Active Disturbance Rejection Control to Consensus of Second-Order Stochastic Multiagent Systems

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***Abstract*—In this article, we attempt for the first time to apply the active disturbance rejection control approach to the disturbance rejection and consensus for a class of second-order stochastic multiagent systems, whose network topology is undirected and connected. Extended state observers are designed for online estimation of the unmeasured states and random total disturbance, which is the effect of external nonrandom disturbance, colored noise, and bounded noise corrupting each agent. Active antidisturbance consensus protocols based on the estimates of unmeasured states and random total disturbance are then designed to guarantee the consensus in both mean square and almost sure practical sense, while random total disturbance of each agent is compensated in real time in the closed loop. Some numerical simulations are performed for a practical system to verify the rationality of the consensus protocols.**

***Index Terms*—Active disturbance rejection control (ADRC), consensus, extended state observer, stochastic multiagent systems (MASs).**

I. Introduction

PROMOTED by various sorts of research topics like coordination, flocking, and formation of mobile autonomous agents, the consensus problem for multiagent systems (MASs) has been attracting considerable attention [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18]. The consensus problem has been widely investigated for different MASs like integrator-type systems [2], [3]; general linear systems [4], [5], [6]; and uncertain nonlinear systems [7], [8], [9]. In particular, since random phenomenon is often ineluctable in practice, the topic of consensus control of stochastic MASs has gained increasing research interests [10]. For instance, the practical consensus tracking and the exponential leader-following consensus problems for stochastic MASs by state feedback have been, respectively, investigated in [11] and [12]. The consensus control of stochastic MASs by dynamic output-feedback has been addressed in [13], the consensus control has been, respectively, developed for MASs with Markovian topologies [14] and the systems with randomly switching topologies and noises [15], the mean square exponential consensus control of second-order stochastic MASs with external stochastic disturbances has been investigated in [16], the adaptive neural network optimal consensus tracking control has been developed for nonlinear MASs subject to stochastic disturbances and actuator bias faults in [17], and the event-triggered consensus of stochastic MASs has been addressed in [18], to name just a few.

On the other hand, disturbance rejection is essential in engineering applications, which could be traced back to [19, p. 228], where it is declared that control should not be affected by both internal uncertainty and external disturbance. Well-known feedback control approaches, such as proportional–integral–derivative (PID) control, robust control, and sliding mode control attenuate disturbances by feedback regulation, which are not direct and rapid enough to cope with mighty disturbances and most often robust but at the cost of ruining the nominal performances, and most of available robust control strategies are worst-case-based designs [20]. These control methods are commonly regarded as the passive antidisturbance control ones [20]. Among many other control methods, the active disturbance rejection control (ADRC) [21] and disturbance observer-based control (DOBC) [20] are two active antidisturbance control methods which estimate disturbances in real time and reject them directly by a compensation control design. Compared with aforementioned feedback control approaches, a disturbance observer-based compensator is added in the design of ADRC and DOBC to improve the robustness and disturbance rejection of the basal feedback control, which is able to attenuate disturbances promptly and is not a worst-case-based design. However, most of the aforementioned literature about consensus control of MASs either has been paying too much attention on plant

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without disturbances or has been adopting passive antidisturbance consensus control methods only by feedback regulation. Recently, the disturbance observer-based consensus control of MASs without considering random factors has been receiving increasing attention (e.g., see [22], [23], [24], [25], [26]).

Nevertheless, the active disturbance rejection consensus control of MASs is still in its early phase, and so far as we know, there are no available studies concerning the random effects. The deterministic counterpart can be found in [27], where the extended state observer (ESO)-based consensus problem has been researched for MASs subject to external disturbance and input and output delays and in [28], where the practical output consensus problem has been addressed by the ADRC approach for nonlinear heterogeneous MASs with limited communication data rate, unknown nonlinearities, and external disturbance. The ADRC proposed in the late 1980s is a mighty estimation/cancellation active antidisturbance control strategy to attenuate external disturbance and system uncertainty via ESO. As the well-known PID, ADRC is almost model free. In the last 30 years, ADRC control technology has been validated efficiently by numerous engineering applications, such as general-purpose control chips produced by Texas Instruments [29], a dc–dc power converter [30], robot control [31], power plants [32], and motor control [33], etc. In addition, in the last 20 years, great attention has also been paid to the theoretical foundation of ADRC on stabilization, output regulation, and performance analysis of controlled plants described by uncertain nonlinear systems [34], [35], [36], [37], [38] and stochastic nonlinear systems [39], [40], among many others.

Motivated by aforementioned state-of-the-art research, in this article, for the first time, we develop the ADRC approach to the consensus problem for a class of second-order stochastic MASs, whose network topology is undirected connected. The main contributions and novelty are as follows: 1) Each agent of the MASs is subject to random uncertainty in large scale including unmeasured velocity signal and external nonlinear couplings of the nonrandom disturbance, colored noise, and bounded noise which are all nonvanishing at any equilibrium; 2) the total effect of all external nonrandom and random disturbances of each agent is estimated by ESOs and rejected by an ESO-based compensator in real time in the proposed ADRC framework, providing a novel active rather than passive disturbance rejection strategy; and 3) not only the mean square practical consensus problem but also the almost sure practical one for the second-order stochastic MASs is resolved.

The rest of this article is organized as follows. The problem formulation and some preliminaries are presented in Section II. The ADRC design and main results of the closed-loop convergence are presented in Section III. Some numerical simulations are carried out to validate the availability of the designed control protocols in Section IV. Finally, Section V concludes this article.

II. Problem Formulation and Preliminaries

Throughout this article, we use some notations as follows. $\otimes$ represents the Kronecker product; $\mathbb{E}X$ represents the expectation of a random variable X; the minimum and maximum eigenvalues of a positive definite matrix X are denoted by $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$, respectively; $\|\cdot\|$ denotes the 2-norm of the matrices or vectors; $\mathbf{1}_m$ and $\mathbf{0}_m$ represent the $m\times 1$ column vector with all components to be one and zero, respectively; $\mathbf{0}_{m\times n}$ represents the $m\times n$ matrix with all zeros and $\mathbb{I}_m$ represents the $m\times m$ identity matrix; and $\mathrm{diag}(p_1,\ldots,p_m)$ denotes a diagonal matrix with diagonal entries p_1 to p_m. $C(\mathbb{R};[0,\infty))$ stands for the family of continuous functions from $\mathbb{R}$ to $[0,\infty)$.

In this article, a network of N agents is considered. The communication topology graph is undirected and is denoted by $\mathcal{G}=\{\mathcal{V},\mathcal{E},\mathcal{A}\}$, where $\mathcal{V}=\{1,\ldots,N\}$ is the node set, $\mathcal{E}=\{(i,j)|i,j=1,\ldots,N\}\subseteq\mathcal{V}\times\mathcal{V}$ is the edge set, and $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ is the weighted adjacency matrix. The edge $(i,j)\in\mathcal{E}$ stands for the information flow from agent j to agent i. For $\mathcal{A}$, $a_{ij}=a_{ji}=1$ if and only if $(i,j)\in\mathcal{E}$ and $a_{ij}=0$ if and only if $(i,j)\notin\mathcal{E}$. Moreover, $a_{ii}=0$ for any $i\in\mathcal{V}$. The set of neighbors of agent i is denoted by $\mathcal{N}_i=\{j|j\in\mathcal{V},(i,j)\in\mathcal{E}\}$. Let $d_i=\sum_{j\in\mathcal{N}_i}a_{ij}$ and $\mathcal{D}=\mathrm{diag}(d_1,\ldots,d_N)$. Then the Laplacian matrix associated with the graph is expressed as $\mathcal{L}=\mathcal{D}-\mathcal{A}$. A path from node j to i is a sequence of edges $(i,i_1),(i_1,i_2),\ldots,(i_k,j)$ in the undirected network with distinct nodes i_l $(l=1,\ldots,k)$. If there exists a path between any pair of distinct nodes of the undirected graph, it is said to be connected.

Let $B(t)=(B_1(t),\ldots,B_N(t))^\top$ and $W(t)=(W_1(t),\ldots,W_N(t))^\top$ be two mutually independent N-dimensional standard Brownian motions defined on a complete probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},P)$ with Ω being a sample space, $\mathcal{F}$ a σ-field, $\{\mathcal{F}_t\}_{t\geq 0}$ a filtration, and P a probability measure. This article addresses ADRC for a class of second-order stochastic MASs driven by both colored noises and bounded noises as

$$\begin{cases}\dot{x}_i(t)=v_i(t)\\ \dot{v}_i(t)=f_i\left(t,x_i(t),v_i(t)\right)+g_i\left(t,w_i(t),\varpi_i(t)\right)+u_i(t)\\ y_i(t)=x_i(t),\ i=1,\ldots,N\end{cases}\tag{1}$$

where $x_i(t)\in\mathbb{R}, v_i(t)\in\mathbb{R}, u_i(t)\in\mathbb{R}$, and $y_i(t)\in\mathbb{R}$ represent the measured position, unmeasured velocity, control protocol, and measurement output of the agent i, respectively, and we set $x(t)=(x_1(t),\ldots,x_N(t))^\top, v(t)=(v_1(t),\ldots,v_N(t))^\top$; $f_i:[0,\infty)\times\mathbb{R}^2\to\mathbb{R}$ is a known system function of the agent i but $f_i(t,x_i(t),v_i(t))$ still represents unknown system dynamics since $v_i(t)$ is unmeasured; The function $g_i:[0,\infty)\times\mathbb{R}^2\to\mathbb{R}$ is unknown, $w_i(t)\in\mathbb{R}$ is the colored noise, and $\varpi_i(t):=\varrho_i(t,B_i(t))$ is the bounded noise defined by an unknown function $\varrho_i:[0,\infty)\times\mathbb{R}\to\mathbb{R}$ satisfying Assumption (A2).

