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DISTRIBUTED ADAPTIVE NONLINEAR CONTROL WITH FUSION LEAST-SQUARES*

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Abstract. This paper is concerned with the adaptive consensus tracking control problem for strict-feedback nonlinear multiagent systems with parameter uncertainty under both fixed and switching topologies. When the topology is fixed, we first propose a novel distributed fusion least-squares algorithm without regressor filtering, which has a clear advantage that the estimate of each agent converges to the true parameter value under a weak cooperative persistent excitation condition. Then, we design a new adaptive consensus tracking control law to guarantee that each agent can asymptotically track the reference trajectory. After that, we generalize the corresponding results to the switching topology case. Finally, two examples are given to demonstrate the theoretical results.

Key words. nonlinear multiagent system, fusion least-squares, adaptive consensus tracking, cooperative persistent excitations

MSC codes. 93C10, 93C40

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1. Introduction.

With the rapid development of computer science and information technologies, control systems evolve from single-agent systems to large-scale networked multiagent systems. The most common control task of multiagent systems is consensus control, focusing on how each agent of multiagent systems keeps its state consistent with others through local communication and decision making. The early studies on consensus control are focused on multiagent systems with exactly known structures (see [1, 8, 16, 20, 29, 40]). Recently, the wide existence of model uncertainties and nonlinear terms in real control systems motivates the exploration of the adaptive consensus control on nonlinear systems. In fact, the adaptive consensus control is full of challenges since it is not just a simple combination of distributed identification and consensus control.

For the adaptive consensus control problem, the primary concern is how to deal with intrinsic model uncertainty such as parameter uncertainty. In general, there are two common schemes, which are direct scheme and indirect scheme, to cope with parameter uncertainty. Most of the existing works about the adaptive consensus control are the direct schemes (see [7, 17, 27, 30, 31, 37]), where the system models are parameterized regarding controller parameters estimated directly without system parameter estimation. For example, Chen et al. in [7] solved the adaptive consensus problem of nonlinear multiagent systems with nonidentical partially unknown control directions by constructing a new Nussbaum function. Huang et al. in [17] investigated

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the adaptive consensus control problem for a class of high-order nonlinear systems with different unknown control directions under the directed graph. Wang, Wen, and Huang in [31] studied the output consensus problem for a class of systems consisting of multiple nonlinear subsystems with intrinsic mismatched unknown parameters. In addition, there are some studies on the adaptive consensus control with indirect schemes (see [6, 23, 24]), in which system parameters are estimated online to calculate controller parameters. Compared with the direct scheme, the indirect scheme can fully explore the internal structure of systems to enhance the overall stability and robustness properties of adaptive control systems. It is worth mentioning that the least-squares (LS) algorithm is an attractive indirect adaptive identification method, since it can adjust adaptive rates online so that all parametric estimates converge approximately with the same speed, resulting in performance and robustness advantages (see [5, 19]). Besides, the LS algorithm has the advantage of averaging out the effect of measurement noises, resulting in less sensitivity to noisy measurement (see [26]).

In recent years, LS-based adaptive control has received significant attention by numerous researchers (see [2, 3, 9, 21, 32]). For instance, Chowdhary and Johnson in [9] presented an approach for combining standard recursive LS-based regression with direct model reference adaptive control by a recursively updated modification term, and this approach was effective in solving adaptive control problems whose uncertainty can be linearly parameterized. In [21], Li and Krstic proposed a new adaptive controller based on a novel LS identification scheme without regressor filtering for stochastic strict-feedback nonlinear systems with unknown parameters. Wang and Liu in [32] developed a data-based output feedback control method for a class of nonlinear systems with unknown mathematical models, where the LS method was used to estimate the corresponding Jacobian matrices. However, all of the above works focus on the LS-based adaptive control of single systems. Due to the good performance of LS-based adaptive control in single systems, we investigate the adaptive consensus tracking control problem of nonlinear multiagent systems based on the distributed LS algorithm in this paper.

In contrast to the previous works, the contributions of this paper include the following:

- This paper is concerned with the LS-based adaptive consensus tracking problem for nonlinear multiagent systems with parameter uncertainty under both fixed and switching topologies. Different from [35], we do not use additional information transmissions such as local control inputs and local neighborhood consensus errors among connected agents.
- A novel distributed fusion LS algorithm is proposed without regressor filtering. On one hand, although the distributed identification scheme is motivated by the discrete-time distributed LS algorithm in [34], the scheme of this paper is not a trivial generalization from discrete-time systems [34] to continuous-time systems. The essential difference between these two algorithms is that the fusion LS does not use the information of differentials of system states, while the differentials of system states are required to be measurable if generalizing [34] to continuous-time systems. On the other hand, a cooperative persistent excitation (PE) condition, weaker than the case that each agent satisfies the traditional PE condition in [18], is proposed for the fusion LS algorithm to guarantee that the estimate converges to the true parameter.
- Besides, a new adaptive consensus tracking law is designed based on the backstepping techniques. Specifically, the adaptive tracking law designed in this paper guarantees that the equilibrium of the closed-loop system is

globally stable and can make each agent track asymptotically the reference trajectory. Compared with [1, 10], this adaptive tracking law is also effective for the systems with parameter uncertainty.

- Different from the direct adaptive consensus schemes in the existing results (see [7, 17, 27, 30, 31, 37]), this paper adopts the indirect schemes to achieve adaptive consensus tracking. The advantage of our indirect design is that it can guarantee the convergence of estimates in addition to achieving the asymptotic consensus tracking. The difficulty in our paper is how to design the estimator and controller that can simultaneously estimate the true parameter and track the reference trajectory.

The remainder of this paper is organized as follows. Section 2 introduces some preliminaries and describes the problem to be studied. Section 3 focuses on the adaptive consensus tracking with fixed topology. Section 4 generalizes the results in section 3 to the switching topology case. Section 5 uses two numerical examples to demonstrate the main results. Section 6 gives some concluding remarks. Appendices A and C collect some useful tools and proofs of lemmas.

2. Preliminaries and problem formulation.

In this section, we first give some basic concepts in matrix and graph theory, and then formulate the adaptive consensus tracking problem to be studied.

2.1. Basic concepts.

We use $x \in \mathbb{R}^n$ and $A \in \mathbb{R}^{m\times n}$ to denote n-dimensional vector and $m \times n$-dimensional real matrix, respectively. Moreover, we denote $\|x\| = \|x\|_2$ and $\|A\| = \left(\lambda_{\max}(AA^T)\right)^{\frac{1}{2}}$ as the Euclidean norm of vector and matrix, respectively, where the notation T denotes the transpose operator, and $\lambda_{\max}(\cdot)$ denotes the largest eigenvalue of the matrix. Correspondingly, $\lambda_{\min}(\cdot)$ denotes the smallest eigenvalue of the matrix. For symmetric matrices $A \in \mathbb{R}^{m\times m}$ and $B \in \mathbb{R}^{m\times m}$, $A \geq B$ represents that $A-B$ is a positive semidefinite matrix. The Kronecker product of the matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$ is defined as $A\otimes B \in \mathbb{R}^{mp\times nq}$. $\mathrm{Tr}\{A\} = \sum_{i=1}^m a_{ii}$ is denoted as the trace of matrix $A = \{a_{ij}\} \in \mathbb{R}^{m\times m}$. Obviously, if A is positive semidefinite matrix, then $\mathrm{Tr}\{A\} \geq \|A\|$. $\mathrm{col}\{\cdot\}$ denotes the vector stacked by the specified vectors, and $\mathrm{diag}\{\cdot\}$ denotes the block matrix formed in a diagonal manner of the corresponding vectors or matrices.

In order to describe the relationship between the agents, an undirected weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is introduced here, where $\mathcal{V} = \{1, 2, \ldots, m\}$ is the set of the agents, $\mathcal{E} \in \mathcal{V}\times\mathcal{V}$ is the edge set describing the communication between the agents, and $\mathcal{A} = \{a_{ij}\} \in \mathbb{R}^{m\times m}$ is the weighted adjacency matrix. The elements of the matrix $\mathcal{A}$ satisfy $0 < a_{ij} < 1$ if $(i, j) \in \mathcal{E}$, $a_{ij} = a_{ji}$ and $\sum_{j=1}^m a_{ij} = 1$ for all $i = 1, \ldots, m$. It is obvious that $\mathcal{A}$ is doubly stochastic. Besides, the Laplacian matrix is defined as $\mathcal{L} = I - \mathcal{A}$, and the neighbor set of the agent i is denoted as $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$ and each agent can only exchange information with its neighbors. Moreover, a path of length ℓ is a sequence of nodes $\{i_1, \ldots, i_\ell\}$ satisfying $(i_j, i_{j+1}) \in \mathcal{E}$ for all $1 \leq j \leq \ell-1$. The graph $\mathcal{G}$ is called connected if for any two agents i and j, there is a path connecting them. The diameter $D(\mathcal{G})$ of the graph $\mathcal{G}$ is defined as the maximum length of the path between any two agents.

2.2. Problem formulation.

Consider a multiagent network consisting of m agents; the dynamic of the ith ($i = 1, \ldots, m$) agent is described by

$$\begin{cases} \dot{x}_{i,j} = x_{i,j+1}, \quad j = 1, \ldots, n-1, \\ \dot{x}_{i,n} = u_i + f_i(x_i) + g_i^T(x_i)\theta, \end{cases} \tag{2.1}$$

where $x_i = [x_{i,1}, \dots, x_{i,n}]^T \in \mathbb{R}^n$ and $u_i \in \mathbb{R}$ are the state and input of agent i, respectively. Besides, $\theta \in \mathbb{R}^p$ is an unknown system parameter. The nonlinear function $f_i(\cdot) : \mathbb{R}^n \to \mathbb{R}$ and $g_i(\cdot) : \mathbb{R}^n \to \mathbb{R}^p$ are continuous and locally Lipschitz. Set $y_r(t) \in \mathbb{R}$ as the reference signal. Then, we give an assumption on the reference signal.

Assumption 2.1. The reference signal y_r is n times differentiable, and there exists a constant M such that $\left|y_r^{(j)}\right| \le M$, $j = 0, 1, \dots, n$, where $y_r^{(0)} = y_r$.

The goal of this paper is to design an adaptive consensus tracking law to achieve all agents' asymptotic tracking. In other words, we design the input u_i to make the state x_i track the reference trajectory $[y_r, \dot{y}_r, \dots, y_r^{(n-1)}]^T$ for all $i = 1, 2, \dots, m$. To achieve this goal, we first design a distributed adaptive estimation algorithm to estimate the unknown parameter. Then, we design an adaptive consensus tracking law to track the reference trajectory based on the parameter estimation. Moreover, the adaptive consensus tracking problem is considered under two topologies in this paper. One is the fixed topology, and the other is the switching topology.

Remark 2.2. Actually, model uncertainties inevitably exist in almost all control problems and there are nonlinear terms of one sort or another in most of the physical systems [26]. Thus, the adaptive consensus control has become a new hot-spot issue in the control field, which has widespread applications in plentiful fields such as mobile robot networks (see [11, 25, 39]), intelligent transportation management [36], and sensor networks [38]. The above-mentioned facts motivate us to investigate the adaptive consensus tracking control problem of nonlinear multiagent systems with parameter uncertainty.

3. The fixed topology. In this section, we consider the adaptive consensus tracking control problem for strict-feedback nonlinear multiagent systems with parameter uncertainty under the fixed topology. To proceed further, we need the following assumption on the topology.

Assumption 3.1. The network graph $\mathcal{G}$ is connected, and the information of the reference signal y_r is available to at least one of the m agents.

Remark 3.2. Assumption 3.1 naturally avoids isolated nodes in the network. Moreover, the connectivity of the topology guarantees all agents can track the reference signal when the reference trajectory is only available to some of the agents.

3.1. Parameter estimation. Based on the diffusion strategy of neighbor estimates and covariances of nonlinear regressors, a distributed LS algorithm is proposed.

ALGORITHM 3.1: FUSION LS ALGORITHM.

For any given agent $i \in \{1, 2, \dots, m\}$, begin with an initial estimate $\bar{\theta}_i(0) \in \mathbb{R}^p$, an initial value $\alpha_i(0) \in \mathbb{R}^p$, and an initial positive definite matrix $\overline{\Gamma}_i(0) \in \mathbb{R}^{p\times p}$; the algorithm is recursively defined as follows:

1. Estimation (generate $\bar{\theta}_i$ by the own information x_i, f_i, g_i, and u_i):

$$\tag{3.1}
\begin{cases}
\bar{\theta}_i(t) = \alpha_i + \overline{\Gamma}_i \displaystyle\int_0^{x_{i,n}} g_i(x_{i,1}, \dots, x_{i,n-1}, \sigma) d\sigma, \\
\dot{\overline{\Gamma}}_i(t) = -\overline{\Gamma}_i g_i g_i^T \overline{\Gamma}_i, \\
\dot{\alpha}_i(t) = -\overline{\Gamma}_i g_i g_i^T \alpha_i - \overline{\Gamma}_i g_i (u_i + f_i) \\
\qquad\qquad - \overline{\Gamma}_i \displaystyle\sum_{j=1}^{n-1} x_{i,j+1} \int_0^{x_{i,n}} \frac{\partial g_i(x_{i,1}, \dots, x_{i,n-1}, \sigma)}{\partial x_{i,j}} d\sigma.
\end{cases}
$$

2. Fusion (generate $\hat{\theta}_i(t)$ by a convex combination of $\overline{\Gamma}_j(t)$ and $\bar{\theta}_j(t)$ for $j \in \mathcal{N}_i$).

