# Event-Based Reliable Dissipative Filtering for T–S Fuzzy Systems With Asynchronous Constraints

Yajuan Liu, Bao-Zhu Guo, Ju H. Park, and Sangmoon Lee

***Abstract*—In this paper, event-triggered reliable dissipative filtering is investigated for a class of Takagi–Sugeno (T–S) fuzzy systems. First, a reliable event-triggered communication scheme is introduced to release sampled measurement outputs only if the variation of the sampled vector exceeds a prescribed threshold condition. Second, an asynchronous premise reconstruct method for T–S fuzzy systems is presented, which relaxes the assumption of the prior work that the premises of the plant and the filter are synchronous. Third, the resulting filtering error system is modeled under consideration of event-triggered communication, sensor failure, and asynchronous premise in a unified framework. By adopting the Lyapunov functional method and integral inequality approach, a delay-dependent criterion is developed to guarantee asymptotic stability for the filtering error systems and achieve strict $(Q, S, R) - \alpha$ dissipativity. Consequently, suitable filters and the event parameters can be derived by solving a set of linear matrix inequalities. Finally, an example is given to show the effectiveness of the proposed method.**

***Index Terms*—Asynchronous constraints, dissipative filtering, event triggering, Takagi–Sugeno (T–S) fuzzy systems.**

## I. Introduction

THE Takagi–Sugeno (T–S) fuzzy systems have been extensively investigated during the past decades because of its wide applications in various fields, such as the suspension vehicle systems [1], the electromagnetic suspension systems [2], and the internal combustion engine systems [3], a few examples among many others. It has been shown that T–S fuzzy models, which are locally linear time-invariant systems connected by IF-THEN rules, can well approximate any smooth nonlinear functions in any compact set within any standard range of accuracy. Therefore, substantial efforts have been made to stability and synthesis for T–S fuzzy systems [4]–[9].

During the past few years, significant research effort has been devoted to the filtering problems due to their practical insights and applications in different areas such as military, astronautics, and signal processing. The aim of filtering is to design a stable filter by using measurement outputs to estimate the system state or a combination of them. Up to now, various methodologies have been developed to deal with the filtering issues for complex systems. Among different filtering techniques, one popular approach is the $\mathcal{H}_\infty$ filter, since it takes the advantage to such a degree that statistical assumptions on the external noise only need to be bounded in energy and the system model is allowed to have uncertainty. As a result, a number of significant results on $\mathcal{H}_\infty$ filtering for T–S fuzzy systems have been reported [10]–[15]. For example, a delay-dependent robust $\mathcal{H}_\infty$ filtering for discrete-time T–S fuzzy systems with time-varying delay was investigated in [10]. The paper [12] discussed fuzzy resilient energy-to-peak filtering for continuous-time nonlinear systems.

In aforementioned works, however, the communication scheme is only concerned with time-triggered scheme where a fixed sampling interval is used, which often leads to oversampling and hence wastes the limited resource and communication bands. In this situation, an event-triggered scheme plays a significant role to mitigate the waste of limited network resource [16], [17]. One significant characteristic of event-triggered schemes is that a task is executed only if a predefined event-triggered condition is violated. Due to its advantage of low-cost communication resources, event-triggered filtering has been focused by many researchers. More recently, event-triggered filtering for discrete T–S fuzzy systems has been discussed in [23]. It should be mentioned that, in practice, the premises of T–S fuzzy systems and the parallel distributed compensation (PDC) fuzzy filter rules are asynchronous under the network environment. For example, the problem of fuzzy tracking controller with the event-triggered communication scheme and the asynchronous operation has been established in [24]. The paper [25] discussed event-triggered $\mathcal{H}_\infty$ control for T–S fuzzy nonlinear systems. The adaptive event-triggered $\mathcal{H}_\infty$ control for T–S fuzzy systems with synchronous fuzzy premises was investigated in [9]. Furthermore, with the existence of those asynchronous premises, the event-triggered communication scheme in [23] cannot theoretically guarantee $\mathcal{H}_\infty$ filtering of a T–S fuzzy system under the network environment. Hence, the problem of event-triggered $\mathcal{H}_\infty$ filtering for networked T–S fuzzy systems with the asyn-

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chronous constraints of the membership functions was investigated in [26]. Furthermore, it has been well recognized that measured outputs of a dynamic system including incomplete observations in practice as contingent failures are possible for all sensors in a system. And this phenomenon makes filter performances degrade and possible hazards happen [27]–[29].

Along with another research frontier, since the concept of dissipative systems was first proposed in [30] and [31], the dissipative property has received much attention by many researchers. A fuzzy dynamic output-feedback $(Q, S, R) - \alpha$ dissipative controller for T–S fuzzy systems with time-varying input delay and output constraints has been investigated in [32]. A two-dimensional dissipative control and filtering for Roesser mode have been investigated in [33]. The dissipative state estimation of the Markov jump neural networks was proposed in [34], to name just a few. Until now, to the best of our knowledge, event-based reliable dissipative filtering problem for T–S fuzzy systems with asynchronous constraints has not been addressed yet, which motivates us for the current study.

Based on the above, event-triggered reliable dissipative for T–S fuzzy systems with asynchronous constraints is first handled in this paper. In order to save the communication resource in a network, an event-triggered scheme that determines whether the sampled data should be transmitted or not is employed. The phenomenon of asynchronous premise and sensor failures are also included. Under these considerations, a new model of filtering error system is proposed. Applying the Lyapunov–Kravoskii functional, integral inequality combining with reciprocal convex technique, a delay-dependent criterion is developed to guarantee the asymptotic stability of the filtering error systems and achieve strict $(Q, S, R) - \alpha$ dissipativity. The filter design conditions are derived in terms of solving a set of linear matrix inequalities (LMIs). A numerical example is given to illustrate the effectiveness of the proposed results.

*Notations*: The notations in this paper are quite standard. $I$ denotes the identity matrix with appropriate dimensions, $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices. $\|\cdot\|$ stands for the Euclidean norm of a given vector. $*$ denotes the elements below the main diagonal of a symmetric block matrix. For symmetric matrices $A$ and $B$, the notation $A > B$ (respectively, $A \geq B$) means that the matrix $A - B$ is positive definite (respectively, nonnegative), and diag$\{\ldots\}$ denotes the block diagonal matrix.

## II. Problem Statement and Preliminaries

Consider a nonlinear system that is described by the fuzzy IF-THEN rules. The $i$th rule of the system is represented in the following form.

Rule 1): IF $\theta_1(t)$ is $M_{i1}, \ldots,$ and $\theta_p(t)$ is $M_{ip}$, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i\omega(t) \\ y(t) = C_i x(t) \\ z(t) = L_i x(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^m$ is the measured output, $\omega(t) \in \mathbb{R}^l$ denotes the external disturbance belonging to $\mathcal{L}_2[0, \infty)$, and $z(t) \in \mathbb{R}^q$ is the signal to be estimated. $A_i$, $B_i$, $C_i$, and $L_i$ are known constant matrices with appropriate dimensions. $M_{ij}$ are the fuzzy sets that are characterized by membership function, $r$ is the number of IF-THEN rules, and $\theta(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_p(t)]$ is the premise variables vector. $\theta_j(t)$ contains unknown variables or uncertain parameters. The fuzzy basis functions are given by

$$h_i(\theta(t)) = \frac{\prod_{j=1}^{p} M_{ij}(\theta_j(t))}{\sum_{i=1}^{r}\prod_{j=1}^{p} M_{ij}(\theta_j(t))} \tag{2}$$

where $M_{ij}(\theta_j(t))$ represents the grade of membership of $\theta_j(t)$ in $M_{ij}$. It can be seen that

$$h_i(\theta(t)) \geq 0, \sum_{i=1}^{r} h_i(\theta(t)) = 1. \tag{3}$$

By employing product-fuzzy inference, a center-average and singleton defuzzifier, the T–S fuzzy system (1) can be rewritten as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))[A_i x(t) + B_i\omega(t)] \\ y(t) = \sum_{i=1}^{r} h_i(\theta(t))C_i x(t) \\ z(t) = \sum_{i=1}^{r} h_i(\theta(t))L_i x(t). \end{cases} \tag{4}$$

