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Parameterized Solution to a Class of Sylvester Matrix Equations

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Abstract: A class of formulas for converting linear matrix mappings into conventional linear mappings are presented. Using them, an easily computable numerical method for complete parameterized solutions of the Sylvester matrix equation $AX - EXF = BY$ and its dual equation $XA - FXE = YC$ are provided. It is also shown that the results obtained can be used easily for observer design. The method proposed in this paper is universally applicable to linear matrix equations.

Keywords: Sylvester matrix equation, parameterized solution, Kronecker product, linear matrix equation, Luenberger observers.

1 Introduction

Sylvester matrix equation has many applications in control theory. Particularly, when a singular control system is considered, it is widely used for designing controls, such as pole placement, tracking, design of Luenberger observers, etc. Due to its importance in practice, it has attracted much attention[1−6].

The Sylvester matrix equation considered in this paper is of the following form:

$$AX - EXF = BY \tag{1}$$

where $A, E \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times r}$, and $F \in \mathbf{R}^{p\times p}$, with unknowns $X \in \mathbf{R}^{n\times p}$ and $Y \in \mathbf{R}^{r\times p}$. We use $\mathbf{R}^{m\times n}$ for the set of $m \times n$ matrices. Sylvester matrix equation (1) and its dual equation play an important role in linear system analysis and control design. Please refer to [7–9] and the references therein for details.

A basic assumption for the solution of (1) is called R-controllable. (E, A, B) is called R-controllable if

$$\mathrm{rank}\begin{bmatrix} sE - A & B \end{bmatrix} = n, \quad \forall\, s \in \mathbf{C}, \quad \mathrm{rank}(B) = r. \tag{2}$$

The following lemma was proved in [8].

Lemma 1. If E, A, and B satisfy R-controllable condition (2), then (1) has rp degree-of-freedom. In other words, (1) has rp linearly independent solutions.

Recently, in [9], a complete general parametric expression for the solution (X, Y) is obtained under the assumption that (E, A, B) is R-controllable.

This paper shows that when the linear matrix equation is expressed as a convenient linear equation, the parameterized solutions can be obtained easily. To begin with, we give some notations and results, which are from [10].

Let $A = (a_{ij}) \in \mathbf{R}^{m\times n}$. Its column stacking form is expressed as

$$cs(A) = (a_{11}, a_{21}, \cdots, a_{m1}, \cdots, a_{1n}, a_{2n}, \cdots, a_{mn})^{\mathrm{T}}. \tag{3}$$

Its row stacking form is

$$rs(A) = (a_{11}, a_{12}, \cdots, a_{1n}, \cdots, a_{m1}, a_{m2}, \cdots, a_{mn})^{\mathrm{T}}. \tag{4}$$

Let $x = (x_i) \in \mathbf{R}^{mn}$. Then,

1)

$$cs^{-1}(x, m) = \begin{bmatrix} x_1 & x_{m+1} & \cdots & x_{(n-1)m+1} \\ x_2 & x_{m+2} & \cdots & x_{(n-1)m+2} \\ \vdots & \vdots & \ddots & \vdots \\ x_m & x_{2m} & \cdots & x_{nm} \end{bmatrix}. \tag{5}$$

2)

$$rs^{-1}(x, n) = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ x_{n+1} & x_{n+2} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{(m-1)n+1} & x_{(m-1)n+2} & \cdots & x_{mn} \end{bmatrix}. \tag{6}$$

Next, we convert a linear matrix mapping into a conventional linear mapping. Define a mapping $\rho : \mathbf{R}^{n\times p} \to \mathbf{R}^{m\times p}$, determined by $X \mapsto AX$, where $X \in \mathbf{R}^{n\times p}$ and $A \in \mathbf{R}^{m\times n}$. We use column stacking form first. Denote $x = cs(X) \in \mathbf{R}^{np}$, $y = cs(AX) \in \mathbf{R}^{mp}$, and consider the matrix mapping ρ as a linear mapping $\rho : \mathbf{R}^{np} \to \mathbf{R}^{mp}$, with its matrix form M^c_ρ:

$$y = cs(AX) = M^c_\rho x. \tag{7}$$

For various linear matrix mappings, we can construct their respective matrix form. The followings are some typical ones.

