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Stability of switched nonlinear systems via extensions of LaSalle's invariance principle

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This paper studies the extension of LaSalle's invariance principle for switched nonlinear systems. Unlike most existing results in which each switching mode in the system needs to be asymptotically stable, this paper allows the switching modes to be only stable. Under certain ergodicity assumptions of the switching signals, two extensions of LaSalle's invariance principle for global asymptotic stability of switched nonlinear systems are obtained using the method of common joint Lyapunov function.

LaSalle's invariance principle, switched systems, weak common Lyapunov function, asymptotically stable

1 Introduction

In recent years, the problem of stability and stabilization of switched systems has attracted considerable attention from the control community[1−3]. They arise in many engineer applications, such as robot manipulators[4], power systems[5], multi-agent models[6−8], etc. The stability of a switched system can be ensured by a common Lyapunov function (CLF) of all switching modes under arbitrary switching law[9,10]. Finding a common Lyapunov function is still an interesting and challenging problem. There is a large amount of literature concerning it. We refer to refs. [2, 11–13] and the references therein for more discussions.

The method of multiple Lyapunov functions is also a useful tool for stability analysis of switched systems. In comparison with common Lyapunov function, it allows each switching mode to have its own Lyapunov function[14]. However, as a compensation, some additional conditions are necessary to ensure that the value of each Lyapunov function on its corresponding mode will decrease.

In practical applications, many switched systems do not share a common Lyapunov function, yet they still may be asymptotically stable under some properly chosen switching laws. Searching certain admissible classes of switching laws is necessary for this kind of problem[15]. Roughly speaking, stability can be ensured if the switching is sufficiently slow. Ref. [15] introduced several concepts to restrict admissible switching signals.

When the derivative of a candidate Lyapunov function with respect to each mode is only non-positive, the function is called a weak Lyapunov function[16]. In order to solve the stability problem in such a case, various extensions of LaSalle's invariance principle for switched systems have been

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investigated. By imposing some restrictions on the admissible trajectories, global asympotic stability results using multiple weak Lyapunov functions are obtained for switched linear systems[15]. Then, it is extended to switched nonlinear systems[17]. A more traditional style extension of LaSalle's invariance principle is proposed in ref. [16]. Its statement is closer in spirit to the classical one. However, it only shows that the solution is attracted to a weakly invariant set M, and the asymptotical stability cannot be obtained unless $M = \{0\}$. Under certain restrictions, another extension of LaSalle's invariance principle for switched nonlinear systems and criteria for asymptotic stability are obtained in ref. [18].

To the best of our knowledge, all of these extensions of LaSalle's invariance principle require each switching mode to be asymptotically stable. Naturally, if we do not impose certain restrictions on the switching signals, each switching mode must be asymptotically stable. Otherwise, when the system stays on a non-asymptotically-stable mode forever, the overall system will not be asymptotically stable.

In this paper, we consider the following switched nonlinear system:

$$\dot{x} = f_{\sigma(t)}(x), \quad x \in \mathbb{R}^n, \tag{1.1}$$

where $\sigma : [0, +\infty) \to \Lambda = \{1, 2, \cdots, N\}$ is a piecewise constant function and continuous from the right, called a switching signal (or switching law). Each $f_i(x)$ is a smooth vector field of $\mathbb{R}^n$ such that $f_i(0) = 0,\ i \in \Lambda$. Lyapunov function approach is a fundamental and powerful tool for stability analysis. It is well known that if there exists a common Lyapunov function, i.e., a positive definite C^1 function $V(x) > 0$, radially unbounded, such that

$$\dot{V}|_i = \nabla V(x) f_i(x) < 0, \quad x \neq 0, \quad i = 1, \cdots, N,$$

then the switched system is globally asymptotically stable. If we ask for globally uniformly asymptotical stability (GUAS), then the existence of a common Lyapunov function becomes necessary and sufficient[9,10].

Different from other results, in this paper, each mode does not need to be asymptotically stable. Under certain ergodicity assumption on the switching signals, we propose two extensions of LaSalle's invariant principle, which are relatively easy to check. In our previous work[7], it was shown that if the switched system is linear, the results are useful for consensus of multi-agent systems.

