Nonfragile Exponential Synchronization of Delayed Complex Dynamical Networks With Memory Sampled-Data Control

Yajuan Liu, Bao-Zhu Guo, Ju H. Park, and Sang-Moon Lee

***Abstract*— This paper considers nonfragile exponential synchronization for complex dynamical networks (CDNs) with time-varying coupling delay. The sampled-data feedback control, which is assumed to allow norm-bounded uncertainty and involves a constant signal transmission delay, is constructed for the first time in this paper. By constructing a suitable augmented Lyapunov function, and with the help of introduced integral inequalities and employing the convex combination technique, a sufficient condition is developed, such that the nonfragile exponential stability of the error system is guaranteed. As a result, for the case of sampled-data control free of norm-bound uncertainties, some sufficient conditions of sampled-data synchronization criteria for the CDNs with time-varying coupling delay are presented. As the formulations are in the framework of linear matrix inequality, these conditions can be easily solved and implemented. Two illustrative examples are presented to demonstrate the effectiveness and merits of the proposed feedback control.**

***Index Terms*— Complex dynamical networks (CDNs), memory sampled-data control, nonfragile synchronization, time-varying coupling delay.**

I. Introduction

IN THE past few decades, much attention has been focused on the study of complex dynamical networks (CDNs) on account of the ubiquity of such real-word systems, such as the Internet, World Wide Web, food chain, scientific citation web, and neural networks, among many others [1]–[4]. CDNs are a set of interconnected nodes in which one node is a basic unit with specific contents or dynamics. As one of the most important collective behaviors, the synchronization problem has attracted unprecedented attention owing to its potential applications in biological systems, physics, communication, and traffic systems [5]–[8]. Various control schemes have been proposed to date to deal with the synchronization problem for CDNs. These include state observer-based control [9], adaptive control [10], impulsive control [11], and pinning control [12], [13]. In addition, time delay occurs commonly in many physical systems, and the existence of time delay may degrade the quality of system and even lead to oscillation, divergence, and instability [14]–[17]. It is, therefore, important to consider the effect of time delays in CDNs.

All the aforementioned works are using continuous-time feedback. In practical implementation, however, control strategy requires digital feedback [18], and the digital control takes merits in speed, small size, accuracy, and low cost during the control process for continuous-time systems. As a result, it is worthwhile to study sampled-data synchronization for CDNs. A crucial issue is that the variation of sampling periods may deteriorate synchronization for the controlled systems. Therefore, it is important to design a sampled-data control, so that the CDNs can be synchronized. In fact, the sampled-data control problem has received considerable attention in the past decades, and numerous results have been reported in the literature. In sampled-data control systems, the input delay approach [19], where the system is modeled as a continuous-time system with a time-varying sawtooth delay in the control input induced by a sampler-and-holder, is popular and has been widely used [20]–[24]. In the framework of the input delay approach [19], sampled-data synchronization for CDNs with time-varying coupling delay was studied in [25] and [26], and several delay-dependent sufficient conditions were developed to guarantee synchronization for CDNs. It should be pointed out that the available information on actual sampling patterns is neglected in [25] and [26], because the input delay induced by sampler-and-holder is simply treated as a bounded fast-varying delay. Therefore, the derived synchronization conditions are conservative to some extent. By introducing some information about actual sampling patterns, some improvements have been made in [27]–[29]. However, the information on the actual sampling pattern has not been fully used in [25]–[29], which may lead to certain degree of conservatism. Although some improved results were obtained in [30], a mistake occurs in constructing Lyapunov function $V(t)$ where P needs to be diagonal, because the existing of augmented vector in $V_1(t)$ and $V(t_k)$ should be no larger than $lim_{t\to t_k^{-1}} V(t_k)$. In this paper, we avoid this problem without using the augmented vector. In addition, stabilization for linear systems is investigated by memory sampled-data control, which means that the updating signal successfully

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transmitted from the sampler to the controller and to the zero-order holder (ZOH) at the instant t_k has experienced a constant signal transmission delay [21], [22]. Therefore, it is necessary to consider the memory sampled-data synchronization for CDNs with time-varying coupling delay, which comprises the first motivation for this paper. It should be also noted that the main difference between the memory and nonmemory controller is whether or not the updating signal successfully transmitted the signal from the sampler to the controller and ZOH at the instant t_k has experienced a constant signal transmission delay η.

In general, an implicit assumption inherent to the control design is that the controller should be implemented exactly. However, in practical situations, the exactly implemented controller struggles to meet the real requirements, because the inaccuracies or uncertainties occurring in controller implementation are inevitable in many industrial applications. Such uncertainties can be attributed to unexpected errors during the controller implementation, such as analog-to-digital and digital-to-analog conversion, round-off errors in numerical computation, and the aging of the components. The uncertainties occurring in the realization of the controller may also lead to deterioration of the performance or even the instability of closed-loop systems. Therefore, nonfragile control approaches that can consider uncertainties emerging in the controller realization have been investigated by many researchers [31]–[35]. However, to the best of our knowledge, a nonfragile sampled-data controller for the synchronization of CDNs with time-varying coupling delay has never been addressed, which is the second motivation for this paper.

In light of the reasons aforementioned, we focus, in this paper, on the design of sampled-data feedback control for CDNs with time-varying coupling delay. As opposed to the controller scheme proposed in [25]–[30], norm-bounded uncertainties and a constant signal transmission delay are considered in the designed sampled-data control. In order to make full use of the available information about the actual sampling pattern, a novel Lyapunov functional is proposed. Based on this modified Lyapunov function, the convex combination technique, and an improved inequality [36] that can provide a more accurate upper bound than Jensen's inequality for dealing with the cross-term, a new criterion is derived to ensure exponential stability for the synchronization error systems. The considered CDNs with time-varying coupling delay can be exponentially synchronized. Furthermore, when the sampled-data controller is free of norm-bound uncertainty, the sufficient conditions of sampled-data synchronization for CDNs with time-varying coupling delay are concluded. The results are formulated in the form of linear matrix inequalities (LMIs) that are easily solvable using standard software packages. Numerical examples are presented to illustrate the effectiveness and reduced conservatism of the proposed method.

Notations: The notations used in this paper are standard. I denotes the identity matrix with appropriate dimensions; $\mathbb{R}^n$ is the n-dimensional Euclidean space; $\mathbb{R}^{m\times n}$ denotes the set of all $m \times n$ real matrices; and $\|\cdot\|$ is the Euclidean norm for given vector. $*$ denotes the elements below the main diagonal of a symmetric block matrix. For symmetric matrices A and B, the notation $A > B$ (respectively, $A \geq B$) means that the matrix $A - B$ is positive definite (respectively, nonnegative), and $\lambda_M(\cdot)$ and $\lambda_m(\cdot)$ stand for the largest and smallest eigenvalues of a given square matrix, respectively. diag$\{\ldots\}$ is used to denote the block diagonal matrix.

II. Problem Statement

Consider the following CDN that consists of N coupled nodes of the form:

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} G_{ij}Ax_j(t-\tau(t)) + u_i(t) \tag{1}$$

where $x_i(t)$ is the state vector; $u_i(t)$ is control input of the node i; $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector-valued function; the scalar constant c denotes the coupling strength; $\tau(t)$ is the time-varying delay satisfying $0 \leq \tau(t) \leq \tau$, $\dot{\tau}(t) \leq \mu$, where $\tau > 0$ and μ are known constants; $A = (a_{ij})_{n\times n}$ is the constant inner-coupling matrix between two connected nodes, and $\mathcal{G} = (G_{ij})_{N\times N}$ is an outer-coupling configuration matrix, where G_{ij} is defined as follows. If there is a connection between node i and node j, then $G_{ij} > 0$; $G_{ij} = 0$, otherwise, and the diagonal elements of matrix G are defined by

$$G_{ii} = -\sum_{j=1, j\neq i}^{N} G_{ij} \quad (i = 1, 2, \ldots, N). \tag{2}$$

The continuous function $f : \mathbb{R}^n \to \mathbb{R}^n$ is a vector-valued function, and satisfies the following sector-bound condition [27]:

$$[f(x) - f(y) - U(x-y)]^\top \times [f(x) - f(y) - V(x-y)] \leq 0 \quad \forall x, y \in \mathbb{R}^n \tag{3}$$

where U and V are known constant matrices of the appropriate dimensions. The nonlinear description in (3) is very general, which includes the Lipschitz condition as a special case.

