Short Papers

Further Results on Stabilization of Chaotic Systems Based on Fuzzy Memory Sampled-Data Control

Yajuan Liu, Ju H. Park, Bao-Zhu Guo, and Yanjun Shu

***Abstract*—This note investigates sampled-data control for chaotic systems. A memory sampled-data control scheme that involves a constant signal transmission delay is employed for the first time to tackle the stabilization problem for Takagi–Sugeno fuzzy systems. The advantage of the constructed Lyapunov functional lies in the fact that it is neither necessarily positive on sampling intervals nor necessarily continuous at sampling instants. By introducing a modified Lyapunov functional that involves the state of a constant signal transmission delay, a delay-dependent stability criterion is derived so that the closed-loop system is asymptotically stable. The desired sampled-data controller can be achieved by solving a set of linear matrix inequalities. Compared with the existing results, a larger sampling period is obtained by this new approach. A simulation example is presented to illustrate the effectiveness and conservatism reduction of the proposed scheme.**

***Index Terms*—Chaotic system, fuzzy memory sampled-data control, Takagi–Sugeno (T–S) fuzzy model.**

I. Introduction

Recently, an increasing attention has been focused on control of chaotic dynamical systems due to its potential applications in power converters, chemical reactions, biological systems, information processing, secure communication, among many others [1]. The chaos, as a major characteristic of complex dynamical systems, possesses irregular and unpredictable behaviors. In many situations, chaos should be avoided or purposely controlled because it could lead systems to undesirable performance-degraded situations [2]. Nowadays, various control approaches have been proposed to synchronize the chaotic systems [3], [4].

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Y. Liu is with the School of Control and Computer Engineering, North China Electric Power University, with Key Laboratory of Systems and Control, Academy of Mathematics and Systems Science, Academia sinica, Beijing 100190, China, and also with the Department of Electrical Engineering,Yeungnam University, Gyeongsan 38541, South Korea (e-mail: yajuan.liu.12@gmail.com).

J. H. Park is with the Department of Electrical Engineering, Yeungnam University, Gyeongsan 38541, South Korea (e-mail: jessie@ynu.ac.kr).

B.-Z. Guo is with the Key Laboratory of Systems and Control, Academy of Mathematics and Systems Science, Academia sinica, Beijing 100190, China, and also with the School of Computer Science and Applied Mathematics, University of the Witwatersrand, Johannesburg 2000, South Africa (e-mail: bzguo@iss.ac.cn).

Y. Shu is with the School of Mathematics and Statistics, Central South University, Changsha 410083, China (e-mail: yanjun198901@126.com).

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Over the past few decades, the fuzzy-model-based control approach has been a promising approach to deal with complex nonlinear systems or even nonanalytic systems. In various fuzzy-model-based control approach, the Takagi–Sugeno (T–S) fuzzy model is an effective way to describe the dynamical behaviors of different types of nonlinear processes [5]–[8]. It has been shown that many chaotic systems can be exactly represented by T–S fuzzy models including chaotic systems. In this situation, a highly complex chaotic system can be systematically analyzed and synthesized by the well-established linear system theory. As a result, various kinds of T–S fuzzy control approaches have been applied to chaotic systems, including fuzzy state-feedback control [9] and fuzzy impulsive control [10].

Implementing the fuzzy controllers using microcontrollers or digital computers, which are available with simplicity, scalability, and cost-effectiveness, can reduce the implementation cost and time [7]. However, the closed-loop control system becomes a sampled-data control system, which contains both continuous and discrete-time signals. More specifically, the inputs and outputs of plants are usually continuous, but the controllers are generally discrete. It is, therefore, difficult to analyze stability of such systems because the discontinuity produced makes system dynamics more complicated. Until now, three approaches have been used to analyze and synthesize sampled-data systems: The first one is based on discrete models [11], the second one is based on impulsive models [12], and the last is input delay approach [13], which is based on the continuous-time model with time-varying input delay. Compared with the other two approaches, the main advantage of the input delay approach is that it does not require the sampling distances to be constant. Furthermore, the sampling holder can be modeled as a delayed control input. The stability and stabilization conditions can then be established by using the Lyapunov–Krasovskii functional method. Owing to this reason, the input delay approach is popular and has been widely used to sampled-data systems and networked control systems [14]–[17]. Very recently, by the input delay approach, the fuzzy sampled-data control for chaotic systems was investigated in [18] and [19]. Further results have been derived by constructing a new Lyapunov functional, which is continuous at sampling times but not necessarily positive definite inside the sampling intervals [20], [21]. Furthermore, stochastic sampled-data robust stabilization for the T–S fuzzy neutral systems with randomly occurring uncertainties and time-varying delays was investigated in [22]. However, these methods [18]–[21] suffer from some drawbacks.

1) The constructed Lyapunov functional in [18]–[21] needs to satisfy the positive condition inside the sampling intervals.
2) The Jensen's inequality is used to estimate the upper bound of some cross terms.
3) The information on the actual sampling pattern has not been fully utilized in [18]–[21], which may lead to some conservatism.

It is, therefore, significant and necessary to improve further the results reported in [18]–[21], which constitutes one of motivations in this study.

