Adaptive $L_2$ disturbance attenuation control of multi-machine power systems with SMES units

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Abstract

Superconducting magnetic energy storage (SMES) units can be used to enhance the stability of power systems. The key to make good use of the SMES units is to design effective controllers. This paper presents a number of results on the analysis and control of multi-machine power systems with such units via Hamiltonian function method. It has been shown that the multi-machine power systems with SMES units can be made to be a port-controlled Hamiltonian (PCH) system by using a state feedback control, and that the stability of the resulting system can be established. Furthermore, this paper proposes a novel energy-based adaptive $L_2$ disturbance attenuation control scheme for the multi-machine systems with SMES units. The control scheme is a decentralized one and consists of two parts: one is an $L_2$ disturbance attenuation excitation controller for the generators, and the other is an adaptive $L_2$ disturbance attenuation controller for the SMES units. Simulations on a six-machine system with one SMES unit show that the proposed control scheme is very effective.

Keywords: Multi-machine power system; SMES; Dissipative PCH system; Stability analysis; Adaptive $L_2$ disturbance attenuation; Energy-based control scheme

1. Introduction

The superconducting magnetic energy storage (SMES) is capable of supplying both active and reactive powers simultaneously and quickly for power systems, and thus capable of enhancing the systems’ stability and reliability dramatically (Banerjee, Chatterjee, & Tripathy, 1990; Irie, Takeo, & Sato, 1992; Jiang & Chu, 2001; Simo & Kamwa, 1995). Primarily, the SMES unit was aimed to store energy during the off-peak load period and release it in the peak load period. It has been shown that the SMES can not only supply the active and reactive power simultaneously, but also help damp the electromechanical oscillations in a network (Simo & Kamwa, 1995; Wu & Lee, 1993). In fact, the unit can also be used as a power system stabilizer, if the control scheme is suitably designed (Irie et al., 1992; Maschowski & Nelles, 1992). The applications of the SMES also include load regulation, transmission stabilization, frequency-oscillation damping, uninterruptible power supply, power compensation, voltage control and improving customer power quality, etc. (Buckles & Hassenzahl, 2000; Juengst, 1998; Luongo, 1996). In the above-mentioned applications, a key to success is to design effective controllers.

In recent years, the port-controlled Hamiltonian (PCH) system, proposed by Maschke and van der Schaft (1992), van der Schaft and Maschke (1995), has been well investigated in van der Schaft (1999), Maschke, Ortega, and van der Schaft (2000), Maschke, Ortega, van der Schaft, and Escobar...
et al. (2000) applied it to the stabilization of single-machine infinite-bus systems, and designed effective control laws, respectively. The realization problem still remains open.

In this paper, we use the PCH system to investigate practical control problems is to transform the system under consideration into a dissipative PCH system, i.e., to obtain the dissipative Hamiltonian realization (Ortega et al., 2002; Wang, Li, & Cheng, 2003). The dissipative Hamiltonian realization of multi-machine systems is a long-standing problem in the energy-based control of power systems. Recently, some significant contributions have been made to the problem (Ortega et al., 2005; Sun et al., 2001; Wang et al., 2003; Xi et al., 2002). It is well worth noticing that Ortega et al. (2005) has set up a useful method to handle the long-standing problem of multi-machine systems with non-negligible transfer conductances. The method is very important, because it can not only provide a PCH structure for the systems but also pave an easy way to analyze the domain of attraction of the systems, which is a key factor in the transient stabilization of power systems. Unfortunately, none of these papers has obtained the final dissipative Hamiltonian realization of multi-machine power systems in general model. The structure of multi-machine power systems in general model is too complicated, so that the realization problem still remains open.

In this paper, we use the PCH system to investigate adaptive $L_2$ disturbance attenuation control of multi-machine power systems with SMES units, and propose an energy-based control design method for the systems (note: it should be pointed out that the model of multi-machine systems used in the paper is not the general one, but it is a standard model. Please refer to Lu, Sun, Xu, & Mochizuki, 1996). First, we transform the multi-machine power systems with SMES units into a PCH system by using a feedback control. Then, we study the stability of the obtained PCH system. Finally, an energy-based adaptive $L_2$ disturbance attenuation controller is designed for the multi-machine power systems with SMES units.

The design method and results proposed in this paper can be used in practice to design energy-based adaptive robust controllers for power systems with SMES units, when there exist external disturbances in the system and parametric uncertainties in the SMES units. In addition, it should be pointed out that the main objective of the paper is to design effective controllers to make good use of the SMES units, and hence the case of generators' parametric uncertainty is not taken into account in the main results of this paper. Although the case is considered in a remark of the paper (see Remark 3 below), further study remains needed to design an effective adaptive controller for the case. This is the practical limitation of the main results in the paper.

The rest of the paper is organized as follows. Section 2 gives the model of the multi-machine power systems with SMES units, and Section 3 transforms the model as a global PCH system. In Section 4, we study the stability of the obtained PCH system. Section 5 investigates the energy-based adaptive $L_2$ disturbance attenuation control design for the multi-machine power systems with SMES units. Section 6 presents some simulation results, which is followed by the conclusion in Section 7.

2. System model

2.1. A model for SMES units

Fig. 1 shows the typical configuration of an SMES unit, which is composed of a 12 pulse thyristor bridge and a superconducting magnet (Jiang et al., 2001; Masahide, Yasunori, & Kiichiro, 1999). The use of self-commuted devices like gate turn off thyristor (GTOs) guarantees a wide control range of active and reactive powers. By using the pulse width modulation (PWM) control with self-commuted converters, one can realize simultaneous control of the active and reactive powers at high MVA levels with less harmonics. The modulation index, $W$, and the phase firing angle, $\alpha$, can be used as the control variables so that it is guaranteed that the SMES is capable of controlling the active power, $P_F$, and the reactive power, $Q_F$, independently (Chu, Jiang, Lai, Wu, & Liu, 2001; Jiang et al., 2001). According to Jiang et al. (2001), Chu et al. (2001),
$P_F$ and $Q_F$ can be expressed as

$$
\begin{align}
P_F &= 1.5 \sqrt{2} W V_F I_F \cos \alpha, \\
Q_F &= 1.5 \sqrt{2} W V_F I_F \sin \alpha,
\end{align}
$$

where $V_F$ is the voltage at the lower side of the transformer connecting to the network (see Fig. 2), $I_F$ is the current of the superconducting coils, and the dynamics of $W$ and $\alpha$ are described by the following first-order inertial equations:

$$
\begin{align}
\dot{W} &= -\frac{1}{T} W + \frac{1}{T} u_W, \\
\dot{\alpha} &= -\frac{1}{T} \alpha + \frac{1}{T} u_{\alpha},
\end{align}
$$

here $T$ is the time constant of the controlled circuit.