Let $r(t) = x_i(t) - s(t)$ be the error vector, where $s(t) \in \mathbb{R}^n$ is the state trajectory of the unforced isolated node $\dot{s}(t) = f(s(t))$. Then, the synchronization error of CDNs can be written as

$$\dot{r}_i(t) = g(r_i(t)) + c\sum_{j=1}^{N} G_{ij}Ar_j(t-\tau(t)) + u_i(t) \tag{4}$$

where $i = 1, 2, \ldots, N$ and $g(r_i(t)) = f(x_i(t)) - f(s(t))$.

Throughout this paper, it is supposed that only the measurements $r(t_k)$ at the sampling instant t_k are available, which are discrete measurements of $r(t)$, and the control signal is assumed to be generated by using a ZOH function with a sequence of holding times

$$0 = t_0 < t_1 < \cdots < \lim_{k\to+\infty} t_k = +\infty. \tag{5}$$

The sampling is not required to be periodic, but the distance between any two consecutive sampling instants is assumed to belong to an interval. Precisely, it is assumed that

$$t_{k+1} - t_k = h_k \leq h \tag{6}$$

for all $k \geq 0$, where $h > 0$.

Consider nonfragile memory sampled-data feedback control in the form of the following:

$$u_i(t) = (K_{1i} + \Delta K_{1i}(t_k))r(t_k) + (K_{2i} + \Delta K_{2i}(t_k))r(t_k - \eta), \quad t_k \le t < t_{k+1} \tag{7}$$

where K_{1i} and K_{2i} are appropriate dimensional control gain matrices to be determined later, and η is a constant signal transmission delay.

The uncertainties $\Delta K_{1i}(t_k)$ and $\Delta K_{2i}(t_k)$ represent the possible controller gain fluctuations. It is assumed that $\Delta K_{1i}(t_k)$ and $\Delta K_{2i}(t_k)$ have the following form:

$$[\Delta K_{1i}(t_k), \Delta K_{2i}(t_k)] = D_i \Delta(t_k)[E_{ai}, E_{bi}] \tag{8}$$

where D_i, E_{ai}, and E_{bi} are known constant matrices with appropriate dimensions, and $\Delta(t_k)$ is an unknown matrix function satisfying $\Delta^\top(t_k)\Delta(t_k) \le I$.

Remark 1: In practical applications, the parameter perturbations are unavoidable, which influence stability and performance of the system if they are not treated appropriately. The nonfragile sampled-data control designed for the synchronization of CDNs is to deal with such perturbations. As opposed to the control scheme proposed in [25]–[29], a constant signal transmission delay η introduced in [21] and [22] is also considered for the first time for the synchronization of CDNs in this paper.

Here, in order to have a simple structure of system equations, we use the Kronecker product to write system (4) as

$$\begin{aligned} \dot{r}(t) &= \bar{g}(r(t)) + c(G \otimes A)r(t - \tau(t)) \\ &\quad + K_1 r(t_k) + K_2 r(t_k - \eta) + Dp(t_k) \\ p(t_k) &= \Delta(t_k) q(t_k) \\ q(t_k) &= E_a r(t_k) + E_b r(t_k - \eta) \end{aligned} \tag{9}$$

where

$$\begin{aligned} r(t) &= \left[r_1^\top(t), r_2^\top(t), \ldots, r_N^\top(t)\right]^\top \\ \bar{g}(r(t)) &= [g^\top(r_1(t)), g^\top(r_2(t)), \ldots, g^\top(r_N(t))]^\top \\ K_1 &= \operatorname{diag}\{K_{11}, K_{12}, \ldots, K_{1N}\} \\ K_2 &= \operatorname{diag}\{K_{21}, K_{22}, \ldots, K_{2N}\} \\ D &= \operatorname{diag}\{D_1, D_2, \ldots, D_N\} \\ E_a &= \operatorname{diag}\{E_{a1}, E_{a2}, \ldots, E_{aN}\} \\ E_b &= \operatorname{diag}\{E_{b1}, E_{b2}, \ldots, E_{bN}\}. \end{aligned}$$

The following definition and lemmas play crucial roles in the proof of the main results.

Definition 1 [27]: The CDN (1) is said to be exponentially synchronized if the error dynamic system (9) is exponentially stable, i.e., there are two constants $\alpha, \beta > 0$, such that

$$\|r(t)\| \le \beta e^{-\alpha t} \sup_{-b \le \theta \le 0} \{\|r(\theta)\|, \|\dot{r}(\theta)\|\} \tag{10}$$

where $b = \max\{\tau, \eta\}$, and α and β are the decay rate and the decay coefficient, respectively.

Lemma 1 [25]: For any matrix $M \in \mathbb{R}^{n\times n}, M = M^T > 0$ and $\beta \le s \le \alpha$, the following inequality holds:

$$-(\alpha - \beta)\int_\beta^\alpha \dot{x}^\top(s) M \dot{x}(s) ds \le -[x(\alpha) - x(\beta)]^\top M [x(\alpha) - x(\beta)].$$

Lemma 2 (Lower Bounds Lemma [37]): Let $f_1, f_2, \ldots, f_N : \mathbb{R}^m \to R$ be in an open subset D of $\mathbb{R}^m$. Then, the reciprocally convex combination of f_i over D satisfies

$$\min_{\{\alpha_i | \alpha_i > 0, \sum_i \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{ij}} \sum_{i \ne j} g_{ij}(t)$$

subject to

$$\left\{ g_{ij} : \mathbb{R}^m \to \mathbb{R}, g_{j,i}(t) \triangleq g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \ge 0 \right\}.$$

Lemma 3 [36]: For any positive definite matrix R and continuously differentiable function $x(t)$ in $[a, b] \in \mathbb{R}^n$, the following inequality holds:

$$-(b - a)\int_a^b \dot{x}^\top(s) R \dot{x}(s) ds \le -[x(b) - x(a)]^T R [x(b) - x(a)] - 3\Omega^\top R \Omega \tag{11}$$

where $\Omega = x(b) + x(a) - (2/(b - a))\int_a^b x(s)ds$.

Lemma 4 [22]: Let $z \in W[a, b)$ and $z(a) = 0$. Then, for any $n \times n$ matrix $R > 0$, the following inequality holds:

$$\int_a^b z^\top(s) R z(s) ds \le \frac{4(b - a)^2}{\pi^2} \int_a^b \dot{z}^\top(s) R \dot{z}(s) ds.$$

Our final objective is to design a nonfragile memory sampled-data control of (7), such that the error system (9) is exponentially stable. Therefore, CDN (1) is exponentially synchronized.

III. Main Results

In this section, we first establish the nonfragile exponential stability for error system (9), and several sufficient conditions will be presented to ensure stability of the system and to synthesize the memory sampled-data control of the form (7).