On the other hand, stabilization for linear systems has been investigated by memory sampled-data control. It means that the updating signal successfully transmitted from the sampler to controller and to zero-order hold (ZOH) at instant t_k has experienced a constant signal transmission delay [23], [24]. Hence, the fuzzy sampled-data control with a constant signal transmission delay is an even more significant type of control compared with the sampled-data proportional control. To the best of the authors' knowledge, the problem of fuzzy memory sampled-data control for chaotic systems has not been studied, which is another motivation in this paper. It should be noted that the main difference between the memory controller and the unmemory one is that whether the updating signal successfully transmitted signal from the sampler to controller and to ZOH at the instant t_k has experienced a constant signal transmission delay η or not.

Motivated by above discussion, our attention focuses on studying the fuzzy sampled-data control problem for chaotic systems, which can be represented by a T–S fuzzy model. A memory sampled-data control scheme that is more general than the conventional sampled-data one is used to deal with this problem. The main advantage of the modified Lyapunov functional is that it enables us to make full use of the information on the piecewise constant input and the actual sampling pattern. By choosing a modified Lyapunov functional that involves the state of the constant delay and employing an inequality given in [25], some less conservative stabilization conditions are presented such that the closed-loop systems is asymptotically stable.

The main contributions of this paper are as follows: 1) A memory sampled-data control scheme is first introduced to the fuzzy sampled-data systems; and 2) a less conservative stabilization design via the fuzzy sampled-data control is obtained for chaotic systems.

Notations: Throughout this paper, $*$ denotes the elements below the main diagonal of a symmetric block matrix, I denotes the identity matrix with appropriate dimensions, $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbf{R}^{m\times n}$ is the set of all $m\times n$ real matrices. For symmetric matrices A and B, the notation $A > B$ (respectively, $A \geq B$) means that the matrix $A - B$ is positive definite (respectively, nonnegative). diag$\{\ldots\}$ denotes the block diagonal matrix.

II. Problem Statement

The T–S fuzzy model is a kind of fuzzy system, which is described by a set of fuzzy IF–THEN rules to represent local linear input–output relations for a nonlinear system. The ith rule of this fuzzy model is of the following form

Rule i: IF $\theta_1(t)$ is M_{i1} and ... and IF $\theta_n(t)$ is M_{in}, THEN

$$\dot{x}(t) = A_i x(t) + B_i u(t) \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state-space vector, $\theta_1(t), \ldots, \theta_n(t)$ are the premise variables, M_{ij} are the fuzzy sets characterized by membership function, $i = 1, 2, \ldots, r$, r is the index number of fuzzy rules, and matrices A_i and B_i are the known constant matrices of appropriate dimensions.

Using singleton fuzzifier, product inference, and center-average defuzzifier, the T–S system (1) can be represented by

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))[A_i x(t) + B_i u(t)] \tag{2}$$

where $h_i(\theta(t))$ denotes the normalized membership function satisfying

$$h_i(\theta(t)) = \frac{w_i(\theta(t))}{\sum_{i=1}^{r} w_i(\theta(t))},\ w_i(\theta(t)) = \prod_{j=1}^{n} M_{ij}(\theta_j(t)) \tag{3}$$

in which $M_{ij}(\theta_j(t))$ is the grade of membership of $\theta_j(t)$ in M_{ij}. It is assumed that

$$w_i(\theta(t)) \geq 0,\ \sum_{i=1}^{r} w_i(\theta(t)) > 0\ \forall t \geq 0. \tag{4}$$

The following conditions are always assumed in fuzzy system modeling:

$$h_i(\theta(t)) \geq 0,\ \sum_{i=1}^{r} h_i(\theta(t)) = 1. \tag{5}$$

In this paper, it is assumed that we only have the measurement $x(t_k)$ at the sampling instant t_k, that is, only discrete measurements of $x(t)$ are available for control purposes, and the control signal is assumed to be generated by using a ZOH function with a sequence of hold times

$$0 = t_0 < t_1 < \ldots < \lim_{k\to+\infty} t_k = +\infty. \tag{6}$$

The stabilization problem is investigated under the parallel distributed compensation (PDC) scheme. In the PDC scheme, fuzzy sampling controllers share the same premise parts with (2), that is, the sampling controller for rule j is given by

Controller Rule j:

IF $\theta_1(t)$ is M_{j1} and ... and IF $\theta_n(t)$ is M_{jn}, THEN

$$\begin{aligned} u(t) =&\ K_{1j}x(t_k) + K_{2j}x(t_k - \eta), t_k \leq t < t_{k+1}, \\ j =&\ 1, 2, \ldots, r \end{aligned} \tag{7}$$

where η is a constant delay.

So, the overall state feedback controller is inferred by

$$\begin{aligned} u(t) =& \sum_{j=1}^{r} h_j(\theta(t_k))[K_{1j}x(t_k) + K_{2j}x(t_k - \eta)], \\ & t_k \leq t < t_{k+1}. \end{aligned} \tag{8}$$

Remark 1: The simplest and most popular way of reconstructing a continuous-time signal from a discrete-time signal in control systems is simply to keep the signal constant until a new sample becomes available. This transforms the discrete-time sequence into a piecewise constant continuous-time signal, and the device performing this transformation is called a ZOH.