The colored noise $w_i(t)$ also known as Ornstein–Uhlenbeck process [41], [42] can be expressed as the solutions of the Itô-type stochastic differential equations (see, e.g., [41, p. 426], [43, p. 101])

$$dw_i(t)=-\mu_i w_i(t)dt+\mu_i\sqrt{2\gamma_i}dW_i(t)\tag{2}$$

where $\mu_i>0$ and $\gamma_i>0$ are constants denoting the correlation time and the noise intensity, respectively; the initial values $w_i(0)\in\mathbb{R}$ are independent of $W(t)$ and $B(t)$; and we set $w(t)=(w_1(t),\ldots,w_N(t))^\top$. It is noted that in control theory, random disturbances are often modeled by white noise, which

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is stationary with zero mean and constant spectral density and is the generalized derivative of the Brownian motion (see, e.g., [44, p. 51, Th. 3.14]). However, the white noise cannot always well model the random disturbances arising in practice because its δ-function correlation is not exactly the one of real processes with finite or long correlation time [41]. A realistic alternative could be the colored noise which is an exponentially correlated process.

For obtaining the disturbance rejection and consensus control objective, we need the following Assumptions (A1)–(A3) and Lemma 2.1.

Assumption (A1): Suppose that $g_i(t, w_i, \varpi_i)$ $(i = 1, \ldots, N)$ have respective first-order continuous partial derivative and second-order continuous partial one with regard to t and (w_i, ϖ_i), and there are known constants $l_i > 0, b_{ij} > 0$ $(i = 1, \ldots, N, j = 1, 2, 3)$ and continuous functions $\sigma_{ij} \in C(\mathbb{R}; [0, \infty))$ $(i = 1, \ldots, N, j = 1, 2)$ such that for $t \geq 0, w_i \in \mathbb{R}, \varpi_i \in \mathbb{R}$

$$|f_i(t, x_i, v_i) - f_i(t, x_i, \hat{v}_i)| \leq l_i \|v_i - \hat{v}_i\| \tag{3}$$

$$\left|\frac{\partial g_i(t, w_i, \varpi_i)}{\partial t}\right| + \left|\frac{\partial g_i(t, w_i, \varpi_i)}{\partial \varpi_i}\right| + \left|\frac{\partial^2 g_i(t, w_i, \varpi_i)}{\partial \varpi_i^2}\right| \leq b_{i1} + b_{i2}|w_i| + \sigma_{i1}(\varpi_i) \tag{4}$$

$$\left|\frac{\partial g_i(t, w_i, \varpi_i)}{\partial w_i}\right| + \left|\frac{\partial^2 g_i(t, w_i, \varpi_i)}{\partial w_i^2}\right| \leq b_{i3} + \sigma_{i2}(\varpi_i). \tag{5}$$

Assumption (A2): The functions $\varrho_i(t, \varsigma_i) : [0, \infty) \times \mathbb{R} \to \mathbb{R}$ $(i = 1, \ldots, N)$ have respective first-order continuous partial derivative and second-order continuous partial one with regard to t and ς_i, and there are known constants $c_i > 0$, such that for $t \geq 0$, $\varsigma_i \in \mathbb{R}$

$$|\varrho_i(t, \varsigma_i)| + \left|\frac{\partial \varrho_i(t, \varsigma_i)}{\partial t}\right| + \left|\frac{\partial \varrho_i(t, \varsigma_i)}{\partial \varsigma_i}\right| + \frac{1}{2}\left|\frac{\partial^2 \varrho_i(t, \varsigma_i)}{\partial \varsigma_i^2}\right| \leq c_i. \tag{6}$$

Remark 2.1: The total effect of all external nonrandom disturbance, colored noise, and bounded noise will be regarded as the random total disturbance of each agent to be estimated by ESOs in mean square sense, which motivates the Assumptions (A1)–(A2). In consideration of the reality that the gain parameters chosen in the domain (11) should be specified, the constants b_{ij}, c_i and nonnegative continuous functions σ_{ij} in Assumptions (A1)–(A2) are supposed to be known. In addition, the bounded noises $\varpi_i(t) := \varrho_i(t, B_i(t))$ $(i = 1, \ldots, N)$ defined by the unknown functions $\varrho_i(\cdot, \cdot)$ satisfying the Assumption (A2) are realistic. This is because this class of random disturbances cover those disturbances without random factors that have been widely addressed via ADRC in literatures like those in [34], [35], [36], [37], [38], where $\varpi_i(t) := \varrho_i(t)$ are defined by the functions with regard to t only, and the condition (6) in Assumption (A2) is degenerated into the conventional condition requiring the boundedness of disturbances and their derivatives; for the random counterpart, the bounded noises with practical background like "$\sin(t + B_i(t))$" and "$\cos(t + B_i(t))$" in [45] and [46] are also covered.

Assumption (A3): The undirected topology graph $\mathcal{G}$ is connected.

Lemma 2.1 [47]: The Laplacian matrix $\mathcal{L}$ of a connected undirected graph $\mathcal{G}$ satisfies: $x^\top \mathcal{L} x = \frac{1}{2}\sum_{i=1}^N \sum_{j=1}^N a_{ij}(x_i - x_j)^2$ for any $x = (x_1, \ldots, x_N)^\top \in \mathbb{R}^N$, which indicates that $\mathcal{L}$ is positive semidefinite, and 0 is a simple eigenvalue of $\mathcal{L}$ with $\mathbf{1}_N$ being the associated eigenvector; The eigenvalues of $\mathcal{L}$ can be expressed by $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$, and $\lambda_2 = \min_{x^\top \mathbf{1}_N = 0, x \neq \mathbf{0_N}} \frac{x^\top \mathcal{L} x}{x^\top x}$.

Now, we introduce the definitions of mean square and almost sure practical consensus of second-order stochastic MASs (1).

Definition 1: The mean square practical consensus problem for second-order stochastic MASs (1) is solvable if for any prescribed $\varepsilon > 0$, there exist a set of control protocols such that: for any initial values, there exists a finite time $T(\varepsilon)$ such that

$$\mathbb{E}|x_i(t) - x_j(t)|^2 \leq \varepsilon, \quad \mathbb{E}|v_i(t) - v_j(t)|^2 \leq \varepsilon$$

for all $t \geq T(\varepsilon)$ and $i, j = 1, \ldots, N$.

Definition 2: The almost sure practical consensus problem for second-order stochastic MASs (1) is solvable if for any prescribed $\varepsilon > 0$, there exist a set of control protocols such that: for any initial values, there exists a finite time $T(\varepsilon)$ such that

$$|x_i(t) - x_j(t)| \leq \Lambda_\omega \varepsilon, \quad |v_i(t) - v_j(t)| \leq \Lambda_\omega \varepsilon \quad a.s.$$

for all $t \geq T(\varepsilon)$ and $i, j = 1, \ldots, N$, where Λ_ω is a positive random variable independent of ε.

The corresponding definitions of mean square and almost sure practical convergence of the following ESOs (8) can be introduced similarly.

The control objective of this article is to design ADRC consensus protocols against large-scale random uncertainty to solve both the mean square practical consensus problem and the almost sure practical one for second-order stochastic MASs (1), which means that the closed-loop states of the collective of agents reach some common position and velocity not only in mean square practical sense but also in almost sure practical one.

III. ADRC Design and Main Results

The random total disturbance affecting the performance of the agent i is

$$z_i(t) := g_i(t, w_i(t), \varpi_i(t)) \tag{7}$$

which is quite general since it combines the unknown nonlinear coupling of the nonrandom disturbance, colored noise, and bounded noise. The random total disturbance, as an extended state of each agent i, is to be estimated online by the following extended state observers (ESOs):

$$\begin{cases} \dot{\hat{x}}_i(t) = \hat{v}_i(t) + k_1 r(y_i(t) - \hat{x}_i(t)) \\ \dot{\hat{v}}_i(t) = f_i(t, x_i(t), \hat{v}_i(t)) + \hat{z}_i(t) + u_i(t) \\ +k_2 r^2(y_i(t) - \hat{x}_i(t)) \\ \dot{\hat{z}}_i(t) = k_3 r^3(y_i(t) - \hat{x}_i(t)), \ i = 1, \ldots, N \end{cases} \tag{8}$$

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where $\hat{x}_i(t), \hat{v}_i(t)$, and $\hat{z}_i(t)$ are the respective estimates of the $x_i(t)$, $v_i(t)$, and $z_i(t)$, and we set $\hat{x}(t) = (\hat{x}_1(t), \ldots, \hat{x}_N(t))^\top, \hat{v}(t) = (\hat{v}_1(t), \ldots, \hat{v}_N(t))^\top$, and $\hat{z}(t) = (\hat{z}_1(t), \ldots, \hat{z}_N(t))^\top$. The parameters $k_i (i = 1, 2, 3)$ are selected to ensure that the matrix

$$U := \begin{bmatrix} -k_1 & 1 & 0 \\ -k_2 & 0 & 1 \\ -k_3 & 0 & 0 \end{bmatrix}_{3\times 3} \tag{9}$$

is Hurwitz, and $r > 0$ is the tuning gain parameter specified in (11). Here and throughout the article, we always drop r for some relevant solutions like $\hat{x}(t), \hat{v}(t), \hat{z}(t)$ by abuse of notations without confusion.