2 Preliminaries

To begin with, we give some rigorous definitions for stabilities.

Definition 2.1. The equilibrium point $x = 0$ of (1.1) is

1) stable if for each $\epsilon > 0$, there is a $\delta = \delta(\epsilon) > 0$ such that

$$||x(0)|| < \delta \Rightarrow ||x(t)|| < \epsilon, \quad \forall\ t \geqslant 0;$$

2) asymptotically stable if it is stable and given a $\eta > 0$, for each $\epsilon > 0$ there exists $T > 0$ such that

$$||x(0)|| < \eta \Rightarrow ||x(t)|| < \epsilon, \quad \forall\ t > T; \tag{2.1}$$

3) globally asymptotically stable if (2.1) holds for all $\eta > 0$.

It is said that the above stabilities hold "uniformly" if they hold for all switching law σ.

Consider a nonlinear system

$$\dot{x} = f(x), \quad x \in \mathbb{R}^n. \tag{2.2}$$

By the well-known LaSalle's invariance principle[19], if there exists a continuously differential, positive definite, radially unbounded function $V(x) : \mathbb{R}^n \to \mathbb{R}$ such that $\dot{V}(x) \leqslant 0$ for all $x \in \mathbb{R}^n$, then every solution of (2.2) converges to the largest invariant set M contained in $Z = \{x \in \mathbb{R}^n \mid \dot{V}(x) = 0\}$. Moreover, if $M = \{0\}$, then the origin of (2.2) is globally asymptotically stable.

Unfortunately, the classical LaSalle's invariance principle cannot be applied to switched systems directly. For switched systems, there are also some extended results of LaSalle's invariance principle as we have mentioned in section 1. Among them, certain restrictions on the switching signals are necessary. A switched system is said to have a nonvanishing dwell time, if there exists a positive time period $\tau_0 > 0$, such that the switching instances $\{\tau_k \mid k = 1, 2, \cdots\}$ satisfy

$$\inf_k (\tau_{k+1} - \tau_k) \geqslant \tau_0. \tag{2.3}$$

Through this paper, we assume

A1. Admissible switching signals have a dwell time $\tau_0 > 0$.

We need to recall another concept: weakly invariant set.

Definition 2.2[16]**.** A compact set M is weakly invariant with respect to (1.1), if for each point $x \in M$, there exists a $\lambda \in \Lambda$, a solution $\varphi(t)$ of the vector field $f_\lambda(x)$ and a real number $b > 0$ such that $\varphi(0) = x$ and $\varphi(t) \in M$ for either $t \in [-b, 0]$ or $t \in [0, b]$.

Now for system (1.1), assume $V(t)$ is the candidate Lyapunov function concerned; we denote $Z_i = \{x \mid \dot{V}(x)|_{f_i} = 0\},\ \forall\ i \in \Lambda$.

With some mild modification, we state Theorem 1 of ref. [16] as

Proposition 2.1[16]**.** Assume system (1.1) has a CWLF,

$$Z = \bigcup_{i \in \Lambda} Z_i,$$

and M is the largest weakly invariant set contained in Z. Then, every solution $\varphi(t, x_0)$ is attracted to M.

This result is the starting point of our following discussion.

Since we only require each mode to be stable, in addition to assumption A1, we need to assume certain ergodicity property for switching signals.

A2. For any $T > 0$ and any $\lambda \in \Lambda$, there exists $t > T$ such that

$$\sigma(t) = \lambda. \tag{2.4}$$

A stronger assumption is

A2′. There exists a $T > 0$, such that for any $t_0 \geqslant 0$,

$$\{t \mid \sigma(t) = \lambda\} \bigcap [t_0, t_0 + T] \neq \emptyset, \forall\ \lambda \in \Lambda. \tag{2.5}$$

Remark.

1) Assumptions A1 and A2 imply that each mode will be active infinite times and the total time length for each mode λ being active is infinity, i.e.,

$$|\{t \mid \sigma(t) = \lambda\}| = \infty, \quad \forall\ \lambda \in \Lambda,$$

where $|\cdot|$ denotes the Lebesgue measure. We call such a switching "ergodic switching".