Remark 2: Different from the control scheme proposed in [18]–[21], a constant signal transmission delay η that is introduced in [23] and [24] is also considered for the first time for chaotic systems represented as T–S fuzzy model in this paper. When $K_{2j} = 0$, the control scheme can be reduced into the sampled-data proportional control one adopted in [18]–[21]. In this sense, the control scheme employed in [18]–[21] is a special case of this paper.

In addition, the sampling is not required to be periodic, and the only assumption is that the distance between any two consecutive sampling instants belongs to an interval. Especially, it is assumed that

$$t_{k+1} - t_k = h_k \leq h \tag{9}$$

for all $k \geq 0$, where $h > 0$.

Substituting (7) into (1), the closed-loop system via sampled-data and state quantized control is formulated as follows:

$$\begin{aligned} \dot{x}(t) =& \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t_k))[A_i x(t) \\ & + B_i K_{1j}x(t_k) + B_i K_{2j}x(t_k - \eta)]. \end{aligned} \tag{10}$$

Now, the following lemma is introduced for the proof of the main results.

Lemma 1 (see [25]): For a given matrix $R > 0$, the following inequality holds for all continuously differentiable function $x(t)$ in $[a, b] \in \mathrm{R}^n$:

$$-(b-a)\int_a^b \dot{x}^T(s) R\dot{x}(s)ds \leq -[x(b)-x(a)]^T R[x(b)-x(a)] - 3\Omega^T R\Omega$$

where $\Omega = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)ds$.

III. Main Results

In this section, a sufficient condition on the stability for chaotic systems with the memory sampled-data controllers in the form of (7) will be derived.

For the sake of simplicity of matrix and vector representation, $I_i \in \mathrm{R}^{6n\times n}(i = 1, 2, \ldots, 6)$ are defined as block entry matrices (for example $I_4 = [0\ 0\ 0\ I\ 0\ 0]^T$). The other notations are defined as

$$\begin{aligned}
\xi^T(t) =& \Big[x^T(t)\ x^T(t_k)\ \frac{1}{t-t_k}\left(\int_{t_k}^t x^T(s)ds\right)\ \dot{x}^T(t)\\
& x^T(t_k-\eta)\ x^T(t-\eta)\Big]\\
\Sigma_1 =& I_1PI_4^T + I_4PI_1^T - [I_1 - I_2]M_2I_2^T\\
& - I_2M_2^T[I_1 - I_2]^T - Y_{1ij}^T W_1 - W_1^T Y_{1ij}\\
& - 3Y_{2ij}^T W_2 - 3W_2^T Y_{2ij} - h_k I_3SI_3^T\\
& - [I_1 - I_2]Q_1[I_1 - I_2]^T\\
& - [I_1 - I_2]Q_2I_5^T - I_5Q_2[I_1 - I_2]^T + \frac{h^2}{2}I_4XI_4^T\\
& + \eta^2 I_4RI_4^T - [I_1 - I_6]R[I_1 - I_6]^T\\
& + h^2 I_4WI_4^T - \frac{\pi^2}{4}[I_6 - I_5]W[I_6 - I_5]^T\\
& + (I_1G_1 + I_4G_2 + I_5G_3)\Phi\\
& + \Phi^T(I_1G_1 + I_4G_2 + I_5G_3)^T\\
\Sigma_2 =& -I_2M_3I_2^T - [I_1\ I_2\ I_5]N[I_1\ I_2\ I_5]^T\\
\Sigma_3 =& [I_4\ I_2]M[I_4\ I_2]^T + [I_1\ I_2\ I_5]N[I_1\ I_2\ I_5]^T\\
& + I_3SI_1^T + I_1SI_3^T + [I_1 - I_2]Q_1I_4^T\\
& + I_4Q_1[I_1 - I_2]^T + I_5Q_2I_4^T + I_4Q_2I_5^T\\
\alpha^T(t) =& [x^T(t)\ x^T(t_k)\ x^T(t_k-\eta)]\\
\tilde{N} =& [N_1\ N_2\ 0\ 0\ N_3\ 0]^T\\
W_1 =& [I\ -I\ 0\ 0\ 0\ 0]\\
W_2 =& [I\ I\ -2I\ 0\ 0\ 0]\\
\Phi =& [A_i\ B_iK_{1j}\ 0\ -I\ B_iK_{2j}\ 0].
\end{aligned}$$

