Nonlinear Decentralized Saturated Controller Design for Power Systems

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***Abstract*—In this paper, a multimachine power system is first represented as the generalized Hamiltonian control system with dissipation. Then, a decentralized saturated steam valving and excitation controller, which is staticly measurable, is proposed based on the Hamiltonian function method. Last, an example of three-machine power system is discussed in detail.**

***Index Terms*—Decentralized control, excitation control, Hamiltonian control system with dissipation, multimachine power system, saturated control, steam valving control.**

I. Introduction

RECENTLY, various advanced nonlinear control technologies have been applied to excitation and steam valving controllers of power systems [2], [7]–[10], [20]. Through careful investigation it is easy to see that most of these are based on differential geometric tools [5], which cancel the inherent system nonlinearities in order to obtain a feedback equivalent linear system. It has been shown in the literature that the dynamics of power systems can be exactly linearized by employing nonlinear feedback and a state transformation. One can then use the conventional linear control theory to design a controller in order to provide good performance [2], [6]–[11], [18]–[20]. However, these controllers suffer some flaws. First, it is well known that the amplitude of the controller is always bounded in the real world. In order to cope with the reality, the simulations in all of these researchers have used the bounded controller, which is the saturation of the original state feedback controller. But the validity of the saturation of the controller cannot be proved theoretically in these papers. Second, since physical limitation on the system structure makes information transfer among subsystems infeasible, decentralized controllers for multimachine power systems must be used in practice. However, the decentralized controller can not be obtained easily by these technologies because the controller must cancel the inherent system nonlinearities and a state transformation must be used in order to obtain a feedback equivalent linear system where the centralized information of the system has to be used. Thus, the following problem is very important in practice: how to design decentralized saturated controllers for multimachine power systems to improve the transient stability? It is an open problem as far as we know. It is also the most important motivation of this work. In order to solve the problem the structure information of the multimachine power systems should be used adequately and sufficiently.

Very recently, port-controlled Hamiltonian systems with dissipation (PCHD) have been extensively studied [3], [4], [12]–[15]. Indeed, the Hamiltonian function in PCHD systems is considered as the total energy (potential and kinetic energy) in the mechanical system and can play the role of a Lyapunov function for the system. In fact, a power system is an energy producing system, so it is convenient and natural to model the power system as a PCHD system. A single-machine infinite bus power system has been represented by a PCHD system [3], [16] and excitation control of a multimachine power system has also been written as a PCHD systems [21]. It has been shown from these researches that the Hamiltonian function method has some advantages.

In this paper, we shall concentrate on enhancing the transient stability of power systems by means of decentralized saturated nonlinear control. The resulting decentralized saturated nonlinear controller can guarantee the overall stability of a large-scale power system. In fact, the most important and difficult step used in the controller design is to represent the considered system as Hamiltonian control system with dissipation, and then a saturated controller is easily obtained. The paper is arranged as follows. The fundamental result about the existence of a saturated controller for Hamiltonian system with dissipation is presented in view of convenience in Section II. In Section III, the applications of the proposed saturated controller to power systems are considered, and a slightly more realistic three-machine example system is also presented to illustrate the effectiveness of the proposed design method. Section IV gives some conclusions.

II. Preliminaries

Consider the following port-controlled Hamiltonian systems with dissipation:

$$\begin{cases} \dot{x} = (J(x) - R(x))\nabla H(x) + g(x)u, \quad x \in R^n,\, u \in R^m \\ y = g^T(x)\nabla H(x) \end{cases} \tag{1}$$

where $J(x)$ is a skew-symmetric structure matrix, $R(x)$ is a positive semidefinite matrix, the smooth function $H(x)$: $R^n \rightarrow$

Manuscript received January 24, 2001; revised November 3, 2002. Manuscript received in final form February 25, 2003. Recommended by Associate Editor A. Ray. This work is supported in part by the National Natural Science Foundation of China under Grant 59837270, the City University of Hong Kong under Grant SRG 7001245, the National Key Project of China under Grants G1998020308 and G1998020300, and the Institute of Systems Science, Academy of Mathematics and Systems Science, Chinese Academy of Sciences.

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Digital Object Identifier 10.1109/TCST.2003.813404

R represents the total stored energy, which is called Hamiltonian function of the system (1), and $u, y \in R^m$ are the port power variables. The notation $\nabla H(x)$ is defined by $\nabla H(x) = (\partial H/\partial x_1, \ldots, \partial H/\partial x_n)^T$.

The following theorem is our main result which is the basis of the paper.

Theorem 1: If the equilibrium point $x^* = 0$ of Hamiltonian control system (1) is a strict local minimum, and system (1) is zero state detectable, then system (1) is stabilizable by a saturated output feedback control

$$u_i = -L_i \mathrm{sat}\left(\frac{K_i y_i}{L_i}\right) \tag{2}$$

where $L_i > 0$ is the magnitude of saturation of the ith control, $K_i > 0$ is feedback gain parameter of the ith control, and the saturated function $\mathrm{sat}(x)$ is defined as follows:

$$\mathrm{sat}(x) = \begin{cases} 1, & \text{if } x \geq 1 \\ x, & \text{if } -1 < x < 1 \\ -1, & \text{if } x \leq -1. \end{cases}$$

Proof: Taking $H(x)$ as a Lyapunov function, so

$$\frac{dH}{dt} = -\nabla H^T R \nabla H + \nabla H^T g u = -\nabla H^T R \nabla H + \sum_{i=1}^{m} y_i u_i.$$

It is easy to see that

$$y_i u_i = \begin{cases} L_i y_i < 0, & \text{if } -K_i y_i \geq L_i \\ -K_i y_i \leq 0, & \text{if } -L_i < -K_i y_i < L_i \\ -L_i y_i < 0, & \text{if } -K_i y_i \leq -L_i \end{cases}$$

and that

$$y_i u_i = 0$$

is equivalent to

$$y_i = 0.$$

So we have

$$\frac{dH}{dt} \leq 0$$

and

$$\frac{dH}{dt} = 0$$

implies

$$y_i = 0, \qquad \forall i = 1, \ldots, m.$$

Then the trajectories of the closed-loop system converge to the largest invariant set contained in

$$A = \{x: y \equiv 0,\, u \equiv 0\}.$$

From the zero state detectability, the closed-loop system is asymptotically stable under the above saturated controller. ■

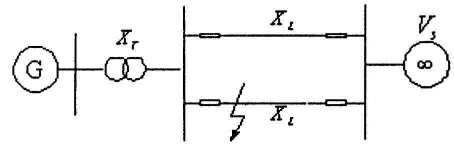

Fig. 1. Single machine infinite bus system.