The control protocols based on ESOs (8) are designed as

$$u_i(t) = \kappa \sum_{j\in\mathcal{N}_i} a_{ij} \Big\{ (x_j(t) - x_i(t)) + (\hat{v}_j(t) - \hat{v}_i(t)) \Big\} - f_i(t, x_i(t), \hat{v}_i(t)) - \hat{z}_i(t),\ i = 1, \ldots, N \tag{10}$$

where a_{ij}'s are the (i, j)th entry of the adjacency matrix $\mathcal{A}$, $\kappa > 0$ is the control gain specified in (11), and $-\hat{z}_i(t)$'s are the ESOs-based compensators providing the "active" disturbance rejection. Compared with passive antidisturbance control methods attenuating disturbances only by feedback regulation, the ESOs-based compensators are designed additionally to achieve the goal that random total disturbance (7) of each agent is estimated in real time by ESOs (8) and rejected directly by the corresponding compensator. It can be found that the control protocols are the ESOs-based output-feedback ones.

The ranges of two tuning gain parameters r and κ are specified as

$$\Pi_1 := \Big\{ r \in [1, \infty) : r > \max\{2\lambda_{\max}(G) \max_{1\le i\le N} l_i + \frac{\kappa\lambda_N^2}{\lambda_2} + \max_{1\le i\le N} l_i^2 + 2,\ 4\lambda_{\max}(G) \max_{1\le i\le N} l_i + 2\} \Big\}$$

$$\Pi_2 := \Big\{ \kappa \in (0, \infty) : \kappa > \frac{4}{\lambda_2} \Big\} \tag{11}$$

where λ_2 and λ_N are the respective smallest positive and maximal eigenvalues of the matrix $\mathcal{L}$ stated in Lemma 2.1, the constant l_i is specified in (3), and the matrix G is specified in (28). All these relative constants and matrices can be specified in practice, see for instance those in the numerical example in Section IV.

It is noteworthy that the tuning gain parameters r and κ are dependent on the smallest positive and maximal eigenvalues of the Laplacian matrix $\mathcal{L}$. This is the main reason why we do not emphasize the designed control protocols (10) in the distributed sense. It seems to be a sophisticated but practical goal to further get rid of the dependence.

Remark 3.2: If the estimated values of measured position $x_i(t)$ $(i = 1, \ldots, N)$ are adopted, that is, the control protocols based on ESOs (8) are designed as

$$u_i(t) = \kappa \sum_{j\in\mathcal{N}_i} a_{ij} \Big\{ (\hat{x}_j(t) - \hat{x}_i(t)) + (\hat{v}_j(t) - \hat{v}_i(t)) \Big\} - f_i(t, \hat{x}_i(t), \hat{v}_i(t)) - \hat{z}_i(t),\ i = 1, \ldots, N$$

then the range of the tuning gain parameter r will be smaller, and $f_i(t, x_i, v_i)$ are required to be Lipshitz with respect to (x_i, v_i) uniformly in $t \in [0, \infty)$.

Set

$$T(r) = \mu_3 r^{\mu_1} + \mu_2 \tag{12}$$

where μ_i $(i = 1, 2, 3)$ are any constants satisfying $\mu_1 > 0, \mu_2 > 0, \mu_3 > 1$.

The mean square practical convergence of the resulting closed-loop comprised of (1), (8), and (10) including mean square practical convergence of the estimation errors of ESOs and mean square practical consensus of agents is generalized as the following Theorem 3.1.

Theorem 3.1: Under Assumptions (A1)–(A3), for any initial values $x(0) \in \mathbb{R}^N, v(0) \in \mathbb{R}^N, w(0) \in \mathbb{R}^N, \hat{x}(0) \in \mathbb{R}^N, \hat{v}(0) \in \mathbb{R}^N, \hat{z}(0) \in \mathbb{R}^N$, there exists globally a unique strong solution $(x^\top(t), v^\top(t), \hat{x}^\top(t), \hat{v}^\top(t), \hat{z}^\top(t))^\top$ to the closed-loop system comprised of (1), (8), and (10), and there exist $r^* \in \Pi_1$ and $\kappa \in \Pi_2$ such that for any $r \ge r^*$, there holds

$$\text{(i)}\mathbb{E}|x_i(t) - \hat{x}_i(t)|^2 \le \frac{\Theta}{r^3},\ \mathbb{E}|v_i(t) - \hat{v}_i(t)|^2 \le \frac{\Theta}{r^2}$$
$$\mathbb{E}|z_i(t) - \hat{z}_i(t)|^2 \le \frac{\Theta}{r} \tag{13}$$

$$\text{(ii)}\mathbb{E}|x_i(t) - x_j(t)|^2 \le \frac{\Upsilon}{r},\ \mathbb{E}|v_i(t) - v_j(t)|^2 \le \frac{\Upsilon}{r} \tag{14}$$

for all $t \ge T(r)$ and $i, j = 1, \ldots, N$, where Θ and Υ are positive constants independent of r specified in (42) and (45), respectively.

Proof: See **" Proof of Theorem 3.1"** in Appendix A. ■

Remark 3.3: It is noteworthy that the mean square practical consensus problem for second-order stochastic MASs (1) in Definition 1 is completely solved by Theorem 3.1. This is because for any prescribed $\varepsilon > 0$, if $\max\{\frac{\Theta}{r^*}, \frac{\Upsilon}{r^*}\} \le \varepsilon$, then the mean square practical consensus of the second-order stochastic MASs (1) can be obtained by the ESOs-based control protocols (10) with the tuning gain of ESOs satisfying $r \ge r^*$, and it can also be obtained by the ESOs-based control protocols (10) with the tuning gain of ESOs satisfying $r \ge \max\{r^*, \frac{\Theta}{\varepsilon}, \frac{\Upsilon}{\varepsilon}\}$ otherwise. In addition, (13) and (14) mean that in practice, if we need high estimation and consensus accuracy, we need to tune high gain parameter r of ESOs, and then the corresponding ESOs-based control protocols (10) are designed. However, the price and peaking values phenomenon near the initial stage of the ESOs-based control protocols caused by high gain should also be considered. Finally, it should be pointed out that not only the steady-state performance but also more important transient one are often required in engineering applications. Equations (13) and (14) in Theorem 3.1 demonstrate exactly the transient estimation and consensus results by giving upper bound of estimation and consensus errors in the transient process $t \in [T(r), \infty)$. As a direct corollary of Theorem 3.1, the corresponding steady-state estimation and consensus results, i.e., the estimation and consensus results when $t \to \infty$ and $r \to \infty$, can be obtained directly.

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Corollary 1: Under Assumptions (A1)–(A3), for any initial values $x(0) \in \mathbb{R}^N, v(0) \in \mathbb{R}^N, w(0) \in \mathbb{R}^N, \hat{x}(0) \in \mathbb{R}^N, \hat{v}(0) \in \mathbb{R}^N, \hat{z}(0) \in \mathbb{R}^N$, the unique strong solution $(x^\top(t), v^\top(t), \hat{x}^\top(t), \hat{v}^\top(t), \hat{z}^\top(t))^\top$ of the closed-loop system comprised of (1), (8), and (10) satisfies

$$\text{(i)} \lim_{\substack{t\to\infty \\ r\to\infty}} \mathbb{E}\left[|x_i(t)-\hat{x}_i(t)|^2 + |v_i(t)-\hat{v}_i(t)|^2 + |z_i(t)-\hat{z}_i(t)|^2\right] = 0$$

$$\text{(ii)} \lim_{\substack{t\to\infty \\ r\to\infty}} \mathbb{E}[|x_i(t)-x_j(t)|^2 + |v_i(t)-v_j(t)|^2] = 0 \quad (15)$$

for $i, j = 1, \ldots, N$.

Proof: It can be concluded directly from Theorem 3.1. ■

On the basis of of Theorem 3.1, the corresponding almost sure convergence of the closed-loop system in the transient process is summarized as follows.

