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Adaptive Control Designed via Deterministic Excitation*

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Abstract: This paper considers the parameter estimation and adaptive stabilization problems for linear discrete-time systems with unknown parameters and bounded isturbances. The a-priori knowledge for designing adaptive controllers is only the order of the system. No assumption is required except controllability and observability of the system. The excitation signals are deterministic, and hence, no external stochastic excitation signal is applied.

Key words: adaptive control; deterministic excitation; stabilization; discrete-time

1 Introduction

Consider the linear single-input single-output discrete-time system

$$A(z)y_n = zB(z)u_n + w_n, \quad \forall\ n \geqslant 0, \tag{1.1}$$

where y_n, u_n and w_n are the system output, input and unknown disturbance, respectively, $A(z)$ and $B(z)$ are polynomials in backward shift operator z

$$A(z) = 1 + a_1 z + \cdots + a_p z^p, \quad p \geqslant 0, \quad a_p \neq 0, \tag{1.2}$$

$$B(z) = b_1 + \cdots + b_q z^{q-1}, \quad q \geqslant 1, \quad b_q \neq 0 \tag{1.3}$$

and

$$\theta = [-a_1 \quad \cdots \quad -a_p \quad b_1 \quad \cdots \quad b_q]^{\mathrm{T}} \tag{1.4}$$

is the unknown parameter of the system. The disturbance w_n is of arbitrary nature: deterministic or stochastic. Assume that $\{w_n\}$ satisfies the following long run average condition

$$\sup_{n\geqslant 0} \frac{1}{n+1} \sum_{j=0}^{n} w_j^2 < \infty, \tag{1.5}$$

or satisfies the more restrictive condition

$$\sup_{n\geqslant 0} |w_n| < \infty. \tag{1.6}$$

The problem of adaptive stabilization consists in designing control aiming at stabilizing the system with unknown parameters. For system (1.1) with $w_n \equiv 0$, the problem was discussed in [1~4] and others. When w_n is not identically equal to zero, the problem is usually solved under conditions more than coprimeness of $A(z)$ and $zB(z)$, which as well-known is sufficient for non-

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adaptive stabilization [5~8]. To the authors' knowledge, under the coprimeness condition only, the problem has first been solved in [9] for system (1. 1) with $\{w_n\}$ being a martingale difference sequence. As in many previous works summarized by Chen and Guo[10], the excitation signals used in [9] are stochastic processes, which, generally speaking, are more difficult to deal with than deterministic ones.

In this paper, under the assumption that $A(z)$ and $zB(z)$ are coprime, we give adaptive controls via deterministic excitation signal such that

$$\sup_{n\geqslant 0}\frac{1}{n+1}\sum_{j=1}^{n}(y_j^2+u_j^2)<\infty \tag{1.7}$$

for the case where (1. 5) holds and

$$\sup_{n\geqslant 0}(|y_n|+|u_n|)<\infty \tag{1.8}$$

for the case where (1. 6) is satisfied.

Through out the paper, for a polynomial $X(z)=\sum_{i=0}^{\mu}x_iz^i$, the norms $\|\cdot\|_1$ and $\|\cdot\|_2$ are defined as follows

$$\|X(z)\|_1=\sum_{i=0}^{\mu}|x_i| \quad \text{and} \quad \|X(z)\|_2=\Big(\sum_{i=0}^{\mu}|x_i|^2\Big)^{1/2}.$$

2 Estimation and Adaptive Control

We estimate the unknown parameter θ by the LS algorithm which recursively defines the estimate θ_n as follows:

$$\theta_{n+1}=\theta_n+\mu_nP_n\varphi_n(y_{n+1}^{\mathrm{T}}-\varphi_n^{\mathrm{T}}\theta_n), \tag{2.1}$$

$$P_{n+1}=P_n-\mu_nP_n\varphi_n\varphi_n^{\mathrm{T}}P_n,\quad \mu_n=(1+\varphi_n^{\mathrm{T}}P_n\varphi_n)^{-1}, \tag{2.2}$$

$$\varphi_n^{\mathrm{T}}=[y_n\ \cdots\ y_{n-p+1}\ u_n\ \cdots\ u_{n-q+1}] \tag{2.3}$$

with $P_0=I$ and arbitrary initial value

$$\theta_0^{\mathrm{T}}=[-a_{10}\ \cdots\ -a_{p0}\ b_{10}\ \cdots\ b_{q0}].$$

For any $n\geqslant 0$ write θ_n in the component form

$$\theta_n^{\mathrm{T}}=[-a_{1n}\ \cdots\ -a_{pn}\ b_{1n}\ \cdots\ b_{qn}]. \tag{2.4}$$