### A. Event-triggered communication scheme

The aim of introducing the event generator is to save the limited communication resource. In addition, an event-triggered communication scheme is proposed to determine whether the current sampled signal should be transmitted or not. First, the output measurement out signal $y(t)$ is first sampled at the time instants $sh(s = 1, 2, \ldots)$ with $h > 0$ being a constant. The sampled signal with its time stamp is encapsulated into a data packet. Releasing of the sampled instant output $y(sh)$ to the filter depends on a predefined event-triggered communication scheme, which is given by

$$\begin{aligned} &[y((t_k + j)h) - y(t_k h)]^\top \Omega[y((t_k + j)h) - y(t_k h)] \\ &\quad \leq \eta y^\top(t_k h)\Omega y(t_k h) \end{aligned} \tag{5}$$

where $j = 1, 2, \ldots,$ $\eta > 0$ is an arbitrary scalar to be determined, and $\Omega > 0$ is a symmetric positive definite event-weighting matrix to de designed. The $(t_k + j)h$ and $t_k h$ are the sampling instants with $h$ being the sampling period. The current sampled data $y^F((t_k + j)h)$ will be released immediately when the event-triggered condition (5) is satisfied.

Under the event-triggered communication scheme (5), the set of transition instants is given by $\{t_0 h, t_1 h, t_2 h, \ldots\}$, where $t_0 h$ denotes the initial transmission instant. It is obvious that $t_0 h < t_1 h < t_2 h < \ldots$ under the assumption that packet disorders and packet dropouts do not occur. For networked uncertainty, we only consider the effect of the transmission delay on the system. In the following, we investigate how to deal with the time varying networked-induced delay $\tau_k$, where $\tau_k$ is the transmission delay of the released data packet $(t_k, y(t_k h))$ transmitted from the data packet processor to zero-order hold (ZOH). It is supposed that $\tau_m \leq \tau_k \leq \tau_M$, where $\tau_m$ and $\tau_M$ are given scalars satisfying $\tau_M > \tau_m \geq 0$. For technique convenience, the following two cases will be considered.

1) If the release interval satisfies $t_{k+1}h - t_k h \le h + \tau_M - \tau_{k+1}$, an interval time-varying delay $\tau(t)$ is defined as follows:

$$\tau(t) = t - t_k h, \quad t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1}). \tag{6}$$

2) If the release interval satisfies $t_{k+1}h - t_k h > h + \tau_M - \tau_{k+1}$, there always exists a scalar $r_M$ satisfying $t_{k+1}h + \tau_{k+1} \in [t_k h + r_M h + \tau_M, t_k h + r_M h + h + h\tau_M)$.

Define

$$\begin{cases} \Gamma_0 = [t_k h + \tau_k, t_k h + h + \tau_M) \\ \Gamma_1 = [t_k h + h + \tau_M, t_k h + 2h + \tau_M) \\ \vdots \\ \Gamma_{r_M} = [t_k h + r_M h + \tau_M, t_{k+1}h + \tau_{k+1}). \end{cases} \tag{7}$$

In order to employ event-triggered condition (5), two piecewise functions $\tau(t)$ and $e_k(t)$ are defined as follows:

$$\tau(t) = \begin{cases} t - t_k h, \quad t \in \Gamma_0 \\ t - t_k h - h, \quad t \in \Gamma_1 \\ \vdots \\ t - t_k h - r_M h, \quad t \in \Gamma_{r_M} \end{cases} \tag{8}$$

$$\delta_k(t) = \begin{cases} y(t_k h) - y(t_k h), \quad t \in \Gamma_0 \\ y(t_k h) - y(t_k h + h), \quad t \in \Gamma_1 \\ \vdots \\ y(t_k h) - y(t_k h + r_M h), \quad t \in \Gamma_{r_M}. \end{cases} \tag{9}$$

It can be seen that $\tau(t) \in [\tau_m, h + \tau_M)$ from definition for $\tau(t)$ in (6) and (8), and $y(t_k h) = \delta_k(t) + y(t - \tau(t))$.

*Remark 1:* From the definition of $\delta_k(t)$ in (9) and $\tau(t)$ in (6) and (8), the event-triggered condition (5), for $t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$, can be expressed as

$$\delta_k^\top(t)\Omega\delta_k(t) \le \eta y^\top(t - \tau(t))\Omega y(t - \tau(t)). \tag{10}$$

### *B. Sensor failure*

The failures cannot be avoided in the sensors. Inspired by the work [29], the following model is used when the sensor failure occurs. More specifically, for the measured output $y(t)$, $y^F(t)$ is adopted to describe the measured signal sent from sensors and

$$y^F(t) = Fy(t_k h), \quad t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1}) \tag{11}$$

where $F = \mathrm{diag}\{f_1, f_2, \ldots, f_m\}$ is the sensor failure function matrix satisfying $0 \le f_{sl} \le f_s \le f_{su} \le 1, \quad s = 1, 2, \ldots, m$, where $f_{sl}$ and $f_{su}$ are known real constants, which characterize the admissible failures of the $s$th sensor.

*Remark 2:* It is easy to observe that when $f_{sl} = f_{su} = 1$, the $sth$ sensor is in complete failure; when $f_{sl} = f_{su} = 0$, there is no failure in $sth$ sensor; and when $0 < f_{sl} < f_{su} < 1$, the partial failures occurs in $sth$ sensor.

Let us denote

$$\begin{aligned} F_0 &= \mathrm{diag}\{f_{01}, f_{02}, \ldots, f_{0m}\} \\ K &= \mathrm{diag}\{k_1, k_2, \ldots, k_m\} \\ G &= \mathrm{diag}\{g_1, g_2, \ldots, g_m\} \end{aligned}$$

where $f_{0s} = \frac{f_{sl}+f_{su}}{2}, g_s = \frac{f_s - f_{0s}}{f_{0s}}$, and $K_s = \frac{f_{su}-f_{sl}}{f_{su}+f_{sl}}$. From the above denotations, it is easy to obtain that

$$F = F_0(I + G), \quad |G| \le K \le I. \tag{12}$$

### *C. Filter*

In the sequence, a filter is proposed as follows:

$$\begin{aligned} \dot{x}_f(t) &= \sum_{j=1}^{r} h_j(\theta(t_k h))[A_{fj}x_f(t) + B_{fj}y^F(t)] \\ z_f(t) &= \sum_{j=1}^{r} h_j(\theta(t_k h))[C_{fj}x_f(t) + D_{fj}y^F(t)] \end{aligned} \tag{13}$$

where $x_f(t)$ denotes the filter state; $z_f(t)$ is an estimate signal of the output $z(t)$; and $A_{fi}$, $B_{fi}$, and $L_{fi} (i = 1, 2, \ldots, r)$ are filter parameter matrices to be designed.

Similar to [9], the following asynchronous constraints on the membership function are adopted:

$$\begin{aligned} &h_j(\theta(t_k h)) = \rho_j h_j(\theta(t)) \\ &|h_j(\theta(t_k h)) - h_j(\theta(t))| \le \Delta_j \end{aligned} \tag{14}$$

where $\Delta_j$ is known positive constant and $\rho_j$ is a parameter related with $h_j(\theta(t))$ and $h_j(\theta(t_k h))$.