Remark 1: As a matter of fact, the controller can be chosen as $u_i = -L_i \mathrm{sgn}(y_i)$ from the proof of above theorem which is a bang-bang control.

III. Application to Power Systems

A. Single Machine Infinite Bus Power Systems

Consider a single machine infinite bus power system in this section. The system shown in Fig. 1 is modeled by the following dynamical system [1], [9]:

$$\begin{cases} \dot{\delta}(t) = \omega(t) - \omega_0 \\ \dot{\omega}(t) = -\dfrac{D}{H}(\omega(t) - \omega_0) + \dfrac{\omega_0}{H}\left(P_H + C_M P_{m0} - \dfrac{V_s}{x'_{ds}} E'_q \sin(\delta(t))\right) \\ \dot{P}_H = -\dfrac{1}{T_{HS}} P_H + \dfrac{C_H}{T_{HS}} P_{m0} + \dfrac{C_H}{T_{HS}} u_H \\ \dot{E}'_q = -\dfrac{1}{T'_d} E'_q + \dfrac{1}{T_{d0}} \dfrac{x_d - x'_d}{x'_{ds}} V_s \cos\delta + \dfrac{1}{T_{d0}} V_f \end{cases} \tag{3}$$

where δ is the power angle between the q-axis electrical potential vector $\vec{E}_q$ and a reference bus voltage vector $\vec{V}_{\mathrm{REF}}$ in the system, in radians; ω is the rotating speed of the generator, in radians/second; P_H is the mechanical power of high-pressure (HP) turbine, in per unit; E'_q are the q-axis internal transient electric potential of the generator, in per unit; P_{m0} is the initial mechanical power of the generator, in per unit; H, C_H are moment of inertia in seconds and the power fraction of HP turbine, respectively; $T_{HS} = T_{Hg} + T_H$ is the equivalent time constant of the HP turbine, T_{Hg} the time constant of oil-servomotor of regulated valve of the HP turbine, T_H the time constant of HP turbine, u_H the electrical control signal from the controller for the regulated valve, T'_d is the time constant of the field winding when the stator circuit is closed, T_{d0} is the excitation circuit time constant, x_d is the d-axis synchronous reactance of a generator, x'_d is the d-axis transient reactance, and V_f is the voltage of the field circuit of a generator.

Let $x_1 = \delta(t)$, $x_2 = \omega(t) - \omega_0$, $x_3 = P_H(t)$, $x_4 = E'_q(t)$, and $a = D/H$, $b = (\omega_0/H)C_M P_{m0}$, $c = (\omega_0/H)(V_s/x'_{ds})$, $d = 1/T_{HS}$, $e = (C_H/T_{HS})P_{m0}$, $h = \omega_0/H$, $p = 1/T'_d$, $q = (1/T_{d0})(x_d - x'_d/x'_{ds})V_s$. Then (3) can be rewritten as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -a x_2 + b + h x_3 - c x_4 \sin x_1 \\ \dot{x}_3 = -d x_3 + e + \dfrac{C_H}{T_{HS}} u_H \\ \dot{x}_4 = -p x_4 + q \cos x_1 + \dfrac{1}{T_{d0}} V_f. \end{cases}$$

That is

$$\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}0 & \frac{cp}{q} & 0 & 0\\ -\frac{cp}{q} & -a\,\frac{cp}{q} & hK & 0\\ 0 & 0 & -dK & 0\\ 0 & 0 & 0 & -p\end{bmatrix}$$
$$\times\begin{bmatrix}\frac{q}{p}\,x_4\sin x_1-\frac{q}{cp}\,b-\frac{q}{p}\,\frac{he}{cd}\\ \frac{q}{cp}\,x_2\\ \frac{1}{K}\left(x_3-\frac{e}{d}\right)\\ x_4-\frac{q}{p}\cos x_1\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\\ \frac{C_H}{T_{HS}}\,u_H & 0\\ 0 & \frac{1}{T_{d0}}\,V_f\end{bmatrix}$$

where $K>0$ is a constant to be determined later.

It is easy to show that the matrix $-(M+M^T)$ is semipositive when

$$K<\frac{4acpd}{h^2q}=\frac{4DT_{d0}}{\omega_0 T_{HS}T'_d(x_d-x'_d)}$$

where

$$M=\begin{bmatrix}0 & \frac{cp}{q} & 0 & 0\\ -\frac{cp}{q} & -a\,\frac{cp}{q} & hK & 0\\ 0 & 0 & -dK & 0\\ 0 & 0 & 0 & -p\end{bmatrix}.$$

Let

$$H(x)=\frac{1}{2}\,\frac{q}{cp}\,x_2^2-\frac{q}{cp}\left(b+\frac{he}{d}\right)x_1-\frac{q}{p}\,x_4\cos x_1+\frac{1}{2K}\left(x_3-\frac{e}{d}\right)^2+\frac{1}{2}x_4^2$$

which is bounded from below since $x_1\in[-\pi,\,\pi]$, and

$$y=g^T\nabla H=\begin{bmatrix}\frac{1}{K}\left(x_3-\frac{e}{d}\right)\\ x_4-\frac{q}{p}\cos x_1\end{bmatrix}. \tag{4}$$