With a straightforward computation shown in Appendix A, we know that the dynamics of $P_F$ and $Q_F$ can be given in the form (Liu, 2002; Xi & Guan, 2001):

$$
\begin{align}
\dot{P}_F &= -\frac{1}{T} P_F + u_P, \\
\dot{Q}_F &= -\frac{1}{T} Q_F + u_Q,
\end{align}
$$

where $u_P$, $u_Q$ are new control inputs of $P_F$ and $Q_F$, respectively.

2.2. A model of multi-machine power systems with SMES units

This paper considers an $n$-machine power system with $m$ SMES units ($1 \leq m \leq n$) interconnected through a transmission network (for simplicity, we denote the system by $n$-M plus $m$-SMES in sequel). We assume that one SMES unit is located close to one generator. Thus, one SMES unit together with its nearby generator can be considered as a new equivalent generator in the network (see Fig. 2). Without loss of generality, we assume that the $m$ SMES units are assigned to the first $m$ generators. Then, the model of the $n$-M plus $m$-SMES system can be given as

**Dynamics of the generators (Lu et al., 1996):**

$$
\begin{align}
\dot{\delta}_i &= \omega_i - \omega_0, \\
\dot{\omega}_i &= \frac{D_i}{M_i} (\omega_i - \omega_0) - \frac{\omega_0}{M_i} P_{ei} + \varepsilon_{i1}, \\
\dot{E}_{qi} &= -\frac{1}{T_{d0i}} E_{qi} + \frac{1}{T_{d0i}} u_{fi} + \varepsilon_{i2}, \\
&\quad i = 1, 2, \ldots, n,
\end{align}
$$

where $\varepsilon_{i1}$, $\varepsilon_{i2}$ are disturbances; $\dot{\delta}_i$ is the power angle of the $i$th generator, in radian; $\delta_0$ the rotor speed of the $i$th generator, in rad/s; $\omega_0 = 2\pi f_0$; $E_{qi}$ the $q$-axis internal transient voltage of the $i$th generator, in per unit (p.u.); $\omega_i$ the internal voltage, in p.u.; $\omega_0$ the field circuit time constant, in seconds; $D_i$ the damping constant, in p.u.; $T_{d0i}$ the field circuit time constant, in seconds; $P_{mi}$ the mechanical power, assumed to be constant, in p.u.; $P_{ei}$ the active electrical power, in p.u.; $I_{di}$ the $d$-axis current, in p.u.; $Y_{ij} = G_{ij} + j B_{ij}$ the admittance of line $i-j$ in p.u.; $Y_{ii} = G_{ii} + j B_{ii}$ the self-admittance of bus $i$, in p.u.; $P_{Fi}$ and $Q_{Fi}$ are, respectively, the active and reactive powers of the $i$th SMES unit (here $i = 1, 2, \ldots, m$), in p.u.; as for $V_{Fi}$, $V_{Ti}$ and $X_{Fi}$, please refer to Fig. 2, and they are all in p.u. It should be noted that the resistance and crosswise voltage excursions of the transformer are neglected here.

**Remark 1.** The nodal susbance matrix of the system (4) is symmetric, and $G_{ij} + j B_{ij}$ (a complex number) is just the element located at $(i, j)$ in the matrix. Thus, we know that $G_{ij} = G_{ji}$ and $B_{ij} = B_{ji}$ hold for all $i, j = 1, 2, \ldots, n$, which is important for our analysis below.

3. A PCH structure of the system

In this section, we transform the $n$-M plus $m$-SMES system into a dissipative PCH system. Assume $(\delta^0_i, \omega_0, E^0_{qi})$, $i = 1, 2, \ldots, n$, is the pre-assigned operating point of the system (4), which should remain unchanged in the control design.
Let
\[ x_{1i} = \delta_i, \quad x_{2i} = \omega_i - \omega_0, \quad x_{3i} = E_0^i, \quad x_{4i} = P_{Fi}, \]
\[ x_{j5} = V_{ij} - V_{i0}, \quad a_i = \frac{\omega_0}{M_i} P_{mi}, \]
\[ b_i = \frac{D_i}{M_i}, \quad c_i = \frac{\omega_0}{M_i} G_{ii}, \]
\[ d_i = \frac{M_i}{T_{d0i}}, \quad e_i = \frac{1}{T_{d0i}}, \]
\[ h_i = \frac{x_{di} - x_{di}^0}{T_{d0i}}, \quad u_i = \frac{1}{T_{d0i}} u_{fi}, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m, \]

then the n-M plus m-SMES system can be rewritten as

\[ \dot{x}_i = x_2, \]
\[ \dot{x}_2 = a_i - b_i x_2 - c_i x_3^2 - d_i x_{1i} \]
\[ - d_i x_{1i} \sum_{s=1,i}^n B_{is} x_s \sin(x_{1s} - x_{1i}) + \varepsilon_{i1}, \]
\[ x_{3i} = -(e_i + h_i B_{ii} x_{3i})^2 \]
\[ + h_i \sum_{s=1,i}^n B_{is} x_s \cos(x_{1s} - x_{1i}) + u_i + \varepsilon_{i2}, \]
\[ x_{4i} = \frac{1}{T_i} x_{4i} + u_{pi} + \varepsilon_{i3}, \]
\[ x_{5i} = - \frac{1}{T_i} x_{5i} + u_{ii} + \varepsilon_{i4}, \quad i = 1, 2, \ldots, m, \]
\[ \dot{x}_{1j} = x_{2j}, \]
\[ \dot{x}_{2j} = a_j - b_j x_{2j} - c_j x_{3j}^2 \]
\[ - d_j x_{3j} \sum_{s=1,j}^n B_{js} x_s \sin(x_{1s} - x_{1j}) + \varepsilon_{j1}, \]
\[ x_{3j} = -(e_j + h_j B_{jj} x_{3j})^2 \]
\[ + h_j \sum_{s=1,j}^n B_{js} x_s \cos(x_{1s} - x_{1j}) + u_j + \varepsilon_{j2}, \]
which is the relationship between the inputs \( u_{Qj} \) and \( u_{ij} \).