From (14), it is easy to know that

$$\beta_1^j \le 1 - \frac{\Delta_j}{h_j(\theta(t))} \le \rho_j \le 1 + \frac{\Delta_j}{h_j(\theta(t))} \le \beta_2^j \tag{15}$$

where $\beta_1^j$ and $\beta_2^j$ are the minimum and maximum values of $\rho_j$ during the operation, which exhibits that

$$\begin{aligned} \frac{v_1^i}{v_2^j} = \frac{\min\{\rho_i\}}{\max\{\rho_j\}} &\le \min\left\{\frac{\rho_i}{\rho_j}\right\} \le \frac{\rho_i}{\rho_j} \\ &\le \max\left\{\frac{\rho_i}{\rho_j}\right\} \le \frac{\max\{\rho_j\}}{\min\{\rho_i\}} = \frac{v_2^i}{v_1^j}. \end{aligned}$$

Letting $v_1 = \min\{v_1^i\}$ and $v_2 = \max\{v_2^i\}$, the following inequality is satisfied:

$$\lambda_1 = \frac{v_1}{v_2} \le \frac{\rho_i}{\rho_j} \le \frac{v_2}{v_1} = \lambda_2. \tag{16}$$

From (14), the filter (13) can be rewritten as

$$\begin{aligned} \dot{x}_f(t) &= \sum_{j=1}^{r} \rho_j h_j(\theta(t))[A_{fj}x_f(t) + B_{fj}y^F(t)] \\ z_f(t) &= \sum_{j=1}^{r} \rho_j h_j(\theta(t))[C_{fj}x_f(t) + D_{fj}y^F(t)]. \end{aligned} \tag{17}$$

*Remark 3:* From the definition of $\lambda_1$ and $\lambda_2$ in (16), it is obvious that $\lambda_2 = \frac{1}{\lambda_1}$. As a special case, if $\lambda_1 = \lambda_2 = 1$, one has $\rho_i = \rho_j = \rho$ and $h_j(\theta(t_k h)) = \rho h_j(\theta(t))(i, j = 1, 2, \ldots, r)$. In fact

$$1 = \sum_{i=1}^{r} h_j(\theta(t_k h)) = \sum_{i=1}^{r} \rho h_j(\theta(t)) = \rho.$$

Then, one has $h_j(\theta(t_k h)) = h_j(\theta(t))$, which means that the fuzzy filter always shares the same membership functions with the considered fuzzy model. In other words, the networked filter is simplified as a point-to-point connected filter as usual case under $\lambda_1 = \lambda_2 = 1$.

### D. Filtering Error System and Problem Formation

Define $\zeta(t) = \left[\, x^\top(t)\; x_f^\top(t) \,\right]^\top$, and $e(t) = z(t) - z_f(t)$. Based on the definition of $\tau(t)$ and $e_k(t)$, the filtering error system can be described by

$$\begin{aligned}\dot{\zeta}(t) =& \sum_{i=1}^{r}\sum_{j=1}^{r} \rho_j h_i(\theta(t))h_j(\theta(t))[\bar{A}_{ij}\zeta(t) + \bar{A}_{0ij}\zeta(t-\tau(t))\\ &+ \bar{B}_i\omega(t) + \bar{B}_{0j}\delta_k(t)]\\ e(t) =& \sum_{i=1}^{r}\sum_{j=1}^{r} \rho_j h_i(\theta(t))h_j(\theta(t))[\bar{C}_{ij}\zeta(t) + \bar{C}_{0ij}\zeta(t-\tau(t))\\ &+ \bar{D}_j\delta_k(t)] \end{aligned} \tag{18}$$

where

$$\begin{aligned}\bar{A}_{ij} =& \begin{bmatrix} A_i & 0 \\ 0 & A_{f_j}\end{bmatrix}, \bar{A}_{0ij} = \begin{bmatrix} 0 & 0 \\ B_{f_j}FC_i & 0\end{bmatrix}\\ \bar{B}_i =& \begin{bmatrix} B_i \\ 0\end{bmatrix}, \bar{B}_{0j} = \begin{bmatrix} 0 \\ B_{f_j}F\end{bmatrix}\\ \bar{C}_{ij} =& \left[\, L_i \; -C_{f_j} \,\right], \bar{C}_{0ij} = \left[\, 0 \; -D_{f_j}FC_i \,\right]\\ \bar{D}_j =& -D_{f_j}F. \end{aligned}$$

*Remark 4:* It is obvious that, under the event-triggered communication scheme, the filtering error system can be regarded as a sampled-data error-dependent time varying system since the definition of $\delta_k(t)$, which is defined as the error between the current sampled-data packet and the latest transmitted data packet.

Before formulating the problem, we first recall the definition of generalized dissipativity.

*Definition 1:* [32] For a given scalar $\alpha > 0$ and constant matrices $Q \leq 0$, $S$ and symmetric $R$, the filtering error system (18) is said to be strictly $(Q, S, R) - \alpha$ dissipative and $\alpha$ is called the dissipativity performance bound, if the following inequality is satisfied under the zero initial condition $\omega(t) \in \mathcal{L}_2[0, \infty)$:

$$\int_0^t J(s)ds \geq \alpha \int_0^t \omega^\top(s)w(s)ds \tag{19}$$

where $J(t) = e^\top(t)Qe(t) + 2e^\top(t)S\omega(t) + \omega^\top(t)R\omega(t)$.

We may assume without loss of generality that $Q \leq 0$ and $-Q = \bar{Q}^\top\bar{Q}$ for some $\bar{Q} \geq 0$.

*Remark 5:* The generalized dissipativity in Definition 1 includes some special cases as follows.

1) When $Q = 0$, $S = -I$, and $R = 2\alpha I$, the strict $(Q, S, R) - \alpha$ dissipativity performance means passive performance.

2) When $Q = -I$, $S = 0$, and $R = (\alpha^2 + \alpha)I$, the strict $(Q, S, R) - \alpha$ dissipativity performance becomes $\mathcal{H}_\infty$ performance.

3) When $Q = -vI$, $S = (1 - v)I$, and $R = ((\alpha^2 - \alpha)v + 2\alpha)I$, the strict $(Q, S, R) - \alpha$ dissipativity performance reduces to mixed passivity/$\mathcal{H}_\infty$ performance. A weighting parameter $v$ provides the possibility of a flexible tradeoff between passivity and $\mathcal{H}_\infty$ performance.

We now formulate our filtering problem as follows.

The problem of designing the event-triggered generalized dissipativity filter in the form of (13) such that the filtering error system (18) is asymptotically stable with $\omega(t) \equiv 0$ and the filtering error system (18) is generalized dissipative in the sense of Definition 1.

The following two lemmas are given for deriving the main result of this paper.

*Lemma 1:* [35] For a given matrix $M > 0$, and a differentiable function $\{x(u)|u \in [a, b]\}$, the following inequality holds

$$\begin{aligned} &-(b-a)\int_a^b \dot{x}^T(s)M\dot{x}(s)ds\\ &\leq\; -\Omega_1^T M\Omega_1 - 3\Omega_2^T M\Omega_2 - 5\Omega_3^T M\Omega_3 \end{aligned}$$

where $\Omega_1 = x(b) - x(a)$, $\Omega_2 = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)ds$, $\Omega_3 = x(b) - x(a) - \frac{6}{b-a}\int_a^b \delta_{a,b}(s)x(s)ds$ with $\delta_{a,b}(u) = 2\left(\frac{u-a}{b-a}\right) - 1$.