Thus, the system consisting of (3) and (4) is a generalized Hamiltonian control system with dissipation

$$\begin{cases}\dot{x}=(J(x)-R(x))\nabla H(x)+g(x)u, & x\in R^4,\ u\in R^2\\ y=g^T(x)\nabla H(x)\end{cases} \tag{5}$$

where

$$J=\frac{M-M^T}{2},\quad R=-\frac{M+M^T}{2},\quad g=\begin{bmatrix}0&0&1&0\\0&0&0&1\end{bmatrix}^T$$
$$x^T=[x_1\quad x_2\quad x_3\quad x_4],\quad u=\begin{pmatrix}u_1\\u_2\end{pmatrix}=\begin{pmatrix}\frac{C_H}{T_{HS}}\,u_H\\ \frac{1}{T_{d0}}\,V_f\end{pmatrix}.$$

Thus, the saturated controller

$$\begin{cases}u_1=-L_1\mathrm{sat}\left[\frac{K_1y_1}{L_1}\right]=-L_1\mathrm{sat}\left[\frac{K_1}{L_1K}\left(x_3-\frac{e}{d}\right)\right]\\ u_2=-L_2\mathrm{sat}\left[\frac{K_2y_2}{L_2}\right]=-L_2\mathrm{sat}\left[\frac{K_2}{L_2}\left(x_4-\frac{q}{p}\cos x_1\right)\right]\end{cases} \tag{6}$$

proposed via Theorem 1 should stabilize the system, where $L_i>0$ is the magnitude of saturation of the ith control, $K_i>0$ is feedback gain parameter of the ith control, $i=1,2$.

In the following, we verify the asymptotical stability of the closed-loop system. In fact, taking $H(x)$ as a Lyapunov function, we have

$$\begin{aligned}\frac{dH}{dt}=&-\frac{aq}{cp}\,x_2^2+h\,\frac{q}{cp}\,x_2\left(x_3-\frac{e}{d}\right)-\frac{d}{K}\left(x_3-\frac{e}{d}\right)^2\\ &-p\left(x_4-\frac{q}{p}\cos x_1\right)^2+y^Tu\\ \le&-\frac{aq}{cp}\left[x_2-\frac{h}{2a}\left(x_3-\frac{e}{d}\right)\right]^2\\ &-\frac{4acpd-Kqh^2}{4acpK}\left(x_3-\frac{e}{d}\right)^2-p\left(x_4-\frac{q}{p}\cos x_1\right)^2.\end{aligned}$$

Then the system is convergent to the biggest invariant subset of the set $A=\{x\colon x_2=0,\, x_3=e/d,\, x_4=(q/p)\cos x_1\}$. From $x_2\equiv 0$, $x_3\equiv e/d$ and $x_4\equiv (q/p)\cos x_1$, we know that $x_1=(1/2)\arcsin((2(bd+he)p)/cdq)$, which is exactly the equilibrium point of the system. Therefore, the system with trajectories contained in A is asymptotically stable. From the *La Salle invariant principle* [17], the system is asymptotically stable. Then the following proposition is true.

Proposition 2: The excitation and steam valving control of single machine infinite bus power system model (3) has the following saturated stabilizing controller:

$$\begin{cases}u_1=L_1\mathrm{sat}\left[-\frac{K_1y_1}{L_1}\right]=L_1\mathrm{sat}\left[-\frac{K_1}{L_1K}\left(x_3-\frac{e}{d}\right)\right]\\ u_2=L_2\mathrm{sat}\left[-\frac{K_2y_2}{L_2}\right]=L_2\mathrm{sat}\left[-\frac{K_2}{L_2}\left(x_4-\frac{q}{p}\cos x_1\right)\right]\end{cases} \tag{7}$$

where $K_1>0$, $K_2>0$, $L_1>0$, and $L_2>0$ are the control gain and, respectively, the magnitude of saturation of the controls.

Remark 2: In fact, the controller (7) have the following form:

$$\begin{cases}u_1=L_1\mathrm{sat}\left[-\frac{K_1}{L_1K}\,(P_H(t)-C_HP_{m0})\right]\\ u_2=L_2\mathrm{sat}\left[-\frac{K_2}{L_2}\left(E'_q(t)-\frac{T_{d0}x'_{ds}}{T'_d(x_d-x'_d)V_s}\cos\delta(t)\right)\right]\end{cases} \tag{8}$$

which can be directly measured.

B. Multimachine Power Systems

Consider an n-machine power system in this section. The system can be modeled by the following equation [1], [9]:

$$\begin{cases} \dot{\delta}_i = \omega_i - \omega_0 \\ \dot{\omega}_i = -\frac{D_i}{H_i}(\omega_i - \omega_0) + \frac{\omega_0}{H_i}(P_{Hi} + C_{Mi}P_{m0i} - P_{ei}) \\ \dot{P}_{Hi} = -\frac{1}{T_{HSi}}P_{Hi} + \frac{C_{Hi}}{T_{HSi}}P_{m0i} + \frac{C_{Hi}}{T_{HSi}}u_{Hi} \\ \dot{E}'_{qi} = \frac{1}{T'_{d0i}}(E_{fi} - E_{qi}) \end{cases} \tag{9}$$