For the sake of simplicity, we use $I_i \in \mathbb{R}^{10n\times n}$ $(i = 1, 2, \ldots, 10)$ to denote block entry matrices, (such as $I_4 = [0, 0, 0, I, 0, 0, 0, 0, 0, 0]^\top$). The notations of others are defined as

$$\begin{aligned} \bar{U} &= \frac{(I \otimes U)^\top (I \otimes V)}{2} + \frac{(I \otimes V)^\top (I \otimes U)}{2} \\ \bar{V} &= -\frac{(I \otimes U)^\top + (I \otimes V)^\top}{2} \\ \upsilon(t) &= \frac{1}{t - t_k}\int_{t_k}^t r(s)ds \\ \beta^\top(t) &= [r^\top(t), r^\top(t_k), r^\top(t_k - \eta)] \\ \gamma^\top(t) &= [\dot{r}^\top(t), 0, 0] \\ \xi^\top(t) &= [r^\top(t), r^\top(t - \tau(t)), r^\top(t - \tau), r^\top(t_k), \upsilon^\top(t), \\ &\quad r^\top(t_k - \eta), r^\top(t - \eta), \bar{g}^\top(r(t)), \dot{r}^\top(t), p^\top(t_k)] \\ \Phi &= [0, cF(G \otimes A), 0, H_1, 0, H_2, 0, F, -F, FD] \\ \Psi &= \varepsilon_2\big(I_4 E_a^\top E_a I_4^\top + I_4 E_a^\top E_b I_6^\top + I_6 E_b^\top E_a I_4^\top \\ &\quad + I_6 E_b^\top E_b I_6^\top - I_{10} I_{10}^\top\big) \\ W_1 &= [I, 0, 0, -I, 0, 0, 0, 0, 0, 0] \\ W_2 &= [I, 0, 0, I, -2I, 0, 0, 0, 0, 0] \\ \tilde{R} &= [R_1, 0, 0, R_2, 0, R_3, 0, 0, 0, 0]^\top \\ \Pi &= [I_1, I_2, I_3] \end{aligned}$$

$$
\begin{aligned}
\Sigma_1 =\ & 2\alpha I_1 P I_1^\top + I_1 P I_9^\top + I_9 P I_1^\top + I_1(Q_1+Q_2)I_1^\top \\
& -(1-\mu)e^{-2\alpha\tau} I_2 Q_1 I_2^\top - e^{-2\alpha\tau} I_3 Q_2 I_3^\top \\
& +\tau^2 I_9 Q_3 I_9^\top + \eta^2 I_9 Q_4 I_9^\top \\
& -e^{-2\alpha\eta}[I_1 - I_7]Q_4[I_1 - I_7]^\top \\
& +e^{-2\alpha\tau}\Pi \begin{bmatrix} -Q_3 & Q_3 - T & T \\ * & -2Q_3 + T + T^\top & Q_3 - T \\ * & * & -Q_3 \end{bmatrix}\Pi^\top \\
& -e^{-2\alpha h}[I_1 - I_4]M_2 I_4^\top - e^{-2\alpha h} I_4 M_2^\top [I_1 - I_4]^\top \\
& -e^{-2\alpha h} Y_1^\top W_1 - e^{-2\alpha h} W_1^\top Y_1 - 3e^{-2\alpha h} Y_2^\top W_2 \\
& -3e^{-2\alpha h} W_2^\top Y_2 + \alpha h^2 [I_1, I_4, I_6] R [I_1, I_4, I_6]^\top \\
& + \frac{h^2}{2} I_9 X I_9^\top + \alpha h^2 I_5 S I_5^\top - h I_5 S I_5^\top \\
& + h^2[\alpha(I_1 - I_6) + I_9] W [\alpha(I_1 - I_6) + I_9]^\top \\
& - \frac{\pi^2}{4} e^{-2\alpha\eta}[I_7 - I_6] W [I_7 - I_6]^\top \\
& - \varepsilon_1 [I_1, I_8] \begin{bmatrix} \bar{U} & \bar{V} \\ * & I \end{bmatrix} [I_1, I_8]^\top + \Psi \\
& +(I_1 + \gamma_1 I_9 + \gamma_2 I_6)\Phi + \Phi^\top (I_1 + \gamma_1 I_9 + \gamma_2 I_6)^\top \\
\Sigma_2 =\ & -e^{-2\alpha h} I_4 M_3 I_4^\top - [I_1, I_4, I_6] R [I_1, I_4, I_6]^\top \\
\Sigma_3 =\ & [I_9, I_4] M [I_9, I_4]^\top + [I_1, I_4, I_6] R [I_1, I_4, I_6]^\top \\
& + I_1 S I_5^\top + I_5 S I_1^\top.
\end{aligned}
$$

Theorem 1: For given any scalars μ, γ_1, γ_2, positive constants h and τ, and diagonal matrices

$$
\begin{aligned}
D &= \operatorname{diag}\{D_1, D_2, \ldots, D_N\} \\
E_a &= \operatorname{diag}\{E_{a1}, E_{a2}, \ldots, E_{aN}\} \\
E_b &= \operatorname{diag}\{E_{b1}, E_{b2}, \ldots, E_{bN}\}
\end{aligned}
$$

the error system (9) is exponentially stable with the decay rate α if there exist matrices $P > 0$, $Q_i > 0$ $(i = 1, 2, 3, 4)$

$$
R = \begin{bmatrix} R_1 & R_2 & R_3 \\ * & R_4 & R_5 \\ * & * & R_6 \end{bmatrix} > 0, \quad M = \begin{bmatrix} M_1 & M_2 \\ * & M_3 \end{bmatrix} > 0
$$

$S > 0$, $W > 0$, $X > 0$, T, Y_1, Y_2, $F = \operatorname{diag}\{F_1, F_2, \ldots, F_N\}$, $H_1 = \operatorname{diag}\{H_{11}, H_{12}, \ldots, H_{1N}\}$, and $H_2 = \operatorname{diag}\{H_{21}, H_{22}, \ldots, H_{2N}\}$ with appropriate dimensions, and scalars $\varepsilon_1, \varepsilon_2 > 0$ satisfying the following LMIs:

$$
\begin{bmatrix} \Sigma_1 + h\Sigma_3 & \frac{h^2}{2}\tilde{R} \\ * & -\frac{h^2}{2}X \end{bmatrix} < 0 \quad (12)
$$

$$
\begin{bmatrix} \Sigma_1 + h\Sigma_2 & hY_1 & 3hY_2 & \frac{h^2}{2}\tilde{R} \\ * & -hM_1 & 0 & 0 \\ * & * & -3hM_1 & 0 \\ * & * & * & -\frac{h^2}{2}X \end{bmatrix} < 0 \quad (13)
$$

$$
\begin{bmatrix} Q_3 & T \\ * & Q_3 \end{bmatrix} > 0. \quad (14)
$$

Furthermore, the desired control gains are given by

$$
K_{1i} = F_i^{-1} H_{1i}, \quad K_{2i} = F_i^{-1} H_{2i}, \quad i = 1, 2, \ldots, N.
$$

Proof: We choose the following Lyapunov functional candidate:

$$
V(t) = \sum_{i=1}^{9} V_i(t), \quad t \in [t_k, t_{k+1}) \quad (15)
$$

where

$$
\begin{aligned}
V_1(t) &= e^{2\alpha t} r^\top(t) P r(t) \\
V_2(t) &= \int_{t-\tau(t)}^{t} e^{2\alpha s} r^\top(s) Q_1 r(s) ds \\
V_3(t) &= \int_{t-\tau}^{t} e^{2\alpha s} r^\top(s) Q_2 r(s) ds \\
V_4(t) &= \tau \int_{-\tau}^{0}\int_{t+\alpha}^{t} e^{2\alpha s} \dot{r}^\top(s) Q_3 \dot{r}(s) ds d\alpha \\
V_5(t) &= \eta \int_{-\eta}^{0}\int_{t+\alpha}^{t} e^{2\alpha s} \dot{r}^\top(s) Q_4 \dot{r}(s) ds d\alpha \\
V_6(t) &= (h - h_k(t)) \int_{t_k}^{t} e^{2\alpha s} \begin{bmatrix} \dot{r}(s) \\ r(t_k) \end{bmatrix}^\top M \begin{bmatrix} \dot{r}(s) \\ r(t_k) \end{bmatrix} ds \\
V_7(t) &= (h - h_k(t)) h_k(t) e^{2\alpha t} \beta^\top(t) R \beta(t) \\
V_8(t) &= (h - h_k(t)) h_k(t) e^{2\alpha t} \upsilon^\top(t) S \upsilon(t) \\
V_9(t) &= h^2 \int_{t_k-\eta}^{t} e^{2\alpha s} \bar{r}^\top(s) W \bar{r}(s) ds \\
& \quad - \frac{\pi^2}{4} \int_{t_k-\eta}^{t-\eta} e^{2\alpha s} (r(s) - r(t_k - \eta))^\top W \\
& \quad \times (r(s) - r(t_k - \eta)) ds.
\end{aligned}
$$

with $h_k(t) = t - t_k$, $\bar{r}(t) = \alpha(r(t) - r(t_k - \eta)) + \dot{r}(t)$.