2) A2′ may be called "finite time ergodic switching". It is easy to see that A2′ implies A2.

3) If both A1 and A2′ hold, then there exists $T > 0$ (replacing original T of A2′ by $T + \tau_0$) such that

$$|\{t \mid \sigma(t) = \lambda\} \bigcap [t_0, t_0 + T]| \geqslant \tau_0,\ \forall\ \lambda \in \Lambda,\ t_0 \geqslant 0. \tag{2.6}$$

Next, we recall a new Lyapunov-like function, called the joint Lyapunov function. The following definition is mimic to the linear case[7].

Definition 2.3. Consider system (1.1).

1) If there exists a positive definite C^1 function $V(x) > 0$, radially unbounded, such that

$$\begin{aligned} &\dot{V}(x)|_{f_i} = \nabla V(x) f_i(x) := Q_i(x) \leqslant 0, \\ &x \neq 0, \quad Q_i(0) = 0, \quad i \in \Lambda, \end{aligned} \tag{2.7}$$

then $V(x)$ is called a common weak Lyapunov function (CWLF) of system (1.1).

2) A common weak Lyapunov function of system (1.1) is called a common joint Lyapunov function (CJLF) if

$$\sum_{i=1}^{N} Q_i(x) < 0, \quad x \neq 0. \tag{2.8}$$

The geometric meaning of (2.8) is that at any point x, at least on one mode, the Lyapunov function is strictly decreasing.

According to the definition, we obtain the following property at once.

Proposition 2.2. For system (1.1), assume there exists a CWLF $V(x) > 0$, then V is a CJLF if and only if

$$\bigcap_{i \in \Lambda} Z_i = \{0\}, \tag{2.9}$$

where $Z_i = \{x \mid Q_i(x) = 0\}$ is the kernel of $Q_i,\ i \in \Lambda$.

Proof. ($\Rightarrow$) Obviously, $0 \in Z_i,\ i \in \Lambda$. If there exists $0 \neq \eta \in \bigcap_{i \in \Lambda} Z_i$, then $Q_i(\eta) = 0,\ \forall\ i \in \Lambda$ which implies $\sum_{i \in \Lambda} Q_i(\eta) = 0$, a contradiction.

($\Leftarrow$) If $V(x)$ is not a CJLF, then there exists $\xi \neq 0$ such that $\sum_{i \in \Lambda} Q_i(\xi) = 0$. Since every $Q_i(x)$ is negative semi-definite, then $Q_i(\xi) = 0,\ \forall\ i \in \Lambda$, that is, $\xi \in Z_i,\ \forall\ i \in \Lambda$, which is a contradiction to (2.9). QED

Unfortunately, under the assumptions of A1 and A2 (or A2′), even for a switched linear system, the existence of a CJLF is not enough to ensure the global asymptotical stability. Ref. [7] gave a counter example.

Therefore, in addition to A1, A2 (A2′) and the existence of CJLF, in the next two sections, we will give some additional conditions to ensure that the system is globally asymptotically stable.

3 LaSalle's invariance principle for disconnected $Z \setminus \{0\}$

Now, we present our first LaSalle type of stability result.

Theorem 3.1. Consider system (1.1). Assume

1) A1, A2 hold;

2) there exists a CJLF;

3) $Z \setminus \{0\}$ is disconnected, where $Z = \bigcup_{i\in\Lambda} Z_i$ and Z_i is the kernel of $Q_i,\ i \in \Lambda$.

Then, system (1.1) is globally asymptotically stable.

Proof. By the common weak Lyapunov function, system (1.1) is stable. Then, we only need to prove the convergence.

For any x_0, construct a nonempty compact set

$$W = \{x \in \mathbb{R}^n \mid V(x) \leqslant V(x_0)\}.$$

Since $\bigcup_{i\in\Lambda} Z_i \backslash \{0\}$ is disconnected, without loss of generality, we assume it is composed of two connected components, denoted by

$$Z_I = \bigcup_{i\in I} Z_i \backslash \{0\}, \quad Z_J = \bigcup_{j\in J} Z_j \backslash \{0\},$$

where $I \cup J = \Lambda$ and $I \cap J = \emptyset$.