(1) For each agent i, set the initial value at time t:

$$\Gamma_{0,i}(t) = \overline{\Gamma}_i(t), \quad \xi_{0,i}(t) = \overline{\Gamma}_i^{-1}(t)\bar{\theta}_i(t). \tag{3.2}$$

(2) Perform the following diffusion process for $l = 1, 2, \ldots, d$, where $d \geq D(\mathcal{G})$:

$$\Gamma_{l,i}^{-1}(t) = \sum_{j \in \mathcal{N}_i} a_{ij}\Gamma_{l-1,j}^{-1}(t), \quad \xi_{l,i}(t) = \sum_{j \in \mathcal{N}_i} a_{ij}\xi_{l-1,j}(t). \tag{3.3}$$

(3) For each agent i, update the estimate $\hat{\theta}_i(t)$ as follows:

$$\hat{\theta}_i(t) = \Gamma_{d,i}(t)\xi_{d,i}(t). \tag{3.4}$$

Remark 3.3. The proposed fusion LS estimator (3.1)–(3.4) in Algorithm 3.1 keeps the same good characteristics as the traditional single-agent case in [19]. Specifically, the time derivative from the parametric model is absorbed into the estimator; thus the need for filtering is removed. Different from [19], we design a new fusion item (3.2)–(3.4) based on the diffusion strategy, whose analogous fusion method has been studied in [4, 12, 22]. Moreover, this fusion method can make up for the shortages of each agent's performance through information exchange to some extent (see Remark 3.13 for details). It is worth noticing that although the fusion item (3.2)–(3.4) is inspired by the discrete-time distributed algorithm [34], there is an essential difference between the two algorithms, which is that the fusion LS does not utilize the differential information of system states but that are required to be measurable if generalizing [34] to continuous-time systems.

Actually, the fusion item does not have to proceed all the time because the estimation item does not depend on it. It implies that the fusion LS algorithm might be used in the sampling and event-triggered adaptive control, i.e., the fusion item is only initiated at the moment of sampling and event-trigger, which can greatly save communication resources and reduce computational complexity. It is a potential advantage of this algorithm.

For convenience of further analysis, we first introduce the following notation: $\Theta = \mathrm{col}\{\theta, \ldots, \theta\} \in \mathbb{R}^{mp\times 1}$, $\bar{\Theta} = \mathrm{col}\{\bar{\theta}_1, \ldots, \bar{\theta}_m\} \in \mathbb{R}^{mp\times 1}$, $\hat{\Theta} = \mathrm{col}\{\hat{\theta}_1, \ldots, \hat{\theta}_m\} \in \mathbb{R}^{mp\times 1}$, $\tilde{\Theta} = \mathrm{col}\{\tilde{\theta}_1, \ldots, \tilde{\theta}_m\} \in \mathbb{R}^{mp\times 1}$ with $\tilde{\theta}_i = \theta - \hat{\theta}_i$, $\overline{\Gamma} = \mathrm{diag}\{\overline{\Gamma}_1, \ldots, \overline{\Gamma}_m\} \in \mathbb{R}^{mp\times mp}$, $\Gamma_d = \mathrm{diag}\{\Gamma_{d,1}, \ldots, \Gamma_{d,m}\} \in \mathbb{R}^{mp\times mp}$, $\mathscr{A}^d = \mathcal{A}^d \otimes I_p \in \mathbb{R}^{mp\times mp}$ with $\mathcal{A}^d = \underbrace{\mathcal{A}\cdot\mathcal{A}\cdot\cdots\cdot\mathcal{A}}_{d} \triangleq \{a_{ij}^{(d)}\}_{m\times m}$, and $\Psi = \mathrm{diag}\{\sum_{j=1}^m a_{1j}^{(d)} g_j g_j^T, \ldots, \sum_{j=1}^m a_{mj}^{(d)} g_j g_j^T\} \in \mathbb{R}^{mp\times mp}$.

Remark 3.4. Since $\mathcal{A}$ is a doubly stochastic matrix, it is easy to see that $\mathcal{A}^d$ is also doubly stochastic, which implies $0 \leq a_{ij}^{(d)} \leq 1$ and $\sum_{j=1}^m a_{ij}^{(d)} = 1$ for $i = 1, \ldots, m$.

LEMMA 3.5. *With the fusion LS estimator* (3.1)–(3.4)*, the following equalities hold for all* $i = 1, \ldots, m$*:*

$$\Gamma_{d,i}^{-1}(t) = \sum_{j=1}^m a_{ij}^{(d)}\overline{\Gamma}_j^{-1}(t), \tag{3.5}$$

$$\hat{\theta}_i(t) = \Gamma_{d,i}(t)\sum_{j=1}^m a_{ij}^{(d)}\overline{\Gamma}_j^{-1}(t)\bar{\theta}_j(t), \tag{3.6}$$

$$\dot{\Gamma}_{d,i} = -\Gamma_{d,i}\sum_{j=1}^m a_{ij}^{(d)} g_j g_j^T \Gamma_{d,i}. \tag{3.7}$$

Proof. The proof is given in Appendix B. ∎

Next, we state the stability results for the fusion LS algorithm.

LEMMA 3.6. *Let the maximal existence interval of solutions of* (2.1) *be* $[0, t_f)$. *Then, with any* $\alpha_i(0) \in \mathbb{R}^p$ *and any positive definite matrix* $\overline{\Gamma}_i(0) \in \mathbb{R}^{p\times p}$ *for* $i \in \{1, \ldots, m\}$, *the functions* $(\hat{\Theta}(t), \Gamma_d(t))$ *generated by the fusion LS algorithm* (3.1)–(3.4) *are bounded.*

Proof. By $\alpha_i = \bar{\theta}_i(t) - \overline{\Gamma}_i \int_0^{x_{i,n}} g_i(x_{i,1}, \ldots, x_{i,n-1}, \sigma)d\sigma$ and (3.1), we have

$$\begin{aligned}\dot{\alpha}_i(t) =& -\overline{\Gamma}_i g_i g_i^T \left(\bar{\theta}_i(t) - \overline{\Gamma}_i \int_0^{x_{i,n}} g_i(x_{i,1}, \ldots, x_{i,n-1}, \sigma)d\sigma\right) - \overline{\Gamma}_i g_i(u_i + f_i)\\ &- \overline{\Gamma}_i \sum_{j=1}^{n-1} x_{i,j+1} \int_0^{x_{i,n}} \frac{\partial g_i(x_{i,1}, \ldots, x_{i,n-1}, \sigma)}{\partial x_{i,j}} d\sigma\\ =& \overline{\Gamma}_i g_i g_i^T \overline{\Gamma}_i \int_0^{x_{i,n}} g_i(x_{i,1}, \ldots, x_{i,n-1}, \sigma)d\sigma - \overline{\Gamma}_i g_i(g_i^T \bar{\theta}_i + u_i + f_i)\\ &- \overline{\Gamma}_i \sum_{j=1}^{n-1} x_{i,j+1} \int_0^{x_{i,n}} \frac{\partial g_i(x_{i,1}, \ldots, x_{i,n-1}, \sigma)}{\partial x_{i,j}} d\sigma. \end{aligned} \tag{3.8}$$

Substituting (3.1) and (3.8) into the derivative of $\bar{\theta}_i(t)$ yields

$$\begin{aligned}\dot{\bar{\theta}}_i(t) =& \dot{\alpha}_i + \dot{\overline{\Gamma}}_i \int_0^{x_{i,n}} g_i(x_{i,1}, \ldots, x_{i,n-1}, \sigma)d\sigma\\ &+ \overline{\Gamma}_i g_i(x_i)\dot{x}_{i,n} + \overline{\Gamma}_i \int_0^{x_{i,n}} \frac{dg_i(x_{i,1}, \ldots, x_{i,n-1}, \sigma)}{dt} d\sigma\\ =& \dot{\alpha}_i - \overline{\Gamma}_i g_i g_i^T \overline{\Gamma}_i \int_0^{x_{i,n}} g_i(x_{i,1}, \ldots, x_{i,n-1}, \sigma)d\sigma + \overline{\Gamma}_i g_i(g_i^T \theta + u_i + f_i)\\ &+ \overline{\Gamma}_i \sum_{j=1}^{n-1} x_{i,j+1} \int_0^{x_{i,n}} \frac{\partial g_i(x_{i,1}, \ldots, x_{i,n-1}, \sigma)}{\partial x_{i,j}} d\sigma\\ =& \overline{\Gamma}_i g_i g_i^T \overline{\Gamma}_i \int_0^{x_{i,n}} g_i(x_{i,1}, \ldots, x_{i,n-1}, \sigma)d\sigma - \overline{\Gamma}_i g_i(g_i^T \bar{\theta}_i + u_i + f_i)\\ &- \overline{\Gamma}_i \sum_{j=1}^{n-1} x_{i,j+1} \int_0^{x_{i,n}} \frac{\partial g_i(x_{i,1}, \ldots, x_{i,n-1}, \sigma)}{\partial x_{i,j}} d\sigma\\ &- \overline{\Gamma}_i g_i g_i^T \overline{\Gamma}_i \int_0^{x_{i,n}} g_i(x_{i,1}, \ldots, x_{i,n-1}, \sigma)d\sigma + \overline{\Gamma}_i g_i(g_i^T \theta + u_i + f_i)\\ &+ \overline{\Gamma}_i \sum_{j=1}^{n-1} x_{i,j+1} \int_0^{x_{i,n}} \frac{\partial g_i(x_{i,1}, \ldots, x_{i,n-1}, \sigma)}{\partial x_{i,j}} d\sigma\\ =& \overline{\Gamma}_i g_i g_i^T(\theta - \bar{\theta}_i). \end{aligned} \tag{3.9}$$

Noting $\tilde{\theta}_i(t) = \theta - \hat{\theta}_i(t)$, (3.1), (3.6)–(3.7), and (3.9), we get

$$\dot{\hat{\theta}}_i(t) = \dot{\Gamma}_{d,i} \sum_{j=1}^{m} a_{ij}^{(d)} \overline{\Gamma}_j^{-1} \bar{\theta}_j(t) + \Gamma_{d,i} \sum_{j=1}^{m} a_{ij}^{(d)} \dot{\overline{\Gamma}}_j^{-1} \bar{\theta}_j(t) + \Gamma_{d,i} \sum_{j=1}^{m} a_{ij}^{(d)} \overline{\Gamma}_j^{-1} \dot{\bar{\theta}}_j(t)$$

$$
\begin{aligned}
&= -\Gamma_{d,i}\sum_{j=1}^{m} a_{ij}^{(d)} g_j g_j^T \Gamma_{d,i}\sum_{j=1}^{m} a_{ij}^{(d)}\overline{\Gamma}_j^{-1}\bar{\theta}_j(t) + \Gamma_{d,i}\sum_{j=1}^{m} a_{ij}^{(d)} g_j g_j^T\bar{\theta}_j(t)\\
&\quad + \Gamma_{d,i}\sum_{j=1}^{m} a_{ij}^{(d)}\overline{\Gamma}_j^{-1}\overline{\Gamma}_j g_j g_j^T(\theta-\bar{\theta}_j(t))\\
&= -\Gamma_{d,i}\sum_{j=1}^{m} a_{ij}^{(d)} g_j g_j^T\hat{\theta}_i(t) + \Gamma_{d,i}\sum_{j=1}^{m} a_{ij}^{(d)} g_j g_j^T\bar{\theta}_j(t) + \Gamma_{d,i}\sum_{j=1}^{m} a_{ij}^{(d)} g_j g_j^T(\theta-\bar{\theta}_j(t))\\
&= \Gamma_{d,i}\sum_{j=1}^{m} a_{ij}^{(d)} g_j g_j^T\tilde{\theta}_i(t),
\end{aligned}
$$

or, equivalently, $\dot{\tilde{\theta}}_i(t) = -\dot{\hat{\theta}}_i(t) = -\Gamma_{d,i}\sum_{j=1}^{m} a_{ij}^{(d)} g_j g_j^T\tilde{\theta}_i(t)$, which implies

$$\dot{\tilde{\Theta}}(t) = -\dot{\hat{\Theta}}(t) = -\Gamma_d\Psi\tilde{\Theta}. \tag{3.10}$$

From $a_{ij}^{(d)}\in[0,1]$ we have $\sum_{j=1}^{m} a_{ij}^{(d)} g_j g_j \geq 0$ and $\Psi\geq 0$. By (3.7) we obtain

$$\dot{\Gamma}_d^{-1} = \Psi, \tag{3.11}$$

which derives $\Gamma_d^{-1}(t)\geq\Gamma_d^{-1}(0)$. Then, from (3.5) and $\overline{\Gamma}_j(0)>0$ for all $j=1,\ldots,m$, we have $\Gamma_{d,i}^{-1}(0)=\sum_{j=1}^{m} a_{ij}^{(d)}\overline{\Gamma}_j^{-1}(0)>0$, i.e., $\Gamma_d^{-1}(0)>0$. Thus, we get $\Gamma_d^{-1}(t)>0$ and $\Gamma_d(t)>0$.