*Lemma 2:* [36] For any given positive definite matrix $M$, suppose that there exists a matrix $X$ with appropriate dimension such that $\begin{bmatrix} M & X \\ X^\top & M\end{bmatrix} > 0$. Then, the following inequality holds:

$$\begin{bmatrix} \frac{1}{\alpha}M & 0 \\ 0 & \frac{1}{1-\alpha}M\end{bmatrix} \geq \begin{bmatrix} M & X \\ X^\top & M\end{bmatrix} \quad \forall\alpha \in (0, 1).$$

## III. Main Results

In this section, a novel method for developing sufficient conditions to guarantee the asymptotic stability of the filtering error system in (18) with the performance described in (19) using Lyapunov–Krasovskii functional approach is proposed. First, suppose that the filter gain matrices are known. We will propose the following performance analysis for the augmented systems (18). For simplicity, we define $\hat{e}_i \in \mathbb{R}^{(30n+2q)\times n}$ to be block entry matrix. For example, $\hat{e}_2 = [0, I, \underbrace{0, \ldots, 0}_{29}, 0]^\top$ and some other scalars and matrices are defined as follows:

$$\begin{aligned}\delta_1(s) =&\; 2\frac{s - t + \tau_1}{\tau_1} - 1, \delta_2(s) = 2\frac{s - t + \tau(t)}{\tau(t) - \tau_1} - 1\\ \delta_3(s) =&\; 2\frac{s - t + \tau_2}{\tau_2 - \tau(t)} - 1, \delta_4(s) = 2\frac{s - t + \tau_2}{\tau_2 - \tau_1} - 1\\ \xi_1^\top(t) =&\; \big[\zeta^\top(t)\;\; \zeta^\top(t-\tau_1)\;\; \zeta^\top(t-\tau(t))\;\; \zeta^\top(t-\tau_2)\\ &\;\dot{\zeta}^\top(t)\big]\\ \xi_2^\top(t) =&\; \frac{1}{\tau_1}\left[\int_{t-\tau_1}^{t} \zeta^\top(s)ds \quad \int_{t-\tau_1}^{t} \delta_1(s)\zeta^\top(s)ds\right] \end{aligned}$$

$$\begin{aligned}
\xi_3^\top(t) =& \ \frac{1}{\tau(t)-\tau_1}\left[\int_{t-\tau(t)}^{t-\tau_1}\zeta^\top(s)ds\right.\\
&\left.\int_{t-\tau(t)}^{t-\tau_1}\delta_2(s)\zeta^\top(s)ds\right]\\
\xi_4^\top(t) =& \ \frac{1}{\tau_2-\tau(t)}\left[\int_{t-\tau_2}^{t-\tau(t)}\zeta^\top(s)ds\right.\\
&\left.\int_{t-\tau_2}^{t-\tau(t)}\delta_3(s)\zeta^\top(s)ds\right]\\
\xi_5^\top(t) =& \ (\tau(t)-\tau_1)\xi_3^\top(t), \xi_6^\top(t) = (\tau_2-\tau(t))\xi_4^\top(t)\\
\xi^\top(t) =& \ \left[\xi_1^\top(t) \ \xi_2^\top(t) \ \xi_3^\top(t) \ \xi_4^\top(t) \ \xi_5^\top(t) \ \xi_6^\top(t)\right.\\
&\left.\omega^\top(t) \ \delta_k^\top(t)\right]\\
e_m =& \ [\hat{e}_{2m-1}, \hat{e}_{2m}], m = 1, 2, \ldots, 15\\
e_{16} =& \ \hat{e}_{31} \ e_{17} = \hat{e}_{32}\\
\bar{R}_2 =& \ \mathrm{diag}\{R_2, 3R_2, 5R_2\}\\
\Pi_1(\tau(t)) =& \ [e_1 \ \ \tau_1 e_6 \ \tau_1 e_7 \ \ e_{14}+e_{12}\\
&(\tau_2-\tau(t))(e_{12}+e_{15})+(\tau(t)-\tau_1)(e_{13}-e_{14})]\\
\Pi_2 =& \ \left[e_5 \ \ e_1-e_2 \ \ e_1+e_2-2e_6 \ \ e_2-e_4\right.\\
&\left.(\tau_2-\tau_1)(e_2+e_4)-2(e_{12}+e_{14})\right]\\
\Pi_3 =& \ [e_2-e_3 \ \ e_2+e_3-2e_8 \ \ e_2-e_3-6e_9]\\
\Pi_4 =& \ [e_3-e_4 \ \ e_3+e_4-2e_{10} \ \ e_3-e_4-6e_{11}]\\
f_1(\tau(t)) =& \ (h(t)-h_1)\begin{bmatrix} e_8^\top \\ e_9^\top \end{bmatrix} - \begin{bmatrix} e_{12}^\top \\ e_{13}^\top \end{bmatrix}\\
f_2(\tau(t)) =& \ (h_2-h(t))\begin{bmatrix} e_{10}^\top \\ e_{11}^\top \end{bmatrix} - \begin{bmatrix} e_{14}^\top \\ e_{15}^\top \end{bmatrix}\\
\Phi_{ij} =& \ [G\bar{A}_{ij} \ \ 0 \ \ G\bar{A}_{0ij} \ \ 0 \ \ -G \ \ \underbrace{0,\ldots,0}_{10}\\
&G\bar{B}_i \ \ G\bar{B}_{0j}]\\
\Psi_{ij} =& \ [\bar{C}_{ij}\bar{Q} \ \ 0 \ \ \bar{C}_{0ij}\bar{Q} \ \ \underbrace{0,\ldots,0}_{13} \ \ \bar{D}_j\bar{Q}]\\
K_1 =& \ [I \ \ 0]
\end{aligned}$$

and $\Sigma_{ij}(\tau(t)) = \Pi_1(\tau(t))P\Pi_2^\top + \Pi_2 P\Pi_1^\top(\tau(t)) + e_1 Q_1 e_1^\top - e_2 Q_1 e_2^\top + e_2 Q_2 e_2^\top - e_4 Q_2 e_4^\top + \tau_1^2 e_5 R_1 e_5^\top - (e_1-e_2)R_1(e_1-e_2)^\top - 3[e_1+e_2-2e_6]R_1[e_1+e_2-2e_6]^\top - 5[e_1-e_2-6e_7]R_1[e_1-e_2-6e_7]^\top + (\tau_2-\tau_1)^2 e_5 R_2 e_5^\top - [\Pi_3 \ \ \Pi_4]\begin{bmatrix}\bar{R}_2 & X\\ * & \bar{R}_2\end{bmatrix}[\Pi_3 \ \Pi_4]^\top + N_1 f_1(\tau(t)) + f_1^\top(\tau(t))N_1^\top + N_2 f_2(\tau(t)) + f_2^\top(\tau(t))N_2^\top + (e_1+\epsilon e_5)\Phi_{ij} + \Phi_{ij}^\top(e_1+\epsilon e_5)^\top - e_{17}\Omega e_{17}^\top + \eta e_3 K_1^\top C_i^\top \Omega C_i K_1 e_3^\top - (e_1\bar{C}_{ij}^\top + e_3\bar{C}_{0ij}^\top + e_{17}\bar{D}_j^\top) S e_{16}^\top - e_{16}S^\top(e_1\bar{C}_{ij}^\top + e_3\bar{C}_{0ij}^\top + e_{17}\bar{D}_j^\top)^\top - e_{16}(R-\alpha I)e_{16}^\top$.