It appears almost impossible to transform system (6) into a Hamiltonian system directly. However, with the following feedback control law the compensated system becomes a PCH system as follows:

\[ \dot{x} = (J - R) \frac{\partial H(x)}{\partial x} + G_1 v + G_2 e, \]

where \( J = -J^T = \text{Diag}[J_1, \ldots, J_m, J_{m+1}, \ldots, J_n], \) \( R = \text{Diag}[R_1, \ldots, R_m, R_{m+1}, \ldots, R_n], \) \( g_1 = \text{Diag}[g_{11}, \ldots, g_{1m}, g_{m+1}, \ldots, g_{mn}], \) \( G_1 = \text{Diag}[G_{11}, \ldots, G_{1m}, G_{m+1}, \ldots, G_{mn}], \)
\[ v = [v_1^T, \ldots, v_m^T, v_{m+1}^T, \ldots, v_n^T], \quad e = [e_1^T, \ldots, e_m^T, e_{m+1}^T, \ldots, e_n^T]^T, \]
\[ x = [x_1^T, \ldots, x_m^T, x_{m+1}^T, \ldots, x_n^T]^T, \]
\[ \dot{e} = \partial H/\partial x = [\dot{x}_1, \ldots, \dot{x}_n]^T, \]

Proof. See Appendix B.

In addition, it can be seen that, to keep the system's pre-assigned operating point unchanged, \( k_i \) and \( u_i \) in the feedback...
(8) should satisfy
\[- (e_i + h_i B_{ii}) E_{q_i}^{(0)} + h_i \sum_{j=1, j \neq i}^{n} B_{ij} E_{q_j}^{(0)} \cos(\delta_i^{(0)} - \delta_j^{(0)})\]
\[- 2c_i h_i \frac{d}{d_t} \delta_i^{(0)} E_{q_i}^{(0)} - k_i E_{q_i}^{(0)} + \ddot{u}_i = 0,\]
where \(i = 1, 2, \ldots, n\).

**Remark 2.** (1) The Hamiltonian realization (9) is a global result, which holds on the whole state space of the power system.

(2) In practice, the variation range of \(x_{i2} (= \omega_i - \omega_0)\) is, in general, ±3%, while that of \(x_{i4}\) (the SMES’ active power) is ±\(P_{\text{max}}\). The two ranges are quite different in value, and \(l_i\) in the feedback control (8) is to make them match in value. This is also the guideline of choosing \(l_i\).

(3) From (16) and the fact that \(x_{i2} \equiv 0\) when the system is stable, it can be seen that the feedback control (8) does not need to consider the system’s operating point.

**Remark 3.** (1) The feedback law (8) admits the parametric uncertainty in the SMES units, that is, the parametric uncertainty in the SMES units does not affect the role of (8) to provide exact compensation for the system.

(2) When the generators’ parameters are not known exactly, the compensation provided by the feedback law (8) will not be exact. In this case, we can modify (8) to obtain the following adaptive feedback law to provide exact compensation for the system:

\[
\begin{align*}
\dot{u}_i &= -\frac{2c_i h_i}{d_i} x_i x_{i3} - k_i x_{i3} + \ddot{u}_i - 2\Phi_i \hat{\eta}_i x_{i1} x_{i3} + \psi_i, \\
\dot{\eta}_i &= 2P_i \Phi_i^T x_{i1} x_{i3} \phi_i(x), \quad i = 1, 2, \ldots, n, \\
\dot{u}_{pj} &= l_j x_{j2} + \psi_j, \\
\dot{u}_{ij} &= \psi_j, \quad j = 1, 2, \ldots, m,
\end{align*}
\]

where \(\Phi_i^T \in \mathcal{R}^m\) (\(s \geq 1\)) is a known vector to assume such that \(c_j h_i / d_i = \frac{\Theta_i h_i^0}{d_i} \Phi_i^T \eta_i; c_j^0, \hat{\eta}_i, h_i^0\) and \(d_i^0\) are the nominal values of \(c_j, h_i, \) and \(d_i, \) respectively; \(\eta \in \mathcal{R}^s\) denotes the unknown element of \(c_j h_i / d_i; \) \(\hat{\eta}\) is the estimate of \(\eta; \) \(P_i \in \mathcal{R}^{m \times s}\) is a positive definite matrix (adaptation gain); and \(\phi_i(x)\) is a function to be determined by Hamiltonian structural properties.

(3) Similar to the proofs of Theorem 1 and Lemma 2 below, we can show that under the control law (17), the \(n\)-M plus \(m\)-SMES system can be transformed into an augmented PCH system with the following function as its Hamiltonian function:

\[
\tilde{H} = H(x) + \frac{1}{2} \sum_{i=1}^{n} (\eta - \hat{\eta})^T P_i^{-1} (\eta - \hat{\eta}),
\]

where \(H(x)\) is given by (15).

(4) When the generators’ parameters are not known exactly, based on the above augmented PCH system, we can use the stability analysis method in Section 4 and the control design method in Section 5 to analyze the stability and design an adaptive \(L_2\) disturbance attenuation controller for the \(n\)-M plus \(m\)-SMES system, respectively, only the processes will be more complicated (see Sections 4 and 5 below).

### 4. Stability analysis

In this section, we analyze the stability of the system (9).

First, we study the properties of the Hamiltonian function \(H(x)\).

Construct two functions:

\[
H_x = \sum_{i=1}^{n} \left( \frac{k_i}{2h_i} x_{i1} + \frac{c_i}{d_i} x_{i1} - \frac{1}{2} \sum_{j=i}^{n} B_{ij} \right) x_{i3}^2
- \frac{a_i}{d_i} x_{i1} + \frac{1}{2} \frac{n}{2} \sum_{j=1}^{n} B_{ij} (|x_{i3}| - |x_{j3}|)^2
- \frac{1}{2} \sum_{i=1}^{n} \frac{2h_i (\eta_i + h_i B_{ii})}{e_i + h_i B_{ii}}\]

\[
H_{\beta} = \sum_{i=1}^{n} \left( \frac{k_i}{2h_i} x_{i1} + \frac{c_i}{d_i} x_{i1} - \frac{1}{2} \sum_{j=i}^{n} B_{ij} \right) x_{i3}^2
- \frac{a_i}{d_i} x_{i1} + \frac{1}{2} \frac{n}{2} \sum_{j=1}^{n} B_{ij} (|x_{i3}| + |x_{j3}|)^2
- \frac{1}{2} \sum_{i=1}^{n} \frac{2h_i (\eta_i^0 + h_i B_{ii})}{e_i + h_i B_{ii}}\]

The difference between \(H_x\) and \(H_{\beta}\) is only in one term: \(H_x\) is with \((|x_{i3}| - |x_{j3}|)^2\), while \(H_{\beta}\) with \((|x_{i3}| + |x_{j3}|)^2\). From (15) and the relation \(B_{ij} = B_{ji}\) (see Remark 1), we can easily obtain

\[
H_x \leq H(x) \leq H_{\beta}
\]

Next, to further study the properties of the Hamiltonian function \(H(x)\), we present a lemma.