Theorem 3.2: Under Assumptions (A1)–(A3), for any initial values $x(0) \in \mathbb{R}^N, v(0) \in \mathbb{R}^N, w(0) \in \mathbb{R}^N, \hat{x}(0) \in \mathbb{R}^N, \hat{v}(0) \in \mathbb{R}^N, \hat{z}(0) \in \mathbb{R}^N$, there exist $r^* \in \Pi_1$ and $\kappa \in \Pi_2$ such that for any $r \geq r^*$, the unique strong solution $(x^\top(t), v^\top(t), \hat{x}^\top(t), \hat{v}^\top(t), \hat{z}^\top(t))^\top$ of the closed-loop system comprised of (1), (8), and (10) satisfies

$$\text{(i)} |x_i(t)-\hat{x}_i(t)| \leq \frac{\Theta_\omega}{r^{\frac{3}{2}}}, \; |v_i(t)-\hat{v}_i(t)| \leq \frac{\Theta_\omega}{r}$$

$$|z_i(t)-\hat{z}_i(t)| \leq \frac{\Theta_\omega}{r^{\frac{1}{2}}} \; a.s. \quad (16)$$

$$\text{(ii)} |x_i(t)-x_j(t)| \leq \frac{\Upsilon_\omega}{r^{\frac{1}{2}}}, \; |v_i(t)-v_j(t)| \leq \frac{\Upsilon_\omega}{r^{\frac{1}{2}}} \; a.s. \quad (17)$$

for all $t \geq T(r)$ and $i, j = 1, \ldots, N$, where Θ_ω and Υ_ω are positive random variables independent of r.

Proof: See **"Proof of Theorem 3.2"** in Appendix B. ■

Similar to the illustrations of Remark 3.3, the almost sure practical consensus problem for second-order stochastic MASs (1) in Definition 2 is completely solved by Theorem 3.2. Compared with the convergence result for the average of the estimation and consensus errors presented in Theorem 3.1, Theorem 3.2 shows the convergence result for every sample path of the estimation and consensus errors.

Similar to Corollary 1, the almost sure convergence of the closed-loop system in the steady-state process is summarized as the following Corollary 2.

Corollary 2: Under Assumptions (A1)–(A3), for any initial values $x(0) \in \mathbb{R}^N, v(0) \in \mathbb{R}^N, w(0) \in \mathbb{R}^N, \hat{x}(0) \in \mathbb{R}^N, \hat{v}(0) \in \mathbb{R}^N, \hat{z}(0) \in \mathbb{R}^N$, the solution $(x^\top(t), v^\top(t), \hat{x}^\top(t), \hat{v}^\top(t), \hat{z}^\top(t))^\top$ of the closed-loop system comprised of (1), (8), and (10) satisfies

$$\text{(i)} \lim_{\substack{t\to\infty \\ r\to\infty}} \left[|x_i(t)-\hat{x}_i(t)|+|v_i(t)-\hat{v}_i(t)|+|z_i(t)-\hat{z}_i(t)|\right] = 0 \, a.s. \quad (18)$$

$$\text{(ii)} \lim_{\substack{t\to\infty \\ r\to\infty}} \left[|x_i(t)-x_j(t)| + |v_i(t)-v_j(t)|\right] = 0 \; a.s. \quad (19)$$

for $i, j = 1, \ldots, N$.

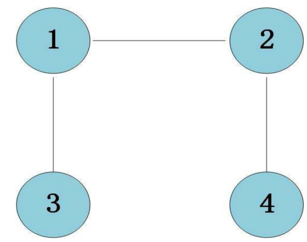

Fig. 1. Communication topology.

Proof: It can be concluded directly from Theorem 3.2. ■

Remark 3.4: Compared with most available studies concerning consensus control of stochastic MASs and theoretical researches of ADRC, the difficulty is the sophisticated stochastic Lyapunov analysis of the closed-loop system resulting from the combination of large-scale random uncertainty of agents and the fact that the control protocol for each agent only uses measurement output and estimates of its neighbors, where the latter leads to the common dynamic decoupling by the ADRC approach [34] is impossible in this scenario. This article makes full use of properties of the undirected connected network topology, Kronecker product, augmented state method of ADRC, and some stochastic analysis methods to overcome the obstacle.

Remark 3.5: It is worth noting that when the random noise vector field does not vanish at the equilibrium at the origin, most available studies only address the input-to-state consensus result with respect to the noise as the input or the boundedness one, resulting from the passive feedback regulation methods. In addition, the more realistic almost sure consensus problem has been relatively less addressed in available studies. Compared with these existing results, in the presence of nonvanishing large-scale random noises, the main results of this article obtain more efficient mean square and almost sure practical convergence of the closed-loop system in both transient and steady-state processes, whose estimation and consensus errors can be arbitrary small by tuning the gain of ESOs, brought by the active real-time estimation/cancellation of the random total disturbance.

IV. Numerical Simulation

For the sake of readability, in this section, some symbols are used again with the same meaning as previous sections. A multiple mechanical system with four one-link manipulators [1], [48] subject to external nonrandom disturbance, colored noise, and bounded noise is considered. Each mechanical system is modeled as

$$\mathcal{M}\ddot{\vartheta}_i(t)+\mathcal{C}\dot{\vartheta}_i(t)+\mathcal{N}\sin(\vartheta_i(t)) = u_i(t)+g_i(t,w_i(t),\varpi_i(t)) \quad (20)$$

where $i = 1, 2, 3, 4$, $\vartheta_i(t), \dot{\vartheta}_i(t)$, and $\ddot{\vartheta}_i(t)$ denote, respectively, the measured link position, unmeasured velocity, and unmeasured acceleration of the ith mechanical system, $\mathcal{M} = 1$, $\mathcal{C} = 0.03$, and $\mathcal{N} = 2$ represent, respectively, the inertia, the inertia coefficient of viscous friction at the joint, and gravity, $u_i(t)$ is the designed torque as a control input, and $g_i(t, w_i(t), \varpi_i(t))$ is the random total disturbance. The communication topology is presented in Fig. 1. The corresponding Laplacian matrix $\mathcal{L}$ is

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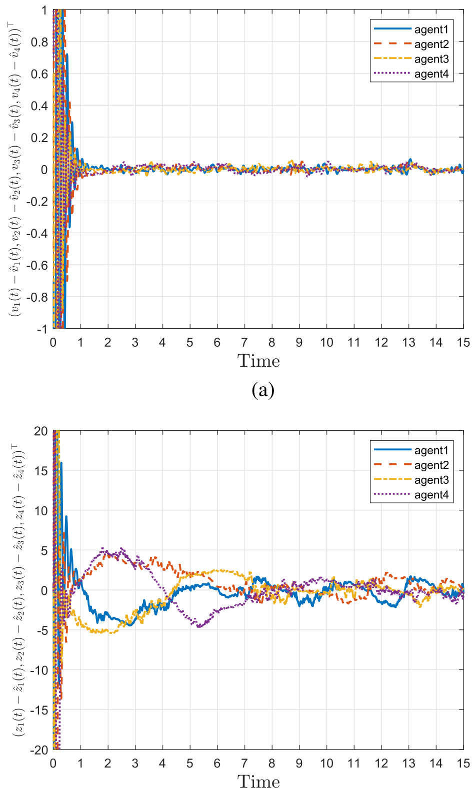
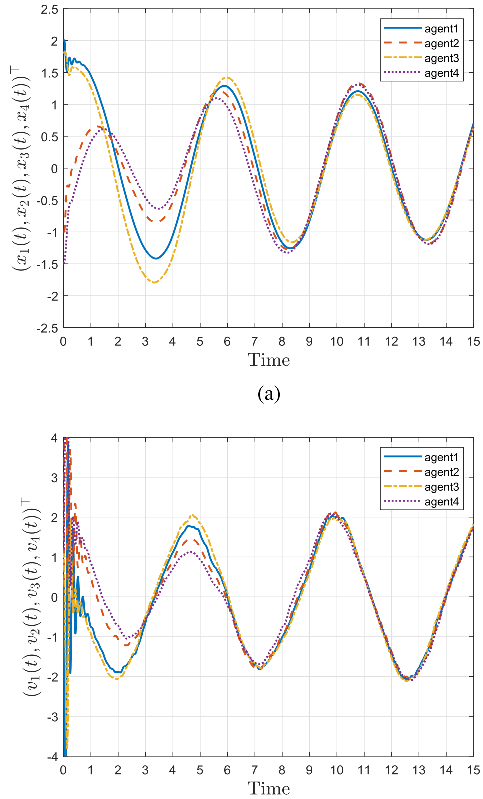


(a)

(b)

Fig. 2. Trajectories of $v_i(t) - \hat{v}_i(t), z_i(t) - \hat{z}_i(t)$.