The time derivative of $V_1(t)$, $V_2(t)$, and $V_3(t)$ can be calculated as

$$
\dot{V}_1(t) = 2\alpha e^{2\alpha t} r^\top(t) P r(t) + 2e^{2\alpha t} r^\top(t) P \dot{r}(t) \quad (16)
$$

$$
\begin{aligned}
\dot{V}_2(t) \le\ & e^{2\alpha t}\{r^\top(t) Q_1 r(t) - (1-\mu) e^{-2\alpha\tau} \\
& \times r^\top(t - \tau(t)) Q_1 r(t - \tau(t))\} \quad (17)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_3(t) \le\ & e^{2\alpha t}\{r^\top(t) Q_2 r(t) - e^{-2\alpha\tau} \\
& \times r^\top(t-\tau) Q_2 r(t-\tau)\}. \quad (18)
\end{aligned}
$$

In view of Lemmas 1 and 2, an upper bound of $\dot{V}_4(t)$ is estimated as

$$
\begin{aligned}
\dot{V}_4(t) \le\ & e^{2\alpha t}\left\{\tau^2 \dot{r}^\top(t) Q_3 \dot{r}(t) - \tau e^{-2\alpha\tau} \int_{t-\tau}^{t} \dot{r}^\top(s) Q_3 \dot{r}(s) ds\right\} \\
\le\ & e^{2\alpha t}\left\{\tau^2 \dot{r}^\top(t) Q_3 \dot{r}(t) - \tau e^{-2\alpha\tau} \int_{t-\tau(t)}^{t} \dot{r}^\top(s) Q_3 \dot{r}(s) ds - \tau e^{-2\alpha\tau} \int_{t-\tau}^{t-\tau(t)} \dot{r}^\top(s) Q_3 \dot{r}(s) ds\right\} \\
\le\ & e^{2\alpha t}\{\tau^2 \dot{r}^\top(t) Q_3 \dot{r}(t) + e^{-2\alpha\tau} \mathcal{X}^\top(t) \mathcal{Q}_3 \mathcal{X}(t)\}
\end{aligned}
$$

where $\mathcal{X}(t) = [r^\top(t), r^\top(t - \tau(t)), r^\top(t - \tau)]^\top$

$$
\mathcal{Q}_3 = \begin{bmatrix} -Q_3 & Q_3 - T & T \\ * & -2Q_3 + T + T^\top & Q_3 - T \\ * & * & -Q_3 \end{bmatrix}.
$$

Based on Lemma 1, an upper bound of $\dot{V}_5(t)$ is given by

$$\begin{aligned}\dot{V}_5(t) &\leq e^{2\alpha t}\Big\{\eta^2\dot{r}^\top(t)Q_4\dot{r}(t)\\&\qquad -\eta e^{-2\alpha\eta}\int_{t-\eta}^{t}\dot{r}^\top(s)Q_4\dot{r}(s)ds\Big\}\\&\leq e^{2\alpha t}\Big\{\eta^2\dot{r}^\top(t)Q_4\dot{r}(t)-e^{-2\alpha\eta}[r(t)-r(t-\eta)]^\top\\&\qquad\times Q_4[r(t)-r(t-\eta)]\Big\}.\end{aligned}$$

Calculating the time-derivative of $V_6(t)$ leads to

$$\begin{aligned}\dot{V}_6(t) &= -\int_{t_k}^{t}e^{2\alpha s}\begin{bmatrix}\dot{r}(s)\\ r(t_k)\end{bmatrix}^\top M\begin{bmatrix}\dot{r}(s)\\ r(t_k)\end{bmatrix}ds\\&\quad+(h-h_k(t))e^{2\alpha t}\begin{bmatrix}\dot{r}(t)\\ r(t_k)\end{bmatrix}^\top M\begin{bmatrix}\dot{r}(t)\\ r(t_k)\end{bmatrix}\\&\leq -e^{2\alpha t}e^{-2\alpha h}\int_{t_k}^{t}\begin{bmatrix}\dot{r}(s)\\ r(t_k)\end{bmatrix}^\top M\begin{bmatrix}\dot{r}(s)\\ r(t_k)\end{bmatrix}ds\\&\quad+(h-h_k(t))e^{2\alpha t}\begin{bmatrix}\dot{r}(t)\\ r(t_k)\end{bmatrix}^\top M\begin{bmatrix}\dot{r}(t)\\ r(t_k)\end{bmatrix}\\&= e^{2\alpha t}\Big\{-e^{-2\alpha h}\int_{t_k}^{t}\dot{r}^\top(s)M_1\dot{r}(s)ds\\&\qquad-2e^{-2\alpha h}r^\top(t_k)M_2^\top[r(t)-r(t_k)]\\&\qquad-(t-t_k)e^{-2\alpha h}r^\top(t_k)M_3r(t_k)\\&\qquad+(h-(t-t_k))\begin{bmatrix}\dot{r}(t)\\ r(t_k)\end{bmatrix}^\top M\begin{bmatrix}\dot{r}(t)\\ r(t_k)\end{bmatrix}\Big\}.\end{aligned}$$

Using Lemma 3, one can obtain

$$\begin{aligned}&-\int_{t_k}^{t}\dot{r}^\top(s)M_1\dot{r}(s)ds\\&\quad\leq\frac{1}{t-t_k}\xi^\top(t)\big[W_1^\top M_1W_1+3W_2^\top M_1W_2\big]\xi(t).\end{aligned}\tag{19}$$

Because, for matrices $Y_i(i=1,2)$, $(1/(t-t_k))(M_1W_i-(t-t_k)Y_i)^\top M_1^{-1}(M_1W_i-(t-t_k)Y_i)\geq 0$, we have

$$-\frac{1}{t-t_k}W_i^\top M_1W_i\leq -Y_i^\top W_i-W_i^\top Y_i+(t-t_k)Y_i^\top M_1^{-1}Y_i.\tag{20}$$

Calculating $\dot{V}_7(t)$ gives

$$\begin{aligned}\dot{V}_7(t) &= 2\alpha(h-h_k(t))h_k(t)e^{2\alpha t}\beta^\top(t)R\beta(t)\\&\quad-e^{2\alpha t}h_k(t)\beta^\top(t)R\beta(t)\\&\quad+(h-h_k(t))e^{2\alpha t}\beta^\top(t)R\beta(t)\\&\quad+2(h-h_k(t))h_k(t)e^{2\alpha t}\beta^\top(t)R\gamma(t)\\&\leq e^{2\alpha t}\{\alpha h^2\beta^\top(t)R\beta(t)-h_k(t)\beta^\top(t)R\beta(t)\\&\quad+2(h-h_k(t))h_k(t)\xi^\top(t)\tilde{R}\dot{r}(t)\\&\quad+(h-h_k(t))\beta^\top(t)R\beta(t)\}.\end{aligned}$$

For any positive matrix X, it is easy to obtain

$$\begin{aligned}&2(h-h_k(t))h_k(t)\xi^\top(t)\tilde{R}\dot{r}(t)\\&\quad\leq\frac{h^2}{2}\Big(\xi^\top(t)\tilde{R}X^{-1}\tilde{R}^\top\xi(t)+\dot{r}^\top(t)X\dot{r}(t)\Big).\end{aligned}$$

Finding the time-derivative of $V_8(t)$ and $V_9(t)$ yields

$$\begin{aligned}\dot{V}_8(t) &\leq e^{2\alpha t}\{\alpha h^2 \upsilon^\top(t)S\upsilon(t)-h\upsilon^\top(t)S\upsilon(t)\\&\qquad+2(h-h_k(t))\upsilon^\top(t)Sr(t)\}\\\dot{V}_9(t) &= e^{2\alpha t}\Big\{h^2\bar{r}^\top(t)W\bar{r}(t)-\frac{\pi^2}{4}e^{-2\alpha\eta}\\&\qquad\times(r(t-\eta)-r(t_k-\eta))^\top W(r(t-\eta)-r(t_k-\eta))\Big\}.\end{aligned}$$

From (3), it follows that:

$$\varepsilon_1e^{2\alpha t}[g(r_i(t))-Ur_i(t)]^\top[g(r_i(t))-Vr_i(t)]\leq 0\tag{21}$$

which is equivalent to

$$-\varepsilon_1e^{2\alpha t}\begin{bmatrix}r(t)\\ \bar{g}(r(t))\end{bmatrix}^\top\begin{bmatrix}\bar{U} & \bar{V}\\ * & I\end{bmatrix}\begin{bmatrix}r(t)\\ \bar{g}(r(t))\end{bmatrix}\geq 0.\tag{22}$$