By (3.7), it can be seen that $\dot{\Gamma}_d\leq 0$, which gives $\Gamma_d(t)\leq\Gamma_d(0)$. Hence,

$$0<\Gamma_d(t)\leq\Gamma_d(0), \tag{3.12}$$

which shows that $\Gamma_d(t)$ is bounded.

Take a Lyapunov function as $V_{\tilde{\Theta}}=\tilde{\Theta}^T\Gamma_d^{-1}\tilde{\Theta}$. Then,

$$
\begin{aligned}
\dot{V}_{\tilde{\Theta}} &= 2\tilde{\Theta}^T\Gamma_d^{-1}\dot{\tilde{\Theta}} + \tilde{\Theta}^T\dot{\Gamma}_d^{-1}\tilde{\Theta}\\
&= -2\tilde{\Theta}^T\Gamma_d^{-1}\Gamma_d\Psi\tilde{\Theta} + \tilde{\Theta}^T\Psi\tilde{\Theta}\\
&= -\tilde{\Theta}^T\Psi\tilde{\Theta}\leq 0. \qquad (3.13)
\end{aligned}
$$

From (3.13) we get $V_{\tilde{\Theta}}(t)\leq V_{\tilde{\Theta}}(0)$, and by $\Gamma_d^{-1}(t)\geq\Gamma_d^{-1}(0)$ we obtain

$$\tilde{\Theta}^T(t)\Gamma_d^{-1}(0)\tilde{\Theta}(t)\leq\tilde{\Theta}^T(t)\Gamma_d^{-1}(t)\tilde{\Theta}(t)\leq\tilde{\Theta}^T(0)\Gamma_d^{-1}(0)\tilde{\Theta}(0),$$

and hence,

$$\left\|\tilde{\Theta}(t)\right\|^2\leq\frac{\lambda_{\max}\left(\Gamma_d^{-1}(0)\right)}{\lambda_{\min}\left(\Gamma_d^{-1}(0)\right)}\left\|\tilde{\Theta}(0)\right\|^2=\frac{\lambda_{\max}\left(\Gamma_d(0)\right)}{\lambda_{\min}\left(\Gamma_d(0)\right)}\left\|\tilde{\Theta}(0)\right\|^2.$$

Thus, $\tilde{\Theta}(t)$ and $\hat{\Theta}(t)$ are bounded.

In summary, the boundedness of $\left(\hat{\Theta}(t),\Gamma_d(t)\right)$ is proved. ∎

3.2. Adaptive consensus tracking control law. In this section, we aim to design a new adaptive consensus tracking law based on neighbor information, which allows each agent to asymptotically track the reference trajectory.

We assume there is a leader node, labeled 0 and described by

$$\begin{cases} x_{0,j} = y_r^{(j-1)}, \quad j = 1, \dots, n-1, \\ x_{0,n} = y_r^{(n-1)}, \end{cases} \tag{3.14}$$

and hence,

$$\begin{cases} \dot{x}_{0,j} = x_{0,j+1}, \quad j = 1, \dots, n-1, \\ \dot{x}_{0,n} = y_r^{(n)}. \end{cases} \tag{3.15}$$

Then, the asymptotic tracking of agent i is achieved if and only if $x_i \to x_0 (t \to \infty)$.

With the aid of the backstepping techniques, we define a new variable $z_i = [z_{i,1}, \dots, z_{i,n}]^T$, $i = 0, 1, \dots, m$, with

$$z_{i,j} = x_{i,j} - \alpha_{i,j}, \quad j = 1, \dots, n, \tag{3.16}$$

where

$$\begin{cases} \alpha_{i,1} = 0, \\ \alpha_{i,2} = -k_1 z_{i,1}, \\ \alpha_{i,j+1} = -k_j z_{i,j} - z_{i,j-1} + \dot{\alpha}_{i,j}, \quad j = 2, \dots, n-1, \end{cases} \tag{3.17}$$

and $k_j > 0$ for all $j = 1, \dots, n-1$. Then, for all $i = 1, \dots, m$ we have

$$\begin{cases} \dot{z}_{i,1} = z_{i,2} - k_1 z_{i,1}, \\ \dot{z}_{i,j} = z_{i,j+1} - k_j z_{i,j} - z_{i,j-1}, \quad j = 2, \dots, n-1, \\ \dot{z}_{i,n} = u_i + f_i(x_i) + g_i^T(x_i)\theta - \dot{\alpha}_{i,n} \end{cases} \tag{3.18}$$

and

$$\begin{cases} \dot{z}_{0,1} = z_{0,2} - k_1 z_{0,1}, \\ \dot{z}_{0,j} = z_{0,j+1} - k_j z_{0,j} - z_{0,j-1}, \quad j = 2, \dots, n-1, \\ \dot{z}_{0,n} = y_r^{(n)} - \dot{\alpha}_{0,n}. \end{cases} \tag{3.19}$$

Remark 3.7. Actually, $\alpha_{i,j}\,(j = 1, \dots, n)$ are a set of virtual controllers, which can make the positive definite Lyapunov function $V_{ij} = \frac{1}{2}\sum_{l=1}^{j}(z_{i,l} - z_{0,l})^2$ tend to zero for all $i = 1, \dots, m$ and $j = 1, \dots, n-1$. Moreover, these virtual controllers and new variables can guarantee that each agent asymptotically tracks the reference trajectory (i.e., $x_i - x_0 \to 0$) only if the nth dimensions of new variables achieve the asymptotic tracking (i.e., $z_{i,n} \to z_{0,n}$), which simplifies the adaptive tracking problem. This point is described in the following lemma.

LEMMA 3.8. *For the system* (3.18), *if*

$$\lim_{t\to\infty}(z_{i,n} - z_{0,n}) = 0 \quad \forall i = 1, \dots, m,$$

then the original system (2.1) *can achieve asymptotic tracking, i.e.,*

$$\lim_{t\to\infty}(x_i - x_0) = 0 \quad \forall i = 1, \dots, m.$$

Proof. The proof is given in Appendix C. ∎

Next, we show the design idea of the adaptive consensus tracking control law. To do so, let $z_{*n} = [z_{1,n}, \ldots, z_{m,n}]^T$, $e_{*n} = [z_{1,n} - z_{0,n}, \ldots, z_{m,n} - z_{0,n}]^T$, and $B = \mathrm{diag}\{b_1, \ldots, b_m\} \in \mathbb{R}^{m\times m}$, where $b_i = 1$ means the agent i can directly obtain the information of the reference signal y_r, and $b_i = 0$ otherwise.

According to Lemma 3.8, we just design the tracking control law u_i to make $z_{i,n} \to z_{0,n}$. It is worth noticing that not all agents can obtain directly the reference signal. Thus, most of the agents have to utilize the state information from their neighbors to track the reference trajectory. Consider $V_{*n} = \frac{1}{2} e_{*n}^T (B + \mathcal{L}) e_{*n}$ and compute the derivative of V_{*n}. Then, the following tracking control law can be proposed:

$$\begin{cases} \bar{u}_i = s_i + \rho_i \,\mathrm{sign}(s_i) - f_i(x_i) + \dot{\alpha}_{i,n} - g_i^T \hat{\theta}_i(t), \\ s_i = \sum_{j\in\mathcal{N}_i} a_{ij}(z_{j,n} - z_{i,n}) + b_i(z_{0,n} - z_{i,n}), \end{cases} \quad i = 1, \ldots, m,$$

where ρ_i is a sufficiently large number making $\rho_i > |\dot{z}_{0,n}|$ for any time. It is noted that the existence of ρ_i is guaranteed by Assumption 2.1, because $\dot{z}_{0,n}$ is a linear function of $y_r, y_r^{(1)}, \ldots, y_r^{(n)}$ from (3.19) and the proof of Lemma 3.8. However, $z_{0,n}$, i.e., the information of x_0, may not be known to each agent. Thus, we estimate ρ_i online as $\dot{\hat{\rho}}_i = \beta_i |s_i|$ with $\beta_i > 0$ for all $i = 1, \ldots, m$.

The adaptive consensus tracking law is summarized as follows.

Algorithm 3.2: Adaptive consensus tracking control law.

$$\begin{cases} u_i = s_i + \hat{\rho}_i \,\mathrm{sign}(s_i) - f_i(x_i) + \dot{\alpha}_{i,n} - g_i^T \hat{\theta}_i(t), \\ \dot{\hat{\rho}}_i = \beta_i |s_i| \end{cases} \tag{3.20}$$

for all $i = 1, \ldots, m$, where $s_i = \sum_{j\in\mathcal{N}_i} a_{ij}(z_{j,n} - z_{i,n}) + b_i(z_{0,n} - z_{i,n})$.

Remark 3.9. It is emphasized that the controller (3.20) in Algorithm 3.2 is realizable though $\dot{\alpha}_{i,n}$ is included. Actually, $\dot{\alpha}_{i,n}$ can be indicated as the function of k_j and $z_{i,j}$ for $j = 1, \ldots, n$ by induction (the proof of this claim is similar to the analysis of (C.3) and so is omitted), so $\dot{\alpha}_{i,n}$ could be expressed as the linear function of $z_{i,j}$ only if the control parameters k_j $(j = 1, \ldots, n)$ are given.

Although the adaptive tracking law in this paper is enlightened by the distributed control laws in [10], there are substantial differences between them. The most essential difference is that the adaptive tracking law (3.20) can deal with the asymptotic tracking problem for the system with parameter uncertainty, while the control law in [10] is only valid for the systems with exactly known structure.

3.3. Main results. In this part, the main results of the adaptive consensus tracking control are established under the fixed topology. For convenience of analysis, let $\Lambda = [\hat{\rho}_1 \,\mathrm{sign}(s_1), \ldots, \hat{\rho}_m \,\mathrm{sign}(s_m)]^T \in \mathbb{R}^{m\times m}$, $G = \mathrm{diag}\{g_1, \ldots, g_m\} \in \mathbb{R}^{mp\times m}$.

In the following theorem, we will state the stability results of the control scheme.

Theorem 3.10. *For the system* (2.1) *with the fusion LS algorithm* (3.1)–(3.4) *and the adaptive tracking law* (3.20), *if Assumptions* 2.1 *and* 3.1 *hold, then the equilibrium* $\tilde{x}_i(t) = 0, \tilde{\Theta}(t) = 0$ *is globally stable. Furthermore,*

$$\lim_{k\to\infty} \tilde{x}_i = 0 \quad \forall i = 1, \ldots, m,$$

which implies that asymptotic tracking is achieved.

Proof. Noting $s_i = \sum_{j\in\mathcal{N}_i} a_{ij}(z_{j,n} - z_{i,n}) - b_i(z_{i,n} - z_{0,n})$ and $\sum_{j=1}^m a_{ij}^{(d)} = 1$, substituting (3.20) into (3.18) yields

$$\begin{aligned}\dot{z}_{i,n} &= u_i + f_i + g_i^T\theta - \dot{\alpha}_{i,n}\\ &= s_i + \hat{\rho}_i \operatorname{sign}(s_i) - f_i(x_i) + \dot{\alpha}_{i,n} - g_i^T\hat{\theta}_i(t) + f_i(x_i) + g_i^T\theta - \dot{\alpha}_{i,n}\\ &= g_i^T\tilde{\theta}_i(t) + \sum_{j\in\mathcal{N}_i} a_{ij}(z_{j,n} - z_{i,n}) - b_i(z_{i,n} - z_{0,n}) + \hat{\rho}_i \operatorname{sign}(s_i).\end{aligned}$$

Then, we have

$$\dot{z}_{*n} = -(B+\mathcal{L})z_{*n} + B\mathbb{1}z_{0,n} + \Lambda + G^T\tilde{\Theta}, \tag{3.21}$$

where $z_{*n} = [z_{1,n}, \ldots, z_{m,n}]^T$ and $\mathbb{1} = [1, 1, \ldots, 1]^T \in \mathbb{R}^m$.