Theorem 1: For a given scalar $h > 0$, the closed sampled-data system (10) of the fuzzy system (1) under controller (7) with gain matrices K_{1j} and K_{2j} is asymptotically stable if there exist symmetric positive matrices $P, R, W, X, M = \begin{bmatrix} M_1 & M_2\\ * & M_3\end{bmatrix}$, symmetric matrices

$$S, Q_1, Q_2, N = \begin{bmatrix} N_1 & N_2 & N_3\\ * & N_4 & N_5\\ * & * & N_6\end{bmatrix}$$

any matrices $G_1, G_2, G_3, Y_{1ij}, Y_{2ij}$ with appropriate dimensions and positive scalars α and β such that the following LMIs hold for all $h_k \in [0, h], i, j = 1, 2, \ldots, r$:

$$\begin{bmatrix} \Sigma_1 + h_k\Sigma_3 & \frac{h^2}{2}\tilde{N}\\ * & -\frac{h^2}{2}X\end{bmatrix} < 0 \quad (11)$$

$$\begin{bmatrix} \Sigma_1 + h_k\Sigma_2 & h_kY_{1ij} & 3h_kY_{2ij} & \frac{h^2}{2}\tilde{N}\\ * & -h_kM_1 & 0 & 0\\ * & * & -3h_kM_1 & 0\\ * & * & * & -\frac{h^2}{2}X\end{bmatrix} < 0. \quad (12)$$

Proof: Consider the following Lyapunov functional:

$$V(t) = \sum_{i=1}^7 V_i(t), t \in [t_k, t_{k+1}) \quad (13)$$

where

$$\begin{aligned}
V_1(t) =& x^T(t)Px(t)\\
V_2(t) =& (h_k - (t-t_k))\int_{t_k}^t \begin{bmatrix}\dot{x}(s)\\ x(t_k)\end{bmatrix}^T M\begin{bmatrix}\dot{x}(s)\\ x(t_k)\end{bmatrix}ds\\
V_3(t) =& (h_k - (t-t_k))(t-t_k)v^T(t)Sv(t)\\
V_4(t) =& (h_k - (t-t_k))[x(t) - x(t_k)]^T[Q_1(x(t) - x(t_k))\\
& + 2Q_2x(t_k-\eta)]\\
V_5(t) =& (h_k - (t-t_k))(t-t_k)\alpha^T(t)N\alpha(t)\\
V_6(t) =& \eta\int_{-\eta}^0\int_{t+\alpha}^t \dot{x}^T(s)R\dot{x}(s)dsd\alpha\\
V_7(t) =& h^2\int_{t_k-\eta}^t \dot{x}^T(s)W\dot{x}(s)ds\\
& - \frac{\pi^2}{4}\int_{t_k-\eta}^{t-\eta}(x(s) - x(x_k-\eta))^TW(x(s) - x(t_k-\eta))ds
\end{aligned}$$

where $v(t) = \frac{1}{t-t_k}\int_{t_k}^t x(s)ds$.

Since $V(t_k) \leq \lim_{t\to t_k^-} V(t)$, it is clear that the Lyapunov functional $V(t)$ is neither necessarily positive on sampling intervals nor necessarily continuous at sampling instants.

The time derivatives of $V_1(t)$ and $V_2(t)$ can be calculated as

$$\dot{V}_1(t) = 2x^T(t)P\dot{x}(t), \quad (14)$$

$$\begin{aligned}
\dot{V}_2(t) =& -\int_{t_k}^t \begin{bmatrix}\dot{x}(s)\\ x(t_k)\end{bmatrix}^T M\begin{bmatrix}\dot{x}(s)\\ x(t_k)\end{bmatrix}ds\\
& + (h_k - (t-t_k))\begin{bmatrix}\dot{x}(t)\\ x(t_k)\end{bmatrix}^T M\begin{bmatrix}\dot{x}(t)\\ x(t_k)\end{bmatrix}\\
=& -\int_{t_k}^t \dot{x}^T(s)M_1\dot{x}(s)ds - 2x^T(t_k)M_2^T[x(t) - x(t_k)]\\
& - (t-t_k)x^T(t_k)M_3x(t_k)\\
& + (h_k - (t-t_k))\begin{bmatrix}\dot{x}(t)\\ x(t_k)\end{bmatrix}^T M\begin{bmatrix}\dot{x}(t)\\ x(t_k)\end{bmatrix}. \quad (15)
\end{aligned}$$

By Lemma 1,

$$-\int_{t_k}^t \dot{x}^T(s)M_1\dot{x}(s)ds \leq -\frac{1}{t-t_k}\xi^T(t)[W_1^TM_1W_1 + 3W_2^TM_1W_2]\xi(t). \quad (16)$$

With matrices Y_{1ij} and Y_{2ij} at hand, it is easy to obtain

$$\frac{1}{t-t_k}(M_1W_1-(t-t_k)Y_{1ij})^TM_1^{-1}(M_1W_1-(t-t_k)Y_{1ij})\geq 0$$
$$\frac{1}{t-t_k}(M_1W_2-(t-t_k)Y_{2ij})^TM_1^{-1}(M_1W_2-(t-t_k)Y_{2ij})\geq 0.$$

Hence

$$-\frac{1}{t-t_k}W_1^TM_1W_1 \leq -Y_{1ij}^TW_1-W_1^TY_{1ij}+(t-t_k)Y_{1ij}^TM_1^{-1}Y_{1ij}, \quad (17)$$
$$-\frac{1}{t-t_k}W_2^TM_1W_2 \leq -Y_{2ij}^TW_2-W_2^TY_{2ij}+(t-t_k)Y_{2ij}^TM_1^{-1}Y_{2ij}. \quad (18)$$