**Lemma 1.** Assume that \(f(x) : \mathcal{R}^n \rightarrow \mathcal{R}\) is a smooth bounded and radially unbounded function. Then there is a local minimum point \(x_0\), with

\[
\frac{\partial f(x)}{\partial x_i}(x_0) = 0, \quad i = 1, \ldots, n.
\]

**Proof.** Since \(f(x)\) is radially unbounded, for any \(l \in \mathcal{R}\)

\[
\mathcal{L} = \{ x | f(x) \leq l \}
\]

is a compact set (we may choose \(l\) large enough to avoid the frustrating case of \(\mathcal{L} = \emptyset\)). On the other hand, \(f(x)\) is lower bounded, then over the compact set \(\mathcal{L}\) we can find a point \(x_0\) such that

\[
f(x_0) = \inf_{x \in \mathcal{L}} f(x).
\]

Since the function is radially unbounded, we can assume that \(x_0\) is not on the boundary (otherwise, we can choose larger \(l\)). By smoothness, \(x_0\) satisfies (19). \(\square\)
Remark 4. In general, $x_0$ is not unique. Thus, $x_0$ is only a minimum point. But if we know that the solution of (19) is unique (an isolated point), it is a strict minimum point (locally strict minimum).

The following corollary is an immediate consequence of Lemma 1.

Corollary 1. Let $V$ be a compact analytic sub-manifold of $\mathbb{R}^d$ (compact is the sense of inherited topology of $\mathbb{R}^d$) and $f(x, y) : V \times \mathbb{R}^m \rightarrow \mathbb{R}$ be smooth, lower bounded and radially unbounded with respect to $y$. Then there is a local minimum point $(x_0, y_0)$ satisfying

\[ \frac{\partial f}{\partial x}(x_0, y_0) = 0, \quad \frac{\partial f}{\partial y}(x_0, y_0) = 0. \]

Now the closed-loop multi-machine system with SMES units is defined on $(S^1)^n \times \mathbb{R}^{2(n+m)}$, where $S^1$ is a circle, and hence $S^n$ is compact. Using the structure of $H_L$ and the property of $x_{i1} \in [-\pi, \pi]$, we know that $H_L$ is lower bounded if $k_i$ is selected to be suitably large, say, $k_i = k_{io}$. Now let $k_i = k_{io}$, and from (18) one can see that $H(x)$ is lower bounded and radially unbounded with respect to $x_{i2}, x_{i3}, x_{i4}$ and $x_{i5}$. From Corollary 1, there exists a working point $x_0$ satisfying $(\partial H/\partial x_{ij})(x_0) = 0$. Note that the equilibrium of the power system is an isolated point, thus the working point is a strict local minimum point. From the properties of the dissipative PCH system (Maschke et al., 2000; Ortega et al., 2002), we know that when $v = 0$ and $\varepsilon = 0$, the system (9) is Lyapunov stable at the working point. Moreover, we have the following result.

Proposition 1. When $v$ and $\varepsilon$ vanish, the system (9) is asymptotically stable at the operating point. In other words, the feedback control (8) is a stabilizer of the $n$-M plus $m$-SMES system.

Proof. See Appendix C.

Remark 5. Eq. (16) with $k_i = k_{io}$ ($i = 1, 2, \ldots, n$) is the condition to determine $k_i$ and $\bar{a}_i$.

5. Adaptive $L_2$ disturbance attenuation of the $n$-M plus $m$-SMES system

In this section, to make good use of the SMES units, we investigate the adaptive $L_2$ disturbance attenuation control design for the $n$-M plus $m$-SMES system. First, we propose a lemma on $L_2$ disturbance attenuation, which will be used in the control design.

Consider a dissipative PCH system as follows:

\[ \begin{cases} \dot{x} = (J(x) - R(x))\nabla H(x) + g_1(x)u + g_2(x)w, \\ \dot{z} = h(x)g_1^T(x)\nabla H(x), \end{cases} \]

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ is the control input, $w \in \mathbb{R}^d$ is the disturbance, $J(x)$ is skew-symmetric, $R(x) \geq 0$, $H(x)$ has a strict local minimum at the system’s equilibrium, $z$ is the penalty function, $h(x)$ is a weighting matrix and $\nabla H = \nabla H/\partial x$. Assume that the system (20) involves parametric perturbations, which are small enough to keep the dissipative structure of the system unchanged, and that the parametric perturbations can be represented by a constant unknown vector $p$. Then, the system (20) becomes (Shen et al., 2000).

\[ \dot{x} = [J(x, p) - R(x, p)]\frac{\partial H}{\partial x}(x, p) + g_1(x)u + g_2(x)w, \]

where $J(x, 0) = J(x)$, $R(x, 0) = R(x)$, $H(x, 0) = H(x)$, $J(x, p)$ is skew-symmetric and $R(x, p)$ is positive semi-definite.

Decompose $J(x, p)$, $R(x, p)$ and $(\nabla H/\partial x)(x, p)$ as follows:

\[ J(x, p) = J(x) + \Delta J(x, p) , \quad R(x, p) = R(x) + \Delta R(x, p), \quad \frac{\partial H}{\partial x}(x, p) = \frac{\partial H}{\partial x}(x) + \Delta H(x, p). \]

Given a disturbance attenuation level $\gamma > 0$, take $z$ as the penalty function. Then, we have the following result.

Lemma 2. For the penalty function $z$ and the given disturbance attenuation level $\gamma > 0$, if

\[ P := R(x, p) + \frac{1}{2\gamma}[g_1(x)g_1^T(x) - g_2(x)g_2^T(x)] \geq 0 \]

and there exists an $s \times m$ matrix $\Phi(x)$ such that

\[ [J(x, p) - R(x, p)]\Delta H(x, p) = g_1(x)\Phi^T(x)\theta \]

holds for all $x$, then an adaptive $L_2$ disturbance attenuation controller of the system (21) can be designed as

\[ \begin{cases} u = -\frac{1}{2}[h^T(x)h(x) + \frac{1}{\gamma^2}I_m]g_1^T(x)\frac{\partial H}{\partial x}(x) - \Phi^T(x)\hat{\theta}, \\ \dot{\theta} = Q(x)g_1^T(x)\frac{\partial H}{\partial x}(x), \end{cases} \]

and $\gamma$-dissipation inequality

\[ \dot{V}(x, \hat{\theta}) + \nabla H(x)P \nabla H(x) \leq \frac{1}{2}[\varepsilon^2||w||^2 - ||z||^2] \]

holds along the trajectories of the closed-loop system (21) with (24), where $\varepsilon \in \mathbb{R}^d$ is an unknown constant parameter vector related to the perturbation $p$, $\hat{\theta}$ is an estimate of $\theta$, $Q > 0$ is a constant adaptation gain matrix, and

\[ V(x, \hat{\theta}) = H(x) + \frac{1}{2}(\theta - \hat{\theta})^TQ^{-1}(\theta - \hat{\theta}). \]