Theorem 1. Assume $A \in \mathbf{R}^{m\times n}$, $B \in \mathbf{R}^{p\times q}$, $C \in \mathbf{R}^{m\times p}$, $D \in \mathbf{R}^{n\times q}$, and $X \in \mathbf{R}^{n\times p}$,

1) If $\rho : X \mapsto AX$, then

$$M^c_\rho = I_p \otimes A. \tag{8}$$

2) If $\rho : X \mapsto XB$, then

$$M^c_\rho = B^{\mathrm{T}} \otimes I_n. \tag{9}$$

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3) If $\rho : X \mapsto CX^{\mathrm{T}}$, then

$$M_\rho^c = (I_n \otimes C)W_{[p,n]}. \tag{10}$$

4) If $\rho : X \mapsto X^{\mathrm{T}}D$, then

$$M_\rho^c = (D^{\mathrm{T}} \otimes I_p)W_{[p,n]}. \tag{11}$$

5) If $\rho : X \mapsto AXB + CX^{\mathrm{T}}D$, then

$$M_\rho^c = (B^{\mathrm{T}} \otimes A) + (D^{\mathrm{T}} \otimes C)W_{[p,n]}. \tag{12}$$

Here, $\otimes$ is the Kronecker product, and $W_{[m,n]}$ is a swap matrix (refer to [10] or [11] for the definition).

Next, we use row stacking form. That is, for $\rho : X \mapsto AX$, we denote $x = rs(X)$, $y = rs(AX)$ and express the matrix form of ρ by M_ρ^r,

$$y = rs(AX) = M_\rho^r x. \tag{13}$$

Similar to Theorem 1, we have the following results.

Theorem 2. Assume that $A \in \mathbf{R}^{m\times n}$, $B \in \mathbf{R}^{p\times q}$, $C \in \mathbf{R}^{m\times p}$, $D \in \mathbf{R}^{n\times q}$, and $X \in \mathbf{R}^{n\times p}$.

1) If $\rho : X \mapsto AX$, then

$$M_\rho^r = A \otimes I_p. \tag{14}$$

2) If $\rho : X \mapsto XB$, then

$$M_\rho^r = I_n \otimes B^{\mathrm{T}}. \tag{15}$$

3) If $\rho : X \mapsto CX^{\mathrm{T}}$, then

$$M_\rho^r = (C \otimes I_n)W_{[n,p]}. \tag{16}$$

4) If $\rho : X \mapsto X^{\mathrm{T}}D$, then

$$M_\rho^r = (I_p \otimes D^{\mathrm{T}})W_{[n,p]}. \tag{17}$$

5) If $\rho : X \mapsto AXB + CX^{\mathrm{T}}D$, then

$$M_\rho^r = (A \otimes B^{\mathrm{T}}) + (C \otimes D^{\mathrm{T}})W_{[n,p]}. \tag{18}$$

2 Parameterized solutions

Using Theorem 1, we can convert (1) into a system of linear equations

$$\begin{bmatrix} I_p \otimes A - F^{\mathrm{T}} \otimes E & -I_p \otimes B \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \tag{19}$$

where $x = cs(X)$ and $y = cs(Y)$.