If $A(z)$ and $zB(z)$ are coprime, then there exist two polynomials

$$G(z)=1+\sum_{j=1}^{q-1}g_jz^j,\quad H(z)=\sum_{j=0}^{p-1}h_jz^j, \tag{2.5}$$

such that

$$A(z)G(z)-zB(z)H(z)=1. \tag{2.6}$$

Replacing a_i, b_j, g_k, h_s by their estimates a_{in}, b_{jn}, g_{kn} and h_{sn} respectively in (1. 2), (1. 3), (2. 5), $i=1,\cdots,p$, $j=1,\cdots,q$, $k=1,\cdots,q-1$, $s=0,\cdots,p-1$, we correspondingly denote $A(z)$, $B(z)$, $G(z)$ and $H(z)$ by $A_n(z)$, $B_n(z)$, $G_n(z)$ and $H_n(z)$, respectively, for example, $A_n(z)=1+a_{1n}z+\cdots+a_{pn}z^p$.

We need the following two lemmas proved in Chen and Zhang[9].

Lemma 1 If $A(z)$ and $zB(z)$ are coprime, then there is a constant $\varepsilon_\theta>0$ such that for any θ_n satisfying $\|\theta_n-\theta\|\leqslant\varepsilon_\theta$, the following Bezout equation

$$A_n(z)G_n(z) - zB_n(z)H_n(z) = 1, \tag{2.7}$$

has a unique solution $(G_n(z), H_n(z))$ satisfying

$$\deg(G_n(z)) \leqslant q-1, \quad \deg(H_n(z)) \leqslant p-1 \tag{2.8}$$

and

$$\|G_n(z)\|_i^i + \|H_n(z)\|_i^i \leqslant 1 + \|G(z)\|_i^i + \|H(z)\|_i^i, \tag{2.9}$$

for $i=1$ or 2.

Lemma 2 Let $\{w_n\}$ in (1.1) be any disturbance (deterministic or stochastic) satisfying (1.5). Then the LS estimate θ_n for θ has the following properties

$$\|\theta_n - \theta\|^2 \leqslant \frac{\|\theta - \theta_0\|^2 + 2nW}{\lambda_{\min}^{(n-1)}}, \quad \forall n \geqslant 0, \tag{2.10}$$

where $W \triangleq \sup\limits_{n\geqslant 0} \frac{1}{n+1}\sum\limits_{j=0}^{n} w_j^2 < \infty$ by condition (1.5) or (1.6), and $\lambda_{\min}^{(n)}$ denoles the minimum eigenvalue of $P_{n+1}^{-1} \triangleq I + \sum\limits_{i=0}^{n} \varphi_i\varphi_i^T$.

From (2.6) it is clear that

$$\begin{aligned} y_n &= A(z)G(z)y_n - zB(z)H(z)y_n \\ &= G(z)[A(z)y_n - zB(z)u_n] + zB(z)[G(z)u_n - H(z)y_n] \\ &= G(z)w_n + zB(z)[G(z)u_n - H(z)y_n] \end{aligned} \tag{2.11}$$

and

$$u_n = H(z)w_n + A(z)[G(z)u_n - H(z)y_n]. \tag{2.12}$$

From this we see that in the case where θ is known and w_n is bounded in the sense (1.5) or (1.6), the system will be stabilized in the sense of (1.7) or (1.8) if u_n is defined from

$$G(z)u_n - H(z)y_n = 0. \tag{2.13}$$

The "certainty equivalence principle" suggests to us defining adaptive control from

$$G_n(z)u_n - H_n(z)y_n = 0. \tag{2.14}$$

However, in the present case the closeness of θ_n to θ is not guaranteed. Consequently, it is not clear if (2.7) is solvable or not. Even if $G_n(z)$ and $H_n(z)$ can be defined from (2.7) we still do not know whether or not they are close to $G(z)$ and $H(z)$ respectively. So it is important that θ_n somehow approximates θ. If this is the case, then adaptive control defined by (2.14) may hopefully stabilize the system. By lemma 2 we see that for first step of approximating θ we may apply an explosive excitation input, by which we mean such an input that yields $\lambda_{\min}^{(n)}/n \xrightarrow[n\to\infty]{} \infty$. However, the stabilization purpose (1.7) or (1.8) does not allow us to apply such an input for a period longer than finite. Thus we need to define stopping times σ_i at which we turn off the explosive excitation input and switch on the control defined by the certainty equivalence principle until τ_i at which the accuracy of the LS estimate θ_n becomes unsatisfactory and we have to apply the explosive excitation input again. After defining stopping times

$$0 \triangleq \tau_0 < \sigma_1 < \tau_1 < \sigma_2 < \tau_2 < \cdots,$$

it is most important to show that there is some integer i such that $\sigma_i < \infty$ and $\tau_i = \infty$, because oth-