Define $N_I = \{x \in W \mid d(x, Z_I) < \epsilon_0\}$, $N_J = \{x \in W \mid d(x, Z_J) < \epsilon_0\}$, and $N_I^c = W \setminus N_I, N_J^c = W \setminus N_J$, where $\epsilon_0 > 0$ can be chosen properly. Then, under subspace topology $N_I,\ N_J$ are open sets containing 0 and $N_I^c,\ N_J^c$ are compact sets.

For any $\epsilon > 0$, let $W_\epsilon = \{x \in W \mid ||x|| < \epsilon\}$. We can choose $\epsilon_0 > 0$ small enough such that $N_I \cap N_J \subset W_\epsilon$ and $\bar{N}_I \backslash W_\epsilon$ and $\bar{N}_J \backslash W_\epsilon$ are disjoint. Let $d = d(\bar{N}_I \setminus W_\epsilon,\ \bar{N}_J \setminus W_\epsilon) > 0$.

Note that when $i \in I$ mode is active, $\dot{V}(x)|_{f_i} < 0,\ \ \forall\ x \in N_I^c$, then there exists a $\delta_I > 0$ such that $\max_{x\in N_I^c,\ i\in I} \dot{V}(x)|_{f_i} = -\delta_I < 0$. Similarly, there exists a $\delta_J > 0$ such that $\max_{x\in N_J^c,\ i\in J} \dot{V}(x)|_{f_j} = -\delta_J < 0$ and $\max_{x\in N_I^c \cap N_J^c,\ i\in\Lambda} \dot{V}(x)|_{f_i} = -\delta < 0$ with $\delta = \max\{\delta_I,\ \delta_J\}$.

We claim that there exists $T > 0$ such that

$$x(t) \in N_I \cap N_J \subset W_\epsilon,\ \forall\ t > T, \tag{3.1}$$

where $x(t)$ is any solution of system (1.1).

We prove it case by case as follows:

(i) If $x(t) \in (N_I \bigcup N_J)^c$, then no matter which mode is active, $V(x)$ decreases strictly, because $\dot{V}(x)|_{f_i} \leqslant -\delta,\ \ \forall\ i \in \Lambda$. Then, we have

$$V(x(t + \triangle t)) \leqslant V(x(t)) - \delta \triangle t. \tag{3.2}$$

(3.2) remains true as long as $x(t)$ stays in $(N_I \bigcup N_J)^c$. Then, $V(x(t + \triangle t)) \to -\infty$ as $\triangle t \to \infty$. Therefore, we assume $x(t)$ will not stay in $(N_I \bigcup N_J)^c$ forever.

(ii) If $x \in N_I \bigcup N_J$, $V(x)$ remains non-increasing. Since the switching set is ergodic, system (1.1) cannot dwell on any one mode forever.

If $x(t)$ enters N_I (same for N_J) only finite times, then after a $T_0 > 0$, the trajectory will stay in N_I^c forever. Then,

$$V(x(t)) < V(x(T_0)) - \delta_I \tau, \tag{3.3}$$

where

$$\tau = |\{T_0 < s < t \mid \sigma(s) \in I\}|.$$

Since as $t \to \infty$, $\tau \to \infty$, we have $V(x(t)) \to -\infty,\ t \to \infty$, a contradiction.

(iii) Assume $x(t)$ travels between $N_I \setminus W_\epsilon$ and $N_J \setminus W_\epsilon$ infinite times. Since $f_i(x)$ is continuous, there exists $b_i > 0$ such that as mode i is active, $\|\dot{x}(t)\| = \|f_i(x)\| \leqslant b_i,\ x \in (N_I \bigcup N_J)^c$. Take $0 < b = \max_{i\in\Lambda} b_i$, then the time $x(t)$ travels between $N_I \setminus W_\epsilon$ and $N_J \setminus W_\epsilon$ satisfies $|\triangle t| \geqslant \frac{d}{b}$. Denote $W_0 = W_\epsilon^c \cap N_I^c \cap N_J^c$. Then, there exists an infinite time sequence $t_1, t_2, \cdots$ at which $x(t)$ goes through the following regions: $N_I \xrightarrow{t_1} W_0 \xrightarrow{t_2} N_J \xrightarrow{t_3} W_0 \xrightarrow{t_4} N_I \xrightarrow{t_5} W_0 \xrightarrow{t_6} \cdots$, with $x(t) \in W_0$ for $t \in [t_{2k-1}, t_{2k}]$ and $t_{2k} - t_{2k-1} \geqslant \frac{d}{b}$. By (3.2)