By (8)

$$p^\top(t_k)p(t_k)\leq q^\top(t_k)q(t_k)\tag{23}$$

and so there exists a positive scalar ε_2 satisfying

$$\begin{aligned}&\varepsilon_2e^{2\alpha t}[q^\top(t_k)q(t_k)-p^\top(t_k)p(t_k)]\\&\quad=e^{2\alpha t}\xi^\top(t)\Psi\xi(t)\geq 0.\end{aligned}$$

From (9), it is easy to know that

$$\begin{aligned}&2e^{2\alpha t}[r^\top(t)F+\gamma_1\dot{r}^\top(t)F+\gamma_2r^\top(t-\eta_k)F][-\dot{r}(t)\\&\quad+c(G\otimes A)r(t-\tau(t))+\bar{g}(r(t))+K_1r(t_k)\\&\quad+K_2r(t_k-\eta)+Dp(t_k)]=0.\end{aligned}\tag{24}$$

Define $FK_1=H$ and $FK_2=L$. Then

$$\dot{V}(t)\leq e^{2\alpha t}\xi^\top(t)\Sigma\xi(t)\tag{25}$$

where $\Sigma=\Sigma_1+(h^2/2)\tilde{R}X^{-1}\tilde{R}^\top+(t-t_k)(\Sigma_2+Y_1^\top M_1^{-1}Y_1+3Y_2^\top M_1^{-1}Y_2)+(h-(t-t_k))\Sigma_3$. As Σ is a convex combination of $t-t_k$ and $h_k-(t-t_k)$, $\Sigma<0$ if and only if (12) and the following inequality holds:

$$\Sigma_1+\frac{h^2}{2}\tilde{R}X^{-1}\tilde{R}^\top+h\Sigma_3<0\tag{26}$$

$$\begin{aligned}&\Sigma_1+h\big(\Sigma_2+Y_1^\top M_1^{-1}Y_1+3Y_2^\top M_1^{-1}Y_2\big)\\&\quad+\frac{h^2}{2}\tilde{R}X^{-1}\tilde{R}^\top<0.\end{aligned}\tag{27}$$

By virtue of the Schur complement, (26) is equivalent to (12), and (27) is equivalent to (13). We then obtain from (12) and (13) that

$$\dot{V}(t)\leq 0,\quad t\in[t_k,t_{k+1}).\tag{28}$$

It follows that for $t\in[t_k,t_{k+1})$:

$$V(t)\leq V(t_k)\leq V(t_{k-1})\leq\cdots\leq V(0).\tag{29}$$

As $V_i(0) = 0 (i = 6, 7, 8)$, we can obtain that

$$\begin{aligned} V(0) &= r^\top(0)Pr(0) + \int_{-\tau(0)}^{0} e^{2\alpha s} r^\top(s)Q_1 r(s)ds \\ &\quad + \int_{-\tau}^{0} e^{2\alpha s} r^\top(s)Q_2 r(s)ds \\ &\quad + \tau \int_{-\tau}^{0}\int_{\alpha}^{0} e^{2\alpha s}\dot{r}^\top(s)Q_3\dot{r}(s)dsd\alpha \\ &\quad + \eta \int_{-\eta}^{0}\int_{\alpha}^{0} e^{2\alpha s}\dot{r}^\top(s)Q_4\dot{r}(s)dsd\alpha \\ &\quad + h^2 \int_{-\eta}^{0} e^{2\alpha s}\bar{r}^\top(s)W\bar{r}(s)ds \\ &\le \lambda_{\max}(P)\|r(0)\|^2 + \tau\lambda_{\max}(Q_1)\sup_{-b\le\theta\le 0}\{\|r(\theta)\|^2\} \\ &\quad + \tau\lambda_{\max}(Q_2)\sup_{-b\le\theta\le 0}\{\|r(\theta)\|^2\} \\ &\quad + (\tau^3\lambda_{\max}(Q_3) + \eta^3\lambda_{\max}(Q_4))\sup_{-b\le\theta\le 0}\{\|\dot{r}(\theta)\|^2\} \\ &\quad + 3a\eta h^2\lambda_{\max}(W)\left(\sup_{-b\le\theta\le 0}\{\|r(\theta)\|, \|\dot{r}(\theta)\|\}\right)^2 \\ &\le \gamma\left(\sup_{-b\le\theta\le 0}\{\|r(\theta)\|, \|\dot{r}(\theta)\|\}\right)^2 \end{aligned} \tag{30}$$

where $a = 6\alpha^2 + 3$ and $\gamma = \lambda_{\max}(P) + \tau(\lambda_{\max}(Q_1) + \lambda_{\max}(Q_2)) + \tau^3\lambda_{\max}(Q_3) + \eta^3\lambda_{\max}(Q_4) + 3a\eta h^2\lambda_{\max}(W)$. On the other hand

$$V(t) \ge e^{2\alpha t}\lambda_{\min}(P)\|r(t)\|^2. \tag{31}$$

By (30) and (31), it follows that:

$$\|r(t)\| \le \sqrt{\frac{\gamma}{\lambda_{\min}(P)}} e^{-\alpha t}\sup_{-b\le\theta\le 0}\{\|r(\theta)\|, \|\dot{r}(\theta)\|\}. \tag{32}$$

In terms of Definition 1, system (9) is exponentially synchronous with the decay rate α. This completes the proof of the theorem. ■

When there is no uncertainty involved, control (7) and the error system of (9) are reduced to

$$u_i(t) = K_{1i}r(t_k) + K_{2i}r(t_k - \eta), t_k \le t < t_{k+1} \tag{33}$$

$$\begin{aligned} \dot{r}(t) &= \bar{g}(r(t)) + c(G\otimes A)r(t - \tau(t)) \\ &\quad + K_1 r(t_k) + K_2 r(t_k - \eta). \end{aligned} \tag{34}$$

By Theorem 1, we have Corollary 1 on the synchronization of CDNs with memory sampled-data control. Here, for notational simplicity, we use $\bar{I}_i \in \mathbb{R}^{9n\times n}$ $(i = 1, 2, \ldots, 9)$ to denote block entry matrices (for example $\bar{I}_4 = [0, 0, 0, I, 0, 0, 0, 0, 0]^\top$). The other notations are defined as