(a)

(b)

Fig. 3. Trajectories of $x_i(t), v_i(t)$ under the random total disturbance $g_i(t, w_i(t), \varpi_i(t)) = \cos(t) + w_i(t) + \varpi_i(t)$.

obtained as

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}.$$

It can be calculated that four eigenvalues of $\mathcal{L}$ are $\lambda_1 = 0, \lambda_2 = 0.5858, \lambda_3 = 2,$ and $\lambda_4 = 3.4142$. Set $y_i(t) = x_i(t) = \vartheta_i(t), v_i(t) = \dot{\vartheta}_i(t)$. Then system (20) is expressed as

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = -0.03 v_i(t) - 2\sin(x_i(t)) + z_i(t) + u_i(t) \\ y_i(t) = x_i(t) \end{cases} \tag{21}$$

where $z_i(t) := g_i(t, w_i(t), \varpi_i(t))$ is the random total disturbance affecting the ith mechanical system. The ESOs are designed for estimation of unmeasured state $v_i(t)$ and random total disturbance $z_i(t)$ of each mechanical system as

$$\begin{cases} \dot{\hat{x}}_i(t) = \hat{v}_i(t) + 3 * 25(y_i(t) - \hat{x}_i(t)) \\ \dot{\hat{v}}_i(t) = -0.03\hat{v}_i(t) - 2\sin(x_i(t)) + \hat{z}_i(t) \\ +u_i(t) + 3 * 25^2(y_i(t) - \hat{x}_i(t)) \\ \dot{\hat{z}}_i(t) = 25^3(y_i(t) - \hat{x}_i(t)) \end{cases} \tag{22}$$

where the gain $r = 25$, and $U = \begin{bmatrix} -3 & 1 & 0 \\ -3 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$ is the Hurwitz matrix. The ESOs-based control protocols are designed as

$$u_i(t) = 10 \sum_{j \in \mathcal{N}_i} a_{ij} \Big\{ (x_j(t) - x_i(t)) + (\hat{v}_j(t) - \hat{v}_i(t)) \Big\}$$
$$+ 0.03\hat{v}_i(t) + 2\sin(x_i(t)) - \hat{z}_i(t) \tag{23}$$

where $\kappa = 10$ is the control gain. A simple calculation shows that

$$G = \begin{bmatrix} 1 & -\frac{1}{2} & -1 \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -1 & -\frac{1}{2} & 4 \end{bmatrix}$$

which is the unique positive definite matrix solution of $GU + U^\top G = -\mathbb{I}_3$.

In the numerical simulations, the initial values are chosen as $x(0) = (2, -1, 1.8, -1.5)^\top, v(0) = (0.5, 0.3, 1, -1.2)^\top$, $\hat{x}(0) = (0.8, 1.1, 0.6, 0.7)^\top$, $\hat{v}(0) = (1.2, 0.8, 0.4, 0.5)^\top$, $\hat{z}(0) = (0.1, 0.15, 0.3, 0.2)^\top$, $w_i(0) = 0.1$.

In Figs. 2 and 3, the random total disturbance is specified as $g_i(t, w_i(t), \varpi_i(t)) = \cos(t) + w_i(t) + \varpi_i(t)$, where $w_i(t)$ is the colored noise defined as that in (2) with $\mu_i = \gamma_i = 1$ $(i = 1, 2, 3, 4)$, and $\varpi_i(t) = \sin(2t + B_i(t))$ is the bounded noise. It is easily found that the Assumptions (A1)–(A3) are all satisfied. It can be observed from Figs. 2 and 3 that the

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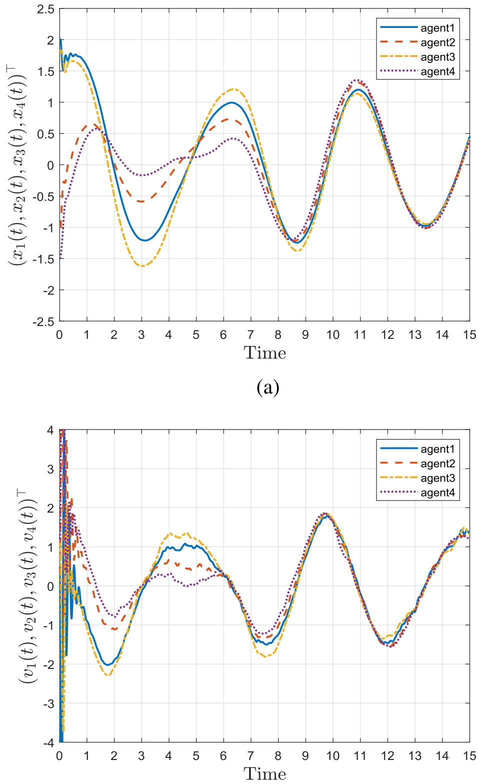

(a)

(b)

Fig. 4. Trajectories of $x_i(t), v_i(t)$ under the random total disturbance $g_i(t, w_i(t), \varpi_i(t)) = 2\cos(2t) + 1.5w_i(t) + 1.2\varpi_i(t)$.

estimation effect for both unmeasured state $v_i(t)$ and random total disturbance $z_i(t)$ and the obtained consensus effect for the multiple mechanical system are very satisfactory after a short time. In Fig. 4, the intensity of the random total disturbance is varied as $g_i(t, w_i(t), \varpi_i(t)) = 2\cos(2t) + 1.5w_i(t) + 1.2\varpi_i(t)$, where the Assumptions (A1)–(A3) are still valid. It can be observed that the corresponding good consensus effect after a short time is still maintained, which reflects the robust performance of the proposed protocols (23) to a certain extent.

V. Conclusion

This article adopts the active disturbance rejection control for the consensus of a class of second-order stochastic MASs. The network topology among agents is supposed to be undirected and connected. Each agent of the multiagent systems is subject to external nonrandom disturbance, colored noise, and bounded noise, whose total effect is considered as a random total disturbance. The random total disturbance of each agent is estimated by ESOs and compensated in the ESOs-based control protocols in real time. Theoretical proofs are presented to prove the consensus of agents in both mean square and almost sure practical sense. The validity of the proposed active disturbance rejection consensus control strategy is validated by some numerical simulations. In the future work, it would be significant to relax the topological structure and introduce event-triggering mechanisms in both the ESOs and ESOs-based control protocols, bringing more efficient utilization of communication/computation resources in networked environment.

Appendix A

Proof of Theorem 3.1: For all $i = 1, \ldots, N$, set

$$\begin{cases} \eta_{i1}(t) = r^2(x_i(t) - \hat{x}_i(t)), \eta_{i2}(t) = r(v_i(t) - \hat{v}_i(t)) \\ \eta_{i3}(t) = z_i(t) - \hat{z}_i(t), \eta_i(t) = (\eta_{i1}(t), \eta_{i2}(t), \eta_{i3}(t))^\top \\ \eta(t) = (\eta_1^\top(t), \ldots, \eta_N^\top(t))^\top \\ \bar{x}(t) = \frac{1}{N}\sum_{i=1}^N x_i(t) = \frac{1}{N}\mathbf{1}_N^\top x(t) \\ \bar{v}(t) = \frac{1}{N}\sum_{i=1}^N v_i(t) = \frac{1}{N}\mathbf{1}_N^\top v(t) \\ \alpha_i(t) = x_i(t) - \bar{x}(t), \ \ \beta_i(t) = v_i(t) - \bar{v}(t) \\ \alpha(t) = (\alpha_1(t), \ldots, \alpha_N(t))^\top, \beta(t) = (\beta_1(t), \ldots, \beta_N(t))^\top \\ \Pi_i(t) = f_i(t, x_i(t), v_i(t)) - f_i(t, x_i(t), \hat{v}_i(t)). \end{cases} \tag{24}$$

Apply Itô's formula to $z_i(t)$ defined in (7) with respect to t along (1) and (2) to yield that

$$\begin{aligned} &dz_i(t) \\ &= \left\{ \frac{\partial g_i(t, w_i(t), \varpi_i(t))}{\partial t} - \frac{\partial g_i(t, w_i(t), \varpi_i(t))}{\partial w_i}\mu_i w_i(t) \right. \\ &\quad + \frac{\partial^2 g_i(t, w_i(t), \varpi_i(t))}{\partial w_i^2}\mu_i^2\gamma_i + \frac{\partial g_i(t, w_i(t), \varpi_i(t))}{\partial \varpi_i} \\ &\quad \left(\frac{\partial \varrho_i(t, B_i(t))}{\partial t} + \frac{1}{2}\frac{\partial^2 \varrho_i(t, B_i(t))}{\partial \varsigma_i^2} \right) \\ &\quad \left. + \frac{1}{2}\frac{\partial^2 g_i(t, w_i(t), \varpi_i(t))}{\partial \varpi_i^2}\left(\frac{\partial \varrho_i(t, B_i(t))}{\partial \varsigma_i} \right)^2 \right\} dt \\ &\quad + \frac{\partial g_i(t, w_i(t), \varpi_i(t))}{\partial \varpi_i}\frac{\partial \varrho_i(t, B_i(t))}{\partial \varsigma_i} dB_i(t) \\ &\quad + \frac{\partial g_i(t, w_i(t), \varpi_i(t))}{\partial w_i}\mu_i\sqrt{2\gamma_i}dW_i(t) \\ &= : \Gamma_{i1}(t)dt + \Gamma_{i2}(t)dB_i(t) + \Gamma_{i3}(t)dW_i(t) \end{aligned}$$

where we use ς_i as the second argument of the function $\varrho_i(\cdot, \cdot)$. It is easily obtained that $\eta_i(t), i = 1, \ldots, N$ satisfy the following Itô-type stochastic systems:

$$\begin{cases} d\eta_{i1}(t) = r\left(\eta_{i2}(t) - k_1\eta_{i1}(t)\right) dt \\ d\eta_{i2}(t) = r\left(\eta_{i3}(t) - k_2\eta_{i1}(t) + \Pi_i(t)\right) dt \\ d\eta_{i3}(t) = -rk_3\eta_{i1}(t)dt + \Gamma_{i1}(t)dt \\ +\Gamma_{i2}(t)dB_i(t) + \Gamma_{i3}(t)dW_i(t). \end{cases} \tag{25}$$