By $e_{*n} = z_{*n} - \mathbb{1}z_{0,n}$, $\mathcal{L}\mathbb{1} = \vec{0} \in \mathbb{R}^m$, and (3.21), we get

$$\begin{aligned}\dot{e}_{*n} &= \dot{z}_{*n} - \mathbb{1}\dot{z}_{0,n} = -(B+\mathcal{L})z_{*n} + (B+\mathcal{L})\mathbb{1}z_{0,n} + \Lambda + G^T\tilde{\Theta} - \mathbb{1}\dot{z}_{0,n}\\ &= -(B+\mathcal{L})e_{*n} - \mathbb{1}\dot{z}_{0,n} + \Lambda + G^T\tilde{\Theta}.\end{aligned} \tag{3.22}$$

From Assumption 3.1 and Lemma A.3, we learn that $B+\mathcal{L}$ is positive definite. We choose the Lyapunov function as

$$V_z = \frac{1}{2}e_{*n}^T(B+\mathcal{L})e_{*n} + \frac{1}{2}\sum_{i=1}^m \beta_i^{-1}(\hat{\rho}_i - \rho_i)^2.$$

Note that

$$\begin{aligned}e_{*n}^T(B+\mathcal{L}) &= z_{*n}^T(B+\mathcal{L}) - z_{0,n}\mathbb{1}^TB = z_{*n}^T\mathcal{L} - (z_{0,n}\mathbb{1} - z_{*n})^TB\\ &= -\left(s_1, \ldots, s_m\right),\\ -e_{*n}^T(B+\mathcal{L})\mathbb{1}\dot{z}_{0,n} &= (s_1, \ldots, s_m)\cdot\begin{pmatrix}\dot{z}_{0,n}\\ \vdots\\ \dot{z}_{0,n}\end{pmatrix} \le \sum_{i=1}^m |s_i||\dot{z}_{0,n}|,\\ e_{*n}^T(B+\mathcal{L})\Lambda &= -\left(s_1, \ldots, s_m\right)\cdot\begin{pmatrix}\hat{\rho}_1\operatorname{sign}(s_1)\\ \vdots\\ \hat{\rho}_m\operatorname{sign}(s_m)\end{pmatrix} = -\sum_{i=1}^m \hat{\rho}_i|s_i|.\end{aligned}$$

Then, by (3.22), $\dot{\hat{\rho}}_i = \beta_i|s_i|$, and $\rho_i > |\dot{z}_{0n}|$, we get

$$\begin{aligned}\dot{V}_z &= e_{*n}^T(B+\mathcal{L})\dot{e}_{*n} + \sum_{i=1}^m \beta_i^{-1}(\hat{\rho}_i - \rho_i)\dot{\hat{\rho}}_i\\ &= -e_{*n}^T(B+\mathcal{L})^2e_{*n} - e_{*n}^T(B+\mathcal{L})\mathbb{1}\dot{z}_{0,n} + e_{*n}^T(B+\mathcal{L})\Lambda\\ &\quad + e_{*n}^T(B+\mathcal{L})G^T\tilde{\Theta} + \sum_{i=1}^m \beta_i^{-1}(\hat{\rho}_i - \rho_i)\cdot\beta_i|s_i|\\ &\le -e_{*n}^T(B+\mathcal{L})^2e_{*n} + e_{*n}^T(B+\mathcal{L})G^T\tilde{\Theta}\\ &\quad - \sum_{i=1}^m |s_i|(\hat{\rho}_i - |\dot{z}_{0,n}|) + \sum_{i=1}^m |s_i|(\hat{\rho}_i - \rho_i)\\ &\le -e_{*n}^T(B+\mathcal{L})^2e_{*n} + e_{*n}^T(B+\mathcal{L})G^T\tilde{\Theta} - \sum_{i=1}^m |s_i|(\rho_i - |\dot{z}_{0,n}|)\\ &\le -e_{*n}^T(B+\mathcal{L})^2e_{*n} + e_{*n}^T(B+\mathcal{L})G^T\tilde{\Theta}.\end{aligned} \tag{3.23}$$

Combining Lemma 8.1.2 in [13] with Assumption 3.1 yields $0 < a_{ij}^{(d)} < 1$ for all $d \geq D(\mathcal{G})$. Then, we have

$$a_{\min} \triangleq \min_{1\leq i,j\leq m} \left\{ a_{ij}^{(d)} \right\} > 0. \tag{3.24}$$

Substituting (3.24) into Ψ gives

$$\Psi = \operatorname{diag}\left\{ \sum_{j=1}^{m} a_{1j}^{(d)} g_j g_j^T, \ldots, \sum_{j=1}^{m} a_{mj}^{(d)} g_j g_j^T \right\} \geq a_{\min} \operatorname{diag}\left\{ \sum_{j=1}^{m} g_j g_j^T, \ldots, \sum_{j=1}^{m} g_j g_j^T \right\}.$$

Thus,

$$\frac{1}{a_{\min}} \Psi - GG^T \geq 0. \tag{3.25}$$

From Assumption 3.1 and Lemma A.3, it can be seen that

$$\gamma_1 \triangleq \lambda_{\min}\left((B + \mathcal{L})^2 \right) > 0. \tag{3.26}$$

Let $V = \frac{1}{a_{\min}} V_{\tilde{\Theta}} + V_z$. From (3.13), (3.23), (3.25), and (3.26) we get

$$\begin{aligned}
\dot{V} \leq & - \frac{1}{a_{\min}} \tilde{\Theta}^T \Psi \tilde{\Theta} - e_{*n}^T (B + \mathcal{L})^2 e_{*n} + e_{*n}^T (B + \mathcal{L}) G^T \tilde{\Theta} \\
\leq & - \frac{1}{2a_{\min}} \tilde{\Theta}^T \Psi \tilde{\Theta} - \frac{1}{2} e_{*n}^T (B + \mathcal{L})^2 e_{*n} - \frac{1}{2} \tilde{\Theta}^T \left(\frac{1}{a_{\min}} \Psi - GG^T \right) \tilde{\Theta} \\
& - \frac{1}{2} [(B + \mathcal{L}) e_{*n} - G^T \tilde{\Theta}]^T [(B + \mathcal{L}) e_{*n} - G^T \tilde{\Theta}] \\
\leq & - \frac{1}{2a_{\min}} \tilde{\Theta}^T \Psi \tilde{\Theta} - \frac{1}{2} e_{*n}^T (B + \mathcal{L})^2 e_{*n} \leq -\frac{1}{2} \gamma_1 e_{*n}^T e_{*n} \leq 0.
\end{aligned}$$

This establishes the global stability.

By $\dot{V} \leq 0$, we know $V(t) \leq V(0)$. This gives that $V(t)$ is bounded, so e_{*n} is bounded and square integrable. By Assumption 3.1 and (3.14), x_0 is bounded. Similar to (C.3) in Appendix C, by (3.16)–(3.17) and induction we can obtain $\dot{\alpha}_{0,j}$ and z_0 is bounded. Hence z_{*n} is bounded from $z_{*n} = e_{*n} + \mathbb{1} z_{0,n}$, implying that $z_{i,n}$ is bounded for all $i = 1, \ldots, m$. Then, by $s_i = \sum_{j\in\mathcal{N}_i} a_{ij}(z_{j,n} - z_{i,n}) + b_i(z_{0,n} - z_{i,n})$, we obtain that s_i is bounded. By (3.19) and the boundedness of $\dot{\alpha}_{0,n}$, $\dot{z}_{0,n}$ is bounded. Moreover, ρ_i is bounded, so Λ is bounded.

Next, we show that G is bounded, i.e., $g_i(x_i)$ is bounded for all $i = 1, \ldots, m$. At first, we obtain that $e_{i,n} = z_{i,n} - z_{0,n}$ is bounded since e_{*n} is bounded. Then, from (C.2) in Appendix C we get $V_i = \frac{1}{2}(e_{i,1}^2 + e_{i,2}^2 + \cdots + e_{i,n-1}^2)$ is bounded, which implies $e_{i,j}$ is bounded. Hence $z_{i,j}$ is bounded. By (3.16)–(3.17) and induction, we obtain that $x_{i,j}$ is bounded, which implies x_i is bounded. Noting that $g_i(x_i)$ is locally Lipschitz on $\mathbb{R}^n$, we conclude that $g_i(x_i)$ is bounded for all $i = 1, \ldots, m$. This completes that the proof that G is bounded.

By Lemma 3.6, we obtain that $\tilde{\Theta}$ is bounded. Based on the above arguments and (3.22), we get that $\dot{e}_{*n}$ is bounded. By the Barbalat lemma we get $\lim_{t\to\infty} e_{*n} = 0$, which shows that $\lim_{t\to\infty} z_{i,n} = z_{0,n}$ for all $i = 1, \ldots, m$. Finally, from Lemma 3.8 we have $\lim_{t\to\infty} x_i = x_0$ for all $i = 1, \ldots, m$.

This completes the proof. ∎

Next, the following theorem describes the convergence properties of the fusion LS algorithm (3.1)–(3.4).

THEOREM 3.11. *If Assumptions* 2.1 *and* 3.1 *hold, then the estimate given by the fusion LS algorithm* (3.1)–(3.4) *has the convergence property with the adaptive tracking law* (3.20)*. Specifically,* $\hat{\Theta}(t)$ *converges to a constant vector. Furthermore, we have*

$$\left\|\tilde{\Theta}(t)\right\|^2 = O\left(\frac{1}{\lambda_{\min}\left(\Gamma_d^{-1}(t)\right)}\right),$$

and $\lim_{t\to+\infty}\hat{\Theta}(t) = \Theta$ *on* $\mathbb{S} = \left\{\lim_{t\to+\infty}\lambda_{\min}\left(\Gamma_d^{-1}(t)\right) = \infty\right\}$*, where* O *is the asymptotic notation.*

Proof. From (3.5) and $\overline{\Gamma}_i(0) > 0$, we have $\Gamma_{d,i}^{-1}(0) = \sum_{j=1}^m a_{ij}^{(d)}\overline{\Gamma}_j^{-1}(0) > 0$, implying $\Gamma_d(0) \triangleq \text{diag}\{\Gamma_{d,1}(0),\ldots,\Gamma_{d,m}(0)\} > 0$. Then, from (3.7) and (3.10) we get $\dot{\Gamma}_d = -\Gamma_d\Psi\Gamma_d, \Gamma_d(0) = \Gamma_d^T(0) > 0$, and $\dot{\tilde{\Theta}} = -\Gamma_d\Psi\tilde{\Theta}$, which implies

$$\tilde{\Theta}(t) = \tilde{\Theta}(0) - \int_0^t \Gamma_d\Psi\tilde{\Theta}ds, \tag{3.27}$$

and $\int_0^t \Gamma_d\Psi\Gamma_d ds = \Gamma_d(0) - \Gamma_d(t) \le \Gamma_d(0)$. From $\Gamma_d > 0$, $\Psi \ge 0$, and Theorem 1.1.6 in [14], we have $\Gamma_d\Psi\Gamma_d \ge 0$, implying $\text{Tr}\{\Gamma_d\Psi\Gamma_d\} \ge \|\Gamma_d\Psi\Gamma_d\|$. Then, we get

$$\int_0^t \|\Gamma_d\Psi\Gamma_d\|\, ds \le \int_0^t \text{Tr}\left\{\Gamma_d\Psi\Gamma_d\right\} ds \le \text{Tr}\left\{\Gamma_d(0)\right\}. \tag{3.28}$$

Next, we will prove $\int_0^t \Gamma_d\Psi\tilde{\Theta}ds < \infty$. By (3.13) and $V_{\tilde{\Theta}}(t) \le V_{\tilde{\Theta}}(0)$, $\int_0^t \tilde{\Theta}^T\Psi\tilde{\Theta}ds \le V_{\tilde{\Theta}}(0) - V_{\tilde{\Theta}}(t) \le V_{\tilde{\Theta}}(0)$ is achieved, which, together with (3.28) and Schwarz inequality, gives

$$\begin{aligned}\left\|\int_0^\infty \Gamma_d\Psi\tilde{\Theta}ds\right\| &\le \int_0^\infty \left\|\Gamma_d\Psi\tilde{\Theta}\right\| ds \le \left(\int_0^\infty \left\|\Gamma_d\Psi^{\frac{1}{2}}\right\|^2 ds\right)^{\frac{1}{2}}\left(\int_0^\infty \left\|\Psi^{\frac{1}{2}}\tilde{\Theta}\right\|^2 ds\right)^{\frac{1}{2}} \\ &= \left(\int_0^\infty \|\Gamma_d\Psi\Gamma_d\|ds\right)^{\frac{1}{2}}\left(\int_0^\infty \tilde{\Theta}^T\Psi\tilde{\Theta}ds\right)^{\frac{1}{2}} \le V_{\tilde{\Theta}}(0)\cdot\text{Tr}\{\Gamma_d(0)\} < \infty.\end{aligned}$$

Thus, by (3.27), there exists a constant vector Θ_0 such that $\lim_{t\to\infty}\hat{\Theta}(t) = \Theta_0$. By (3.12) we obtain that $\Gamma_d(t)$ is positive semidefinite and $\left\|\Gamma_d^{\frac{1}{2}}(t)\right\|^2 = \|\Gamma_d(t)\|$. From (3.13) we have $V_{\tilde{\Theta}}(t) = \tilde{\Theta}^T\Gamma_d^{-1}\tilde{\Theta} \le V_{\tilde{\Theta}}(0)$, implying that $\sup_{t\ge0}\left\|\Gamma_d^{-\frac{1}{2}}\tilde{\Theta}\right\|^2 < \infty$. Hence, $\left\|\tilde{\Theta}(t)\right\|^2 \le \|\Gamma_d(t)\|\left\|\Gamma_d^{-\frac{1}{2}}\tilde{\Theta}\right\|^2 = O\left(\|\Gamma_d(t)\|\right) = O\big(\frac{1}{\lambda_{\min}\left(\Gamma_d^{-1}(t)\right)}\big)$.

This completes the proof of this theorem. ∎

From Theorem 3.11, we learn that the estimate given by (3.1)–(3.4) converges to the true parameter if $\lambda_{\min}\left(\Gamma_d^{-1}(t)\right) \to \infty$. Thus, the system nonlinear function g_i is an important factor that affects the algorithm performance. the algorithm performance. Next, we explore the relationship between the system nonlinear function g_i and the convergence of parameter estimation. To proceed, we need the following assumption.