$$\begin{aligned} \bar{\xi}^\top(t) &= [r^\top(t), r^\top(t-\tau(t)), r^\top(t-\tau), r^\top(t_k), \upsilon^\top(t) \\ &\qquad r^\top(t_k-\eta), r^\top(t-\eta), \bar{g}^\top(r(t)), \dot{r}^\top(t)] \\ \bar{\Phi} &= [0, cF(G\otimes A), 0, H_1, 0, H_2, 0, F, -F] \\ \bar{W}_1 &= [I, 0, 0, -I, 0, 0, 0, 0, 0] \\ \bar{W}_2 &= [I, 0, 0, I, -2I, 0, 0, 0, 0] \\ \bar{R} &= [R_1, 0, 0, R_2, 0, R_3, 0, 0, 0]^\top \\ \bar{\Pi} &= [\bar{I}_1, \bar{I}_2, \bar{I}_3] \\ \bar{\Sigma}_1 &= 2\alpha\bar{I}_1 P\bar{I}_1^\top + \bar{I}_1 P\bar{I}_9^\top + \bar{I}_9 P\bar{I}_1^\top + \bar{I}_1(Q_1+Q_2)\bar{I}_1^\top \\ &\quad - (1-\mu)e^{-2\alpha\tau}\bar{I}_2 Q_1\bar{I}_2^\top - e^{-2\alpha\tau}\bar{I}_3 Q_2\bar{I}_3^\top + \tau^2\bar{I}_9 Q_3\bar{I}_9^\top \\ &\quad + \eta^2\bar{I}_9 Q_4\bar{I}_9^\top - e^{-2\alpha\eta}[\bar{I}_1 - \bar{I}_7]Q_4[\bar{I}_1 - \bar{I}_7]^\top \\ &\quad + e^{-2\alpha\tau}\bar{\Pi}\begin{bmatrix} -Q_3 & Q_3 - T & T \\ * & -2Q_3 + T + T^\top & Q_3 - T \\ * & * & -Q_3 \end{bmatrix}\bar{\Pi}^\top \\ &\quad - e^{-2\alpha h}[\bar{I}_1 - \bar{I}_4]M_2\bar{I}_4^\top - e^{-2\alpha h}\bar{I}_4 M_2^\top[\bar{I}_1 - \bar{I}_4]^\top \\ &\quad - e^{-2\alpha h}Y_1^\top\bar{W}_1 - e^{-2\alpha h}\bar{W}_1^\top Y_1 - 3e^{-2\alpha h}Y_2^\top\bar{W}_2 \\ &\quad - 3e^{-2\alpha h}\bar{W}_2^\top Y_2 + \alpha h^2[\bar{I}_1, \bar{I}_4, \bar{I}_6]R[\bar{I}_1, \bar{I}_4, \bar{I}_6]^\top \\ &\quad + \frac{h^2}{2}\bar{I}_9 X\bar{I}_9^\top + \alpha h^2\bar{I}_5 S\bar{I}_5^\top - h\bar{I}_5 S\bar{I}_5^\top \\ &\quad + h^2[\alpha(\bar{I}_1 - \bar{I}_6) + \bar{I}_9]W[\alpha(\bar{I}_1 - \bar{I}_6) + \bar{I}_9]^\top \\ &\quad - \frac{\pi^2}{4}e^{-2\alpha\eta}[\bar{I}_7 - \bar{I}_6]W[\bar{I}_7 - \bar{I}_6]^\top \\ &\quad - \varepsilon_1[\bar{I}_1, \bar{I}_8]\begin{bmatrix} \bar{U} & \bar{V} \\ * & I \end{bmatrix}[\bar{I}_1, \bar{I}_8]^\top \\ &\quad + (\bar{I}_1 + \gamma_1\bar{I}_9 + \gamma_2\bar{I}_6)\bar{\Phi} + \bar{\Phi}^\top(\bar{I}_1 + \gamma_1\bar{I}_9 + \gamma_2\bar{I}_6)^\top \\ \bar{\Sigma}_2 &= -e^{-2\alpha h}\bar{I}_4 M_3\bar{I}_4^\top - [\bar{I}_1, \bar{I}_4, \bar{I}_6]R[\bar{I}_1, \bar{I}_4, \bar{I}_6]^\top \\ \bar{\Sigma}_3 &= [\bar{I}_9, \bar{I}_4]M[\bar{I}_9, \bar{I}_4]^\top + [\bar{I}_1, \bar{I}_4, \bar{I}_6]R[\bar{I}_1, \bar{I}_4, \bar{I}_6]^\top \\ &\quad + I_1 S I_5^\top + I_5 S I_1^\top. \end{aligned}$$

Corollary 1: For given any scalars μ, γ_1, γ_2 and positive constants h and τ, the error system (34) is exponentially stable with the decay rate α if there exist matrices $P > 0$, $Q_i > 0$ $(i = 1, 2, 3, 4)$

$$R = \begin{bmatrix} R_1 & R_2 & R_3 \\ * & R_4 & R_5 \\ * & * & R_6 \end{bmatrix} > 0, \quad M = \begin{bmatrix} M_1 & M_2 \\ * & M_3 \end{bmatrix} > 0,$$

$S > 0$, $W > 0$, $X > 0$, T, Y_1, Y_2, $F = \mathrm{diag}\{F_1, F_2, \ldots, F_N\}$, $H_1 = \mathrm{diag}\{H_{11}, H_{12}, \ldots, H_{1N}\}$, and $H_2 = \mathrm{diag}\{H_{21}, H_{22}, \ldots, H_{2N}\}$ with appropriate dimensions and a scalar $\varepsilon_1 > 0$ satisfying the following LMIs:

$$\begin{bmatrix} \bar{\Sigma}_1 + h\bar{\Sigma}_3 & \frac{h^2}{2}\bar{R} \\ * & -\frac{h^2}{2}X \end{bmatrix} < 0 \tag{35}$$

$$\begin{bmatrix} \bar{\Sigma}_1 + h\bar{\Sigma}_2 & hY_1 & 3hY_2 & \frac{h^2}{2}\bar{R} \\ * & -hM_1 & 0 & 0 \\ * & * & -3hM_1 & 0 \\ * & * & * & -\frac{h^2}{2}X \end{bmatrix} < 0 \tag{36}$$

$$\begin{bmatrix} Q_3 & T \\ * & Q_3 \end{bmatrix} > 0. \tag{37}$$

Furthermore, the desired control gains are given as

$$K_{1i} = F_i^{-1}H_{1i}, \quad K_{2i} = F_i^{-1}H_{2i}, \quad i = 1, 2, \ldots, N.$$

Remark 2: For sampled-data synchronization of CDNs with time-varying delay, the choice of the sampling interval is very important for designing suitable feedback control. It is obvious that a longer sampling period will result in lower communication channel occupation, fewer actuations of the controller, and less signal transmission [24]. To achieve this objective, the memory sampled-data controller is employed here, which may provide a larger sampling period than that from the conventional sampled-data controllers proposed in [25]–[30].

Remark 3: Unlike the Lyapunov–Krasovskii function constructed in [25]–[29], $V_7(t)$, $V_8(t)$, and $V_9(t)$ have more information with the actual sampling pattern. In the proof of Theorem 1, an improved integral inequality [36] that provides more accuracy than those based on Jensen's inequality is employed to estimate the derivative of the Lyapunov function $V_6(t)$.

Remark 4: When $K_2 = 0$ in control (33), the control scheme for the synchronization of delayed CDNs is reduced to the conventional control scheme studied in [25]–[29]. Removing the term $r(t_k - \eta)$ in the constructed Lyapunov function (15), we have Corollary 2 induced from Corollary 1 directly. Once again for the simplicity of matrix representation, block entry matrices $\hat{I}_i \in \mathbb{R}^{7n\times n}(i = 1, 2, \ldots, 7)$ are defined (for example, $\hat{I}_4 = [0, 0, 0, I, 0, 0, 0]^\top$). The notations of several matrices are defined as

$$\begin{aligned}
\hat{\xi}^\top(t) &= [r^\top(t), r^\top(t-\tau(t)), r^\top(t-\tau), r^\top(t_k),\\
&\qquad \frac{1}{t-t_k}\left(\int_{t_k}^{t} r(s)ds\right)^\top, \bar{g}^\top(r(t)), \dot{r}^\top(t)]\\
\hat{\Phi} &= [0, cF(G\otimes A), 0, H_1, 0, F, -F]\\
\hat{W}_1 &= [I, 0, 0, -I, 0, 0, 0]\\
\hat{W}_2 &= [I, 0, 0, I, -2I, 0, 0]\\
\hat{R} &= [R_1, 0, 0, R_2, 0, 0, 0]^\top\\
\hat{\Pi} &= [\hat{I}_1, \hat{I}_2, \hat{I}_3]\\
\hat{\Sigma}_1 &= 2\alpha\hat{I}_1 P\hat{I}_1^\top + \hat{I}_1 P\hat{I}_7^\top + \hat{I}_7 P\hat{I}_1^\top + \hat{I}_1(Q_1+Q_2)\hat{I}_1^\top\\
&\quad -(1-\mu)e^{-2\alpha\tau}\hat{I}_2 Q_1\hat{I}_2^\top - e^{-2\alpha\tau}\hat{I}_3 Q_2\hat{I}_3^\top + \tau^2\hat{I}_7 Q_3\hat{I}_7^\top\\
&\quad + e^{-2\alpha\tau}\hat{\Pi}\begin{bmatrix} -Q_3 & Q_3 - T & T\\ * & -2Q_3 + T + T^\top & Q_3 - T\\ * & * & -Q_3\end{bmatrix}\hat{\Pi}^\top\\
&\quad - e^{-2\alpha h}[\hat{I}_1 - \hat{I}_4]M_2\hat{I}_4^\top - e^{-2\alpha h}\hat{I}_4 M_2^\top[\hat{I}_1 - \hat{I}_4]^\top\\
&\quad - e^{-2\alpha h}Y_1^\top\hat{W}_1 - e^{-2\alpha h}\hat{W}_1^\top Y_1 - 3e^{-2\alpha h}Y_2^\top\hat{W}_2\\
&\quad - 3e^{-2\alpha h}\hat{W}_2^\top Y_2 + \alpha h^2[\hat{I}_1, \hat{I}_4]R[\hat{I}_1, \hat{I}_4]^\top\\
&\quad + \frac{h^2}{2}\hat{I}_7 X\hat{I}_7^\top + \alpha h^2\hat{I}_5 S\hat{I}_5^\top - h\hat{I}_5 S\hat{I}_5^\top\\
&\quad - \varepsilon_1[\hat{I}_1, \hat{I}_6]\begin{bmatrix}\bar{U} & \bar{V}\\ * & I\end{bmatrix}[\hat{I}_1, \hat{I}_6]^\top\\
&\quad + (\hat{I}_1 + \gamma_1\hat{I}_7)\hat{\Phi} + \hat{\Phi}^\top(\hat{I}_1 + \gamma_1\hat{I}_7)^\top\\
\hat{\Sigma}_2 &= -e^{-2\alpha h}\hat{I}_4 M_3\hat{I}_4^\top - [\hat{I}_1, \hat{I}_4]R[\hat{I}_1, \hat{I}_4]^\top\\
\hat{\Sigma}_3 &= [\hat{I}_7, \hat{I}_4]M[\hat{I}_7, \hat{I}_4]^\top + [\hat{I}_1, \hat{I}_4]R[\hat{I}_1, \hat{I}_4]^\top\\
&\quad + \hat{I}_5 S\hat{I}_1^\top + \hat{I}_1 S\hat{I}_5^\top.
\end{aligned}$$