and

$$\begin{aligned} E_{qi} &= E'_{qi} + (x_{di} - x'_{di})I_{di} \\ E_{fi} &= k_{ci}u_{fi} \\ I_{di} &= -\sum_{j=1}^{n} E'_{qi}B_{ij}\cos(\delta_i - \delta_j) \\ I_{qi} &= \sum_{j=1}^{n} B_{ij}E'_{qj}\sin(\delta_i - \delta_j) \\ P_{ei} &= E'_{qi}\sum_{j=1}^{n} B_{ij}E'_{qj}\sin(\delta_i - \delta_j) \\ Q_{ei} &= -E'_{qi}\sum_{j=1}^{n} B_{ij}E'_{qj}\cos(\delta_i - \delta_j) \\ E_{qi} &= x_{adi}I_{fi} \\ E_{qi} &= V_{ti} + \frac{Q_{ei}x_{di}}{V_{ti}} \end{aligned}$$

where

δ_i	power angle between the q-axis electrical potential vector $\vec{E}_{qi}$ and a reference bus voltage vector $\vec{V}_{REF}$ in the system in rad;
ω_i	rotating speed of the ith generator, in rad/s;
P_{Hi}	mechanical power of high-pressure (HP) turbine, in per unit;
E'_{qi}	q-axis internal transient electric potential of the ith generator, in per unit;
P_{m0i}	initial mechanical power of the ith generator, in per unit;
H_i and C_{Hi}	moment of inertia in second and the power fraction of HP turbine, respectively;
B_{ij}	ith row and jth column element of the nodal susceptance matrix, which is symmetric, at the internal nodes after eliminating all physical buses, in per unit;
P_{ei}	electric power;
$T_{HSi}=T_{Hgi}+T_{Hi}$	equivalent time constant of HP turbine;
T_{Hgi}	time constant of oil-servomotor of regulated valve of HP turbine;
T_{Hi}	time constant of HP turbine;
u_{Hi}	electrical control signal from the controller for the regulated valve;
E_{qi}	EMF in the quadrature axis in per unit;
E_{fi}	equivalent EMF in the excitation coil in per unit.
T'_{d0i}	direct axis transient short circuit time constant in seconds;
x_{di}	direct axis reactance of the ith generator in per unit;
x'_{di}	direct axis transient reactance of the ith generator, in per unit;
Q_{ei}	reactive power, in per unit
I_{fi}	excitation current, in per unit;
I_{di}	direct axis current, in per unit;
I_{qi}	quadrature axis current, in per unit;
k_{ci}	gain of the excitation amplifier, in per unit;
u_{fi}	input of the SCR amplifier of the ith generator, in per unit;
x_{adi}	mutual reactance between the excitation coil and the stator coil of the ith generator, in per unit;
x_{Ti}	transformer reactance;
x_{ij}	transmission line reactance between the ith generator and the jth generator;
V_{ti}	terminal voltage of the ith generator.

Remark 3: In power systems, P_{ei}, Q_{ei}, and V_{ti} are readily measured. So E_{qi} can be calculated using these available variables.

Denote $a_i = D_i/H_i$, $b_i = \omega_0/H_i$, $c_i = (\omega_0/H_i)C_{Mi}P_{m0i}$, $d_i = (x_{di} - x'_{di})/T'_{d0i}$, $e_i = 1/T_{HSi}$, $k_i = (C_{Hi}/T_{HSi})P_{m0i}$, $h_i = 1/T'_{d0i}$. Let $x_{i1} = \delta_i(t)$, $x_{i2} = \omega_i(t) - \omega_0$, $x_{i3} = P_{Hi}(t)$, and $x_{i4} = E'_{qi}(t)$ as state variables and $u_{i1} = (C_{Hi}/T_{HSi})u_{Hi}$ and $u_{i2} = (1/T'_{d0i})E_{fi}$ as controls. Then (9) can be rewritten as follows:

$$\begin{cases} \dot{x}_{i1} = x_{i2}, \\ \dot{x}_{i2} = -a_ix_{i2} + c_i + b_ix_{i3} - b_ix_{i4}\sum_{j=1}^{n} B_{ij}x_{j4} \\ \qquad \times \sin(x_{i1} - x_{j1}) \\ \dot{x}_{i3} = -e_ix_{i3} + k_i + u_{i1} \\ \dot{x}_{i4} = -h_ix_{i4} + d_i\sum_{j=1}^{n} B_{ij}x_{j4}\cos(x_{i1} - x_{j1}) + u_{i2}. \end{cases} \tag{10}$$

It is easy to see that the system (10) can be represented as

$$\frac{d}{dt}\begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \\ x_{i4} \end{bmatrix} = \begin{bmatrix} 0 & b_i & 0 & 0 \\ -b_i & -a_ib_i & b_iK_i & 0 \\ 0 & 0 & -e_iK_i & 0 \\ 0 & 0 & 0 & -d_i \end{bmatrix} \nabla_{x_i}H + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u_{i1} \\ u_{i2} \end{bmatrix}$$

where $x_i^T = (x_{i1}, x_{i2}, x_{i3}, x_{i4})$, $K_i > 0$ is a constant and

$$H = \sum_{i=1}^{n} \left[\frac{1}{2b_i} x_{i2}^2 - \left(\frac{c_i}{b_i} + \frac{k_i}{e_i} \right) x_{i1} + \frac{1}{2K_i} \left(x_{i3} - \frac{k_i}{e_i} \right)^2 - \frac{1}{2} x_{i4} \sum_{j=1}^{n} B_{ij} x_{j4} \cos(x_{i1} - x_{j1}) + \frac{h_i}{2d_i} x_{i4}^2 \right].$$

Given an operation point $(x_{i1}^e, 0, x_{i3}^e, x_{i4}^e)$, one must insert constant controls $\overline{u}_{i1}$ and $\overline{u}_{i2}$ such that

$$\begin{cases} x_{i2}^e = 0 \\ c_i + b_i x_{i3}^e = b_i x_{i4}^e \sum_{j=1}^{n} B_{ij} x_{j4}^e \sin(x_{i1}^e - x_{j1}^e) \\ x_{i3}^e = \dfrac{k_i + \overline{u}_{i1}}{e_i} \\ x_{i4}^e = \dfrac{d_i}{h_i} \sum_{j=1}^{n} B_{ij} x_{j4}^e \cos(x_{i1}^e - x_{j1}^e) + \overline{u}_{i2}. \end{cases} \tag{11}$$