Proof. Substituting (24) into (21) and using the condition (23) yield

\[ \begin{cases} \dot{x} = [J(x, p) - R(x, p)]\frac{\partial H}{\partial x}(x) + g_1(x)\hat{\theta} + g_2(x)w, \\ \dot{\theta} = Q(x)g_1^T(x)\frac{\partial H}{\partial x}(x), \end{cases} \]

which can be changed into an augmented dissipative PCH system as follows:

\[ \dot{X} = [\dot{J}(X) - \dot{R}(X)]\nabla V(X) + \dot{G}_1(X)(\hat{\theta} + \dot{\theta}) + \dot{G}_2(X)w, \]

(27)
where \( X = [x^T, \hat{\theta}^T]^T \), \( \bar{G}_1(X) = [g_1^T, 0]^T \), \( \bar{G}_2(X) = [g_2^T, 0]^T \) with proper dimensions,
\[
\bar{J}(X) = \begin{bmatrix} J(x, p) & (Q \Phi(x) g_1^T)^T \\
Q \Phi(x) g_1^T & 0 \end{bmatrix},
\]
\[
\bar{R}(X) = \text{Diag}(R(x, p), 0), \nabla V(X) = (\partial^T V / \partial x, \partial^T V / \partial \hat{\theta})^T \text{ and}
\]
\[
\bar{u} = -\frac{1}{2} \{ (h^T \bar{h} + \frac{1}{2} I_m) g_1^T \} \frac{\partial H}{\partial x}(x),
\]
\[
= -\frac{1}{2} \{ (h^T \bar{h} + \frac{1}{2} I_m) g_1^T \} \bar{G}_1^T(X) \frac{\partial V}{\partial x}.
\]
(28)

Rewriting the penalty function \( z \) leads to
\[
z = h \bar{g}_1^T(x) \nabla H(x) = h \bar{G}_1^T(X) \nabla V.
\]
(29)

With the system (27), the control law (28) and the penalty function (29), it can be seen from Wang et al. (2003) that
\[
\dot{V} + \frac{\partial V^T}{\partial x} [\bar{R}(X) + \frac{1}{2} \{ (\bar{G}_1 \bar{G}_1^T - \bar{G}_2 \bar{G}_2^T) \} \frac{\partial V}{\partial x}]
\leq \frac{1}{2} \{ \gamma^2 \| w \|^2 - \| z \|^2 \},
\]
which implies that (25) holds, and thus the proof is completed. \( \square \)

**Remark 6.** Lemma 2 is motivated by Shen et al. (2000). When \( g_1(x) \equiv g_2(x) \), Lemma 2 degenerates to the result of Shen et al. (2000).

In the following, we apply Lemma 2 to design an energy-based adaptive \( L_2 \) disturbance attenuation control scheme for the \( n \)-M plus \( m \)-SMES system.

Let a disturbance attenuation level \( \gamma > 0 \) be given. Assume that the time constants of the SMES units have perturbations: \( T_i \to T_i + p_i \), where \( p_i \) are unknown and satisfy \( |p_i| < T_i, i = 1, 2, \ldots, m \). Then, it is easy to see from Section 3 that in the system (9) the Hamiltonian function \( H(x, p) \) and the matrices \( R_i \) \( (i = 1, 2, \ldots, m) \) involve the perturbation \( p := [p_1, \ldots, p_m]^T \), and they will take the forms of \( H(x, p) \) and \( R_i(x, p) \), respectively. A straightforward computation shows that in this case the system (9) becomes
\[
\dot{x} = [J - R(x, p)] \frac{\partial H}{\partial x}(x, p) + G_1 v + G_2 \epsilon,
\]
(30)

where \( R(x, p) = \text{Diag}(R_1(x, p), \ldots, R_n(x, p), R_{m+1}, \ldots, R_m), \)
\( (\partial H / \partial x)(x, p) = \{ (\partial^2 H / \partial x^2)(x, p), 0, (\partial^2 H / \partial x \partial \hat{\theta})(x, p),
(\partial^2 H / \partial x \partial \hat{\theta})(x, p), \ldots, (\partial^2 H / \partial x \partial \hat{\theta})(x, p) \}^T, \)
\[
\left\{ R_i(x, p) = R_i + \Delta R_i(x, p), \right. \\
\frac{\partial H}{\partial x_i}(x, p) = \frac{\partial H}{\partial x_i}(x) + \Delta_i H(x, p), \quad i = 1, 2, \ldots, m,
\]
and
\[
\Delta R_i(x, p) = \text{Diag} \left\{ 0, 0, 0, \frac{-p_i T_i}{T_i (T_i + p_i)} \right\},
\]
(31)
\[
\Delta_i H(x, p) = \left[ 0, 0, 0, \frac{-p_i x_{i5}}{T_i (T_i + p_i)} \right]^T.
\]
(32)

To design the control scheme for the system, we take
\[
z = r(x) G_1^T V H(x)
\]
(33)
as the penalty function, where \( r(x) = \text{Diag}(r_{11}, r_{12}, r_{13}, \ldots, r_{m1}, r_{m2}, r_{m3}, r_{m+1, 1}, r_{m+1, 2}, \ldots, r_{m+1, n}) > 0 \) is a constant weighting matrix, \( z = [z_1, z_2, \ldots, z_{m+1}, \ldots, z_{m}]^T, z_i = [z_{i1}, z_{i2}, z_{i3}]^T (i = 1, 2, \ldots, m) \), and \( z_j := z_{j1} (j = m + 1, 1, \ldots, n) \).

Now, we check whether the condition (23) holds for the system (30). Choose \( \Phi_i(x) = [0, 0, x_{i5}] \) and \( \theta_i = p_i / (T_i (T_i + p_i)) \), \( i = 1, 2, \ldots, m \), then it is easy to see from (30)–(32) that
\[
[J_i - R_i(x, p)] \Delta_i H(x, p) = g_1 \Phi_1^T(x) \theta_i,
\]
(34)

where \( \Delta_i H(x, p) = [(\Delta_1 H(x, p))^T, \ldots, (\Delta_m H(x, p))^T]^T, 0, \ldots, 0]^T \). Thus, (23) is satisfied.