*Theorem 1:* For given scalars $\alpha$, $\tau_2$, $\tau_1$, $\lambda_1$, and $\lambda_2$, the filtering error system (18) is asymptotically stable with strict $(Q,S,R)-\alpha$ dissipative, if there exist matrices $P>0$, $Q_1>0$, $Q_2>0$, $R_1>0$, $R_2>0$, $X$, $G$, $N_1$, $N_2$, $A_{fj}$, $B_{fj}$, $C_{fj}$, $D_{fj}$ with appropriate dimensions such that the following LMIs hold with $i,j=1,\ldots,r, i<j$

$$\begin{bmatrix}\Sigma_{ii}(\tau(t)) & \Psi_{ii}^\top\\ * & -I\end{bmatrix} < 0 \tag{20}$$

$$\begin{bmatrix}\Sigma_{ij}(\tau(t)) + \lambda_1\Sigma_{ji}(\tau(t)) & \Psi_{ij}^\top & \lambda_1\Psi_{ji}^\top\\ * & -I & 0\\ * & * & -\lambda_1 I\end{bmatrix} < 0 \tag{21}$$

$$\begin{bmatrix}\Sigma_{ij}(\tau(t)) + \lambda_2\Sigma_{ji}(\tau(t)) & \Psi_{ij}^\top & \lambda_2\Psi_{ji}^\top\\ * & -I & 0\\ * & * & -\lambda_2 I\end{bmatrix} < 0 \tag{22}$$

$$\begin{bmatrix}\bar{R}_2 & X\\ * & \bar{R}_2\end{bmatrix} > 0 \tag{23}$$

where $\tau(t) \in \{\tau_1, \tau_2\}$.

*Proof:* Construct the following Lyapunov functional candidate

$$V(t) = \sum_{i=1}^{5} V_i(t) \tag{24}$$

where

$$\begin{aligned}
V_1(t) =& \ \bar{\zeta}^\top(t)P\bar{\zeta}(t)\\
V_2(t) =& \ \int_{t-\tau_1}^{t}\zeta^\top(s)Q_1\zeta(s)ds\\
V_3(t) =& \ \int_{t-\tau_2}^{t-\tau_1}\zeta^\top(s)Q_2\zeta(s)ds\\
V_4(t) =& \ \tau_1\int_{t-\tau_1}^{t}\int_{\theta}^{t}\dot{\zeta}^\top(s)R_1\dot{\zeta}(s)dsd\theta\\
V_5(t) =& \ (\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\int_{\theta}^{t}\dot{\zeta}^\top(s)R_2\dot{\zeta}(s)dsd\theta
\end{aligned}$$

with $\bar{\zeta}^\top(t) = [\zeta^\top(t) \ \int_{t-\tau_1}^{t}\zeta^\top(s)ds \ \int_{t-\tau_1}^{t}\delta_1(s)\zeta^\top(s)ds \ \int_{t-\tau_2}^{t-\tau_1}\zeta^\top(s)ds \ \ (\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\delta_4(s)\zeta^\top(s)ds]$.

The time derivative of $V_1(t)$ along the trajectory of (18) is computed as

$$\dot{V}_1(t) = \ 2\bar{\zeta}^\top(t)P\hat{\zeta}(t) \tag{25}$$

where

$$\bar{\zeta}(t) = \begin{bmatrix}\zeta(t)\\ \int_{t-\tau_1}^{t}\zeta(s)ds\\ \int_{t-\tau_1}^{t}\delta_1(s)\zeta(s)ds\\ \int_{t-\tau_2}^{t-\tau(t)}\zeta(s)ds + \int_{t-\tau(t)}^{t-\tau_1}\zeta(s)ds\\ \tilde{\zeta}_1(t)\end{bmatrix}$$

and

$$\hat{\zeta}(t) = \begin{bmatrix} \dot{\zeta}(t) \\ \zeta(t) - \zeta(t-\tau_1) \\ \zeta(t) + \zeta(t-\tau_1) - \frac{2}{\tau_1}\int_{t-\tau_1}^{t}\zeta(s)ds \\ \zeta(t-\tau_1) - \zeta(t-\tau_2) \\ \tilde{\zeta}_2(t) \end{bmatrix}$$

with $\tilde{\zeta}_1(t) = (\tau_2 - \tau(t))(\int_{t-\tau(t)}^{t-\tau_1}\zeta(s)ds + \int_{t-\tau_2}^{t-\tau(t)}\delta_3(s)\zeta(s)ds) + (\tau(t) - \tau_1)(\int_{t-\tau(t)}^{t-\tau_1}\delta_2(s)\zeta(s)ds - \int_{t-\tau_2}^{t-\tau(t)}\zeta(s)ds)$ and $\tilde{\zeta}_2(t) = (\tau_2 - \tau_1)(\zeta(t-\tau_1) + \zeta(t-\tau_2)) - 2(\int_{t-\tau_2}^{t-\tau(t)}\zeta(s)ds + \int_{t-\tau(t)}^{t-\tau_1}\zeta(s)ds)$.

The computation of $V_2(t) - V_5(t)$ yields to

$$\dot{V}_2(t) = \zeta^\top(t)Q_1\zeta(t) - \zeta^\top(t-\tau_1)Q_1\zeta(t-\tau_1) \tag{26}$$

$$\dot{V}_3(t) = \zeta^\top(t-\tau_1)Q_2\zeta(t-\tau_1) - \zeta^\top(t-\tau_2)Q_2\zeta(t-\tau_2) \tag{27}$$

$$\dot{V}_4(t) = \tau_1^2\dot{\zeta}^\top(t)R_1\dot{\zeta}(t) - \tau_1\int_{t-\tau_1}^{t}\dot{\zeta}^\top(s)R_1\dot{\zeta}(s)ds \tag{28}$$

$$\dot{V}_5(t) = (\tau_2-\tau_1)^2\dot{\zeta}^\top(t)R_2\dot{\zeta}(t) - (\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{\zeta}^\top(s)R_2\dot{\zeta}(s)ds. \tag{29}$$

Based on Lemma 1, one has

$$-\tau_1\int_{t-\tau_1}^{t}\dot{\zeta}^\top(s)R_1\dot{\zeta}(s)ds \leq -\gamma_1^T R_1\gamma_1 - 3\gamma_2^T R_1\gamma_2 - 5\gamma_3^T R_1\gamma_3 \tag{30}$$

where $\gamma_1 = \zeta(t) - \zeta(t-\tau_1), \gamma_2 = \zeta(t) + \zeta(t-\tau_1) - \frac{2}{\tau_1}\int_{t-\tau_1}^{t}\zeta(s)ds, \gamma_3 = \zeta(t) - \zeta(t-\tau_1) - \frac{6}{\tau_1}\int_{t-\tau_1}^{t}\delta_1(s)\zeta(s)ds$.

By adopting Lemmas 1 and 2, one can get

$$\begin{aligned} &-(\tau_2-\tau_1)\int_{t-\tau(t)}^{t-\tau_1}\dot{\zeta}^\top(s)R_2\dot{\zeta}(s)ds \\ &= -(\tau_2-\tau_1)\int_{t-\tau(t)}^{t-\tau_1}\dot{\zeta}^\top(s)R_2\dot{\zeta}(s)ds \\ &\quad - (\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau(t)}\dot{\zeta}^\top(s)R_2\dot{\zeta}(s)ds \\ &\leq -\frac{\tau_2-\tau_1}{\tau(t)-\tau_1}\bar{\gamma}_1^\top\bar{R}_2\bar{\gamma}_1 - \frac{\tau_2-\tau_1}{\tau_2-\tau(t)}\bar{\gamma}_2^\top\bar{R}_2\bar{\gamma}_2 \\ &\leq -\begin{bmatrix}\bar{\gamma}_1\\ \bar{\gamma}_2\end{bmatrix}^\top\begin{bmatrix}\bar{R}_2 & X\\ * & \bar{R}_2\end{bmatrix}\begin{bmatrix}\bar{\gamma}_1\\ \bar{\gamma}_2\end{bmatrix} \end{aligned} \tag{31}$$