The derivatives of $V_3(t)$, $V_4(t)$, and $V_5(t)$ can be estimated by

$$\dot{V}_3(t)=-h_kv^T(t)Sv(t)+2(h_k-(t-t_k))v^T(t)Sx(t) \quad (19)$$
$$\begin{aligned}\dot{V}_4(t)&=-[x(t)-x(t_k)]^T[Q_1(x(t)-x(t_k))+2Q_2x(t_k-\eta)]\\&\quad+2(h_k-(t-t_k))[(x(t)-x(t_k))^TQ_1\\&\quad+x^T(t_k-\eta)Q_2]\dot{x}(t)\end{aligned} \quad (20)$$
$$\begin{aligned}\dot{V}_5(t)&=-(t-t_k)\alpha^T(t)N\alpha(t)+(h_k-(t-t_k))\alpha^T(t)N\alpha(t)\\&\quad+2(h_k-(t-t_k))(t-t_k)\xi^T(t)\tilde{N}\dot{x}(t).\end{aligned} \quad (21)$$

For any positive matrix X, it is easy to obtain

$$2(h_k-(t-t_k))(t-t_k)\xi^T(t)\tilde{N}\dot{x}(t)\leq \frac{h^2}{2}\left(\xi^T(t)\tilde{N}X^{-1}\tilde{N}^T\xi(t)+\dot{x}^T(t)X\dot{x}(t)\right). \quad (22)$$

An upper bound of $\dot{V}_6(t)$ is given by

$$\dot{V}_6(t)\leq \eta^2\dot{x}^T(t)R\dot{x}(t)-[x(t)-x(t-\eta)]^TR[x(t)-x(t-\eta)]. \quad (23)$$

Calculating the time derivative of $V_7(t)$ gives

$$\dot{V}_7(t)=h^2\dot{x}^T(t)W\dot{x}(t)-\frac{\pi^2}{4}(x(t-\eta)-x(t_k-\eta))^TW\times(x(t-\eta)-x(t_k-\eta)). \quad (24)$$

Based on system (10), it is easy to have that

$$\begin{aligned}&2\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(t))h_j(\theta(t_k))[e^T(t)G_1+\dot{e}^T(t)G_2\\&+e^T(t_k-\eta)G_3]\times[-\dot{x}(t)+A_ix(t)+B_iK_{1j}x(t_k)\\&+B_iK_{2j}x(t_k-\eta)]=0\end{aligned} \quad (25)$$

and hence

$$\dot{V}(t)\leq\xi^T(t)\Sigma\xi(t) \quad (26)$$

where $\Sigma=\Sigma_1+\frac{h^2}{2}\tilde{N}X^{-1}\tilde{N}^T+(t-t_k)(\Sigma_2+Y_{1ij}^TM_1^{-1}Y_{1ij}+3Y_{2ij}^TM_1^{-1}Y_{2ij})+(h_k-(t-t_k))\Sigma_3$. Since Σ is a convex combination of $t-t_k$ and $h_k-(t-t_k)$, by using the Schur complement, $\Sigma<0$ if and only if (11) and (12) hold. This completes the proof of the theorem. ■

By Theorem 1, the memory sampled-data controller design method for system (10) is provided by Theorem 2. For the sake of simplicity of matrix, the following notations are used:

$$\begin{aligned}\bar{\Phi}&=[A_iG\ B_iT_{1j}\ 0\ -I\ B_iT_{2j}\ 0]\\\hat{N}&=[G^TN_1G\ G^TN_2G\ 0\ 0\ G^TN_3G\ 0]^T\\\bar{\Sigma}_1&=I_1\bar{P}I_4^T+I_4\bar{P}I_1^T-[I_1-I_2]\bar{M}_2I_2^T-I_2\bar{M}_2^T[I_1-I_2]^T\\&\quad-\bar{Y}_{1ij}^TW_1-W_1^T\bar{Y}_{1ij}-3\bar{Y}_{2ij}^TW_2-3W_2^T\bar{Y}_{2ij}\\&\quad-h_kI_3\bar{S}I_3^T-[I_1-I_2]\bar{Q}_1[I_1-I_2]^T\\&\quad-[I_1-I_2]\bar{Q}_2I_5^T-I_5\bar{Q}_2[I_1-I_2]^T+\frac{h^2}{2}I_4\bar{X}I_4^T\\&\quad+\eta^2I_4\bar{R}I_4^T-[I_1-I_6]\bar{R}[I_1-I_6]^T\\&\quad+h^2I_4\bar{W}I_4^T-\frac{\pi^2}{4}[I_6-I_5]\bar{W}[I_6-I_5]^T\\&\quad+(I_1+\alpha I_4+\beta I_5)\bar{\Phi}+\bar{\Phi}^T(I_1+\alpha I_4+\beta I_5)^T\\\bar{\Sigma}_2&=-I_2\bar{M}_3I_2^T-[I_1\ I_2\ I_5]\bar{N}[I_1\ I_2\ I_5]^T\\\bar{\Sigma}_3&=[I_4\ I_2]\bar{M}[I_4\ I_2]^T+[I_1\ I_2\ I_5]\bar{N}[I_1\ I_2\ I_5]^T\\&\quad+I_3\bar{S}I_1^T+I_1\bar{S}I_3^T+[I_1-I_2]\bar{Q}_1I_4^T+I_4\bar{Q}_1[I_1-I_2]^T\\&\quad+I_5\bar{Q}_2I_4^T+I_4\bar{Q}_2I_5^T.\end{aligned}$$