Then, we check whether the condition (22) holds for the system (30), i.e., whether \( R(x, p) + (1/2 \gamma^2) [G_1 G_1^T - G_2 G_2^T] \geq 0 \) holds for all \( x \) and the above \( p, \gamma \). Let
\[
P_i := R_i(x, p) + \frac{1}{2 \gamma^2} g_1 g_1^T - \frac{1}{2 \gamma^2} g_1 g_1^T
\]
\[
= \text{Diag}(0, b_i d_i, h_i, \frac{l_i}{T_i + p_i}, 1) - \text{Diag}(0, \frac{1}{2 \gamma^2}, 0, 0, 0)
\]
\[
i = 1, 2, \ldots, m,
\]
\[
P_j := R_j(x) + \frac{1}{2 \gamma^2} g_1 g_1^T - \frac{1}{2 \gamma^2} g_1 g_1^T
\]
\[
= \text{Diag}(0, b_j d_j, h_j) - \text{Diag}(0, \frac{1}{2 \gamma^2}, 0, 0)
\]
\[
j = m + 1, \ldots, n.
\]

It can be seen that \( P_i \geq 0 \) when \( 1/2 \gamma^2 \leq b_i d_i = \omega_0 D_i / M_i^2 \), \( i = 1, 2, \ldots, n \) (see Section 3). Now we choose
\[
\gamma \geq \gamma_0 := \max_{1 \leq i \leq n} \left\{ \frac{M_i}{\sqrt{2 \omega_0 D_i}} \right\}
\]
then \( P_i \geq 0 \), which implies \( R(x, p) + (1/2 \gamma^2) [G_1 G_1^T - G_2 G_2^T] = \text{Diag}(P_1, \ldots, P_n) \geq 0 \). Thus, (22) holds for the system (30) when \( \gamma \geq \gamma_0 \).

Therefore, all the conditions of Lemma 2 can be satisfied. According to Lemma 2, when \( \gamma \geq \gamma_0 \), an adaptive \( L_2 \) disturbance attenuation controller of the system (30) can be designed as
\[
\left\{ \begin{array}{l}
\dot{v} = -\frac{1}{2} \bar{r}^T(x) r(x) G_1^T \frac{\partial H}{\partial x}(x) - \frac{1}{2 \gamma^2} G_1^T \frac{\partial H}{\partial x}(x) - \Phi_1^T(x) \dot{\theta}, \\
\dot{\theta} = Q \Phi(x) G_1^T \frac{\partial H}{\partial x}(x),
\end{array} \right.
\]
(35)

where \( \dot{\theta} \) is the estimate of \( \theta \), and \( Q = \text{Diag}(q_1, \ldots, q_n) > 0 \) is a constant adaptation gain matrix.
Fig. 3. A six-machine system with one SMES unit.

From (35) and (8), we have

\[
\begin{align*}
    u_i &= -2c_i h_i \frac{d_i}{d_i} x_{i1} x_{i3} - k_0 x_{i3} + \tilde{u}_i - \frac{1}{2} (r_{i1} + \frac{1}{r_{i1}^2}) z_{i1}, \\
    i &= 1, 2, \ldots, n, \\
    u_{Pj} &= l_j x_{j2} - \frac{1}{2} (r_{j2} + \frac{1}{r_{j2}^2}) z_{j2}, \\
    u_{Qj} &= -\frac{1}{2} (r_{j3} + \frac{1}{r_{j3}^2}) z_{j3} - x_{j3} \tilde{\theta}_j, \\
    \dot{\tilde{\theta}}_j &= \frac{q_j}{T_j} x_{j3}^2, \\
    j &= 1, 2, \ldots, m,
\end{align*}
\]

where \( \tilde{\theta}_j \) is the \( j \)-th component of \( \tilde{\theta} \), \( j = 1, 2, \ldots, m \) (note: \( \tilde{\theta}_i = 0, i = m+1, \ldots, n \)). Rewrite (36) with the original notations of the variables and parameters, then we obtain the desired adaptive \( L_2 \) disturbance attenuation controller of the \( n \)-M plus \( m \)-SMES system as follows:

\[
\begin{align*}
    u_{fi} &= -2G_{ii}(x_{di} - x_{d_i}') \delta_i E_{q_i}' - k_0 T_{doi} E_{q_i}' + T_{doi} \tilde{u}_i \\
    &= -\frac{1}{2} T_{doi} (r_{i1} + \frac{1}{r_{i1}^2}) z_{i1}, \\
    i &= 1, 2, \ldots, n, \\
    u_{Pj} &= l_j (\omega_j - \omega_0) - \frac{1}{2} (r_{j2} + \frac{1}{r_{j2}^2}) 1 T_j P_{Fj}, \\
    u_{Qj} &= -\frac{V_{Fj}}{2T_j X_{Tj}} (r_{j3} + \frac{1}{r_{j3}^2} + 2T_j \hat{\theta}_j) \Delta V_{ij} \\
    &= -\frac{V_{Fj}}{T_j X_{Tj}} (V_{Fj} - V_{ij0}) - \frac{V_{Fj}^2 - Q_{Fj} X_{Tj} V_{Fj}}{V_{Fj} X_{Tj}}, \\
    \dot{\tilde{\theta}}_j &= \frac{q_j}{T_j} (\Delta V_{ij})^2, \\
    j &= 1, 2, \ldots, m,
\end{align*}
\]

where \( \Delta V_{ij} = V_{ij} - V_{ij0} \), and \( r_{i1}, r_{j2}, r_{j3}, q_j \) are adjustable positive constants.

Summarizing the above leads to the following result.

**Theorem 2.** For the penalty function (33) and the given disturbance attenuation level \( \gamma > 0 \), if

\[
\gamma \geq \gamma_0 = \max_{1 \leq i \leq n} \left[ \frac{M_i}{\sqrt{2} \omega_i D_i} \right]
\]

holds, then an adaptive \( L_2 \) disturbance attenuation controller of the \( n \)-M plus \( m \)-SMES system with the parametric perturbation \( p \) can be given as (37).

**Remark 7.**

1. It is noted that the control scheme (37) is a decentralized one.
2. The control scheme (37) consists of two parts: the first part is an \( L_2 \) disturbance attenuation excitation controller for the \( n \) generators, and the second part is an adaptive \( L_2 \) disturbance attenuation controller for the \( m \) SMES units.

6. Simulation

A six-machine system (Lu et al., 1996) is chosen as an example to demonstrate the effectiveness of the control (37). The system is shown in Fig. 3, where one SMES unit is connected into the system. As for the system’s data, see Lu et al. (1996).

In this example, Generator No. 6 is a synchronous condenser and Generator No. 1 is chosen as the reference machine. The simulation is completed by the PSASP package, which is a professional testing system for power systems designed by China Electrical Power Research Institute.