Set $C_1 = (0, 0, 1)^\top$, $C_2 = (0, 1, 0)^\top$, $\Gamma_1(t) = (C_1^\top\Gamma_{11}(t), \ldots, C_1^\top\Gamma_{N1}(t))^\top$, $\Gamma_2(t) = \text{diag}\{C_1\Gamma_{12}, C_1\Gamma_{22}, \ldots, C_1\Gamma_{N2}\} \in \mathbb{R}^{3N\times N}$, $\Gamma_3(t) = \text{diag}\{C_1\Gamma_{13}, C_1\Gamma_{23}, \ldots, C_1\Gamma_{N3}\} \in \mathbb{R}^{3N\times N}$ and $\Pi(t) = (C_2^\top\Pi_1(t), \ldots, C_2^\top\Pi_N(t))^\top$. By (25), it is obtained that $\eta(t)$ satisfies

$$d\eta(t) = r(I_N \otimes U)\eta(t)dt + \Gamma_1(t)dt + r\Pi(t)dt$$

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$$+\Gamma_2(t)dB(t)+\Gamma_3(t)dW(t). \quad (26)$$

Let $D=\mathbb{I}_N-\frac{1}{N}\mathbf{1}_N\mathbf{1}_N^\top\in\mathbb{R}^{N\times N}$. We then have

$$\begin{cases}\alpha(t)=x(t)-\frac{1}{N}\mathbf{1}_N\mathbf{1}_N^\top x(t)=Dx(t)\\ \beta(t)=v(t)-\frac{1}{N}\mathbf{1}_N\mathbf{1}_N^T v(t)=Dv(t).\end{cases}$$

We further set $\varphi(t)=(x^\top(t),v^\top(t))^\top$ and $\Phi(t)=(\alpha^\top(t),\beta^\top(t))^\top$. We then have $\Phi(t)=\mathbb{I}_2\otimes D\varphi(t)$. In addition, it follows from Assumption (A3) that $\mathcal{L}$ is symmetrical and hence $D\mathcal{L}=\mathcal{L}D$. Therefore, it can be concluded that $\Phi(t)$ satisfies

$$d\Phi(t)=(C_3\,C_4^\top\otimes\mathbb{I}_N-\kappa C_4C_3^\top\otimes\mathcal{L})\Phi(t)dt$$
$$-\kappa C_4C_4^\top\otimes\mathcal{L}\hat{\Phi}(t)dt+C_4\otimes D\tilde{z}(t)dt+C_4\otimes D\Pi^*(t)dt$$

where $\hat{\varphi}(t):=(\hat{x}^\top(t),\hat{v}^\top(t))^\top, C_3:=(1,0)^\top, C_4:=(0,1)^\top, \hat{\Phi}(t):=\mathbb{I}_2\otimes D\hat{\varphi}(t), \tilde{z}(t):=(z_1(t)-\hat{z}_1(t),\ldots,z_N(t)-\hat{z}_N(t))^\top, \Pi^*(t)=(\Pi_1(t),\ldots,\Pi_N(t))^\top$. We thus conclude that the closed-loop system comprised of (1), (8), and (10) is equivalent to

$$\begin{cases}d\Phi(t)=(C_3\,C_4^\top\otimes\mathbb{I}_N-\kappa C_4C_3^\top\otimes\mathcal{L})\Phi(t)dt\\ -\kappa C_4C_4^\top\otimes\mathcal{L}\hat{\Phi}(t)dt+C_4\otimes D\tilde{z}(t)dt\\ +C_4\otimes D\Pi^*(t)dt\\ d\eta(t)=r(I_N\otimes U)\eta(t)dt+\Gamma_1(t)dt+r\Pi(t)dt\\ +\Gamma_2(t)dB(t)+\Gamma_3(t)dW(t).\end{cases} \quad (27)$$

It follows from (2) that the colored noises $w_i(t)$ $(i=1,\ldots,N)$ can be referred to as the extended state variables of (27), and the bounded noises $\varpi_i(t)$ $(i=1,\ldots,N)$ with deterministic bound satisfy Assumption (A2). These, together with Assumption (A1), yield that the drift and diffusion terms of (27) actually satisfy the local Lipschitz and linear growth conditions. From the existence-and-unique theorem of the Itô-type stochastic systems (see, e.g., [43, p.58, Th. 3.6]), there exist a unique global solution $(\Phi^\top(t),\eta^\top(t))^\top$ to the equivalent closed-loop system, and the unique global solution $(x^\top(t),v^\top(t),\hat{x}^\top(t),\hat{v}^\top(t),\hat{z}^\top(t))^\top$ to the closed-loop system that is comprised of (1), (8), and (10) can be concluded directly. Choose the Lyapunov candidates as

$$V_1(\eta(t))=\eta^\top(t)(\mathbb{I}_N\otimes G)\eta(t),\ \ V_2(\Phi(t))=\frac{1}{2}\Phi^\top(t)Q\Phi(t)$$

where G is the unique positive definite matrix satisfying

$$GU+U^\top G=-\mathbb{I}_3 \quad (28)$$

with U given in (9) and $Q=\begin{bmatrix}2\kappa\mathcal{L} & \mathbb{I}_N\\ \mathbb{I}_N & \mathbb{I}_N\end{bmatrix}$. By Lemma 2.1, it follows that:

$$V_2(\Phi(t))=\kappa\alpha^\top(t)\mathcal{L}\alpha(t)+\beta^\top(t)\alpha(t)+\frac{1}{2}\beta^\top(t)\beta(t)$$
$$\geq\kappa\lambda_2\alpha^\top(t)\alpha(t)+\beta^\top(t)\alpha(t)+\frac{1}{2}\beta^\top(t)\beta(t)$$

where it can be obtained that $V_2(\Phi(t))\geq 0$ and $V_2(\Phi(t))=0$ if and only if $\Phi(t)=\mathbf{0}_{2N\times 1}$ since $\kappa\in\Pi_2$. Define

$$V(\eta(t),\Phi(t))=V_1(\eta(t))+V_2(\Phi(t)).$$

By using Itô's formula, one has

$$\begin{aligned}&dV(\eta(t),\Phi(t))\\
&=\Bigg(r\eta^\top(t)(\mathbb{I}_N\otimes(GU+U^\top G))\eta(t)+2\eta^\top(t)(\mathbb{I}_N\otimes G)\cdot\\
&\quad(\Gamma_1(t)+r\Pi(t))+\sum_{i=1}^N\Gamma_{i2}^2(t)C_1^\top GC_1\\
&\quad+\sum_{i=1}^N\Gamma_{i3}^2(t)C_1^\top GC_1\Bigg)dt+2\eta^\top(t)(I_N\otimes G)\Gamma_2(t)dB(t)\\
&\quad+2\eta^\top(t)(I_N\otimes G)\Gamma_3(t)dW(t)\\
&\quad+\Bigg\{\frac{1}{2}\Phi^\top(t)\Bigg(Q\left(C_3\,C_4^\top\otimes I_N-\kappa C_4\mathbf{1}_2^\top\otimes\mathcal{L}\right)\\
&\quad+\left(C_3\,C_4^\top\otimes\mathbb{I}_N-\kappa C_4\mathbf{1}_2^\top\otimes\mathcal{L}\right)^\top Q\Bigg)\Phi(t)\\
&\quad+\Phi^\top(t)Q\Bigg(\kappa C_4C_4^\top\otimes\mathcal{L}\tilde{\Phi}(t)+C_4\otimes D\tilde{z}(t)\\
&\quad+C_4\otimes D\Pi^*(t)\Bigg)\Bigg\}dt\\
&=\Bigg(-r\|\eta(t)\|^2+2\eta^\top(t)(I_N\otimes G)(\Gamma_1(t)+r\Pi(t))\\
&\quad+\sum_{i=1}^N\Gamma_{i2}^2(t)C_1^\top GC_1^\top+\sum_{i=1}^N\Gamma_{i3}^2(t)C_1^\top GC_1\Bigg)dt\\
&\quad+2\eta^\top(t)(I_N\otimes G)\Gamma_2(t)dB(t)\\
&\quad+2\eta^\top(t)(I_N\otimes G)\Gamma_3(t)dW(t)\\
&\quad+\Bigg(\Phi^\top(t)(C_4\,C_4^\top\otimes\mathbb{I}_N-\mathbb{I}_2\otimes\kappa\mathcal{L})\Phi(t)\\
&\quad+\kappa\Phi^\top(t)((C_4C_4^\top+C_3C_4^\top)\otimes\mathcal{L})\tilde{\Phi}(t)\\
&\quad+\Phi^\top(t)(\mathbf{1}_2\otimes D)\tilde{z}(t)+\Phi^\top(t)(\mathbf{1}_2\otimes D)\Pi^*(t)\Bigg)dt\end{aligned} \quad (29)$$

where $\tilde{\Phi}(t):=\Phi(t)-\hat{\Phi}(t)$. By Assumptions (A1)–(A2), it follows that there are known positive constants π_j $(j=1,2)$ independent of the tuning gain parameter r such that for all $t\geq 0$:

$$\mathbb{E}\|(I_N\otimes G)\Gamma_1(t)\|^2\leq\pi_1$$
$$\mathbb{E}\sum_{i=1}^N(\Gamma_{i2}^2(t)+\Gamma_{i3}^2(t))\leq\pi_2 \quad (30)$$

and by (3) in Assumption (A1), we also have

$$\|\Pi(t)\|=\|\Pi^*(t)\|\leq\frac{\max_{1\leq i\leq N}l_i}{r}\|\eta(t)\|. \quad (31)$$