Theorem 2: For given scalars $h>0,\alpha>0$, and $\beta>0$, system (10) is asymptotically stable if there exist symmetric positive matrices $\bar{P},\bar{R},\bar{W},\bar{X},\bar{M}=\begin{bmatrix}\bar{M}_1 & \bar{M}_2\\ * & \bar{M}_3\end{bmatrix}$, symmetric matrices

$$\bar{S},\bar{Q}_1,\bar{Q}_2,\bar{N}=\begin{bmatrix}\bar{N}_1 & \bar{N}_2 & \bar{N}_3\\ * & \bar{N}_4 & \bar{N}_5\\ * & * & \bar{N}_6\end{bmatrix}$$

and any matrices $G,\bar{Y}_{1ij},\bar{Y}_{2ij},T_{1j},T_{2j}$ with appropriate dimensions such that the following LMIs hold for all $h_k\in[0,h],i,j=1,2,\ldots,r$:

$$\begin{bmatrix}\bar{\Sigma}_1+h_k\bar{\Sigma}_3 & \frac{h^2}{2}\hat{N}\\ * & -\frac{h^2}{2}\bar{X}\end{bmatrix}<0 \quad (27)$$

$$\begin{bmatrix}\bar{\Sigma}_1+h_k\bar{\Sigma}_2 & h_k\bar{Y}_{1ij} & 3h_k\bar{Y}_{2ij} & \frac{h^2}{2}\hat{N}\\ * & -h_k\bar{M}_1 & 0 & 0\\ * & * & -3h_k\bar{M}_1 & 0\\ * & * & * & -\frac{h^2}{2}\bar{X}\end{bmatrix}<0 \quad (28)$$

where the controller gain matrices K_{1j} and K_{2j} in (7) are given by $K_{1j}=T_{1j}G^{-1}$ and $K_{2j}=T_{2j}G^{-1}$.

Proof: Define

$$\begin{aligned}&G_1=G^{-1},G_2=\alpha G^{-1},G_3=\beta G^{-1},\\&T_{1j}=K_{1j}G,T_{2j}=K_{2j}G,\bar{P}=G^TPG,\\&\bar{R}=G^TRG,\bar{W}=G^TWG,\bar{S}=G^TSG,\\&\bar{X}=G^TXG,\bar{Q}_1=G^TQ_1G,\bar{Q}_2=G^TQ_2G,\\&\bar{M}=\mathrm{diag}\{G^T,G^T\}M\mathrm{diag}\{G,G\},\\&\bar{N}=\mathrm{diag}\{G^T,G^T,G^T\}N\mathrm{diag}\{G,G,G\},\\&\bar{Y}_{1ij}=\mathrm{diag}\{G^T,G^T,G^T,G^T,G^T,G^T\}Y_{1ij}G,\\&\bar{Y}_{2ij}=\mathrm{diag}\{G^T,G^T,G^T,G^T,G^T,G^T\}Y_{2ij}G.\end{aligned}$$

Pre- and post-multiplying (11) by $\mathrm{diag}\{G,G,G,G,G,G,G\}^T$ and $\mathrm{diag}\{G,G,G,G,G,G,G\}$ gives (27). Pre- and post-multiplying (12) by $\mathrm{diag}\{G,G,G,G,G,G,G,G,G\}^T$ and $\mathrm{diag}\{G,G,G,G,G,G,G,G,G\}$ yields (28). This completes the proof of the theorem. ■

Remark 3: It should be pointed out that considering the sampled-data control design problem under a bigger sampling period is important since a longer sampling period will lead to lower communication channel occupation, fewer actuation of the controller, and less signal transmission [18]–[21].

Remark 4: Different from the Lyapunov–Krasovskii function constructed in [18]–[21], $V_4(t)$–$V_7(t)$ in (13), which fully capture the characteristic of the sampling-data systems and make full use of the sampling pattern, may provide a larger sampling period. In other words, the memory sampled-data control may provide less conservative result, which will be demonstrated in the simulation example.

When $K_{2j} = 0$, the memory sampled-data controller is reduced to the sampled-data controller considered in [21]. We can easily obtain the following corollary from Theorem 2 by letting $Q_2 = R = W = 0$ and removing the term of $x(t_k - \eta)$ in the Lyapunov functional (13).