where $\bar{\gamma}_1 = [\gamma_4^\top \;\; \gamma_5^\top \;\; \gamma_6^\top]^\top, \bar{\gamma}_2 = [\gamma_7^\top \;\; \gamma_8^\top \;\; \gamma_9^\top]^\top, \gamma_4 = \zeta(t-\tau_1) - \zeta(t-\tau(t)), \gamma_5 = \zeta(t-\tau_1) + \zeta(t-\tau(t)) - \frac{2}{\tau(t)-\tau_1}\int_{t-\tau(t)}^{t-\tau_1}\zeta(s)ds, \gamma_6 = \zeta(t-\tau_1) - \zeta(t-\tau(t)) - \frac{6}{\tau(t)-\tau_1}\int_{t-\tau(t)}^{t-\tau_1}\delta_2(s)\zeta(s)ds, \gamma_7 = \zeta(t-\tau(t)) - \zeta(t-\tau_2), \gamma_8 = \zeta(t-\tau(t)) + \zeta(t-\tau_2) - \frac{2}{\tau_2-\tau(t)}\int_{t-\tau_2}^{t-\tau(t)}\zeta(s)ds, \gamma_9 = \zeta(t-\tau(t)) - \zeta(t-\tau_2) - \frac{6}{\tau_2-\tau(t)}\int_{t-\tau_2}^{t-\tau(t)}\delta_3(s)\zeta(s)ds$.

Since $\xi_5^\top(t) = (\tau(t)-\tau_1)\xi_3^\top(t)$ and $\xi_6^\top(t) = (\tau_2-\tau(t))\xi_4^\top(t)$, for any matrices $N_1$ and $N_2$ with appropriate dimensions, the following inequality holds:

$$2\xi^\top(t)(N_1 f_1(\tau(t)) + N_2 f_2(\tau(t)))\xi(t) = 0. \tag{32}$$

From the error system (18), one has

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\rho_j h_i(\theta(t))h_j(\theta(t))[\zeta^\top(t)G + \dot{\zeta}^\top(t)G][\bar{A}_{ij}\zeta(t) + \bar{A}_{0ij}\zeta(t-\tau(t)) + \bar{B}_i\omega(t) + \bar{B}_{0j}\delta_k(t)] = 0. \tag{33}$$

By combining the event-triggered condition (5), (25)–(33), and adding $-e^\top(t)Qe(t) - 2e^\top(t)S\omega(t) - \omega^T(t)[R-\alpha I]\omega(t)$ on both sides, we have

$$\begin{aligned} &\dot{V}(t) - e^\top(t)Qe(t) - 2e^\top(t)S\omega(t) \\ &\quad - \omega^\top(t)[R-\alpha I]\omega(t) \\ &\leq \xi^\top(t)\sum_{i=1}^{r}\sum_{j=1}^{r}\rho_j h_i(\theta(t))h_j(\theta(t))\bar{\Sigma}_{ij}(\tau(t))\xi(t) \\ &\leq \xi^\top(t)\left(\sum_{i=1}^{r}\sum_{j>i}\rho_j h_i(\theta(t))h_j(\theta(t))(\bar{\Sigma}_{ij}(\tau(t)) + \frac{\rho_i}{\rho_j}\bar{\Sigma}_{ji}(\tau(t))) + \sum_{i=1}^{r}\rho_i h_i^2(\theta(t))\bar{\Sigma}_{ii}(\tau(t))\right)\xi(t) \end{aligned} \tag{34}$$

where $\bar{\Sigma}_{ij}(\tau(t)) = \Sigma_{ij}(\tau(t)) - \Psi_{ij}^\top Q\Psi_{ij}$.

If the LMIs in (20)–(22) are satisfied, based on Schur complement, one has

$$\bar{\Sigma}_{ii}(\tau(t)) < 0 \tag{35}$$

$$\bar{\Sigma}_{ij}(\tau(t)) + \lambda_1\bar{\Sigma}_{ji}(\tau(t)) < 0 \tag{36}$$

$$\bar{\Sigma}_{ij}(\tau(t)) + \lambda_2\bar{\Sigma}_{ji}(\tau(t)) < 0. \tag{37}$$

Define

$$\varepsilon_1 = \frac{\lambda_2 - \frac{\rho_i}{\rho_j}}{\lambda_2 - \lambda_1},\ \varepsilon_2 = \frac{\frac{\rho_i}{\rho_j} - \lambda_1}{\lambda_2 - \lambda_1}. \tag{38}$$

It follows from (36) and (37) that

$$\varepsilon_1(\bar{\Sigma}_{ij}(\tau(t)) + \lambda_1\bar{\Sigma}_{ji}(\tau(t))) + \varepsilon_2(\bar{\Sigma}_{ij}(\tau(t)) + \lambda_2\bar{\Sigma}_{ji}(\tau(t))) < 0 \tag{39}$$

which yields

$$\bar{\Sigma}_{ij}(\tau(t)) + \frac{\rho_i}{\rho_j}\bar{\Sigma}_{ji}(\tau(t)) < 0. \tag{40}$$

Based on (34), (35), and (40), we have

$$\dot{V}(t) - E(t) \leq 0 \tag{41}$$

where $E(t) = e^\top(t)Qe(t) + 2e^\top(t)S\omega(t) + \omega^\top(t)[R-\alpha I]\omega(t)$.

Integrating both sides of (41) from 0 to $t$ yields

$$V(t) - V(0) \leq \int_0^t E(s)ds \tag{42}$$

which implies that $\int_0^t E(s)ds > 0$ under the zero initial condition. Therefore, based on definition 1, we can conclude that the closed-loop system (18) is strictly $(Q,S,R)-\alpha$ dissipative. Furthermore, when $\omega(t)=0$, $\dot{V}(t) \le e^\top(t)Qe(t) \le 0$, which means that the closed-loop system (18) is asymptotically stable.

In the following, we will give a method to design the filter gain parameters and the triggered matrix based on Theorem 1. Let

$$G = \begin{bmatrix} G_1 & G_2 \\ G_3 & G_4 \end{bmatrix}.$$

Then, pre- and postmultiply $G$ by diag$\{I \ \ G_2G_4^{-1}\}$ to obtain

$$\begin{bmatrix} I & 0 \\ 0 & G_2G_4^{-1} \end{bmatrix} G \begin{bmatrix} I & 0 \\ 0 & G_4^{-1}G_2^T \end{bmatrix} = \begin{bmatrix} G_1 & G_2G_4^{-1}G_2^T \\ G_2G_4^TG_3 & G_2G_4^{-1}G_2^T \end{bmatrix}. \tag{43}$$

Consequently, the $G$ matrix can be defined, without loss of generality, as $G = \begin{bmatrix} G_1 & G_2 \\ G_3 & G_2 \end{bmatrix}$. By a simple matrix calculation, it is straightforward to derive that

$$G\bar{A}_{ij} = \begin{bmatrix} G_1A_i & G_2A_{fj} \\ G_3A_i & G_2A_{fj} \end{bmatrix}$$

$$G\bar{A}_{0ij} = \begin{bmatrix} G_2B_{fj}FC_i & 0 \\ G_2B_{fj}FC_i & 0 \end{bmatrix}$$

$$G\bar{B}_j = \begin{bmatrix} G_1B_i \\ G_3B_i \end{bmatrix}, \ G\bar{B}_{0j} = \begin{bmatrix} G_2B_{fj}F \\ G_2B_{fj}F \end{bmatrix}. \tag{44}$$

Define a set of variables as

$$\hat{A}_{fj} = G_2A_{fj}, \hat{B}_{fj} = G_2B_{fj}. \tag{45}$$

Based on the definition of (45), the following equation can be rewritten as:

$$G\bar{A}_{ij} = \begin{bmatrix} G_1A_i & \hat{A}_{fj} \\ G_3A_i & \hat{A}_{fj} \end{bmatrix}, \ G\bar{B}_i = \begin{bmatrix} G_1B_i \\ G_3B_i \end{bmatrix}$$

$$G\bar{A}_{0ij} = \begin{bmatrix} \hat{B}_{fj}FC_i & 0 \\ \hat{B}_{fj}FC_i & 0 \end{bmatrix}, \ G\bar{B}_{0j} = \begin{bmatrix} \hat{B}_{fj}F \\ \hat{B}_{fj}F \end{bmatrix}. \tag{46}$$

Then, based on Theorem 1 and (46), the following theorem can be obtained.