Assumption 3.12 (cooperative PE condition). For the bounded signal $g_i \in \mathbb{R}^p$ $(i = 1,\ldots,m)$, there exist two positive constants T and δ such that

$$\int_t^{t+T}\sum_{i=1}^m g_i(\tau)g_i^T(\tau)d\tau \ge \delta I_p \quad \forall t \ge 0. \tag{3.29}$$

Remark 3.13. In the traditional single-agent case (where m = 1), the cooperative PE condition could reduce to the well-known PE condition in [18].

PE condition: the bounded signal $g(t) \in \mathbb{R}^p$ is of PE if there exist two positive constants T and α such that

$$\int_t^{t+T} g(\tau)g^T(\tau)d\tau \geq \alpha I_p \quad \forall t \geq 0. \tag{3.30}$$

Comparing (3.29) with (3.30), it can be seen that the cooperative PE condition could be satisfied if each agent follows the PE condition. Meanwhile, the cooperative PE condition could be satisfied even if each agent does not meet the PE condition. A simple example is given to show it. For $m = 2$, let $g_1 = [1, 0]^T$ and $g_2 = [0, 1]^T$. It is not difficult to verify that neither g_1 nor g_2 can satisfy the PE condition, but they can cooperate to satisfy the cooperative PE condition. The comparison between the above two conditions also reflects indirectly that the distributed algorithm can make up for the deficiencies of individual performance through information exchange between individuals to some extent.

THEOREM 3.14. *With the adaptive tracking law* (3.20)*, if Assumptions* 2.1*,* 3.1*, and* 3.12 *hold, then the estimate given by the fusion LS algorithm* (3.1)–(3.4) *has the following property:* $\lim_{t\to+\infty} \hat{\theta}_i(t) = \theta$ *for all* $i = 1, \ldots, m$*.*

Proof. Actually, we just need to prove $\lim_{t\to+\infty} \lambda_{\min}\left(\Gamma_d^{-1}(t)\right) = \infty$ according to the conclusion of Theorem 3.11.

From (3.11) we get $\dot{\Gamma}_d^{-1} = \Psi \triangleq \operatorname{diag}\left\{ \sum_{j=1}^m a_{1j}^{(d)} g_j g_j^T, \ldots, \sum_{j=1}^m a_{mj}^{(d)} g_j g_j^T \right\}$, which implies

$$\Gamma_d^{-1}(t) = \Gamma_d^{-1}(0) + \int_0^t \Psi(\tau)d\tau. \tag{3.31}$$

Let $\lfloor x \rfloor = \max\{a \in \mathbb{Z} | a \leq x\}$ for $x \in \mathbb{R}$. Then, by Assumption 3.12 and (3.24) we get $\int_0^t \Psi(\tau)d\tau \geq \sum_{l=1}^{\lfloor \frac{t}{T} \rfloor} \int_{(l-1)T}^{lT} \Psi(\tau)d\tau \geq \sum_{l=1}^{\lfloor \frac{t}{T} \rfloor} a_{\min} \int_{(l-1)T}^{lT} I_m \otimes \sum_{j=1}^m g_j(\tau)g_j^T(\tau)d\tau \geq a_{\min}\delta \left\lfloor \frac{t}{T} \right\rfloor I_{mp}$. This, together with (3.31) and $\Gamma_d^{-1}(0) > 0$, leads to

$$\lambda_{\min}\left(\Gamma_d^{-1}(t)\right) \geq a_{\min}\delta \left\lfloor \frac{t}{T} \right\rfloor. \tag{3.32}$$

Thus, we have $\lim_{t\to+\infty} \lambda_{\min}\left(\Gamma_d^{-1}(t)\right) = \infty$. This completes the proof. ∎

Remark 3.15. It should be emphasized that the fusion LS algorithm (3.1)–(3.4) is not a simple superposition of individual estimation algorithms. This distributed algorithm states that each agent could finish the estimation task through information exchange among agents even if any individual cannot due to lacking suitable excitation. Moreover, this distributed algorithm has fewer requirements on the performance of each system, and the results can be applied to a wider range. Furthermore, the distributed algorithm could broaden the convergence condition from PE condition to cooperative PE condition, which is the reason that the d times fusion has been introduced in the fusion LS algorithm.

Remark 3.16. It is worth noticing that the adaptive tracking control law (3.20) can still achieve the asymptotic tracking when the fusion times of the fusion LS algorithm is $d = 1$ and each agent has a self-loop in the connected topology. Meanwhile, the fusion LS algorithm with $d = 1$ is able to estimate accurately the unknown parameter only if

$$\min_{1\le i\le m}\left\{\lambda_{\min}\left(\sum_{j\in\mathcal{N}_i} a_{ij}\int_t^{t+T} g_j g_j^T d\tau\right)\right\} > 0. \tag{3.33}$$

In this section, we analyze the properties of the fusion LS algorithm and the effect of the adaptive tracking law under fixed topology. However, in most real communication processes, the communication links among the agents often change in time. For example, in the flocking and vehicle formation control, the communication topology depends on the environment of the flocking and the relative positions of the vehicles, which are usually changing over time. Thus, it will be interesting to consider the adaptive asymptotic tracking problem under the switching topology.

4. The switching topology. In this section, the adaptive consensus tracking control problem is studied for strict-feedback nonlinear multiagent systems with parameter uncertainty under the switching topology.

At first, we illustrate some notation for the switching topology. Let $\sigma(t) : [0,\infty) \to \mathfrak{L}_{\mathcal{T}}$ represent a switching signal that determines the communication topology. $\mathcal{T}$ is a set of graphs with a common vertex set $\mathcal{V}$. Because a graph with vertex set $\mathcal{V}$ has $\frac{m(m+1)}{2}$ edges at most, the set $\mathcal{T}$ is finite and is denoted as $\mathcal{T} = \{\mathcal{G}_1,\ldots,\mathcal{G}_N\}$, where N represents the total number of graphs in $\mathcal{T}$ and $\mathfrak{L}_{\mathcal{T}} = \{1,2,\ldots,N\}$. Define switching topology as $\mathcal{G}_{\sigma(t)} = \big(\mathcal{V},\mathcal{E}_{\sigma(t)},\mathcal{A}_{\sigma(t)}\big)$, where $\mathcal{A}_{\sigma(t)} = \{a_{ij}(t)\}$. Then, we have $\mathcal{G}_{\sigma(t)} \in \mathcal{T} = \{\mathcal{G}_1,\ldots,\mathcal{G}_N\}$. Let $\mathcal{N}_i(t)$ be the neighbors set of the agent i at time t, and $B_{\sigma(t)} = \mathrm{diag}\{b_1(t),\ldots,b_m(t)\}$, where $b_i(t) = 1$ represents that agent i can obtain the information of reference signal $y_r(t)$ at time t, otherwise $b(t) = 0$. Then, we obtain that $B_{\sigma(t)} \in \{B_1,\ldots,B_L\}$, where L is finite and represents the total number of matrices B_j. In addition, let $D_{\max} = \max\{D(\mathcal{G}_1),\ldots,D(\mathcal{G}_N)\}$, where $D(\mathcal{G}_j)$ is the diameter of the graph $\mathcal{G}_j \in \mathcal{T}$.

To facilitate analysis, the following assumption of the switching topology is presented.

Assumption 4.1. Graph $\mathcal{G}_{\sigma(t)}$ is connected, and the information of the reference signal $y_r(t)$ is available to at least one of the m agents at time t.

Remark 4.2. More specifically, $\mathcal{G}_{\sigma(t)}$ is connected, meaning $\mathcal{G}_j$ is connected for all $j \in \mathfrak{L}_{\mathcal{T}}$. And the reference signal $y_r(t)$ is available to at least one of the m agents at time t, implying there always exists $j \in \{1,\ldots,m\}$ making $b_j(t) > 0$ at time t.

4.1. Identification and control law. Due to the effect of the switching topology, the estimation algorithm and adaptive tracking law designed in section 3 are invalid in this section, which needs to be recomposed.

We redesign the fusion LS algorithm as follows.

First, the estimate of each agent is the same as (3.1) because this step is independent of the topology structure.

Second, the fusion item is redesigned as follows:

(1) For each $i \in \{1,\ldots,m\}$, set the initial value at time t as

$$\Gamma_{0,i}(t) = \overline{\Gamma}_i(t), \quad \xi_{0,i}(t) = \overline{\Gamma}_i^{-1}(t)\bar{\theta}_i(t), \tag{4.1}$$

where $\overline{\Gamma}_i(t)$ and $\bar{\theta}_i(t)$ are given in (3.1).

(2) Perform the following diffusion process for $l = 1, 2, \ldots, d$ ($d \geq D_{\max}$),

$$\Gamma_{l,i}^{-1}(t) = \sum_{j\in\mathcal{N}_i(t)} a_{ij}\,(t)\,\Gamma_{l-1,i}^{-1}(t), \quad \xi_{l,i}(t) = \sum_{j\in\mathcal{N}_i(t)} a_{ij}\,(t)\,\xi_{l-1,j}(t). \tag{4.2}$$

(3) For each agent i, update the estimate $\hat{\theta}_i(t)$ as

$$\hat{\theta}_i(t) = \Gamma_{d,i}(t)\xi_{d,i}(t). \tag{4.3}$$

Then, we redesign the adaptive tracking law (3.20) as

$$\begin{cases} u_i(t) = s_i(t) + \hat{\rho}_i\,\mathrm{sign}(s_i(t)) - f_i(x_i) + \dot{\alpha}_{i,n}(t) - g_i^T\hat{\theta}_i(t), \\ \dot{\hat{\rho}}_i(t) = \beta_i|s_i(t)| \end{cases} \tag{4.4}$$

for all $i = 1, \ldots, m$, where

$$s_i(t) = \sum_{j\in\mathcal{N}_i(t)} a_{ij}\,(t)\,(z_{j,n} - z_{i,n}) + b_i\,(t)\,(z_{0,n} - z_{i,n}). \tag{4.5}$$

For convenience, let $\Psi_{\sigma(t)} = \mathrm{diag}\left\{\sum_{j=1}^m a_{1j}^{(d)}\,(t)\,g_jg_j^T, \ldots, \sum_{j=1}^m a_{mj}^{(d)}\,(t)\,g_jg_j^T\right\}$, $\mathscr{A}_{\sigma(t)}^d = \mathcal{A}_{\sigma(t)}^d \otimes I_p$ with

$$\mathcal{A}_{\sigma(t)}^d \triangleq \left\{a_{ij}^{(d)}\,(t)\right\}_{m\times m} = \underbrace{\mathcal{A}_{\sigma(t)} \cdot \mathcal{A}_{\sigma(t)} \cdot \cdots \cdot \mathcal{A}_{\sigma(t)}}_{d},$$

and $B_{\sigma(t)} = \mathrm{diag}\,\{b_1\,(t)\,, \ldots, b_m\,(t)\}$. Similar to (3.5)–(3.7), from (4.1)–(4.3) we have $\Gamma_{d,i}^{-1}(t) = \sum_{j=1}^m a_{ij}^{(d)}\,(t)\,\overline{\Gamma}_j^{-1}(t)$ and $\hat{\theta}_i(t) = \Gamma_{d,i}(t)\sum_{j=1}^m a_{ij}^{(d)}\,(t)\,\bar{\Gamma}_j^{-1}(t)\bar{\theta}_j(t)$. From (3.18), (4.5), and $\sum_{j=1}^m a_{ij}^{(d)}(t) = 1$, we obtain

$$\begin{aligned} \dot{z}_{i,n} &= u_i + f_i + g_i^T\theta - \dot{\alpha}_{i,n} = s_i + \hat{\rho}_i\,\mathrm{sign}(s_i) - g_i^T\hat{\theta}_i(t) + g_i^T\theta \\ &= g_i^T\tilde{\theta}_i + \sum_{j\in\mathcal{N}_i(t)} a_{ij}\,(t)\,(z_{j,n} - z_{i,n}) - b_i\,(t)\,(z_{i,n} - z_{0,n}) + \hat{\rho}_i\,\mathrm{sign}(s_i). \end{aligned}$$

Denote $z_{*n} = [z_{1,n}, \ldots, z_{m,n}]^T$; we have

$$\dot{z}_{*n} = -\left(\mathcal{L}_{\sigma(t)} + B_{\sigma(t)}\right) z_{*n} + B_{\sigma(t)}\mathbb{1}z_{0n} + \Lambda + G^T\tilde{\Theta}, \tag{4.6}$$

where $\mathcal{L}_{\sigma(t)} = I - \mathcal{A}_{\sigma(t)}$ is the Laplacian matrix of the switching graph $\mathcal{G}_{\sigma(t)}$.

4.2. Main results. Based on Lemma A.3 and the fact that N and L are finite, under Assumption 4.1 we have

$$\lambda_{\min}\left\{\mathcal{L}_{\sigma(t)} + B_{\sigma(t)}\right\} \geq \mu^* \triangleq \min_{1\leq i\leq N, 1\leq j\leq L} \lambda_{\min}\left\{\mathcal{L}_i + B_j\right\} > 0, \tag{4.7}$$

where $\mathcal{L}_i$ is the Laplacian matrix of Graph $\mathcal{G}_i \in \{\mathcal{G}_1, \ldots, \mathcal{G}_N\}$ and $B_j \in \{B_1, \ldots, B_L\}$.

The main results of this section are stated as follows.