Corollary 2: For given any scalars μ, γ_1 and positive constants h and τ, the error system (34) is exponentially stable with the decay rate α, if there exist matrices $P > 0$, $Q_i > 0 (i = 1, 2, 3)$

$$R = \begin{bmatrix} R_1 & R_2\\ * & R_4\end{bmatrix} > 0,\quad M = \begin{bmatrix} M_1 & M_2\\ * & M_3\end{bmatrix} > 0,$$

$S > 0$, $X > 0$, T, Y_1, Y_2, $F = \mathrm{diag}\{F_1, F_2, \ldots, F_N\}$, and $H_1 = \mathrm{diag}\{H_{11}, H_{12}, \ldots, H_{1N}\}$ with appropriate dimensions and a scalar $\varepsilon_1 > 0$ satisfying the following LMIs:

$$\begin{bmatrix} \hat{\Sigma}_1 + h\hat{\Sigma}_3 & \frac{h^2}{2}\hat{R}\\ * & -\frac{h^2}{2}X\end{bmatrix} < 0 \tag{38}$$

$$\begin{bmatrix} \hat{\Sigma}_1 + h\hat{\Sigma}_2 & hY_1 & 3hY_2 & \frac{h^2}{2}\hat{R}\\ * & -hM_1 & 0 & 0\\ * & * & -3hM_1 & 0\\ * & * & * & -\frac{h^2}{2}X\end{bmatrix} < 0 \tag{39}$$

$$\begin{bmatrix} Q_3 & T\\ * & Q_3\end{bmatrix} > 0. \tag{40}$$

Furthermore, the desired control gains are given as

$$K_{1i} = F_i^{-1}H_{1i},\quad i = 1, 2, \ldots, N.$$

Remark 5: It is indicated that Theorem 1 and Corollaries 1 and 2 only can be applied to time-varying delay when μ is known and less than one. When μ is unknown or the time-varying delay $\tau(t)$ is not differentiable, setting $Q_1 = 0$, the corresponding results of Theorem 1 and Corollaries 1 and 2 can be applied to address these cases.

IV. Numerical Examples

In this section, the validity of the proposed design method will be illustrated by two numerical examples.

Example 1: A CDN, including three nodes (1), is considered in this example. The outer-coupling matrix is assumed to be $G = (G_{ij})_{N\times N}$ with

$$G = \begin{bmatrix} -1 & 0 & 1\\ 0 & -1 & 1\\ 1 & 1 & -2\end{bmatrix}.$$

The inner-coupling matrix A is given as $A = \begin{bmatrix} 1 & 0\\ 0 & 1\end{bmatrix}$. $f(\cdot)$ is the nonlinear function, and is taken as

$$f(x_i(t)) = \begin{bmatrix} -0.5x_{i1} + \tanh(0.2x_{i1}) + 0.2x_{i2}\\ 0.95x_{i2} - \tanh(0.75x_{i2})\end{bmatrix}$$

which implies that $f(\cdot)$ satisfies (3) with

$$U = \begin{bmatrix} -0.5 & 0.2\\ 0 & 0.95\end{bmatrix},\quad V = \begin{bmatrix} -0.3 & 0.2\\ 0 & 0.2\end{bmatrix}.$$

The other parameter matrices for uncertainties are given as

$$D_i = \begin{bmatrix} 1 & 0\\ 0 & 1\end{bmatrix},\quad E_{ai} = \begin{bmatrix} 0.2 & 0\\ 0 & 0.2\end{bmatrix},\quad E_{bi} = \begin{bmatrix} 0.5 & 0\\ 0 & 0.5\end{bmatrix}.$$

For the case where there is no uncertainty, that is $\Delta(t_k) = 0$ and the convergence rate α is not zero, the exponential synchronized condition in [25], [26], [28], and [29] is not considered. The exponential synchronized condition proposed in these papers is not applicable anymore. Taking $\tau = 0.25$, $\mu = 0.5$, and applying Theorem 1 with $c = 0.2$, Table I shows the maximal sampling period with different α and η values. In addition, applying Corollaries 1 and 2 with $c = 0.8$, for

TABLE I

MAXIMUM SAMPLING PERIOD h FOR VARIOUS α AND η VALUES FOR $c = 0.2$

α/η	0.05	0.1	0.2
0.05	0.4254	0.4073	0.3745
0.1	0.3932	0.3772	0.3469

TABLE II

MAXIMUM SAMPLING PERIOD h FOR $c = 0.8$

α	0.2	0.3	0.4
[27]	0.5622	0.4755	0.4191
Corollary 2	0.6151	0.5003	0.4162
Corollary 1($\eta = 0.0001$)	0.6588	0.5432	0.4530

TABLE III

MAXIMUM SAMPLING PERIOD h FOR VARIOUS c VALUES

c	0.50	0.75	1.00
[25]	0.5409	0.1633	infeasible
[26]	0.5573	0.2277	infeasible
[27]	0.9016	0.8957	0.7316
[28]	0.9225	0.7530	0.5880
[29]	0.9353	0.9062	0.8692
Corollary 2	1.0428	1.0172	0.9513
Corollary 1($\eta = 0.0001$)	1.0666	1.0277	0.9988

different α values, the maximal sampling period is obtained and summarized in Table II. On the other hand, when $\alpha = 0$, $\tau = 0.25$, and $\mu = 0.5$, the comparison of sampling intervals for a different coupling strength c by Corollaries 1 and 2 with the results in [25]–[29] is shown in Table III. From the results of Tables II and III, we can see that the proposed method in this paper can give larger delay bounds than those in [25]–[29].

To show the effect of the perturbations in controller, taking $\Delta(t_k) = \sin(t_k)$, $\alpha = 0.1$, $\eta = 0.05$, $c = 0.2$, $\tau(t) = 0.125 + 0.125\sin(4t)$, $\beta_1 = 1.2$, $\beta_2 = 0.1$, and solving the LMIs (12)-(14), the maximum value of h is 0.3932, and the corresponding nonfragile sampled-data controllers gains are

$$K_{11} = \begin{bmatrix} -1.2786 & -0.1299 \\ 0.0207 & -2.1688 \end{bmatrix}$$
$$K_{12} = \begin{bmatrix} -1.2786 & -0.1299 \\ 0.0207 & -2.1688 \end{bmatrix}$$
$$K_{13} = \begin{bmatrix} -1.2082 & -0.0946 \\ 0.0272 & -1.9546 \end{bmatrix}$$
$$K_{21} = \begin{bmatrix} -0.1928 & -0.0845 \\ 0.0060 & -0.1055 \end{bmatrix}$$
$$K_{22} = \begin{bmatrix} -0.1928 & -0.0845 \\ 0.0060 & -0.1055 \end{bmatrix}$$
$$K_{23} = \begin{bmatrix} -0.1958 & -0.0987 \\ 0.0036 & -0.1343 \end{bmatrix}.$$

For the initial condition $x_1(0) = [7, -4]^\top$, $x_2(0) = [3, -9]^\top$, $x_3(0) = [-6, 5]^\top$, and $s(0) = [0, -1]^\top$, the response curves of error systems without control input are shown in Fig. 1. Using the above-mentioned controller gains, the response curves of the error systems and control input are given in Figs. 2 and 3, respectively. From Fig. 2, it is seen that the synchronization is convergent to zero.