Assume that U is a nonsingular matrix such that

$$U^{-1}FU = J$$

where J is the Jordan canonical form of F. We define $\tilde{X} = XU$ and $\tilde{Y} = YU$. Then, (1) can be expressed equivalently as

$$A\tilde{X} - E\tilde{X}J = B\tilde{Y}. \tag{20}$$

Correspondingly, (19) becomes

$$\begin{bmatrix} I_p \otimes A - J^{\mathrm{T}} \otimes E & -I_p \otimes B \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = 0 \tag{21}$$

where $\tilde{x} = cs(\tilde{X})$ and $\tilde{y} = cs(\tilde{Y})$. Now, (21) has a block lower triangular form:

$$\begin{bmatrix} A-\lambda_1 E & -B & 0 & \cdots & & 0 \\ * & & A-\lambda_2 E & -B & \cdots & 0 \\ \vdots & & & & & \\ * & & * & & \cdots & A-\lambda_p E \quad -B \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = 0 \tag{22}$$

where λ_i, $i = 1, \cdots, p$ are eigenvalues of F. Equations (21)–(22) and the following proposition have been proved in [12].

Proposition 1. Equation (1) has solutions of (minimum) degree-of-freedom rp, if and only if

$$\mathrm{rank}(\lambda E - A \quad B) = n, \quad \forall\, \lambda \in \sigma(F). \tag{23}$$

Obviously, Lemma 1 is a special case of Proposition 1, because (2) ensures (23). Hereinafter, we assume that (23) holds.

Then, from (19), we have rp linearly independent solutions

$$\begin{bmatrix} x^1 \\ y^1 \end{bmatrix}, \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}, \cdots, \begin{bmatrix} x^{rp} \\ y^{rp} \end{bmatrix}. \tag{24}$$

Then, the set of rp linearly independent solutions of (1) are

$$\begin{cases} X^i = cs^{-1}(x^i, n), \\ Y^i = cs^{-1}(y^i, r), \end{cases} \quad i = 1, 2, \cdots, rp. \tag{25}$$

It follows that the parameterized solution is

$$\begin{cases} X = \sum\limits_{i=1}^{rp} \mu_i cs^{-1}(x^i, n) \\ Y = \sum\limits_{i=1}^{rp} \mu_i cs^{-1}(y^i, r) \end{cases} \tag{26}$$

where $\mu = (\mu_1, \cdots, \mu_{rp})^{\mathrm{T}}$ are parameters. $\mu \neq 0$ corresponds to non-zero solution.

Remark 1. It is obvious that the set of solutions has rp degree-of-freedom, if and only if the coefficient matrix of (19) has full row rank. That is,

$$\mathrm{rank}\Big(I_p \otimes A - F^{\mathrm{T}} \otimes E \quad -I_p \otimes B\Big) = pn. \tag{27}$$

This fact is important in finding independent solutions. An easy way to find the solutions is to choose arbitrary rp rows, equivalently, an $rp \times (r+n)p$ matrix Φ, such that

$$\Psi = \begin{bmatrix} I_p \otimes A - F^{\mathrm{T}} \otimes E & -I_p \otimes B \\ \Phi & \end{bmatrix} \tag{28}$$

is non-singular. Then, the last rp columns of Ψ^{-1} form (24), the set of rp linearly independent solutions of (19).

In the design of Luenberger observer, we have to solve the dual equation of (1)[9]. Precisely, it is

$$XA - FXE = YC \tag{29}$$

where $A, E \in \mathbf{R}^{n\times n}$, $C \in \mathbf{R}^{m\times n}$, and $F \in \mathbf{R}^{p\times p}$, with unknowns $X \in \mathbf{R}^{p\times n}$ and $Y \in \mathbf{R}^{p\times m}$.

Using Theorems 2, we can convert (29) into a system of linear equations:

$$\begin{bmatrix} I_p \otimes A^{\mathrm{T}} - F \otimes E^{\mathrm{T}} & -I_p \otimes C^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \tag{30}$$

where $x = rs(X)$ and $y = rs(Y)$.