THEOREM 4.3. *For the system* (2.1) *with the fusion LS algorithm* (3.1), (4.1)–(4.3), *and the adaptive tracking law* (4.4)–(4.5), *if Assumptions* 2.1 *and* 4.1 *hold, then the following conclusions hold.*

(1) *Let the maximal existence interval of solutions about the system* (2.1) *be* $[0, t_f)$. *Then, with any* $\alpha_i(0) \in \mathbb{R}^p$ *and any positive definite matrix* $\overline{\Gamma}_i(0) \in \mathbb{R}^{p\times p}$ *for* $i \in \{1,\ldots,m\}$, *the functions* $(\hat{\Theta}(t), \Gamma_d(t))$ *generated by* (3.1), (4.1)–(4.3) *are bounded.*

(2) *The equilibrium* $\tilde{x}_i = 0, \tilde{\Theta} = 0$ *is globally stable. Furthermore,*

$$\lim_{k\to\infty} \tilde{x}_i = 0 \quad \forall i = 1,\ldots,m,$$

which implies that the asymptotic tracking is achieved.

(3) *The estimate generated by* (3.1), (4.1)–(4.3) *has a convergence property, i.e.,* $\hat{\Theta}(t)$ *converges to a constant vector* Θ_0. *Besides,*

$$\left\|\tilde{\Theta}(t)\right\|^2 = O\left(\frac{1}{\lambda_{\min}\left(\Gamma_d^{-1}(t)\right)}\right).$$

Furthermore, if Assumption 3.12 *holds, then the estimate converges to its true parameter,*

$$\lim_{t\to+\infty} \hat{\theta}_i(t) = \theta \quad \forall i = 1,\ldots,m.$$

Proof. Conclusions (1) and (3) can be proved similarly to those in Lemma 3.6, Theorem 3.10, and Theorem 3.14 in section 3, so the detailed proofs are omitted.

Next, we will prove conclusion (2).

Let $e_{*n}(t) = z_{*n}(t) - \mathbb{1}z_{0,n}(t)$. Then, by (4.6) and $\mathcal{L}_{\sigma(t)}\mathbb{1} = \vec{0} \in \mathbb{R}^m$ we get

$$\begin{aligned}\dot{e}_{*n}(t) &= \dot{z}_{*n}(t) - \mathbb{1}\dot{z}_{0,n}(t) + \left(\mathcal{L}_{\sigma(t)} + B_{\sigma(t)}\right)\mathbb{1}z_{0,n} + \Lambda + G^T\tilde{\Theta} - \mathbb{1}\dot{z}_{0,n}\\ &= -\left(\mathcal{L}_{\sigma(t)} + B_{\sigma(t)}\right)e_{*n}(t) - \mathbb{1}\dot{z}_{0,n}(t) + \Lambda + G^T\tilde{\Theta}.\end{aligned} \tag{4.8}$$

From (4.7), we obtain that $\mathcal{L}_{\sigma(t)} + B_{\sigma(t)}$ is positive definite. We choose the Lyapunov function as

$$V_z(t) = \frac{1}{2}e_{*n}^T(t)\left(\mathcal{L}_{\sigma(t)} + B_{\sigma(t)}\right)e_{*n}(t) + \frac{1}{2}\sum_{i=1}^m \beta_i^{-1}(\hat{\rho}_i(t) - \rho_i)^2.$$

Noting that

$$-e_{*n}^T(\mathcal{L}_{\sigma(t)} + B_{\sigma(t)})\mathbb{1}\dot{z}_{0,n} = (s_1,\ldots,s_m)\cdot\begin{pmatrix}\dot{z}_{0,n}\\ \vdots\\ \dot{z}_{0,n}\end{pmatrix} \le \sum_{i=1}^m |s_i||\dot{z}_{0,n}|,$$

$$e_{*n}^T(\mathcal{L}_{\sigma(t)} + B_{\sigma(t)})\Lambda = -(s_1,\ldots,s_m)\cdot\begin{pmatrix}\hat{\rho}_1\,\mathrm{sign}(s_1)\\ \vdots\\ \hat{\rho}_m\,\mathrm{sign}(s_m)\end{pmatrix} = -\sum_{i=1}^m \hat{\rho}_i|s_i|,$$

from (4.4), (4.6), and $\rho_i > |\dot{z}_{0,n}|$ we have

$$\begin{aligned}\dot{V}_z(t) =& e_{*n}^T\left(\mathcal{L}_{\sigma(t)} + B_{\sigma(t)}\right)\dot{e}_{*n} + \sum_{i=1}^m \beta_i^{-1}(\hat{\rho}_i - \rho_i)\dot{\hat{\rho}}_i\\ =& -e_{*n}^T(t)\left(\mathcal{L}_{\sigma(t)} + B_{\sigma(t)}\right)^2 e_{*n}(t) - e_{*n}^T(t)\left(\mathcal{L}_{\sigma(t)} + B_{\sigma(t)}\right)\mathbb{1}\dot{z}_{0,n}(t)\\ &+ e_{*n}^T(t)\left(\mathcal{L}_{\sigma(t)} + B_{\sigma(t)}\right)\Lambda + e_{*n}^T(t)\left(\mathcal{L}_{\sigma(t)} + B_{\sigma(t)}\right)G^T\tilde{\Theta}\\ &+ \sum_{i=1}^m \beta_i^{-1}(\hat{\rho}_i - \rho_i)\beta_i|s_i(t)|\\ \le& -e_{*n}^T(t)\left(\mathcal{L}_{\sigma(t)} + B_{\sigma(t)}\right)^2 e_{*n}(t) + e_{*n}^T(t)\left(\mathcal{L}_{\sigma(t)} + B_{\sigma(t)}\right)G^T\tilde{\Theta}\end{aligned}$$

$$
\begin{aligned}
& -\sum_{i=1}^{m}|s_i(t)|(\hat{\rho}_i(t)-|\dot{z}_{0,n}(t)|)+\sum_{i=1}^{m}|s_i(t)|(\hat{\rho}_i(t)-\rho_i) \\
\leq & -e_{*n}^T(t)\left(\mathcal{L}_{\sigma(t)}+B_{\sigma(t)}\right)^2 e_{*n}(t)+e_{*n}^T(t)\left(\mathcal{L}_{\sigma(t)}+B_{\sigma(t)}\right)G^T\tilde{\Theta} \\
& -\sum_{i=1}^{m}|s_i(t)|\left(\rho_i-|\dot{z}_{0,n}(t)|\right) \\
\leq & -e_{*n}^T(t)\left(\mathcal{L}_{\sigma(t)}+B_{\sigma(t)}\right)^2 e_{*n}(t)+e_{*n}^T(t)\left(\mathcal{L}_{\sigma(t)}+B_{\sigma(t)}\right)G^T\tilde{\Theta}.
\end{aligned} \tag{4.9}
$$

Let $V_{\tilde{\Theta}}(t)=\tilde{\Theta}^T(t)\Gamma_d^{-1}(t)\tilde{\Theta}(t)$. Similar to the proof of Lemma 3.6, we can get $\dot{V}_{\tilde{\Theta}}(t)=-\tilde{\Theta}(t)^T\Psi_{\sigma(t)}\tilde{\Theta}(t)\leq 0$. Noticing Lemma 8.1.2 in [13] and Assumption 4.1, we get $0<a_{ij}^{(d)}(t)<1$ for all $d\geq D_{\max}$. Let $a_{\min}^{\sigma(t)}=\min_{1\leq i,j\leq m}\{a_{ij}^{(d)}(t)\}$. Then, we have $a_{\min}^{\sigma(t)}>0$, which implies $\frac{1}{a_{\min}^{\sigma(t)}}\Psi_{\sigma(t)}-GG^T>0$. Choosing $V(t)=\frac{1}{a_{\min}^{\sigma(t)}}V_{\tilde{\Theta}}(t)+V_z(t)$, from $\Psi_{\sigma(t)}\geq 0$ and (4.7) we get

$$
\begin{aligned}
\dot{V}\leq & -\frac{1}{a_{\min}^{\sigma(t)}}\tilde{\Theta}^T\Psi_{\sigma(t)}\tilde{\Theta}-e_{*n}^T\left(\mathcal{L}_{\sigma(t)}+B_{\sigma(t)}\right)^2 e_{*n}+e_{*n}^T\left(\mathcal{L}_{\sigma(t)}+B_{\sigma(t)}\right)G^T\tilde{\Theta} \\
\leq & -\frac{1}{2a_{\min}^{\sigma(t)}}\tilde{\Theta}^T\Psi_{\sigma(t)}\tilde{\Theta}-\frac{1}{2}e_{*n}^T\left(\mathcal{L}_{\sigma(t)}+B_{\sigma(t)}\right)^2 e_{*n} \\
& -\frac{1}{2}\left[\left(\mathcal{L}_{\sigma(t)}+B_{\sigma(t)}\right)e_{*n}-G^T\tilde{\Theta}\right]^T\left[\left(\mathcal{L}_{\sigma(t)}+B_{\sigma(t)}\right)e_{*n}-G^T\tilde{\Theta}\right] \\
& -\frac{1}{2}\tilde{\Theta}^T\left(\frac{1}{a_{\min}^{\sigma(t)}}\Psi_{\sigma(t)}-GG^T\right)\tilde{\Theta} \\
\leq & -\frac{1}{2a_{\min}^{\sigma(t)}}\tilde{\Theta}^T\Psi_{\sigma(t)}\tilde{\Theta}-\frac{1}{2}e_{*n}^T\left(\mathcal{L}_{\sigma(t)}+B_{\sigma(t)}\right)^2 e_{*n} \\
\leq & -\frac{1}{2}\mu^* e_{*n}^T e_{*n}\leq 0,
\end{aligned}
$$

from which the global stability is established.

By $\dot{V}\leq 0$, we know $V(t)\leq V(0)$, which means $V(t)$ is bounded, so $e_{*n}(t)$ is bounded and square integrable. Similar to the proof of Theorem 3.10, we get that z_0, z_{*n}, $\dot{\alpha}_{0,n}$, $\dot{z}_{0,n}$, and G are bounded. So, by $s_i=\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)(z_{j,n}-z_{i,n})+b_i(t)(z_{0,n}-z_{i,n})$, we conclude that s_i is bounded for all $i=1,\ldots,m$, which implies Λ is bounded.

From conclusion (1), we obtain that $\tilde{\Theta}$ is bounded, which together with (4.7) implies that $\dot{e}_{*n}(t)$ is bounded. By the Barbalat lemma,

$$
\lim_{t\to\infty}e_{*n}(t)=0.
$$

In other words, we get $\lim_{t\to\infty}z_{i,n}(t)=z_{0,n}(t)$ for all $i=1,\ldots,m$. Finally, by Lemma 3.8, the conclusion (2) is proved. ∎

5. Simulation examples. In this section, we give two simulation examples, containing the adaptive consensus tracking control of single-link robot arms, to show the effectiveness of the fusion LS algorithm and the adaptive tracking law developed under both fixed and switching topologies, respectively.

Example 5.1. The network topology is composed of $m=20$ agents whose dynamics obey (2.1) with $n=4$ and $p=5$, where the unknown parameter is $\theta=$

$[3, -1.5, 2, 0, -1]^T$, the nonlinear functions of agent i $(i = 1, \dots, 20)$ are

$$g_i(x_i) = \begin{cases} [x_{i1} + 3, 0, 0, 0, 0]^T & \text{if } \operatorname{mod}(i, 5) = 1; \\ [0, x_{i2} + 3, 0, 0, 0]^T & \text{if } \operatorname{mod}(i, 5) = 2; \\ [0, 0, x_{i3} + 3, 0, 0]^T & \text{if } \operatorname{mod}(i, 5) = 3; \\ [0, 0, 0, x_{i4} + 3, 0]^T & \text{if } \operatorname{mod}(i, 5) = 4; \\ [0, 0, 0, 0, x_{i1} + 3]^T & \text{if } \operatorname{mod}(i, 5) = 0 \end{cases}$$

and

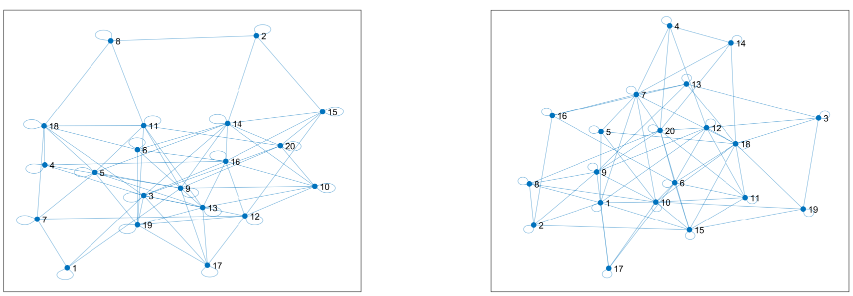
$$f_i(x_i) = \begin{cases} x_{i,1}^2 + x_{i,3}^2 & \text{if } \operatorname{mod}(i, 2) = 1; \\ x_{i,2}^2 + x_{i,4}^2 & \text{if } \operatorname{mod}(i, 2) = 0, \end{cases}$$

where $\operatorname{mod}(p, q) = r$ is that the remainder of p divided by q is r for positive integers r, p, and q. Besides, set the reference signal as $y_r = \sin t$.