*Theorem 2:* For given scalars $\alpha$, $\tau_2$, $\tau_1$, $\lambda_1$, and $\lambda_2$, the filtering error system (18) is asymptotically stable with strict $(Q,S,R)-\alpha$ dissipative, if there exist matrices $P>0$, $Q_1>0$, $Q_2>0$, $R_1>0$, $R_2>0$, $X$, $N_1$, $N_2$, $\hat{A}_{fj}$, $\hat{B}_{fj}$, $\hat{C}_{fj}$, $\hat{D}_{fj}$, and $G = \begin{bmatrix} G_1 & G_2 \\ G_2^\top & G_2 \end{bmatrix}$ with appropriate dimensions such that the LMIs (20)–(23) hold. And the filtering matrices can be given as $A_{fj} = G_2^{-1}\hat{A}_{fj}, B_{fj} = G_2^{-1}\hat{B}_{fj}, C_{fj} = \hat{C}_{fj}$, and $D_{fj} = \hat{D}_{fj}$.

*Remark 6:* The proposed Theorems 1 and 2 have some advantages by using Lemmas 1 and 2. One the one hand, the inequality in Lemma 1 encompasses the Wirtinger-based inequality. The derived Theorems 1 and 2 are of less conservatism than the ones using Wirtinger-based inequality. On the other hand, Lemma 1 has fewer slack variables to estimate the upper bounds of some cross terms than those employing the free-weighting matrix approach.

*Remark 7:* It should be noted that (33) developed a new method for the filter design problem, which is different from the traditional design approach [10]–[15], [18]–[29]. The advantage of the proposed method is that it can remove the enforcement of the constraints on the Lyapunov function variables $P$. More slack variables in the matrix $G$ and augmented vector of $p$ can be used to get some improved results [14].

*Remark 8:* For the results of our paper, the filter parameters can be determined by solving a set of LMIs. One problem of the LMI method is the computational issue especially when the size of LMI becomes large. Fortunately, with the help of the MATLAB LMI toolbox, solving the LMIs (20)–(23) will not be a big deal. When the LMIs (20)–(23) have a solution, the filter gain can be derived directly and the filter to the estimation system can be implemented.

*Remark 9:* The event-triggered fuzzy filter for a class of nonlinear networked systems was studied in [21]. The paper [26] investigated the event-triggered $\mathcal{H}_\infty$ filtering for networked T–S fuzzy systems with asynchronous constraints. However, the asynchronous constraints of the membership functions were not considered in [21]. The sensor failures were not investigated in [26]. And the reliable phenomenon was not taken into consideration in [21] and [26]. The problem of event-based reliable dissipative filtering for T–S fuzzy systems with asynchronous constraints is studied in this paper, which means that the topic considered here is more general.

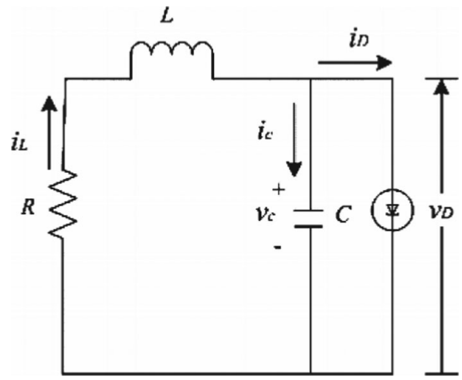


Fig. 1. Tunnel diode circuit [11], [21].

## IV. Numerical Example and Simulation

In this section, an example will be given to illustrate the proposed design method.

Consider the following tunnel diode circuit system shown in Fig. 1 [11], [21], where the tunnel diode is presented by

$$i_D(t) = 0.002v_D(t) + 0.01v_D^3(t).$$

Taking $x_1(t) = v_c(t)$ and $x_2(t) = i_L(t)$ as the state variables, the circuits can be governed by the following state equations:

$$\begin{aligned} C\dot{x}_1(t) &= -0.002x_1(t) - 0.01x_1^3(t) + x_2(t) \\ L\dot{x}_2(t) &= -x_1(t) - Mx_2(t) + \omega(t) \\ y(t) &= x_1(t) \\ z(t) &= x_1(t) \end{aligned} \tag{47}$$

where $x_1(t)$ is the capacitor voltage and $x_2(t)$ represents inductance current, $\omega(t)$ is the disturbance noise input, $y(t)$ denotes the measurement output, and $z(t)$ is the controlled output. Let $C = 20\,\mathrm{mF}$, $L = 1\,\mathrm{H}$, and $M = 10\,\Omega$, the tunnel diode circuit system (47) can be rewritten as

$$\begin{aligned} \dot{x}_1(t) &= -0.1x_1(t) - 0.5x_1^3(t) + 50x_2(t) \\ \dot{x}_2(t) &= -x_1(t) - 10x_2(t) + \omega(t) \\ y(t) &= x_1(t) \\ z(t) &= x_1(t). \end{aligned} \tag{48}$$

It is assumed that $|x_1(t)| \leq 3$, and the nonlinear networked system (48) can be expressed by T–S fuzzy system (4), and the parameters of the system (4) are given as

$$A_1 = \begin{bmatrix} -0.1 & 50 \\ -1 & -10 \end{bmatrix}, \ A_2 = \begin{bmatrix} -4.6 & 50 \\ -1 & -10 \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$L_1 = L_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The membership functions are given as

$$h_1(\theta(t)) = \begin{cases} \dfrac{x_1(t)+3}{3}, & -3 < x_1(t) < 0 \\ \dfrac{3 - x_1(t)}{3}, & 0 < x_1(t) < 2 \\ 0, & \text{otherwise} \end{cases}$$

and $h_2(\theta(t)) = 1 - h_1(\theta(t))$.

Let $\tau_1 = 0.05$, $\tau_2 = 0.1$, $\epsilon = 0.1$, $F = 0.8$, $\eta = 0.5$, $\lambda_1 = 0.5$, and $\lambda_2 = 2$. Then, by solving the conditions in Theorem 2, the derived results for the desired filtering cases are as follows.

1) $\mathcal{H}_\infty$ performance case: Set $Q = -I$, $S = 0$, $R = (\alpha^2 + \alpha)I$, $\eta = 0.5$, and $h = 0.1$, by solving LMIs in (20)–(23), the obtained performance index $\gamma$ is 0.836, and the corresponding filter matrices and $\Omega$ are given as follows:

$$A_{f1} = \begin{bmatrix} -9.3352 & 33.6646 \\ -1.4849 & -8.5538 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} -4.1736 \\ 0.1740 \end{bmatrix}$$

$$C_{f1} = \begin{bmatrix} -1.0790 & 1.2101 \end{bmatrix}, \quad D_{f1} = -0.3017$$

$$A_{f2} = \begin{bmatrix} -12.6744 & 36.4132 \\ -1.7155 & -7.7338 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} -3.0100 \\ 0.3034 \end{bmatrix}$$

$$C_{f2} = \begin{bmatrix} -1.1175 & 2.4546 \end{bmatrix}, \quad D_{f2} = -0.2022$$

$$\Omega = 0.2205.$$

Associate with the above event-triggered gain matrices, $z(t)$ and $z_f(t)$ are shown in Fig. 2 under the disturbance $\omega(t) = e^{-t}\sin(t)$. From this figure, it is obvious that the event-triggered $\mathcal{H}_\infty$ filter produces a good estimation of $z(t)$. Moreover, the transmission instants and transmission intervals are given in Fig. 3. In addition, for given $h = 0.1$, different minimum $\mathcal{H}_\infty$ levels $\gamma_{\min}$ is listed in Table I. And for given $\eta = 0.2$, Table II shows the achieved minimum $\mathcal{H}_\infty$ levels $\gamma_{\min}$. From Table I, for a given $h$, a smaller value of $\eta$ leads to a better $\mathcal{H}_\infty$ performance. And from Table II, for a given $\eta$, a small $h$ can provide better $\mathcal{H}_\infty$ performance.