For the sake of simplicity of matrix and vector representation, $\hat{I}_i \in \mathrm{R}^{4n\times n} (i = 1, 2, \ldots, 4)$ are defined as block entry matrices (for instance, $\hat{I}_4 = [0\ 0\ 0\ I]^T$). The other notations are defined as follows:

$$\begin{aligned}
\hat{\xi}^T(t) &= \left[x^T(t)\ x^T(t_k)\ \frac{1}{t-t_k}\int_{t_k}^{t} x^T(s)ds\ \dot{x}^T(t)\right]\\
\hat{W}_1 &= [\hat{I}\ \ -\hat{I}\ 0\ 0]\\
\hat{W}_2 &= [\hat{I}\ \hat{I}\ \ -2\hat{I}\ 0]\\
\check{N} &= [G^T N_1 G\ G^T N_2 G\ 0\ 0]^T\\
\hat{\Phi} &= [A_i G\ B_i T_{1j}\ 0\ -\hat{I}]\\
\hat{\Sigma}_1 &= \hat{I}_1\bar{P}\hat{I}_4^T + \hat{I}_4\bar{P}\hat{I}_1^T - [\hat{I}_1 - \hat{I}_2]\bar{M}_2\hat{I}_2^T - \hat{I}_2\bar{M}_2^T[\hat{I}_1 - \hat{I}_2]^T\\
&\quad - \bar{Y}_{1ij}^T\hat{W}_1 - \hat{W}_1^T\bar{Y}_{1ij} - 3\bar{Y}_{2ij}^T\hat{W}_2 - 3\hat{W}_2^T\bar{Y}_{2ij}\\
&\quad - h_k\hat{I}_3 S\hat{I}_3^T - [\hat{I}_1 - \hat{I}_2]\bar{Q}[\hat{I}_1 - \hat{I}_2]^T + \frac{h^2}{2}\hat{I}_4\bar{X}\hat{I}_4^T\\
&\quad + (\hat{I}_1 + \alpha\hat{I}_4)\hat{\Phi} + \hat{\Phi}^T(\hat{I}_1 + \alpha\hat{I}_4)^T\\
\hat{\Sigma}_2 &= -\hat{I}_2\bar{M}_3\hat{I}_2^T - [\hat{I}_1\ \hat{I}_2]\bar{N}[\hat{I}_1\ \hat{I}_2]^T\\
\hat{\Sigma}_3 &= [\hat{I}_4\ \hat{I}_2]\bar{M}[\hat{I}_4\ \hat{I}_2]^T + [\hat{I}_1\ \hat{I}_2]\bar{N}[\hat{I}_1\ \hat{I}_2]^T\\
&\quad + \hat{I}_3\bar{S}\hat{I}_1^T + \hat{I}_1\bar{S}\hat{I}_3^T + [\hat{I}_1 - \hat{I}_2]\bar{Q}\hat{I}_4^T + \hat{I}_4\bar{Q}[\hat{I}_1 - \hat{I}_2]^T.
\end{aligned}$$

Corollary 1: For given scalars $h > 0$ and $\alpha > 0$, system (10) is asymptotically stable if there exist symmetric positive matrices $\bar{P}, \bar{X}, \bar{M} = \begin{bmatrix} \bar{M}_1 & \bar{M}_2 \\ * & \bar{M}_3 \end{bmatrix}$, symmetric matrices $\bar{S}, \bar{Q}, \bar{N} = \begin{bmatrix} \bar{N}_1 & \bar{N}_2 \\ * & \bar{N}_4 \end{bmatrix}$, and any matrices $G, \bar{Y}_{1ij}, \bar{Y}_{2ij}, T_{1j}$ with appropriate dimensions such that the following LMIs hold for all $h_k \in [0, h], i, j = 1, 2, \ldots, r$:

$$\begin{bmatrix} \hat{\Sigma}_1 + h_k\hat{\Sigma}_3 & \frac{h^2}{2}\check{N} \\ * & -\frac{h^2}{2}\check{X} \end{bmatrix} < 0 \qquad (29)$$

$$\begin{bmatrix} \hat{\Sigma}_1 + h_k\hat{\Sigma}_2 & h_k\bar{Y}_{1ij} & 3h_k\bar{Y}_{2ij} & \frac{h^2}{2}\check{N} \\ * & -h_k\bar{M}_1 & 0 & 0 \\ * & * & -3h_k\bar{M}_1 & 0 \\ * & * & * & -\frac{h^2}{2}\bar{X} \end{bmatrix} < 0. \qquad (30)$$

And the controller gain matrix K_{1j} is given by $K_{1j} = T_{1j}G^{-1}$.

Remark 5: For the fuzzy sampled-data control scheme of chaotic systems used in [18]–[21], the less conservative criteria provided in this paper rely on the constructed Lyapunov functional and its derivative estimation. First, a new Lyapunov functional that is neither necessarily positive on sampling intervals nor necessarily continuous at sampling instants is constructed. Second, Lemma 1 is used to estimate the upper bound of the cross term instead of using Jensen's inequality.

TABLE I
MAXIMUM ALLOWABLE BOUND h

	[19]	[20]	[21]	Corollary 1	Theorem 2($\eta = 0.01$)
h	0.0534	0.0692	0.0736	0.0818	0.0959

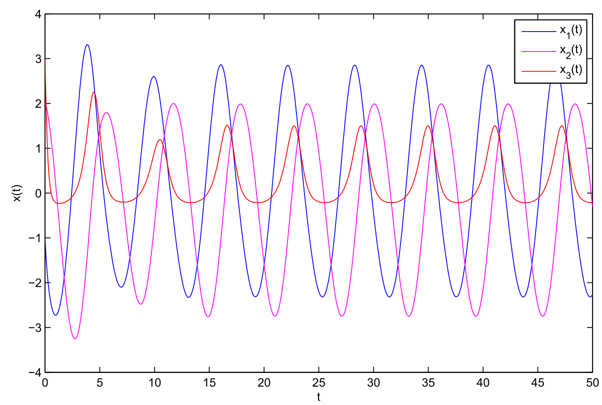

Fig. 1. State response of the uncontrolled system (1).