In order to illustrate the difference of effectiveness of different control schemes, we made comparisons of the following control configurations:

**Controller I:** Generators No. 2–5 are equipped with the power system stabilizer (PSS) and the SMES unit is equipped with the PID controller, where the parameters of the PID controller are tuned as follows (Lu & Sun, 1993): for \( u_P \), \( P = -0.01 \), \( I = -0.02 \) and \( D = 0 \); for \( u_Q \), \( P = -100.01 \), \( I = -10.2 \) and \( D = 0 \) (note: (i) PSS and PID controllers are widely used in
practical power systems; (ii) Since Generator No.1 is used as the reference and Generator No.6 is a synchronous condenser, neither of them is equipped with the PSS).

Controller II: Generators No. 2–5 are equipped with our $L_2$ disturbance attenuation controller (the first part in (37)) and the SMES unit is equipped with our adaptive $L_2$ disturbance attenuation controller (the second part in (37)), where we let $\gamma=2$ (note: in this example $\gamma_0=79.5/\sqrt{2} \times 314 \times 3=1.8316$), $T_1=0.02$, $r_{11}=r_{12}=r_{13}=0.2$, $l_1=1$ and $q_1=1$.

In simulation, to study the robustness of the controllers, $T_1$ is given 50% perturbation and the following two faults are considered, respectively:

Fault 1: A three-phase temporary short-circuit fault is assumed to occur at point K (see Fig. 3) during the time period 0–0.15 s. In this case, the simulation results are showed in Figs. 4 and 5, which are the responses of the rotor angles under the above two control configurations, respectively, where $\delta_{11} = \delta_i \ - \delta_1$ $(i = 2, 3, 4, 5)$.

Fault 2: A three-phase temporary short-circuit fault is assumed to occur at point L (see Fig. 3) during the time period 0–0.15 s. In this case, the simulation results are showed in Figs. 6 and 7, which indicate the responses of the rotor angles under the above two control configurations, respectively.

From Figs. 4–7, we can see that under our adaptive $L_2$ disturbance attenuation control scheme (37) the system remains stable when Faults 1 and 2 occur. While when the PSS+PID control scheme is used, the system falls out of synchronism soon after the faults occur.

The simulations show that the adaptive $L_2$ disturbance attenuation control scheme proposed in the paper is more effective and has better robustness against disturbances and parametric perturbations.

7. Conclusion

This paper has presented a number of results on the analysis and control of multi-machine power systems with SMES units. We have transformed the multi-machine power systems with SMES units into a dissipative PCH system by using a state feedback, and also analyzed the stability of the resulting system. As the main result, this paper has proposed a novel energy-based adaptive $L_2$ disturbance attenuation control scheme for the multi-machine systems with SMES units. The control scheme is a decentralized one and consists of two parts: one is an $L_2$ disturbance attenuation excitation controller for the generators, and the other is an adaptive $L_2$ disturbance attenuation controller for the SMES units. Simulations show that the proposed control scheme is very effective.
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Appendix A. The derivation from (1)–(3)

The inductance of the superconducting coils is very large, and $I_F$ changes very very slow. In practical control design, the dynamics of $I_F$ can be ignored, see, e.g., [Jiang et al., 2001; Liu, 2002]. With this, it is easy to know from (1) and (2) that

$$
\dot{P}_F = -\frac{1}{T}P_F + 1.5\sqrt{2}W\dot{V}_F I_F \cos \pi 
+ \frac{1}{T}(1.5\sqrt{2}V_F I_F u_w \cos \pi + Q_F x - Q_F u_w),
$$

$$
\dot{Q}_F = -\frac{1}{T}Q_F + 1.5\sqrt{2}W\dot{V}_F I_F \sin \pi 
+ \frac{1}{T}(1.5\sqrt{2}V_F I_F u_w \sin \pi - P_F x + P_F u_w).
$$

Choose new controls

$$
u_F = 1.5\sqrt{2}W\dot{V}_F I_F \cos \pi 
+ \frac{1}{T}(1.5\sqrt{2}V_F I_F u_w \cos \pi + Q_F x - Q_F u_w),
$$

$$
u_Q = 1.5\sqrt{2}W\dot{V}_F I_F \sin \pi 
+ \frac{1}{T}(1.5\sqrt{2}V_F I_F u_w \sin \pi - P_F x + P_F u_w),
$$

then we obtain (3).

Appendix B. The proof of Theorem 1

Substituting (8) into (6) leads to

$$
\begin{bmatrix}
\dot{x}_{11} \\
\dot{x}_{12} \\
\dot{x}_{13} \\
\dot{x}_{14} \\
\dot{x}_{15}
\end{bmatrix}
= 
\begin{bmatrix}
0 & d_i & 0 & 0 & 0 \\
-d_i & -b_i d_i & 0 & -d_i l_i & 0 \\
0 & 0 & -h_j & 0 & 0 \\
0 & d_i l_i & 0 & -\frac{h_j}{l_i} & 0 \\
0 & 0 & 0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
x_i \\
d_i \\
l_i \\
c_i \\
1
\end{bmatrix} + g_i v_i + g_{i2} \xi_i,

i = 1, 2, \ldots, m,
$$

$$
\begin{bmatrix}
\dot{x}_{j1} \\
\dot{x}_{j2} \\
\dot{x}_{j3} \\
\dot{x}_{j4} \\
\dot{x}_{j5}
\end{bmatrix}
= 
\begin{bmatrix}
0 & d_j & 0 & 0 & 0 \\
-d_j & -b_j d_j & 0 & -d_j l_j & 0 \\
0 & 0 & -h_j & 0 & 0 \\
0 & d_j l_j & 0 & -\frac{h_j}{l_j} & 0 \\
0 & 0 & 0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
x_j \\
d_j \\
l_j \\
c_j \\
1
\end{bmatrix} + g_j v_j + g_{j2} \xi_j,

j = m + 1, \ldots, n,
$$

where $j = m + 1, \ldots, n$; $g_{i1}$, $g_{i2}$, $g_{j1}$, $g_{j2}$, $v_i$, $v_j$, $\xi_i$ and $\xi_j$ are defined by (11), (12) and (14), respectively; and

$$
f_w^{(1)} := -\frac{a_w}{d_w} + c_w \frac{x_{w3}^2}{d_w} 
+ x_{w3} \sum_{s=1, s \neq w}^n B_{ws} x_{s3} \sin(x_{w1} - x_{s1}),
$$

$$
f_w^{(2)} := \frac{e_w}{h_w} + \frac{h_w B_{w3}}{h_w} x_{w3} + 2c_w \frac{x_{w1} x_{w3}}{d_w} 
+ \frac{k_w x_{w3} - \bar{u}_w}{h_w} \sum_{s=1, s \neq w}^n B_{ws} x_{s3} \cos(x_{w1} - x_{s1}),
$$