It yields from Lemma 2.1 that

$$-\kappa\alpha^\top(t)\mathcal{L}\alpha(t)\leq-\kappa\lambda_2\alpha^\top(t)\alpha(t)$$
$$-\kappa\beta^\top(t)\mathcal{L}\beta(t)\leq-\kappa\lambda_2\beta^\top(t)\beta(t). \quad (32)$$

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By Young's inequality and the fact that the maximum eigenvalue of $\mathcal{L}^2$ is λ_N^2, we have

$$\begin{aligned}&\kappa\Phi^\top(t)((C_4C_4^\top + C_3C_4^\top)\otimes\mathcal{L})\tilde{\Phi}(t)\\ &\le \frac{1}{2}\kappa\lambda_2\|\Phi(t)\|^2 + \frac{\kappa}{\lambda_2}\tilde{\Phi}^\top(t)(C_4C_4^\top\otimes\mathcal{L}^2)\tilde{\Phi}(t)\\ &\le \frac{1}{2}\kappa\lambda_2\|\Phi(t)\|^2 + \frac{\kappa\lambda_N^2}{\lambda_2}\|\tilde{\Phi}(t)\|^2 \qquad (33)\end{aligned}$$

$$\begin{aligned}&\Phi^\top(t)(\mathbf{1}_2\otimes D)\tilde{z}(t) \le \frac{1}{2}\|\Phi(t)\|^2 + \tilde{z}^\top(t)D^2\tilde{z}(t)\\ &\le \frac{1}{2}\|\Phi(t)\|^2 + \|\tilde{z}(t)\|^2 \qquad (34)\end{aligned}$$

and also

$$\Phi^\top(t)(\mathbf{1}_2\otimes D)\Pi^*(t) \le \frac{1}{2}\|\Phi(t)\|^2 + \|\Pi^*(t)\|^2. \qquad (35)$$

Furthermore, by (24), for all $t\ge 0$

$$\begin{aligned}&\|\tilde{\Phi}(t)\|^2 = \|\tilde{\alpha}(t)\|^2 + \|\tilde{\beta}(t)\|^2 \le \frac{\|\eta(t)\|^2}{r^2}\\ &\|\tilde{z}(t)\|^2 \le \|\eta(t)\|^2 \qquad (36)\end{aligned}$$

where $\tilde{\alpha}(t) := D\tilde{x}(t), \tilde{\beta}(t) := D\tilde{v}(t)$ with $\tilde{x}(t) := (\tilde{x}_1(t),\ldots,\tilde{x}_N(t))^\top, \tilde{x}_i(t) = x_i(t) - \hat{x}_i(t), \tilde{v}(t) = (\tilde{v}_1(t),\ldots,\tilde{v}_N(t))^\top, \tilde{v}_i(t) = v_i(t) - \hat{v}_i(t), i = 1,\ldots,N$. Choose $r^*\in\Pi_1$ and $\kappa\in\Pi_2$. Then

$$\begin{aligned}&\theta_1 := r^* - 2\lambda_{\max}(G)\max_{1\le i\le N} l_i - \frac{\kappa\lambda_N^2}{\lambda_2 r^{*2}}\\ &- \frac{\max_{1\le i\le N} l_i^2}{r^{*2}} - 2 > 0\\ &\theta_2 := \frac{\kappa\lambda_2}{2} - 2 > 0.\end{aligned}$$

By regarding the colored noises $w_i(t)$ $(i = 1,\ldots,N)$ as the extended state variables of (27) again, it follows from Assumptions (A1)–(A2) that the drift and diffusion terms of (27) satisfy linear growth condition. Hence, it can be easily concluded (see, e.g., [43, p.51, Lemma 3.2]) that for any $t\ge 0$, $\int_0^t 2\eta^\top(s)(I_N\otimes G)\Gamma_2(s)dB(s) + \int_0^t 2\eta^\top(s)(I_N\otimes G)\Gamma_3(s)dW(s)$ is a martingale so that its expectation becomes zero. This, together with (29)–(36), yields further that

$$\begin{aligned}&\frac{d\mathbb{E}(V(\eta(t),\Phi(t)))}{dt}\\ =&-r\mathbb{E}\|\eta(t)\|^2 + 2\mathbb{E}\eta^\top(t)(I_N\otimes G)(\Gamma_1(t) + r\Pi(t))\\ &+\mathbb{E}\sum_{i=1}^N(\Gamma_{i2}^2(t) + \Gamma_{i3}^2(t))C_1^\top GC_1\\ &+\mathbb{E}\Phi^\top(t)(C_4C_4^\top\otimes\mathbb{I}_N - \mathbb{I}_2\otimes\kappa\mathcal{L})\Phi(t)\\ &+\kappa\mathbb{E}\Phi^\top(t)((C_4C_4^\top + C_3C_4^\top)\otimes\mathcal{L})\tilde{\Phi}(t)\\ &+\mathbb{E}\Phi^\top(t)(\mathbf{1}_2\otimes D)\tilde{z}(t) + \mathbb{E}\Phi^\top(t)(\mathbf{1}_2\otimes D)\Pi^*(t)\\ \le&-r\mathbb{E}\|\eta(t)\|^2 + \mathbb{E}\|\eta(t)\|^2 + \pi_1\\ &+2\lambda_{\max}(G)\max_{1\le i\le N} l_i\mathbb{E}\|\eta(t)\|^2 + \pi_2 C_1^\top GC_1\\ &+(1-\kappa\lambda_2)\mathbb{E}\|\Phi(t)\|^2 + \frac{1}{2}\kappa\lambda_2\mathbb{E}\|\Phi(t)\|^2 + \frac{\kappa\lambda_N^2}{\lambda_2 r^2}\mathbb{E}\|\eta(t)\|^2\\ &+\mathbb{E}\|\Phi(t)\|^2 + \mathbb{E}\|\eta(t)\|^2 + \frac{\max_{1\le i\le N} l_i^2}{r^2}\mathbb{E}\|\eta(t)\|^2\\ =&-(r - 2\lambda_{\max}(G)\max_{1\le i\le N} l_i - \frac{\kappa\lambda_N^2}{\lambda_2 r^2} - \frac{\max_{1\le i\le N} l_i^2}{r^2} - 2)\cdot\\ &\mathbb{E}\|\eta(t)\|^2 - (\frac{\kappa\lambda_2}{2} - 2)\mathbb{E}\|\Phi(t)\|^2 + \pi_1 + \pi_2C_1^\top GC_1\\ \le&-\theta_1\mathbb{E}\|\eta(t)\|^2 - \theta_2\mathbb{E}\|\Phi(t)\|^2 + \theta_3, \qquad (37)\end{aligned}$$

where

$$\theta_3 := \pi_1 + \pi_2 C_1^\top GC_1.$$

Set

$$\theta_4 = \min\left\{\frac{\theta_1}{\lambda_{\max}(G)}, \frac{2\theta_2}{\lambda_{\max}(Q)}\right\}.$$

It follows from (37) that:

$$\frac{d\mathbb{E}V(\eta(t),\Phi(t))}{dt} \le -\theta_4\mathbb{E}V(\eta(t),\Phi(t)) + \theta_3$$

which yields

$$\mathbb{E}V(\eta(t),\Phi(t)) \le e^{-\theta_4 t}\mathbb{E}V(\eta(0),\Phi(0)) + \frac{\theta_3}{\theta_4}$$

for all $t\ge 0$. In addition

$$\begin{aligned}&\mathbb{E}V(\eta(0),\Phi(0)) = \mathbb{E}V_1(\eta(0)) + V_2(\Phi(0))\\ &\le \lambda_{\max}(G)(r^4\|\tilde{x}(0)\|^2 + r^2\|\tilde{v}(0)\|^2 + \mathbb{E}\|\tilde{z}(0)\|^2)\\ &\quad + V_2(\Phi(0)).\end{aligned}$$