Define $\tilde{X} = U^{-1}X$ and $\tilde{Y} = U^{-1}Y$. Similar argument to Proposition 1 yields the following corollary:

Corollary 1. Equation (29) has solutions of (minimum) degree-of-freedom rp, if and only if

$$\operatorname{rank} \begin{bmatrix} \lambda E - A \\ C \end{bmatrix} = n, \quad \forall\, \lambda \in \sigma(F). \tag{31}$$

Now, assume that (31) holds, and the linearly independent solutions of (30) have the form of (24). Then, the set of rp linearly independent solutions of (29) are

$$\begin{cases} X^i = rs^{-1}(x^i, n), \\ Y^i = rs^{-1}(y^i, m), \end{cases} \quad i = 1, 2, \cdots, rp. \tag{32}$$

The parameterized solution is

$$\begin{cases} X = \sum\limits_{i=1}^{rp} \mu_i rs^{-1}(x^i, n) \\ Y = \sum\limits_{i=1}^{rp} \mu_i rs^{-1}(y^i, m). \end{cases} \tag{33}$$

3 The algorithm

According to the results in Section 2, an algorithm for solving the Sylvester matrix equation (1) is constructed.

Step 1. Convert system (1) into the form of (19)

$$\begin{bmatrix} I_p \otimes A - F^{\mathrm{T}} \otimes E & -I_p \otimes B \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0.$$

Step 2. We have assumed that (23) holds. Then, choose an $rp \times (r+n)p$ matrix Φ such that (28)

$$\Psi := \begin{bmatrix} I_p \otimes A - F^{\mathrm{T}} \otimes E & -I_p \otimes B \\ \multicolumn{2}{c}{\Phi} \end{bmatrix}$$

is nonsingular.

Step 3. Compute Ψ^{-1}. Then, the last rp columns of Ψ^{-1}

$$\begin{bmatrix} x^1 \\ y^1 \end{bmatrix}, \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}, \cdots, \begin{bmatrix} x^{rp} \\ y^{rp} \end{bmatrix}$$

form the set of rp linearly independent solutions of (19).

Step 4. Compute

$$\begin{cases} X = \sum\limits_{i=1}^{rp} \mu_i cs^{-1}(x^i, n) \\ Y = \sum\limits_{i=1}^{rp} \mu_i cs^{-1}(y^i, r) \end{cases}$$

that are the parameterized solutions of (1).

4 An illustrative example

As an application example, we consider the following singular linear system[9]

$$\begin{cases} E\dot{x} = Ax + Bu, & x \in \mathbf{R}^n,\ u \in \mathbf{R}^r \\ y = Cx, & y \in \mathbf{R}^m. \end{cases} \tag{34}$$

To ensure the uniqueness of the solution, we assume that (E, A) is a normal pair (or system (34) is normal), that is, there exists $s \in \mathbf{C}$ such that

$$\det(sE - A) \neq 0. \tag{35}$$

The system is called R-observable if

$$\operatorname{rank} \begin{bmatrix} sE - A \\ C \end{bmatrix} = n, \quad \forall\, s \in \mathbf{C}. \tag{36}$$

The Luenberger observer has the following form:

$$\begin{cases} \dot{z} = Fz + Gy + Su, & z \in \mathbf{R}^p \\ \omega = Mz + Ny, & \omega \in \mathbf{R}^r. \end{cases} \tag{37}$$

The design purpose is to find parameter matrices $F \in \mathbf{R}^{p\times p}$, $G \in \mathbf{R}^{p\times m}$, $S \in \mathbf{R}^{p\times r}$, $M \in \mathbf{R}^{r\times p}$, and $N \in \mathbf{R}^{r\times m}$, such that for a certain $K \in \mathbf{R}^{r\times n}$, any initial $x(0)$, $z(0)$ and arbitrary input $u(t)$, we have

$$\lim_{t\to\infty} (Kx(t) - \omega(t)) = 0. \tag{38}$$

We refer to [9] or [13] for the following result.

Theorem 3. Assume system (34) is normal and R-observable. Then, system (37) is a Kx observer, if and only if there exist matrices F, T, G, S, M, and N, satisfying

$$\begin{cases} S = TB \\ TA - FTE = GC \\ K = MTE + NC \\ \mathrm{Re}[\sigma(F)] < 0, \text{ i.e., } F \text{ is Hurwitz.} \end{cases} \tag{39}$$

Next, we use the same example in [9] to show how convenient our approach is.