The network topology is shown in Figure 1 (a). Here we use the Metropolis rule in [33] to construct the weights, i.e.,

$$a_{ij} = \begin{cases} 1 - \sum_{l \neq i} a_{il} & \text{if } j = i; \\ 1/(\max\{n_i, n_j\}) & \text{if } j \in \mathcal{N}_i/\{i\}, \end{cases} \tag{5.1}$$

where n_i is the degree of the node i, i.e., the number of its neighbors. Besides, only the 2nd and 15th agents can obtain the information of the reference signal y_r.

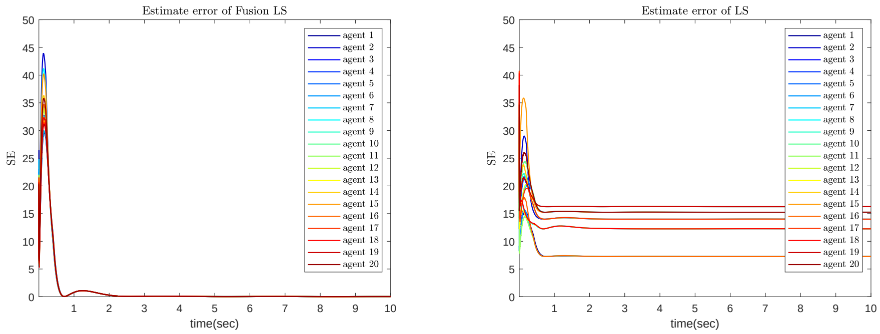


(a) (b)

FIG. 1. *The network topologies.*

It is not difficult to verify that none of nonlinear items g_i $(i = 1, \dots, m)$ of 20 agents can satisfy the PE condition (3.30), but they can cooperate to satisfy the cooperative PE condition (3.29). We carry out the simulation with the control parameters $k_1 = k_2 = k_3 = 1$ and the same initial values.

Figure 2 (a) gives the responses of the fusion LS algorithm, and Figure 2 (b) gives the responses of the noncooperative LS algorithm (i.e., the algorithm (3.1)). It can be seen that the square error (SE) of each agent cannot converge to zero when using the noncooperative LS algorithm to estimate θ. However, the SE of each agent can converge to zero with the fusion LS algorithm, which means that the estimation task can be still fulfilled through exchanging the information between agents even though any individual agent cannot. Besides, Figure 3 gives the responses of all agents' states, which shows that all agents can track asymptotically the reference trajectory.



(a) SE of Fusion LS algorithm (b) SE of LS algorithm

FIG. 2. *The comparison of cooperative and noncooperative algorithms.*

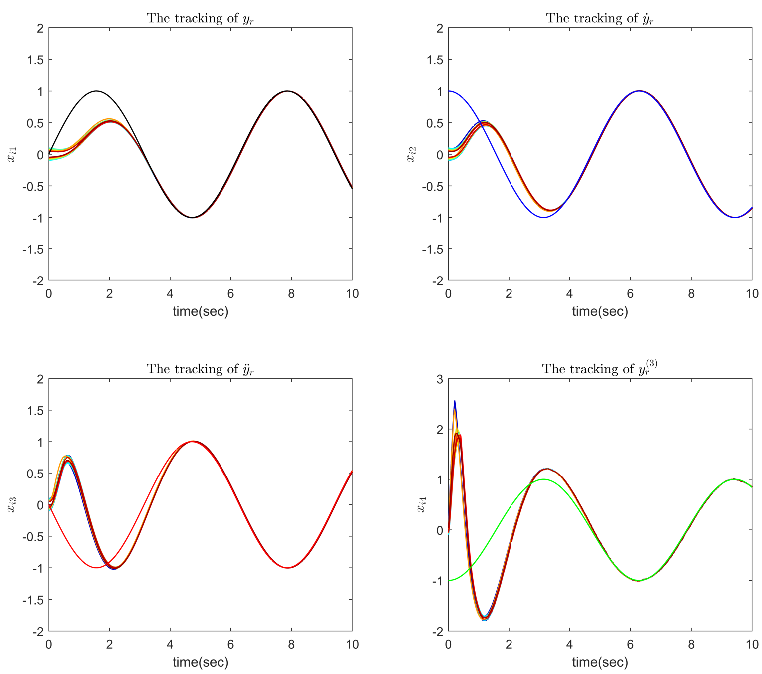


FIG. 3. *The responses of all agents'.*

Example 5.2. The network topology is composed of $m = 20$ agents. Each agent is a single-link robot arm as shown in Figure 4, which consists of a rigid link coupled through a gear train to a dc motor. The dynamic of the single-link robot arm is described as [28]

$$J\ddot{q}_i + B\ddot{q}_i + Mg_0 l \sin(q_i) = u_i, \quad i = 1, \ldots, 20, \tag{5.2}$$

where the state q_i is the angle of the link, J is the total rotational inertias of the link and the motor, B is the overall damping coefficient, M is the total mass of the

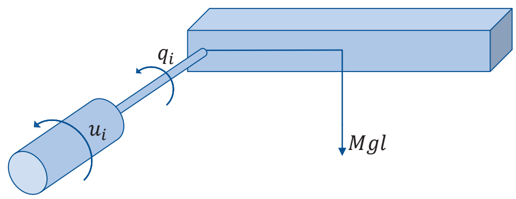


FIG. 4. *Single-link robot arm of the agent i.*

link, g_0 is the gravitational acceleration, and l is the distance from the joint axis to the link center of mass. In this example, $J = 1$, $M = 0.35$, $g_0 = 9.8$, and the follower dynamical coefficients B and l are assumed to be unknown. To obtain a state model for the single-link robot arm, denote $\theta = [B, l]^T$ and take the state variables as $x_{i,1} = q_i$ and $x_{i,2} = \dot{q}_i$. Then, from (5.2) we get the state-space form as

$$\begin{cases} \dot{x}_{i,1} = x_{i,2}, \\ \dot{x}_{i,2} = u_i - [x_{i,2}, 3.43 \sin(x_{i,1})] \cdot \theta, \end{cases} \qquad i = 1, \ldots, 20. \tag{5.3}$$

In addition, set the reference signal as $y_r = \sin t$.

The switching network topologies are shown in Figure 1, where Figure 1 (a) is used in $t \in [2k, 2k+1]$ and Figure 1 (b) is used in $t \in [2k+1, 2k+2]$ for $k = 0, 1, 2, \ldots$. Similarly, here we also use the Metropolis rule in (5.1) to construct the weights. Besides, only the 2nd and 15th agents are able to obtain the information of the reference signal when $t \in [2k, 2k+1]$ and the 4th, 10th, and 16th agents can obtain the information when $t \in [2k+1, 2k+2]$ for $k = 0, 1, 2, \ldots$.

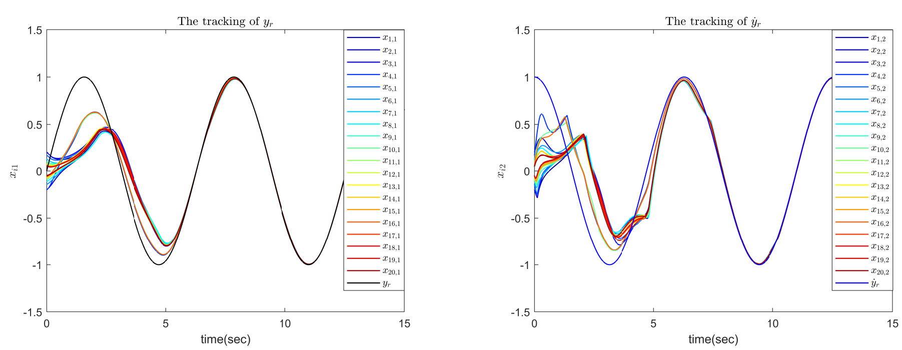
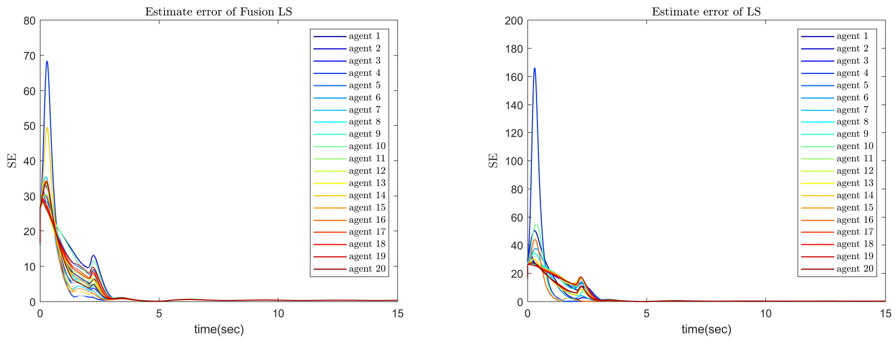


FIG. 5. *The responses of all agents' states.*

We proceed with the simulation by use of the fusion LS algorithm (3.1), (4.1)–(4.3) with $d = 1$ and the adaptive consensus tracking control law (4.4) with the control parameters $k_1 = 2, \beta = 1$, and the same initial values. Then, Figure 5 gives the responses of all agents' states and Figure 6 gives the responses of the fusion LS algorithm and the noncooperative LS algorithm (i.e., the algorithm (3.1)). From Figure 5, we learn that each agent is able to asymptotically track the reference trajectory under the switching topology, which confirms Remark 3.15 to some extent. Besides, Figure 6 shows that both the fusion LS algorithm and LS algorithm can estimate the



(a) SE of Fusion LS algorithm (b) SE of LS algorithm

FIG. 6. *The responses of the cooperative and noncooperative algorithms.*

true parameter even if the fusion LS algorithm only fuses once. It is because all of the nonlinear items g_i of 20 agents can satisfy the PE condition (3.30) and (3.33) in Remark 3.15.

6. Conclusions. In this paper, we have solved the adaptive consensus tracking control problems for strict-feedback nonlinear systems with parameter uncertainty based on the distributed LS algorithm under both fixed and switching topologies. Under the fixed connected topology, we propose a novel fusion LS algorithm without regressor filtering. With such an estimation algorithm, a new adaptive consensus tracking control law is presented based on the backstepping techniques. The adaptive tracking law guarantees the asymptotic tracking ability of each agent. Moreover, the estimate given by the proposed algorithm is convergent to a constant vector. In addition, we show the estimate converges to its true parameter if system nonlinear functions satisfy the cooperative PE condition. Furthermore, the corresponding results are generalized to the switching topology case.

For the LS-based adaptive control of nonlinear multiagent systems, many important issues are still open and worth investigating, such as the LS-based adaptive control under communication limitations or under time-varying directed topologies, the sampling adaptive control, the event-triggered adaptive control, and so on.

Appendix A. Useful tools. In this appendix, three lemmas are collected, which are frequently used in the controller design and stability analysis.

LEMMA A.1 (see [10]). *For the system* (3.18)*, if there is a bounded time-varying function* d_j $(j = 1, \dots, n)$ *such that* $\lim_{t\to\infty}(z_{i,j} - d_j) = 0$ *for all* $i = 1, \dots, m$*, then there is a bounded time-varying function* c_j $(j = 1, \dots, n)$ *such that the original system* (2.1) *is convergent, i.e.,* $\lim_{t\to\infty}(x_{i,j} - c_j) = 0$ *for all* $i = 1, \dots, m$.

LEMMA A.2 (see [10]). *For the system* (3.18)*, if there is a bounded time-varying function* d_n *such that* $\lim_{t\to\infty}(z_{i,n} - d_n) = 0$*, then there is a bounded time-varying function* d_j $(j = 1, \dots, n-1)$ *such that* $\lim_{t\to\infty}(z_{i,j} - d_j) = 0$ *for all* $i = 1, \dots, m$.

LEMMA A.3 (see [15]). *If the graph* $\mathcal{G}$ *is connected, then* $\mathcal{L}+H$ *has* m *positive real eigenvalues, where* $\mathcal{L}$ *is the Laplacian matrix of the graph* $\mathcal{G}$, $H = \operatorname{diag}\{h_1, \dots, h_m\} \geq 0$*, and* $H \neq 0$.