For any given $\mu_1 > 0$, we set

$$\begin{aligned}\theta_5 = \sup_{r\ge r*} e^{-\theta_4 r^{\mu_1}}\Big(&\lambda_{\max}(G)(r^4\|\tilde{x}(0)\|^2 + r^2\|\tilde{v}(0)\|^2\\ &+\mathbb{E}\|\tilde{z}(0)\|^2) + V_2(\Phi(0))\Big).\end{aligned}$$

Since $\lim_{r\to\infty} e^{-\theta_4 r^{\mu_1}} r^j = 0$ $(j = 2,4)$, we can conclude that θ_5 is a positive constant independent of r. Therefore, for all $t\ge r^{\mu_1}$, one has

$$\begin{aligned}&\mathbb{E}V(\eta(t),\Phi(t)) \le e^{-\theta_4 t}\mathbb{E}V(\eta(0),\Phi(0)) + \frac{\theta_3}{\theta_4}\\ &\le e^{-\theta_4 r^{\mu_1}}V(\eta(0),\Phi(0)) + \frac{\theta_3}{\theta_4}\\ &\le \theta_5 + \frac{\theta_3}{\theta_4} =: \theta_6. \qquad (38)\end{aligned}$$

Based on the above analysis (37)–(38) and $r\ge r^*\in\Pi_1$, it yields that for all $t\ge r^{\mu_1}$

$$\begin{aligned}\frac{d\mathbb{E}V_1(\eta(t))}{dt} \le& -r\mathbb{E}\|\eta(t)\|^2 + \mathbb{E}\|\eta(t)\|^2 + \pi_1\\ &+2\lambda_{\max}(G)\max_{1\le i\le N} l_i\mathbb{E}\|\eta(t)\|^2 + \pi_2C_1^\top GC_1\end{aligned}$$

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$$\begin{aligned} &\le -\left(r - 2\lambda_{\max}(G)\max_{1\le i\le N} l_i - 1\right)\mathbb{E}\|\eta(t)\|^2 + \theta_3 \\ &\le -\frac{r}{2}\mathbb{E}\|\eta(t)\|^2 + \theta_3 \\ &\le -\frac{r}{2\lambda_{\max}(G)}\mathbb{E}V_1(\eta(t)) + \theta_3. \end{aligned} \tag{39}$$

By (38), for all $t \ge r^{\mu_1} + \mu_2$ with μ_2 being any positive constant, it holds

$$\begin{aligned} \mathbb{E}V_1(\eta(t)) &\le e^{-\frac{r(t-r^{\mu_1})}{2\lambda_{\max}(G)}}\mathbb{E}V_1(\eta(r^{\mu_1})) + \int_{r^{\mu_1}}^{t} e^{-\frac{r(t-s)}{2\lambda_{\max}(G)}}\theta_3 ds \\ &\le e^{-\frac{\mu_2 r}{2\lambda_{\max}(G)}}\theta_6 + \frac{2\theta_3\lambda_{\max}(G)}{r} \le \frac{\theta_7}{r} \end{aligned} \tag{40}$$

where

$$\theta_7 := \sup_{r\ge r^*} e^{-\frac{\mu_2 r}{2\lambda_{\max}(G)}} r\theta_6 + 2\theta_3\lambda_{\max}(G)$$

is independent of r. Therefore, for $t \ge r^{\mu_1} + \mu_2$ and $i = 1,\ldots,N$, it holds

$$\begin{aligned} \mathbb{E}|x_i(t) - \hat{x}_i(t)|^2 &\le \frac{\mathbb{E}V_1(\eta(t))}{\lambda_{\min}(G)r^2} \le \frac{\Theta}{r^3} \\ \mathbb{E}|v_i(t) - \hat{v}_i(t)|^2 &\le \frac{\mathbb{E}V_1(\eta(t))}{\lambda_{\min}(G)r} \le \frac{\Theta}{r^2} \\ \mathbb{E}|z_i(t) - \hat{z}_i(t)|^2 &\le \frac{\mathbb{E}V_1(\eta(t))}{\lambda_{\min}(G)} \le \frac{\Theta}{r} \end{aligned} \tag{41}$$

where

$$\Theta := \frac{\theta_7}{\lambda_{\min}(G)}. \tag{42}$$

This gives (13). Similarly to (37) and by (40), for all $t \ge r^{\mu_1} + \mu_2$, we have

$$\begin{aligned} \frac{d\mathbb{E}V_2(\Phi(t))}{dt} &\le -\left(\frac{\kappa\lambda_2}{2} - 2\right)\mathbb{E}\|\Phi(t)\|^2 \\ &+ \left(1 + \frac{\kappa\lambda_N^2}{\lambda_2 r^2} + \frac{\max_{1\le i\le N} l_i^2}{r^2}\right)\mathbb{E}\|\eta(t)\|^2 \\ &\le -\frac{\kappa\lambda_2 - 4}{\lambda_{\max}(Q)}\mathbb{E}V_2(\Phi(t)) + \frac{\theta_8}{r} \end{aligned} \tag{43}$$

where

$$\theta_8 := \frac{(1 + \frac{\kappa\lambda_N^2}{\lambda_2 r^{*2}} + \frac{\max_{1\le i\le N} l_i^2}{r^{*2}})\theta_7}{\lambda_{\min}(G)}.$$

By (38), for all $t \ge T(r) := \mu_3 r^{\mu_1} + \mu_2$ defined as that in (12) with $\mu_3 > 1$, one has

$$\begin{aligned} \mathbb{E}\|\Phi(t)\|^2 &\le \frac{2\mathbb{E}V_2(\Phi(t))}{\lambda_{\min}(Q)} \\ &\le \frac{2}{\lambda_{\min}(Q)}e^{-\frac{\kappa\lambda_2-4}{\lambda_{\max}(Q)}(t-r^{\mu_1}-\mu_2)}\mathbb{E}V_2(\Phi(r^{\mu_1}+\mu_2)) \\ &+ \frac{2}{\lambda_{\min}(Q)}\int_{r^{\mu_1}+\mu_2}^{t} e^{-\frac{\kappa\lambda_2-4}{\lambda_{\max}(Q)}(t-s)}\frac{\theta_8}{r}ds \\ &\le \frac{2\theta_6}{\lambda_{\min}(Q)}e^{-\frac{\kappa\lambda_2-4}{\lambda_{\max}(Q)}(\mu_3-1)r^{\mu_1}} + \frac{2\theta_8\lambda_{\max}(Q)}{r(\kappa\lambda_2-4)\lambda_{\min}(Q)} \\ &\le \frac{\Upsilon}{4r} \end{aligned} \tag{44}$$

where

$$\Upsilon := \sup_{r\ge r^*} 4\left(\frac{2\theta_6 r}{\lambda_{\min}(Q)}e^{-\frac{\kappa\lambda_2-4}{\lambda_{\max}(Q)}(\mu_3-1)r^{\mu_1}} + \frac{2\theta_8\lambda_{\max}(Q)}{(\kappa\lambda_2-4)\lambda_{\min}(Q)}\right) \tag{45}$$

is a constant independent of gain parameter r. The (44) yields

$$\mathbb{E}|x_i(t) - \bar{x}(t)|^2 = \mathbb{E}|\alpha_i(t)|^2 \le \mathbb{E}\|\Phi(t)\|^2 \le \frac{\Upsilon}{4r}$$

$$\mathbb{E}|v_i(t) - \bar{v}(t)|^2 = \mathbb{E}|\beta_i(t)|^2 \le \mathbb{E}\|\Phi(t)\|^2 \le \frac{\Upsilon}{4r}$$

for $t \ge T(r)$ and $i = 1,\ldots,N$. This further yields that

$$\mathbb{E}|x_i(t) - x_j(t)|^2 \le \frac{\Upsilon}{r},\ \mathbb{E}|v_i(t) - v_j(t)|^2 \le \frac{\Upsilon}{r}$$

for all $i, j = 1,\ldots,N$. The proof is then completed. ■

Appendix B

Proof of Theorem 3.2: We only prove that the first inequality in (17) holds, and others can be obtained similarly. By Theorem 3.1 and Chebyshev's inequality ([43, p.5]), there exist $r^* \in \Pi_1$ and $\kappa \in \Pi_2$ such that for any $r \ge r^*$ and all $t \ge T(r)$, it holds that

$$P\left\{|x_i(t) - x_j(t)| \ge \frac{m\sqrt{\Upsilon}}{r^{\frac{1}{2}}}\right\} \le \frac{r}{m^2\Upsilon}\mathbb{E}|x_i(t) - x_j(t)|^2 \le \frac{1}{m^2} \tag{46}$$

for $m \in \mathbb{Z}^+$ and $i, j = 1,\ldots,N$. From the Borel–Cantelli's lemma ([43, p.7]), for almost all $\omega \in \Omega$, there exists a random variable $m^*(\omega)$ such that whenever $m \ge m^*(\omega)$, we have

$$|x_i(t) - x_j(t)| \le \frac{m\sqrt{\Upsilon}}{r^{\frac{1}{2}}} \tag{47}$$

for all $t \ge T(r)$ and $i, j = 1,\ldots,N$. Define $\Upsilon_\omega = m^*(\omega)\sqrt{\Upsilon} > 0$ which is an r-independent random variable. The proof is then completed. ■

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