IV. Numerical Example

The Rossler's system with an input term is given as follows:

$$\begin{cases} \dot{x}_1(t) = -x_2(t) - x_3(t) \\ \dot{x}_2(t) = x_1(t) + ax_2(t) \\ \dot{x}_3(t) = bx_1(t) - (c - x_1(t))x_3(t) + u(t). \end{cases} \qquad (31)$$

It is noted that the Lorenz system (31) with $x_1(t) \in [c - d, c + d]$ can be represented in the T–S fuzzy system (10) with

$$A_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & -d \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & d \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

and the membership functions $h_1(x_1(t)) = \frac{c+d-x_1(t)}{2d}$ and $h_2(x_1(t)) = 1 - h_1(x_1(t))$. In this example, we choose $a = 0.3, b = 0.5, c = 5$, and $d = 10$.

By using the fuzzy sampled-data control, the results on the largest sampling interval h for different methods are listed in Table I. From Table I, we see that the maximal value of the sampling period is $h = 0.0818$ by applying Corollary 1. While by applying the design methods proposed in [19]–[21], we can find the largest sampling interval h ensuring the stabilization of chaotic systems (1) are $0.0534, 0.0692$, and 0.0736, respectively. By using only the sampled-data proportional control, our proposed approach is able to achieve less conservative results compared to those presented in [19]–[21].

According to the structure of the memory sampled-data controller (7) and choosing $\alpha = 0.08$ and $\beta = 0.08$, we can find that the largest sampling interval is 0.0959, which preserves chaotic systems (10) to be stable, and the corresponding connection weights are given as follows:

$$\begin{aligned}
K_{11} &= \left[6.5828\ 1.5310\ -11.9482\right]\\
K_{12} &= \left[-0.0269\ -0.0044\ -0.1698\right]\\
K_{21} &= \left[6.5828\ 1.5310\ -11.9482\right]\\
K_{22} &= \left[-0.0269\ -0.0044\ -0.1698\right].
\end{aligned}$$

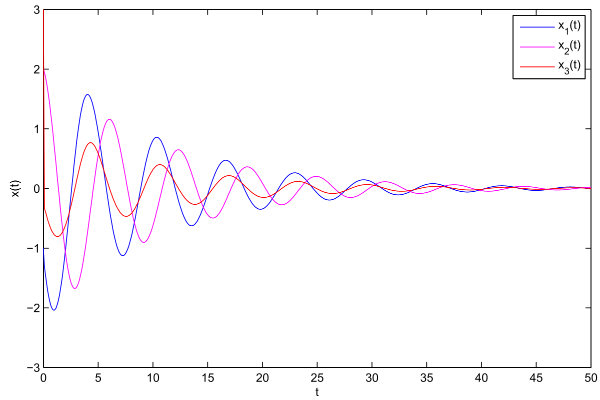

Fig. 2. State response of the controlled system (1).

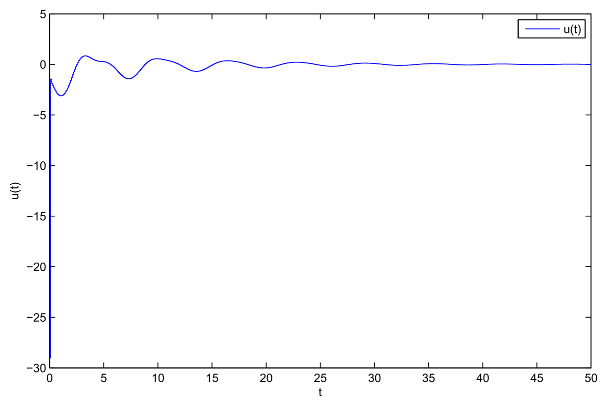

Fig. 3. Control input (7).

With the above-obtained connection weights and the initial conditions $x(0) = [-1\ 2\ 3]^T$, the state responses of the open-loop system are given in Fig. 1. The state responses of the closed-loop system are shown in Fig. 2, which shows that the controlled system is stable. The control input $u(t)$ is shown in Fig. 3.

V. Conclusion

In this note, a sampled-data control for chaotic systems is investigated by developing a memory sampled-data control scheme. Combining the Lyapunov functional method and the integral inequality, we present a new stabilization criterion. The results proposed in this note fully make use of the available information about the actual sampling pattern and thus have less conservatism than the existing ones. A numerical example is carried out to show that the proposed memory sampled-data controller can provide a longer sampling period than that in the existing literature. The performance of the sampled-data control for T–S fuzzy systems will be taken into account in our future work. Moreover, the new sampled-data approach is expected to be extended to multiagent systems [26], [27].

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