing, China. His research interests include quantized control and adaptive control.

Yanjun Zhang received his B.S. degree in mathematics and applied mathematics in 2010, and M.S. degree in applied mathematics in 2013, both from Qufu Normal University, China, and his Ph.D. degree in control theory and control engineering in 2017 from Nanjing University of Aeronautics and Astronautics, China. From Jun. 2019 to Jun. 2021, he was a postdoc in the Institute of Systems Science, Academy of Mathematics and Systems Science, Chinese Academy of Sciences. He is currently an associate professor in the School of Automation, Beijing Institute of Technology. His current research interests include adaptive control, quantized control, nonlinear systems, and their applications.

Ji-Feng Zhang received the B.S. degree in mathematics from Shandong University, China, in 1985 and the Ph.D. degree from the Institute of Systems Science (ISS), Chinese Academy of Sciences (CAS), China, in 1991. He is now with the ISS, Academy of Mathematics and Systems Science, CAS. His current research interests include system modeling, adaptive control, stochastic systems, and multi-agent systems.

He is an IEEE Fellow, IFAC Fellow, CAA Fellow, CSIAM Fellow, member of the European Academy of Sciences and Arts, and Academician of the International Academy for Systems and Cybernetic Sciences. He received the Second Prize of the State Natural Science Award of China in 2010 and 2015, respectively. He is a Vice-President of the Chinese Mathematical Society and the Chinese Association of Automation. He was a Vice-Chair of the IFAC Technical Board, member of the Board of Governors, IEEE Control Systems Society; Convenor of Systems Science Discipline, Academic Degree Committee of the State Council of China; Vice-President of the Systems Engineering Society of China. He has served as Editor-in-Chief, Deputy Editor-in-Chief, Senior Editor or Associate Editor for more than 10 journals, including *Science China Information Sciences*, *National Science Review*, *IEEE Transactions on Automatic Control* and *SIAM Journal on Control and Optimization* etc.