Let

$$H_e = \sum_{i=1}^{n} \left[\frac{1}{2b_i} x_{i2}^2 - \left(\frac{c_i}{b_i} + \frac{k_i + \overline{u}_{i1}}{e_i} \right) x_{i1} + \frac{1}{2K_i} \left(x_{i3} - \frac{k_i + \overline{u}_{i1}}{e_i} \right)^2 + \frac{h_i}{2d_i} x_{i4}^2 - \frac{\overline{u}_{i2}}{d_i} x_{i4} - \frac{1}{2} x_{i4} \sum_{j=1}^{n} B_{ij} x_{j4} \cos(x_{i1} - x_{j1}) \right]$$

which is bounded from below because of $x_{i1} \in [-\pi, \pi]$, and

$$y_i = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \nabla_{x_i} H_e = \begin{pmatrix} \dfrac{1}{K_i} \left(x_{i3} - \dfrac{k_i + \overline{u}_{i1}}{e_i} \right) \\ \dfrac{h_i}{d_i} x_{i4} - \sum_{j=1}^{n} B_{ij} x_{j4} \cos(x_{i1} - x_{j1}) - \dfrac{\overline{u}_{i2}}{d_i} \end{pmatrix}. \tag{12}$$

Then the forced system (10)–(12) with the feedback control

$$u_{ij} = \overline{u}_{ij} + v_{ij}, \qquad i = 1, \ldots, n,\ j = 1, 2$$

can be represented as

$$\frac{d}{dt} \begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \\ x_{i4} \end{bmatrix} = \begin{bmatrix} 0 & b_i & 0 & 0 \\ -b_i & -a_i b_i & b_i K_i & 0 \\ 0 & 0 & -e_i K_i & 0 \\ 0 & 0 & 0 & -d_i \end{bmatrix} \nabla_{x_i} H_e + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{i1} \\ v_{i2} \end{bmatrix}$$
$$y_i = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \nabla_{x_i} H_e. \tag{13}$$

It is easy to see that

$$R_i = -\frac{1}{2} \left(\begin{bmatrix} 0 & b_i & 0 & 0 \\ -b_i & -a_i b_i & b_i K_i & 0 \\ 0 & 0 & -e_i K_i & 0 \\ 0 & 0 & 0 & -d_i \end{bmatrix} + \begin{bmatrix} 0 & b_i & 0 & 0 \\ -b_i & -a_i b_i & b_i K_i & 0 \\ 0 & 0 & -e_i K_i & 0 \\ 0 & 0 & 0 & -d_i \end{bmatrix}^T \right) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a_i b_i & -\dfrac{b_i K_i}{2} & 0 \\ 0 & -\dfrac{b_i K_i}{2} & e_i K_i & 0 \\ 0 & 0 & 0 & d_i \end{pmatrix} \geq 0$$

when $K_i < 4a_i e_i / b_i = 4D_i / \omega_0 T_{HSi}$. That is to say, when $K_i < 4a_i e_i / b_i$ the system (13) is a generalized Hamiltonian control system with dissipation.

Then the following saturated controller is designed:

$$\begin{cases} v_{i1} = -L_{i1} \mathrm{sat}\left(\dfrac{l_{i1} y_{i1}}{L_{i1}} \right) = -L_{i1} \mathrm{sat}\left(\dfrac{l_{i1}}{L_{i1} K_i} \left(x_{i3} - \dfrac{k_i + \overline{u}_{i1}}{e_i} \right) \right) \\ v_{i2} = -L_{i2} \mathrm{sat}\left(\dfrac{l_{i2} y_{i2}}{L_{i2}} \right) = -L_{i2} \mathrm{sat}\left(\dfrac{l_{i2}}{L_{i2}} \left(\dfrac{h_i}{d_i} x_{i4} - \dfrac{\overline{u}_{i2}}{d_i} \right) - \dfrac{l_{i2}}{L_{i2}} \sum_{j=1}^{n} B_{ij} x_{j4} \cos(x_{i1} - x_{j1}) \right), \\ \qquad i = 1, \ldots, n \end{cases}$$

where $l_{i1} > 0$ and $l_{i2} > 0$ are control gains which can be arbitrarily selected and $L_{i1} > 0$ and $L_{i2} > 0$ are magnitude of saturation of the controls which can be arbitrarily selected also. Selecting $H_e(x)$ as a Lyapunov function, we have

$$\frac{dH}{dt} = -\sum_{i=1}^{n} \left(\nabla_{x_i} H_e^T R_i \nabla_{x_i} H_e + y_{i1} v_{i1} + y_{i2} v_{i2} \right) \leq -\sum_{i=1}^{n} \nabla_{x_i} H_e^T R_i \nabla_{x_i} H_e.$$

Then, the closed-loop system is convergent to the largest invariant set contained in

$$A = \left\{ x\colon x_{i2}^e = 0,\ x_{i3}^e = \frac{k_i + \overline{u}_{i1}}{e_i},\ x_{i4}^e = \frac{d_i}{h_i} \sum_{j=1}^{n} B_{ij} x_{j4}^e \cos(x_{i1}^e - x_{j1}^e) + \overline{u}_{i2} \right\}.$$

From $x_{i2} \equiv 0$, $x_{i3} \equiv k_i/e_i$, and $x_{i4}^e = (d_i/h_i)\sum_{j=1}^n B_{ij}x_{j4}^e\cos(x_{i1}^e - x_{j1}^e) + \overline{u}_{i2}$, we know that $c_i + b_i x_{i3}^e = b_i x_{i4}^e \sum_{j=1}^n B_{ij}x_{j4}^e \sin(x_{i1}^e - x_{j1}^e)$, which is exactly the equilibrium point of the closed-loop system. Therefore, the closed-loop system with trajectories contained in A is asymptotically stable. From the La Salle invariant principle [17], the closed-loop system is asymptotically stable. So, the following proposition is true.