Appendix B. Proof of Lemma 3.5. From (3.3) we can get

$$\begin{pmatrix} \Gamma_{l,1}^{-1}(t) \\ \Gamma_{l,2}^{-1}(t) \\ \vdots \\ \Gamma_{l,m}^{-1}(t) \end{pmatrix} = \begin{pmatrix} a_{11}I_p & a_{12}I_p & \cdots & a_{1m}I_p \\ a_{21}I_p & a_{22}I_p & \cdots & a_{2m}I_p \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}I_p & a_{m2}I_p & \cdots & a_{mm}I_p \end{pmatrix} \cdot \begin{pmatrix} \Gamma_{l-1,1}^{-1}(t) \\ \Gamma_{l-1,2}^{-1}(t) \\ \vdots \\ \Gamma_{l-1,m}^{-1}(t) \end{pmatrix},$$

$$\begin{pmatrix} \xi_{l,1}(t) \\ \xi_{l,2}(t) \\ \vdots \\ \xi_{l,m}(t) \end{pmatrix} = \begin{pmatrix} a_{11}I_p & a_{12}I_p & \cdots & a_{1m}I_p \\ a_{21}I_p & a_{22}I_p & \cdots & a_{2m}I_p \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}I_p & a_{m2}I_p & \cdots & a_{mm}I_p \end{pmatrix} \cdot \begin{pmatrix} \xi_{l-1,1}(t) \\ \xi_{l-1,2}(t) \\ \vdots \\ \xi_{l-1,m}(t) \end{pmatrix},$$

which implies

$$\text{(B.1)} \qquad \operatorname{vec}\left\{\Gamma_l^{-1}(t)\right\} = \mathscr{A} \operatorname{vec}\left\{\Gamma_{l-1}^{-1}(t)\right\}, \quad \xi_l(t) = \mathscr{A}\xi_{l-1}(t),$$

where vec$\{\cdot\}$ denotes the operator that stacks the blocks of a block diagonal matrix on top of each other, $\mathscr{A} = \mathcal{A} \otimes I_n$, $\Gamma_l^{-1}(t) = \operatorname{diag}\left\{\Gamma_{l,1}^{-1}(t), \ldots, \Gamma_{l,m}^{-1}(t)\right\}$, and $\xi_l(t) = \operatorname{col}\left\{\xi_{l,1}(t), \ldots, \xi_{l,m}(t)\right\}$ for $l = 0, 1, \ldots, d$.

By (3.3) and (B.1) we have

$$\text{(B.2)} \qquad \begin{cases} \operatorname{vec}\left\{\Gamma_d^{-1}(t)\right\} = \mathscr{A} \operatorname{vec}\left\{\Gamma_{d-1}^{-1}(t)\right\} = \cdots = \mathscr{A}^d \operatorname{vec}\left\{\overline{\Gamma}^{-1}(t)\right\}, \\ \xi_d(t) = \mathscr{A}\xi_{d-1}(t) = \cdots = \mathscr{A}^d \xi_0(t) = \mathscr{A}^d \overline{\Gamma}^{-1} \bar{\Theta}(t), \end{cases}$$

where the last equalities are obtained by the use of $\Gamma_0^{-1}(t) = \operatorname{diag}\left\{\overline{\Gamma}_1^{-1}(t), \ldots, \overline{\Gamma}_m^{-1}(t)\right\}$ and $\xi_0(t) = \operatorname{col}\left\{\overline{\Gamma}_1^{-1}(t)\bar{\theta}_1(t), \ldots, \overline{\Gamma}_m^{-1}(t)\bar{\theta}_m(t)\right\} = \overline{\Gamma}^{-1}(t)\bar{\Theta}(t)$.

Then, from (3.4) we get

$$\begin{pmatrix} \hat{\theta}_1(t) \\ \hat{\theta}_2(t) \\ \vdots \\ \hat{\theta}_m(t) \end{pmatrix} = \begin{pmatrix} \Gamma_{d,1}(t) & 0 & \cdots & 0 \\ 0 & \Gamma_{d,2}(t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Gamma_{d,m}(t) \end{pmatrix} \cdot \begin{pmatrix} \xi_{d,1}(t) \\ \xi_{d,2}(t) \\ \vdots \\ \xi_{d,m}(t) \end{pmatrix},$$

and by (B.2), $\hat{\Theta}(t) = \Gamma_d(t)\xi_d(t) = \Gamma_d(t)\mathscr{A}^d\overline{\Gamma}^{-1}\bar{\Theta}(t)$. This, together with (B.2), yields

$$\begin{aligned} \begin{pmatrix} \hat{\theta}_1(t) \\ \hat{\theta}_2(t) \\ \vdots \\ \hat{\theta}_m(t) \end{pmatrix} &= \begin{pmatrix} \Gamma_{d,1}(t) & 0 & \cdots & 0 \\ 0 & \Gamma_{d,2}(t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Gamma_{d,m}(t) \end{pmatrix} \\ &\quad \cdot \begin{pmatrix} a_{11}^{(d)}I_p & a_{12}^{(d)}I_p & \cdots & a_{1m}^{(d)}I_p \\ a_{21}^{(d)}I_p & a_{22}^{(d)}I_p & \cdots & a_{2m}^{(d)}I_p \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}^{(d)}I_p & a_{m2}^{(d)}I_p & \cdots & a_{mm}^{(d)}I_p \end{pmatrix} \cdot \begin{pmatrix} \overline{\Gamma}_1^{-1}(t)\bar{\theta}_1(t) \\ \overline{\Gamma}_2^{-1}(t)\bar{\theta}_2(t) \\ \vdots \\ \overline{\Gamma}_m^{-1}(t)\bar{\theta}_m(t) \end{pmatrix} \end{aligned}$$

and

$$\begin{pmatrix} \Gamma_{d,1}^{-1}(t) \\ \Gamma_{d,2}^{-1}(t) \\ \vdots \\ \Gamma_{d,m}^{-1}(t) \end{pmatrix} = \begin{pmatrix} a_{11}^{(d)}I_p & a_{12}^{(d)}I_p & \cdots & a_{1m}^{(d)}I_p \\ a_{21}^{(d)}I_p & a_{22}^{(d)}I_p & \cdots & a_{2m}^{(d)}I_p \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}^{(d)}I_p & a_{m2}^{(d)}I_p & \cdots & a_{mm}^{(d)}I_p \end{pmatrix} \cdot \begin{pmatrix} \overline{\Gamma}_1^{-1}(t) \\ \overline{\Gamma}_2^{-1}(t) \\ \vdots \\ \overline{\Gamma}_m^{-1}(t) \end{pmatrix},$$

Thus, we have $\hat{\theta}_i(t) = \Gamma_{d,i}(t)\sum_{j=1}^m a_{ij}^{(d)}\overline{\Gamma}_j^{-1}(t)\bar{\theta}_j(t)$ and $\Gamma_{d,i}^{-1}(t) = \sum_{j=1}^m a_{ij}^{(d)}\overline{\Gamma}_j^{-1}(t)$ for all $i = 1, \dots, m$, which implies that (3.5) and (3.6) hold.

From $\Gamma\Gamma^{-1} = I$, we have $\dot{\Gamma}^{-1} = -\Gamma^{-1}\dot{\Gamma}\Gamma^{-1}$ and $\dot{\Gamma} = -\Gamma\dot{\Gamma}^{-1}\Gamma$. Then, by (3.1) and (3.5), we obtain that $\dot{\overline{\Gamma}}_i^{-1}(t) = -\overline{\Gamma}_i^{-1}\dot{\overline{\Gamma}}_i(t)\overline{\Gamma}_i^{-1} = -\overline{\Gamma}_i^{-1}\cdot(-\overline{\Gamma}_i g_i g_i^T\overline{\Gamma}_i)\cdot\overline{\Gamma}_i^{-1} = g_i g_i^T$ and $\dot{\Gamma}_{d,i}^{-1} = \sum_{j=1}^m a_{ij}^{(d)}\dot{\overline{\Gamma}}_j^{-1} = \sum_{j=1}^m a_{ij}^{(d)} g_j g_j^T$. Hence, we have

$$\dot{\Gamma}_{d,i} = -\Gamma_{d,i}\dot{\Gamma}_{d,i}^{-1}\Gamma_{d,i} = -\Gamma_{d,i}\sum_{j=1}^m a_{ij}^{(d)} g_j g_j^T\Gamma_{d,i}.$$

This completes the proof of Lemma 3.5. ∎

Appendix C. Proof of Lemma 3.8. Define $e_{i,j} = z_{i,j} - z_{0,j}$ for $i = 1, \dots, m$ and $j = 1, \dots, n$. Then, from (3.17) we have

$$\text{(C.1)} \qquad \begin{cases} \dot{e}_{i,1} = -k_1 e_{i,1} + e_{i,2}, \\ \dot{e}_{i,j} = -k_j e_{i,j} - e_{i,j-1} + e_{i,j+1}, \quad j = 2, \dots, n-2, \\ \dot{e}_{i,n-1} = -k_{n-1}e_{i,n-1} - e_{i,n-2} + e_{i,n}. \end{cases}$$

Next, we will prove the system (C.1) is asymptotically stable subjected to the disturbance $e_{i,n}$.

Let $V_i = \frac{1}{2}(e_{i,1}^2 + e_{i,2}^2 + \cdots + e_{i,n-1}^2)$ for $i = 1, \dots, m$. Then, we have

$$\begin{aligned} \dot{V}_i(t) &= e_{i,1}\dot{e}_{i,1} + \cdots + e_{i,n-1}\dot{e}_{i,n-1} = -k_1 e_{i,1}^2 - \cdots - k_{n-1}e_{i,n-1}^2 + e_{i,n-1}e_{i,n} \\ &\le -k_1 e_{i,1}^2 - \cdots - \frac{1}{2}k_{n-1}e_{i,n-1}^2 + \frac{1}{2k_{n-1}}e_{i,n}^2 \le -cV_i(t) + \frac{1}{2k_{n-1}}e_{i,n}^2, \end{aligned}$$

where $c = \min\left\{k_1, \dots, k_{n-2}, \frac{1}{2}k_{n-1}\right\} > 0$. Hence, we have

$$\text{(C.2)} \qquad V_i(t) \le V_i(0)e^{-ct} + \frac{1}{2k_{n-1}}\int_0^t e^{-c(t-\tau)}e_{i,n}^2(\tau)d\tau.$$

Noting that $\lim_{t\to\infty} e_{i,n}(t) = 0$ implies $\lim_{t\to\infty}\int_0^t e^{-c(t-\tau)}e_{i,n}^2(\tau)d\tau = 0$, from (C.2) we obtain that $\lim_{t\to\infty} V_i(t) = 0$ holds if $\lim_{t\to\infty} e_{i,n}(t) = 0$ holds for $i = 1, \dots, m$, which shows that the above claim is true.

We note that $e_{i,n} = z_{i,n} - z_{0,n}$ converges to zero, so $\lim_{t\to\infty} e_{i,j} = 0$ for all $i = 1, \dots, m$ and $j = 1, \dots, n-1$. Therefore, we have $\lim_{t\to\infty} z_{i,j} = z_{0,j}$.

By (3.16) and (3.17), we get $x_{i,1} = z_{i,1} \to z_{0,1} = x_{0,1}$ as $t \to \infty$.

Noting $\alpha_{i,2} = -k_1 z_{i,1} \to -k_1 z_{0,1} = \alpha_{0,2}$, we have $x_{i,2} = z_{i,2} + \alpha_{i,2} \to z_{0,2} + \alpha_{0,2} = x_{0,2}$ as $t \to \infty$.

Similarly, as $t \to \infty$, we get

$$\begin{aligned} &\dot{\alpha}_{i,2} = -k_1\dot{z}_{i,1} = -k_1(z_{i,2} - k_1 z_{i,1}) \to -k_1(z_{0,2} - k_1 z_{0,1}) = -k_1\dot{z}_{0,1} = \dot{\alpha}_{0,2}, \\ &\alpha_{i,3} = -k_2 z_{i,2} - z_{i,1} + \dot{\alpha}_{i,2} \to -k_2 z_{0,2} - z_{0,1} + \dot{\alpha}_{0,2} = \alpha_{0,3}, \\ &x_{i,3} = z_{i,3} + \alpha_{i,3} \to z_{0,3} + \alpha_{0,3} = x_{0,3}; \end{aligned}$$

$$\begin{aligned}
\ddot{\alpha}_{i,2} &= \dot{z}_{i,2} - k_1\dot{z}_{i,1} = (z_{i,3} - k_2 z_{i,2} - z_{i,1}) - k_1(z_{i,2} - k_1 z_{i,1}) \\
&\to (z_{0,3} - k_2 z_{0,2} - z_{0,1}) - k_1(z_{0,2} - k_1 z_{0,1}) = \dot{z}_{0,2} - k_1\dot{z}_{0,1} = \ddot{\alpha}_{0,2}, \\
\dot{\alpha}_{i,3} &= -k_2\dot{z}_{i,2} - \dot{z}_{i,1} + \ddot{\alpha}_{i,2} = -k_2(z_{i,3} - k_2 z_{i,2} - z_{i,1}) - (z_{i,2} - k_1 z_{i,1}) + \ddot{\alpha}_{i,2} \\
&\to -k_2(z_{0,3} - k_2 z_{0,2} - z_{0,1}) - (z_{0,2} - k_1 z_{0,1}) + \ddot{\alpha}_{0,2} \\
&= -k_2\dot{z}_{0,2} - \dot{z}_{0,1} + \ddot{\alpha}_{0,2} = \dot{\alpha}_{0,3}, \\
\alpha_{i,4} &= -k_3 z_{i,3} - z_{i,2} + \dot{\alpha}_{i,3} \to -k_3 z_{0,3} - z_{0,2} + \dot{\alpha}_{0,3} = \alpha_{0,4}, \\
x_{i,4} &= z_{i,4} + \alpha_{i,4} \to z_{0,4} + \alpha_{0,4} = x_{0,4}; \\
&\vdots
\end{aligned} \tag{C.3}$$

As shown above, by induction, we can get $\lim_{t\to\infty} x_{i,j} = x_{0,j}$ for all $i = 1, \ldots, m$ and $j = 1, \ldots, n$. Hence, we obtain $\lim_{t\to\infty}(x_i - x_0) = 0$ for all $i = 1, \ldots, m$.

This completes the proof of Lemma 3.8. ∎

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