$$\begin{aligned} V(x(t_{2k})) &\leqslant V(x(t_{2k-1})) - \delta\frac{d}{b} \leqslant V(x(t_{2k-3})) - 2\delta\frac{d}{b} \\ &\leqslant \cdots \leqslant V(x(t_1)) - k\delta\frac{d}{b} \to -\infty,\ k \to \infty \end{aligned}$$

is a contradiction.

Therefore, after a finite time, the trajectory of $x(t)$ will stay in $N_I \bigcap N_J$ forever, which means (3.1) holds. The conclusion follows. QED

Taking Proposition 2.2 into consideration, condition 2 can be replaced by CWLF, because CWLF plus condition 3 implies CJLF.

Also note that when $N = 2$, we have $Z_1 \bigcap Z_2 = \{0\}$, so condition 3 is automatically satisfied. This observation leads to

Corollary 3.1. Theorem 3.1 remains true if the last condition is replaced by $N = 2$.

Taking Proposition 2.1 into consideration, we have the following stronger result.

Corollary 3.2. Let M be the largest weakly invariant set contained in Z. Then, Theorem 3.1 remains true if in the last condition $Z \setminus \{0\}$ is replaced by $M \setminus \{0\}$.

4 LaSalle's invariance principle for a class of f_i

In this section, we impose certain constrains on system (1.1). We need some preparations first.

Lemma 4.1. Consider system (1.1). Assume every switching mode is stable. Denote $K_i = ker(f_i) = \{x \mid f_i(x) = 0\}$, $K = \bigcap_{i\in\Lambda} K_i$, and let $y \in K$. Assume the switching signal satisfies A1 and A2$'$, then for any $R > 0$, there exists $r > 0$, such that if $x_0 \in B_r(y)$, then

$$\varphi(t, x_0) \in B_R(y), \quad 0 \leqslant t \leqslant T, \qquad (4.1)$$

where $\varphi(t, x_0)$ is the solution of system (1.1) with $\varphi(0, x_0) = x_0$ and T is the same as in A2$'$.

Proof. Since every switching mode is stable, $y \in K$ is a stable equilibrium of every subsystem. Then, for any $R > 0$, we can find $r_i > 0 (i \in \Lambda)$, associated with every subsystem of (1.1), such that as long as $\|x_0 - y\| < r_i, \|\varphi(t, x_0) - y\| < R, t \geqslant 0$.

Now, suppose the switching moments over $[0, T]$ are $t_i, i = 1, 2, \cdots, s$. Denote $x_i = \varphi(t_i, x_0), i = 1, 2, \cdots, s$. Since every switching mode is stable, for any $R > 0$, there exists $0 < R_s < R$ such that $\|x_s - y\| < R_s$ implies $\|\varphi(t, x_s) - y\| < R,\ t_s \leqslant t \leqslant T$. For $R_s > 0$, there exists $0 < R_{s-1} < R_s$ such that $\|x_{s-1} - y\| < R_{s-1}$ implies $\|\varphi(t, x_{s-1}) - y\| < R_s,\ t_{s-1} \leqslant t \leqslant t_s$. Continuing this argument, then for $R_1 > 0$, there exists $0 < r < R_1$ such that $\|x_0 - y\| < r$ implies $\|\varphi(t, x_0) - y\| < R_1$, $0 \leqslant t \leqslant t_1$. From the above procedure, it follows that as long as $x_0 \in B_r(y)$, (4.1) holds.

Lemma 4.2. $\ker(f_i) \subset \ker(Q_i),\ \forall\ i \in \Lambda$.