2) Strictly dissipative case: Let $Q = -0.4$, $R = 1$, $S = 0.5$, $\eta = 0.1$, and $h = 0.2$, by solving LMIs in (20)–(23), optimal

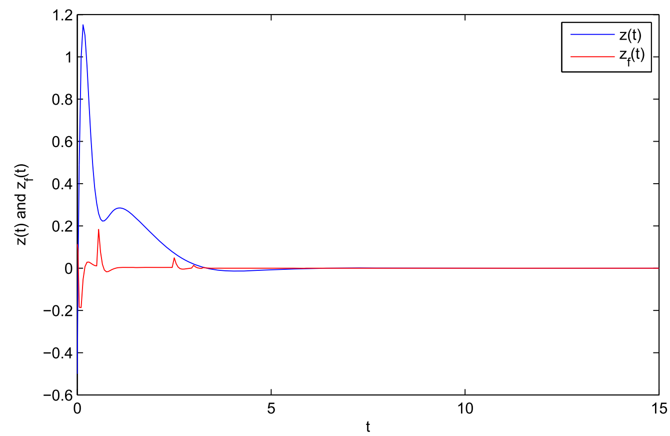


Fig. 2. $z(t)$ and its estimation $z_f(t)$ in the case of 1).

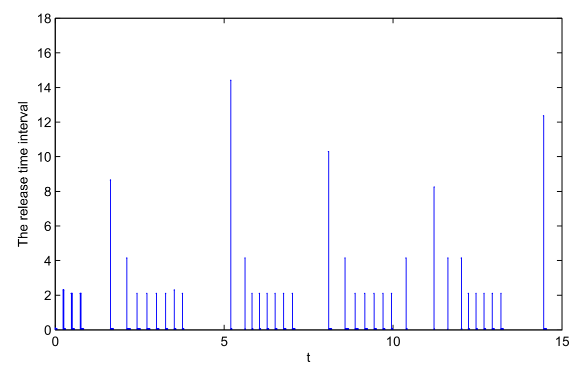


Fig. 3. Event-based release instants and release interval in the case of 1).

TABLE I
ALLOWABLE MINIMUM $\mathcal{H}_\infty$ LEVEL $\gamma_{\min}$ FOR $h = 0.1$ WITH DIFFEREN VALUES OF $\eta$

| $\eta$ | 0.05 | 0.1 | 0.3 | 0.5 | 0.8 |
|---|---|---|---|---|---|
| $\gamma$ | 0.752 | 0.774 | 0.814 | 0.836 | 0.859 |

TABLE II
ALLOWABLE MINIMUM $\mathcal{H}_\infty$ LEVEL $\gamma_{\min}$ FOR $\eta = 0.2$ WITH DIFFEREN VALUES OF $h$

| $h$ | 0.02 | 0.1 | 0.2 | 0.4 | 0.7 |
|---|---|---|---|---|---|
| $\gamma$ | 0.679 | 0.798 | 0.883 | 0.954 | 0.981 |

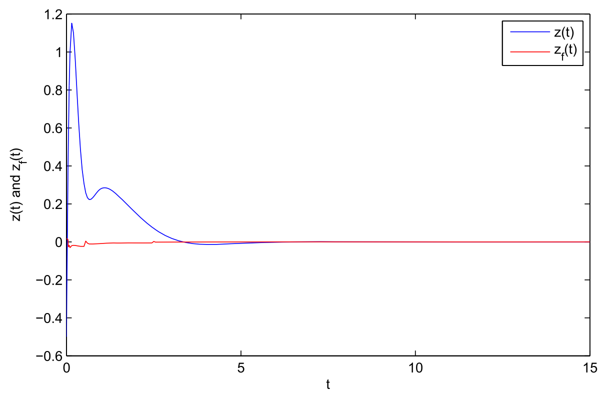


Fig. 4. $z(t)$ and its estimation $z_f(t)$ in the case of 2).

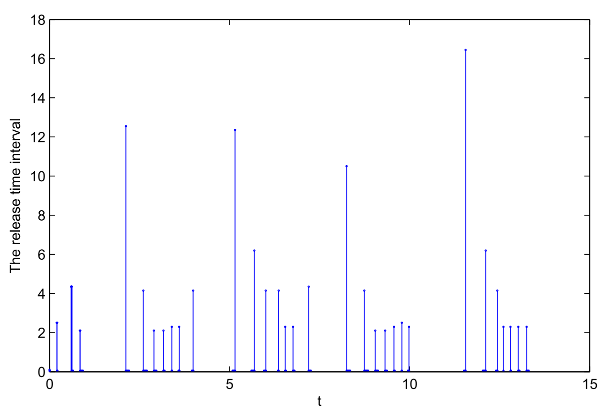


Fig. 5. Event-based release instants and release interval in the case of 2).

dissipative performance bound is computed as 0.717, and the corresponding filter matrices and $\Omega$ are given as follows:

$$\begin{aligned}
A_{f1} &= \begin{bmatrix} -3.7189 & 2.8251 \\ -1.3459 & -14.2167 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} 0.0385 \\ 0.3639 \end{bmatrix} \\
C_{f1} &= \begin{bmatrix} -0.5051 & 1.7312 \end{bmatrix}, \quad D_{f1} = -0.0490 \\
A_{f2} &= \begin{bmatrix} -8.6863 & -3.9287 \\ -1.3802 & -14.7656 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} 0.7409 \\ 0.2812 \end{bmatrix} \\
C_{f2} &= \begin{bmatrix} -0.4526 & 1.9681 \end{bmatrix}, \quad D_{f2} = -0.0288 \\
\Omega &= 0.0453.
\end{aligned}$$

Associate with the above event-triggered gain matrices, $z(t)$ and $z_f(t)$ are shown in Fig. 4 with the same initial condition and disturbance. Moreover, the release time intervals are given in Fig. 5. From these figures, the effectiveness of the designed filter is confirmed.

## V. Conclusion

The problem of event-triggered reliable dissipative filtering has been addressed for a class of nonlinear systems represented as T–S fuzzy models. A new event-triggered communication scheme including sensor failures has been proposed to reduce the utilization of network bandwidth. Under the event-triggered communication scheme, the filtering error system has been modeled as a time-delay system. Based on this model, a sufficient condition has been derived to guarantee the asymptotic stability of the filtering error systems and achieve strict $(Q, S, R) - \alpha$ dissipativity by LMI approach. A simulation example has been shown to demonstrate the effectiveness of the proposed method. In future, the distributed filtering for fuzzy systems is an interesting topic work that will be investigated, and the energy-efficient filtering algorithms in [37] would provide some insights to solve this problem. Moreover, future applications on the remote estimation of nonlinear circuits will be considered.

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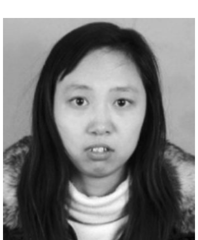

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