$$
w = i, j.
$$

Set Hamiltonian functions as

$$
H_i = -\frac{a_i}{d_i} x_{i1} + \frac{c_i}{d_i} x_{i1}^2 x_{i3} + \frac{1}{2d_i^2} x_{i1}^2 
- x_{i3} \sum_{s=1, s \neq i}^n B_{is} x_{s3} \cos(x_{i1} - x_{s1}) 
+ e_i + h_i B_{ii} + k_i x_{i3}^2 \frac{\bar{u}_i}{h_i} x_{i1} + \frac{1}{2l_i} x_{i4}^2 + \frac{1}{2T_i} x_{i5}^2,
$$

$$
i = 1, 2, \ldots, m,
$$

$$
H_j = -\frac{a_j}{d_j} x_{j1} + \frac{c_j}{d_j} x_{j1}^2 x_{j3} + \frac{x_{j2}^2}{2d_j} + \frac{e_j + h_j B_{jj} + k_j x_{j3}^2}{2h_j} 
- x_{j3} \sum_{s=1, s \neq j}^n B_{js} x_{s3} \cos(x_{j1} - x_{s1}) - \frac{\bar{u}_j}{h_j} x_{j3},
$$

$$
j = m + 1, \ldots, n,
$$

then (A.1)–(A.2) become

$$
\dot{x}_i = (J_i - R_i) \frac{\partial H_i}{\partial x_i} + g_{i1} v_i + g_{i2} \xi_i, \quad i = 1, 2, \ldots, n, \quad (A.3)
$$

where $J_i$ and $R_i$ are defined by (10) and (13).

It is noted that the above structure does not provide a Hamiltonian structure for the overall system, because in each individual subsystem the cross-variables are frozen as constants. In the following, we search for a Hamiltonian function for the overall system, which is considered as the total energy of the
whole system. Let
\[
H(x) = \sum_{i=1}^{n} H_i + \frac{1}{2} \sum_{i=1}^{n} x_{i3}^2 \sum_{j=1, j \neq i}^{n} B_{ij} x_{j3} \cos(x_{i1} - x_{j1})
\]
\[
= \sum_{i=1}^{n} \left\{ -\frac{a_i}{d_i} x_{i1} + \frac{c_i}{d_i} x_{i1} x_{i3}^2 + \frac{1}{2d_i} x_{i2}^2 + e_i + h_i B_{ii} + k_i \right\} \frac{x_{i3}}{h_i} - \frac{1}{2} \sum_{j=1, j \neq i}^{n} B_{ij} x_{j3} \cos(x_{i1} - x_{j1})
\]
\[
+ \frac{1}{2} \sum_{i=1}^{m} \left[ \frac{1}{4} x_{i4}^2 + \frac{1}{4} x_{i5}^2 \right].
\]

Using the relation \( B_{ij} = B_{ji} \) (see Remark 1), we can show that:
\[
\frac{\partial H(x)}{\partial x_i} = \partial H_i = i, 1, 2, \ldots, n. \tag{A.4}
\]

Eq. (A.4) indicates that \( H(x) \) can serve as the Hamiltonian function for the overall system. Thus, from (A.3) and (A.4) the overall system can be expressed as
\[
\dot{x} = (J - R) \frac{\partial H(x)}{\partial x} + G_1 v + G_2 \epsilon,
\]
where \( J, R, G_1, G_2, v \) and \( \epsilon \) are defined as in Theorem 1. \( \Box \)

Appendix C. The proof of Proposition 1

When \( v = 0 \) and \( \epsilon = 0 \), from (9) and (15) we obtain
\[
\frac{dH}{dr} = -\sum_{i=1}^{n} \frac{\partial H_i}{\partial x_i} R_i \frac{\partial H}{\partial x_i}
\]
\[
= -\sum_{i=1}^{n} h_i \frac{d_i}{x_{i2}} - \sum_{i=1}^{m} \frac{1}{T_i} x_{i4}^2 + \frac{1}{T_i} x_{i5}^2 - \sum_{i=1}^{n} h_i \left\{ \frac{e_i}{h_i} B_{ii} x_{i3} + \frac{c_i}{d_i} x_{i1} x_{i3} + \frac{k_i}{h_i} x_{i3} \right\} - \frac{u_i}{h_i} + \left( \sum_{j=1, j \neq i}^{n} B_{ij} x_{j3} \cos(x_{i1} - x_{j1}) \right)^2 \leq 0.
\]

Because the system (9) with \( v = 0 \) and \( \epsilon = 0 \) is stable at the working point (see Section 4), it can be seen from the dynamic system theory that the system converges to the largest invariant set contained in
\[
S = \{ x : \frac{dH}{dr} = 0 \} = \{ x : x_{i2} = 0, f_i = 0, x_{j4} = 0, x_{j5} = 0, \forall t \geq 0 \}
\]
\[
= \{ x : f_i = 0, x_{j4} = 0, x_{j5} = 0, \forall t \geq 0, i = 1, \ldots, n, j = 1, \ldots, m \}.
\]

where
\[
f_i = e_i + h_i B_{ii} x_{i3} - \sum_{j=1, j \neq i}^{n} B_{ij} x_{j3} \cos(x_{i1} - x_{j1})
\]
\[
+ \frac{2c_i}{d_i} x_{i1} x_{i3} + \frac{k_i}{h_i} x_{i3} - \frac{u_i}{h_i}.
\]

From \( x_{i2} = 0 \) and \( x_{j4} = 0 \), we can conclude that (see the system (6))
\[
a_i - c_i x_{i3}^2 - d_i x_{j3} \sum_{j=1, j \neq i}^{n} B_{ij} x_{j3} \sin(x_{i1} - x_{j1}) = 0,
\]
\[
i = 1, 2, \ldots, n.
\]

Thus, the points in the largest invariant set satisfy
\[
\left\{ a_i - c_i x_{i3}^2 - d_i x_{j3} \sum_{j=1, j \neq i}^{n} B_{ij} x_{j3} \sin(x_{i1} - x_{j1}) = 0,
\right.
\]
\[
x_{i2} = 0, f_i = 0, x_{j4} = 0, x_{j5} = 0,
\]
\[
i = 1, 2, \ldots, n; j = 1, 2, \ldots, m,
\]
which is exactly the condition that the equilibrium satisfies. Therefore, there exists a suitably small neighborhood, \( \Omega \), of the operating point such that the largest invariant set in \( \Omega \) contains only one point, that is, the operating point. From the LaSalle’s invariance principle (Khalil, 1996), the system (9) with \( v = 0 \) and \( \epsilon = 0 \) is asymptotically stable. \( \Box \)

References
Y. Wang et al. / Automatica 42 (2006) 1121–1132


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