Proof. For any $x_0 \in \ker(f_i)$, we have $f_i(x_0) = 0$. Then, $Q_i(x_0) = \dot{V}(x_0)|_{f_i} = \nabla V(x_0) f_i(x_0) = 0$. The conclusion follows. QED

Denote by M the largest weakly invariant set contained in $Z = \bigcup_{i\in\Lambda} Z_i$, and let

$$V_i = M \cap Z_i, \quad i \in \Lambda.$$

It is easy to see that $\ker(f_i)$ itself is a weakly invariant set contained in $Z_i \subset Z$, hence $\ker(f_i) \subset V_i$. Next, we give one more assumption.

A3. $\ker(f_i) = V_i, i \in \Lambda$.

The next proposition was obtained in ref. [16], which gives a property of the ω-limit set.

Proposition 4.1[16]**.** Let $\varphi(t, x_0)$ be a solution of system (1.1) with dwell time τ_0. $\Omega(x_0)$ is its ω-limit set. Then, $\Omega(x_0)$ is a weakly invariant set contained in Z.

Now, we are ready to state our second main result.

Theorem 4.1. Consider system (1.1). Assume A1, A2$'$, and A3 hold and there exists a CJLF. Then system (1.1) is globally asymptotically stable.

Proof. Let $x(t) = \varphi(t, x_0)$ be any solution of system (1.1) with $\varphi(0, x_0) = x_0$. Since $V(x)$ is monotonically not increasing and bounded, we have

$$\lim_{t\to\infty} V(x(t)) = V_0.$$

If $V_0 = 0$, we are done. Thus, we assume $V_0 > 0$ and will draw a contradiction.

Since $x(t)$ is bounded, there exists an infinite sequence $\{t_k\}$ such that

$$x_k := x(t_k) \to y, \quad t \to \infty,$$

and $\lim_{k\to\infty} V(x(t_k)) = V(y) = V_0$. Now, since y is a ω-limit point, by Proposition 4.1, we have $y \in M \subset Z$ and by the assumption $V_0 > 0, y \neq 0$.

Split Λ into two disjoint subsets, $I \subset \Lambda$ and $J = \Lambda \setminus I$, satisfying

$$y \in Z_i, \forall\ i \in I, \quad y \notin Z_j, \forall\ j \in J.$$

Since $y \in M$, thus $I \neq \emptyset$ and $y \in V_i, \forall\ i \in I$. According to Proposition 2.2, $J \neq \emptyset$.

Denote

$$d = \min_{j\in J} d(y, Z_j) > 0, \qquad (4.2)$$

we can choose $0 < R < d/2$ and define a ball $B_R(y) = \{x \mid ||x - y|| < R\}$. Then, we have

$$d(x, Z_j) > R, \quad \forall\, x \in B_R(y), \quad j \in J. \tag{4.3}$$

For any $x \in \bar{B}_R(y)$, the closure of $B_R(y)$, when mode $j \in J$, is active,

$$\dot{V}(x(t))|_{f_j} < 0.$$

Since $\bar{B}_R(y)$ is compact, there exists an $\alpha > 0$ such that $\max_{x \in \bar{B}_R(y), j \in J} \dot{V}(x(t))|_{f_j} = -\alpha < 0$.

Now, assume $0 < R_1 < R$ is small enough such that as $x_0 \in B_{R_1}(y), x(t) \in B_R(y), \forall\, t \in [t_0, t_0 + \tau_0]$. Then, when $x_0 \in B_{R_1}(y)$ and t_0 is the moment when mode $j \in J$ becomes active, we have

$$V(x(t_0 + \tau_0)) < V(x_0) - \alpha\tau_0. \tag{4.4}$$

On the other hand, using Lemma 4.1 associated with assumption A3, we can find $0 < r < R_1$ such that when $x_0 \in B_r(y)$ and only modes $i \in I$ are active, we have

$$\varphi(t, x_0) \in B_{R_1}(y), \quad 0 \leqslant t \leqslant T. \tag{4.5}$$

Since y belongs to the ω-limit set, there exists $N > 0$ such that $x_k \in B_r(y)$ for all $k > N$. Recalling assumption A2′, the finite time ergodic property, on every interval $[t_k, t_k + T]$, all the modes will be active at least once. Let $t'_k \in [t_k, t_k + T]$ be the moment when a $j \in J$ mode is triggered. Then by (4.5), $\varphi(t'_k, x_k) \in B_{R_1}(y)$. According to (4.4), we obtain