Proposition 3: The multimachine power system model (9) has the following saturated stabilizing controller:

$$\begin{cases} u_{Hi} = \dfrac{T_{HSi}}{C_{Hi}}\left[\overline{u}_{i1} - L_{i1}\text{sat}\left(\dfrac{l_{i1}}{L_{i1}K_i}\left(x_{i3} - \dfrac{k_i + \overline{u}_{i1}}{e_i}\right)\right)\right] \\ E_{fi} = T'_{d0i}\left[\overline{u}_{i2} - L_{i2}\text{sat}\left(\dfrac{l_{i2}}{L_{i2}}\left(\dfrac{h_i}{d_i}x_{i4} - \dfrac{\overline{u}_{i2}}{d_i}\right),\right.\right. \\ \qquad \left.\left. - \dfrac{l_{i2}}{L_{i2}}\displaystyle\sum_{j=1}^n B_{ij}x_{j4}\cos(x_{i1} - x_{j1})\right)\right], \\ \qquad i = 1, \ldots, n \end{cases} \tag{14}$$

around a prescribed operation point $(x_{i1}^e, 0, x_{i3}^e, x_{i4}^e)$, where $l_{ij} > 0$ $(i = 1, \ldots, n, j = 1, 2)$ are the control gains and $L_{ij} > 0$ $(i = 1, \ldots, n, j = 1, 2)$ are the magnitude of saturation of the control.

Remark 4: It is easy to see that

$$\begin{cases} u_{Hi} = \dfrac{T_{HSi}}{C_{Hi}}\left[\overline{u}_{i1} - L_{i1}\text{sat}\left(\dfrac{l_{i1}}{L_{i1}K_i}(P_{Hi} - C_{Hi}P_{m0i} \right.\right. \\ \qquad \left.\left. - T_{HSi}\overline{u}_{i1})\right)\right] \\ E_{fi} = T'_{d0i}\left[\overline{u}_{i2} - L_{i2}\text{sat}\left(\dfrac{l_{i2}}{L_{i2}}\left(\dfrac{1}{x_{di} - x'_{di}}E'_{qi}\right.\right.\right. \\ \qquad \left.\left.\left. - \dfrac{T'_{d0i}}{x_{di} - x'_{di}}\overline{u}_{i2} - \displaystyle\sum_{j=1}^n B_{ij}E'_{qj}\cos(\delta_i - \delta_j)\right)\right)\right] \\ \quad = T'_{d0i}\left[\overline{u}_{i2} - L_{i2}\text{sat}\left(\dfrac{l_{i2}}{L_{i2}}\left(\dfrac{1}{x_{di} - x'_{di}}E_{qi}\right.\right.\right. \\ \qquad \left.\left.\left. - \dfrac{T'_{d0i}}{x_{di} - x'_{di}}\overline{u}_{i2}\right)\right)\right] \\ \quad = T'_{d0i}\left[\overline{u}_{i2} - L_{i2}\text{sat}\left(-\dfrac{l_{i2}}{L_{i2}}\dfrac{T'_{d0i}}{x_{di} - x'_{di}}\overline{u}_{i2}\right.\right. \\ \qquad \left.\left. + \dfrac{l_{i2}}{L_{i2}}\dfrac{1}{x_{di} - x'_{di}}\left(V_{ti} + \dfrac{Q_{ei}x_{di}}{V_{ti}}\right)\right)\right] \\ \quad = T'_{d0i}\left[\overline{u}_{i2} - L_{i2}\text{sat}\left(\dfrac{l_{i2}}{L_{i2}(x_{di} - x'_{di})}\right.\right. \\ \qquad \left.\left. \times\left(V_{ti} + \dfrac{Q_{ei}x_{di}}{V_{ti}} - T'_{d0i}\overline{u}_{i2}\right)\right)\right] \\ \qquad i = 1, \ldots, n \end{cases} \tag{15}$$

which is decentralized and can be directly measured. That is, the controller (15) is decentralized, saturated, and measurable.

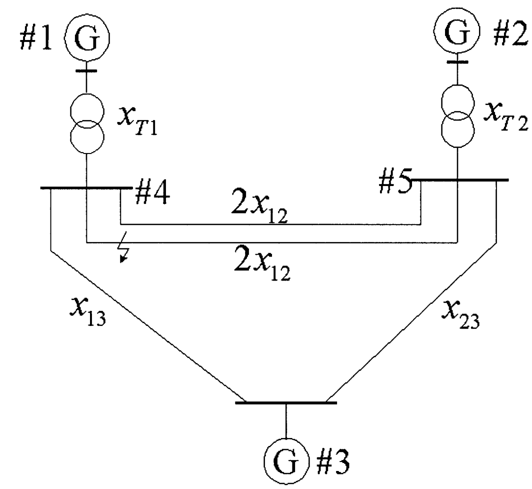

Fig. 2. Three-machine example system.

C. Three-Machine Power System Example

A three-machine example system shown in Fig. 2 is chosen to demonstrate the effectiveness of the proposed decentralized saturated controller.