Example 1. Consider system (34). Assume

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

As in [9], we want to design a Luenberger observer to track Kx, where

$$K = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}.$$

F can be any stable matrix. Now, following [9], we choose

$$F = \begin{bmatrix} 0 & -2 \\ 1 & -2 \end{bmatrix}.$$

Consider the second equation of (39) first. Using (30), a straightforward computation shows that it can be written as

$$\begin{bmatrix} -5 & 0 & 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} t \\ g \end{bmatrix} = 0 \tag{40}$$

where $t = rs(T)$ and $g = rs(G)$. Similar to (28), we can choose a Φ and construct the $(n+r)p \times (n+r)p$ matrix Ψ as

$$\Psi = \begin{bmatrix} -5 & 0 & 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Then, the last four columns of Ψ^{-1} form the linearly independent set of solutions of (40):

$$\begin{bmatrix} t^1 & t^2 & t^3 & t^4 \\ g^1 & g^2 & g^3 & g^4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \\ -2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & -5 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & -3 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Then,

$$T_1 = \begin{bmatrix} 0 & 2 & -2 \\ 0 & 1 & 0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

$$T_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad T_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 2 & 0 \\ -3 & 0 \end{bmatrix},$$

$$G_3 = \begin{bmatrix} -5 & 0 \\ -1 & 0 \end{bmatrix}, \quad G_4 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Using (33), the parameterized solutions are

$$T = \begin{bmatrix} \mu_3 & 2\mu_1 + \mu_4 & -2\mu_1 \\ \mu_2 & \mu_1 & \mu_4 \end{bmatrix}$$

$$G = \begin{bmatrix} 2\mu_2 - 5\mu_3 & 2\mu_1 + \mu_4 \\ -3\mu_2 - \mu_3 & \mu_1 \end{bmatrix}.$$

It follows that

$$S = TB = \begin{bmatrix} \mu_3 & 2\mu_1 + \mu_4 \\ \mu_2 & \mu_1 \end{bmatrix}.$$

Then, we solve the third equation of (39). Denoting $\alpha = cs(M)$ and $\beta = cs(N)$, we have

$$\begin{bmatrix} E^{\rm T}T^{\rm T} \otimes I_3 & C^{\rm T} \otimes I_2 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = cs(K). \tag{41}$$

Equation (41) can be rewritten as

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}. \tag{42}$$

Its general solution is

$$\begin{bmatrix} 0 & -1 & \alpha_1 & \alpha_2 & 0 & 1 & 1 & 0 \end{bmatrix}^{\rm T}.$$

where $\alpha_1, \alpha_2 \in \mathbf{R}$ are parameters.

Using (5), we have

$$M = \begin{bmatrix} 0 & \alpha_1 \\ -1 & \alpha_2 \end{bmatrix}, \quad N = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Letting $\mu_1 = \mu_2 = \mu_3 = 0$, $\mu_4 = 1$, and $\alpha_1 = \alpha_2 = 0$, we have the particular solution given in [9]:

$$T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad S = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$M = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, \quad N = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

5 Conclusion

Following the pioneering works[7, 9, 12, 14, 15] of Duan et al., we considered the parameterized solutions of Sylvester equation and its dual equation. An elegant dual relation has been revealed. The formulas for parameterized numerical solutions are obtained, and an algorithm is constructed. The conditions and algorithms provided in this paper are neat and simple. Moreover, the method proposed in this paper is generally applicable to solving general linear matrix equations.

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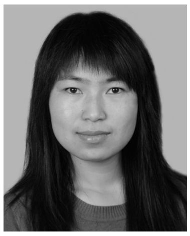

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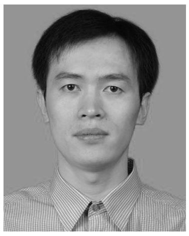

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