$$V(x(t'_k + \tau_0)) < V(x(t'_k)) - \alpha\tau_0, \quad \forall\, k > N.$$

Then,

$$\begin{aligned} V(x(t'_{N+l} + \tau_0)) &\leqslant V(x(t'_{N+l})) - \alpha\tau_0 \\ &\leqslant V(x(t'_{N+l-1})) - 2\alpha\tau_0 \\ &\leqslant \cdots \leqslant V(x(t'_{N+1}) - l\alpha\tau_0 \\ &\quad \to -\infty, \quad l \to \infty, \end{aligned}$$

which is a contradiction. QED

In general, it is not straightforward to verify A3. We thus give a sufficient condition here.

Proposition 4.2. If $\ker(f_i) = Z_i, i \in \Lambda$, then A3 is satisfied.

Proof. If $\ker(f_i) = Z_i$, then $V_i \subset \ker(f_i)$. The conclusion follows.

Remark. In general, for switched nonlinear systems, it is not easy to obtain the global asymptotic stability result. Sometimes, we only need the local stability. If the Lyapunov function is defined on a neighborhood of the origin which is a compact set, then the conclusions of Theorem 3.1 and Theorem 4.1 hold locally.

Before ending this paper, we give an example to illustrate the effectiveness of our theorems.

Example 4.1. Consider the following switched system:

$$\dot{x} = f_{\sigma(t)}(x), \quad x \in \mathbb{R}^4, \tag{4.6}$$

where $\sigma(t) \in \Lambda = \{1, 2, 3\}$,

$$f_1(x) = \begin{pmatrix} -x_1^5 \\ x_1^3 x_2 - x_2^3 \\ 0 \\ -2x_4^3 - x_3^2 x_4 \end{pmatrix}, \quad f_2(x) = \begin{pmatrix} 0 \\ 0 \\ -x_3^3 \\ 2x_3^2 - 3x_4 \end{pmatrix}, \quad f_3(x) = \begin{pmatrix} 0 \\ -2x_2^3 + x_2 x_3^2 \\ -x_3^3 \\ 0 \end{pmatrix}.$$

Choosing $V(x) = \frac{1}{2}\sum_{i=1}^4 x_i^2$, we have

$$\begin{aligned} & Q_1(x) := \dot{V}(x)|_{f_1} \\ &= -(x_1^3 - \frac{1}{2}x_2^2)^2 - \frac{3}{4}x_2^4 - 2x_4^4 - x_3^2 x_4^2 \leqslant 0, \\ & Q_2(x) := \dot{V}(x)|_{f_2} \\ &= -(x_3^2 - x_4)^2 - 2x_4^2 \leqslant 0, \\ & Q_3(x) := \dot{V}(x)|_{f_3} \\ &= -2(x_2^2 - \frac{1}{4}x_3^2)^2 - \frac{7}{8}x_3^4 \leqslant 0. \end{aligned}$$

Obviously, $\sum_{i=1}^3 Q_i(x) < 0, \forall\, x \neq 0$. Thus, V is a CJLF. In a addition,

$$\begin{aligned} \ker(f_1) &= Z_1 = \{x \mid x_1 = x_2 = x_4 = 0\}, \\ \ker(f_2) &= Z_2 = \{x \mid x_3 = x_4 = 0\}, \\ \ker(f_3) &= Z_3 = \{x \mid x_2 = x_3 = 0\}. \end{aligned}$$

One can see easily that $\bigcup_{i \in \Lambda} Z_i \setminus \{0\}$ is disconnected. According to Theorem 3.1 (or 4.1), we conclude that system (4.6) is globally asymptotically stable if the switching signals satisfy A1 and A2 (or A2′).

5 Conclusion

In this paper, we investigated the stability of switched nonlinear systems. By introducing common joint Lyapunov function, two extensions of the LaSalle's invariance principle were obtained. Unlike traditional extensions, our results do not require individual switching modes to be asymptotically stable, while certain ergodicity restrictions are imposed on the switching signals. It has been shown that in a practical dynamic process, such as joint connection of multi-agent systems[6,8], ergodicity assumption is reasonable.

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