The system parameters used in the simulation are as follows [20]:

System parameters	Generator #1	Generator #2
x_d (per unit)	1.863	2.36
x'_d (per unit)	0.257	0.319
x_T (per unit)	0.129	0.11
D (per unit)	5	3
T'_{d0} (per unit)	6.9	7.96
H (s)	8	10.2
x_{ad} (s)	1.712	1.712
k_c	1	1
ω_0 (rad/s)	314.159	
C_M	0.7	0.72
C_H	0.3	0.29
P_{m0}	0.82	0.8
$T_{H\sum i}$	0.398	0.4
x_{12}	0.55	
x_{13}	0.53	
x_{23}	0.6	

In the example system, the generator #3 is chosen as slack bus. In simulation the control parameters are selected as $l_{11} = 0.02$, $l_{12} = 0.01$, $l_{21} = 0.03$, $l_{22} = 0.02$, $K_1 = 0.05$, and $K_2 = 0.01$. Suppose the operation point is given as (0.2700, 0, 0.2460, 2.2257, 0.2500, 0, 0.2320, 3.4379), then one can obtain $\overline{u}_{11} = 0$, $\overline{u}_{12} = 0.5720$, $\overline{u}_{21} = 0$, and $\overline{u}_{22} = 1.2190$. The control input limitations are supposed to be

$$\begin{aligned} -0.05 &\le u_{i1} - \overline{u}_{i1} \le 0.05, \\ -0.01 &\le u_{i2} - \overline{u}_{i2} \le 0.01, \qquad i = 1, 2. \end{aligned}$$

The simulation results concerning the dynamic behavior of the three-machine system under the proposed control law are shown in Figs. 3–6, where a typical symmetrical three-phase short-circuit fault occurring on the transmission line near bus 4 is considered.

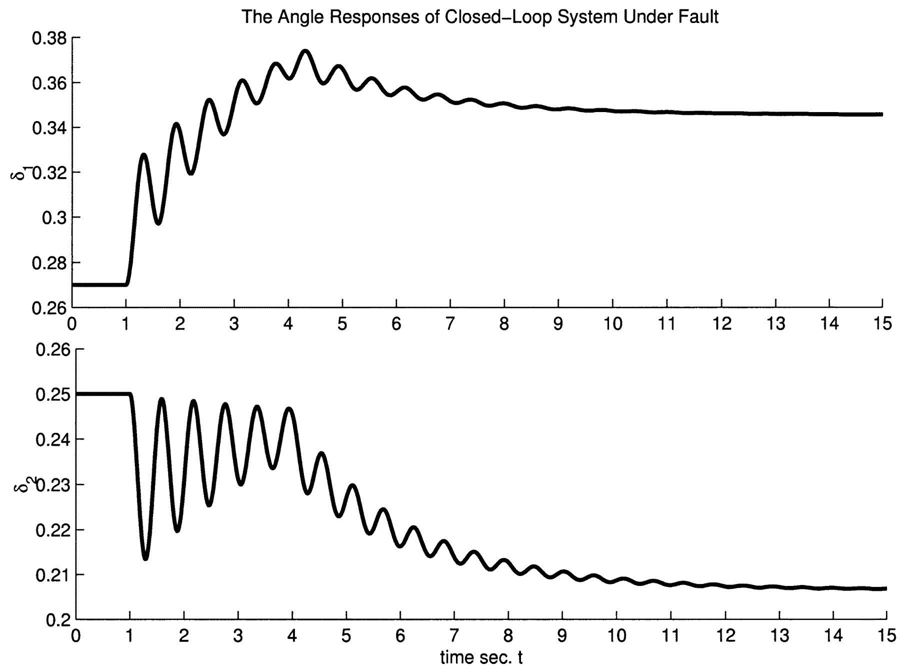

Fig. 3. Responses of angles δ_1 and δ_2 under the fault that occurs at 1 s and is cleared at 4 s.

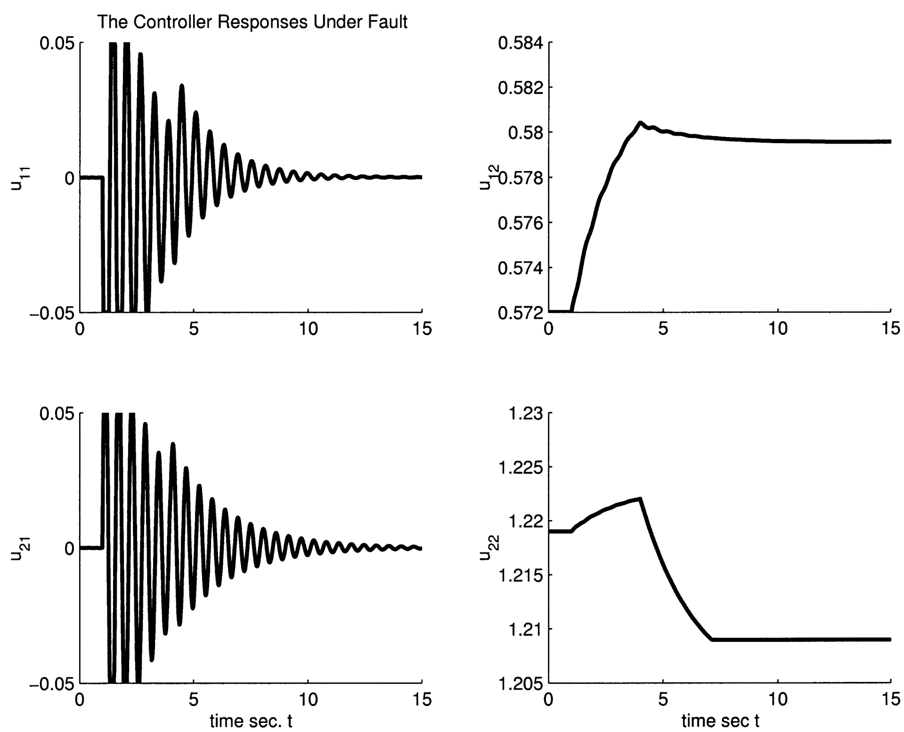

Fig. 4. Responses of the controller u_{11}, u_{12}, u_{21}, and u_{22} under the fault that occurs at 1 s and is cleared at 4 s.

It can be observed from Figs. 3 and 4 that with the proposed saturated decentralized control law the dynamic system is stabilized under the fault that occurs at 1 s and is cleared at 4 s.

It can be further observed from Figs. 5 and 6 that with the proposed saturated decentralized control law the dynamic system is also stabilized under the fault that occurs at 1 s, is cleared at 4 s, and restored at 5 s.

Comparing Figs. 3 and 4 with Figs. 5 and 6, one notices that the resulting operation point can be restored if the transmission line is restored. This indicates that restoring transmission line is useful for the dynamical performance of closed-loop system.

IV. CONCLUSION

In this paper, the controller design of power systems have been considered. Using Hamiltonian function method a decentralized, saturated, and measurable controller has been obtained. In fact, in the literature decentralized saturated controllers are

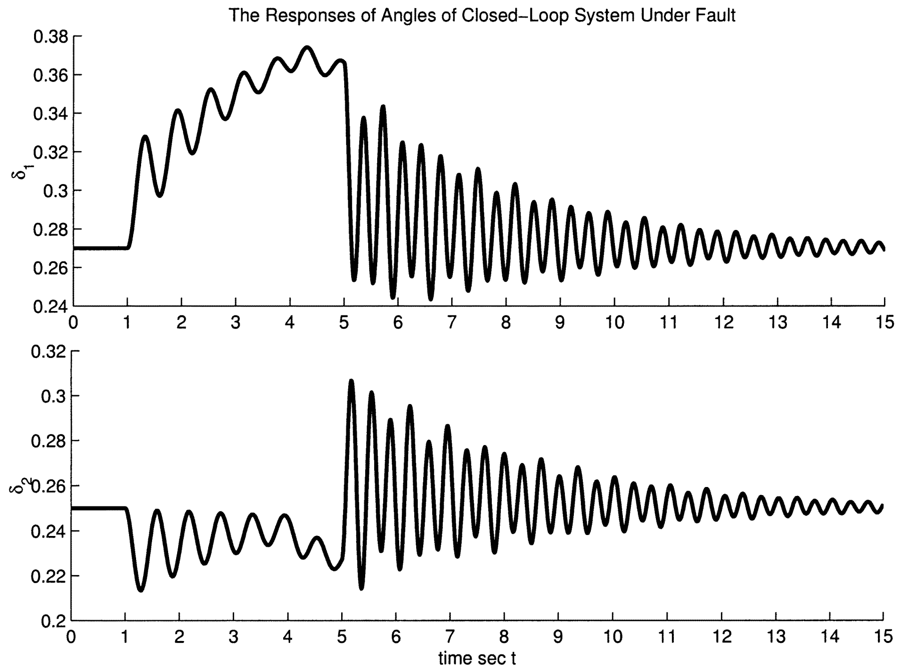

Fig. 5. The responses of angles δ_1 and δ_2 under the fault that occurs at 1 s, cleared at 4 s, and restored at 5 s.

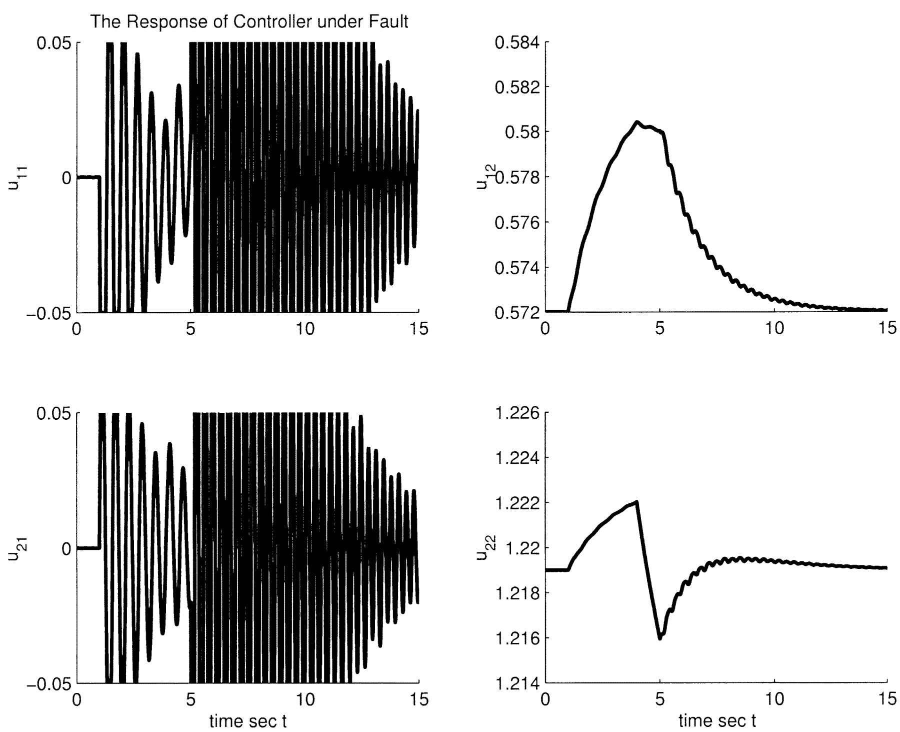

Fig. 6. Responses of the controller u_{11}, u_{12}, u_{21}, and u_{22} under the fault that occurs at 1 s, cleared at 4 s,and restored at 5 s.

widely used for power systems, but no theoretic foundation is given. The result of this paper have provided theoretic foundation. Finally, a three-machine power system has been presented to illustrate the proposed controller design.

ACKNOWLEDGMENT

The authors wish to acknowledge the help of Dr. M. Bodson, Editor-in-Chief, and W. Johnson. The authors wish to thank the anonymous referees for careful reading of the manuscript. Their helpful comments are greatly appreciated.

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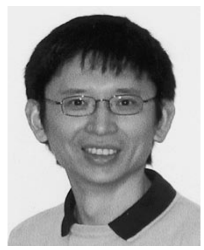

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