Example 2: In this example, Chua's circuit is considered as an unforced isolated node of CDN (1), which is expressed as

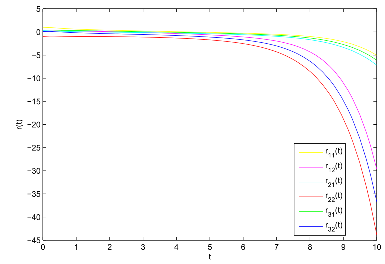

Fig. 1. State response of error system without control input in Example 1.

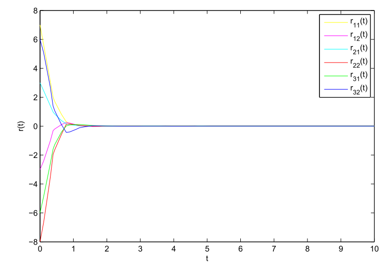

Fig. 2. State response of error system in Example 1.

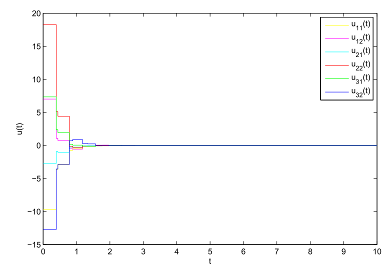

Fig. 3. Control inputs in Example 1.

the following equation:

$$\begin{cases} \dot{s}_1(t) = \sigma_1(-s_1(t) + s_2(t) - \upsilon(s_1(t))) \\ \dot{s}_2(t) = s_1(t) - s_2(t) + s_3(t) \\ \dot{s}_3(t) = -\sigma_2 s_2(t) \end{cases} \quad (41)$$

where $\sigma_1 = 10, \sigma_2 = 14.87$, $\upsilon(s_1) = bs_1 + 0.5(a-b)r(s_1)$, $a = -1.27, b = -0.68$, and $r(s_1) = (|s_1 + 1| - |s_1 - 1|)$. Denote $s = [s_1, s_2, s_3]^\top$

$$f(s) = \begin{bmatrix} -\sigma_1 - \sigma_1 b & \sigma_1 & 0 \\ 1 & -1 & 1 \\ 0 & -\sigma_2 & 0 \end{bmatrix} + \begin{bmatrix} -0.5\sigma_1(a-b)r(s_1) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

It can be found that $f(s)$ satisfies (3) with

$$U = \begin{bmatrix} 2.7 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} -3.2 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix}.$$

And the inner-coupling matrix A is given as

$$A = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}.$$

TABLE IV
MAXIMUM SAMPLING PERIOD h WITH $c = 1$

Method	[26]	[28]	[29]	Corollary 2	Corollary 1 ($\eta = 0.0001$)
h	0.0711	0.1120	0.1327	0.1580	0.1659

The outer-coupling matrix $G = (G_{ij})_{N\times N}$ is assumed to be

$$G = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}.$$

The other parameter matrices for uncertainties are given as

$$D_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_{ai} = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}$$

$$E_{bi} = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.8 \end{bmatrix}.$$

When there is no uncertainty, that is $\Delta(t_k) = 0$, and $\alpha = 0$, $\tau = 0.04$, and $\mu = 0.01$, the compared results of maximal sampling period h for various c are listed in Table IV. From Table IV, it is seen that the results calculated based on the criteria given in this paper are less conservative than those reported in the existing literature, which shows the advantage of the proposed method.

To show the effect of the perturbations in the controller, letting $\Delta(t_k) = \cos(t_k), \alpha = 0.1, \eta = 0.1, \tau(t) = 0.05|\sin(t)|, c = 0.5, \beta_1 = 0.2$, and $\beta_2 = 0.1$, we solve the LMIs (12)–(14), the maximum sampling interval h is 0.1022, and the corresponding nonfragile sampled-data controller gains

$$K_{11} = \begin{bmatrix} -7.4822 & -8.3744 & -0.5858 \\ 0.0441 & -2.9294 & -0.4713 \\ 3.1257 & 11.0609 & -4.3728 \end{bmatrix}$$

$$K_{12} = \begin{bmatrix} -7.6639 & -8.3190 & -0.6299 \\ 0.0427 & -3.1465 & -0.3562 \\ 3.2397 & 11.4472 & -4.3751 \end{bmatrix}$$

$$K_{13} = \begin{bmatrix} -7.6639 & -8.3190 & -0.6299 \\ 0.0427 & -3.1465 & -0.3562 \\ 3.2397 & 11.4472 & -4.3751 \end{bmatrix}$$

$$K_{21} = \begin{bmatrix} -0.3050 & 0.0142 & 0.1783 \\ 0.1703 & -0.1066 & -0.1719 \\ 0.0746 & 0.5122 & -0.1634 \end{bmatrix}$$

$$K_{22} = \begin{bmatrix} -0.3259 & 0.0248 & 0.2213 \\ 0.1759 & -0.1285 & -0.1993 \\ 0.0675 & 0.4179 & -0.2664 \end{bmatrix}$$

$$K_{23} = \begin{bmatrix} -0.3259 & 0.0248 & 0.2213 \\ 0.1759 & -0.1285 & -0.1993 \\ 0.0675 & 0.4179 & -0.2664 \end{bmatrix}.$$

The initial values of the dynamical networks are set to be $x_1(0) = [2, -3, 5]^\top, x_2(0) = [5, -7, 1]^\top, x_3(0) = [2, -2, 4]^\top$, and $s(0) = [1, 0, -2]^\top$. The response curves of the error systems without control input are shown in Fig. 4. Using the above-mentioned controller gain matrices, the trajectory curves of the error systems and the control input are given in Figs. 5 and 6, respectively. From Fig. 5, it can be seen that the synchronization error is tending to zero, which implies that the synchronization of the CDNs can be achieved by the designed nonfragile memory sampled-data controller.

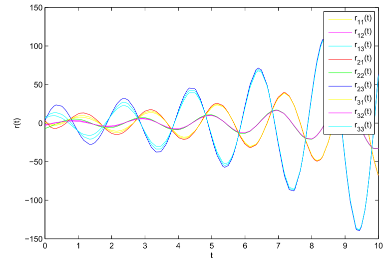

Fig. 4. State response of error system without control input in Example 2.

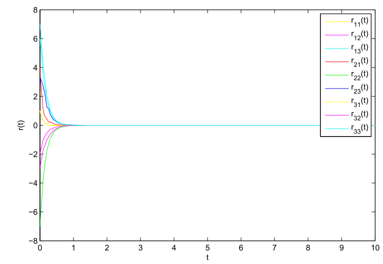

Fig. 5. State response of error system in Example 2.

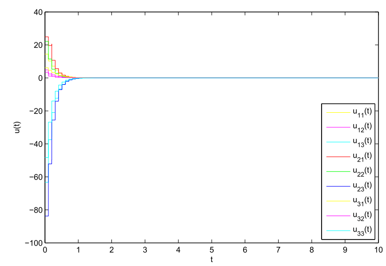

Fig. 6. Control inputs in Example 2.

V. CONCLUSION

In this paper, a nonfragile memory sampled-data control for the synchronization of CDNs with time-varying delay was investigated. A new Lyapunov function has been constructed for synchronization error systems where the information about the actual sampling is fully considered. Furthermore, in the case of no uncertainty, a sampled-data synchronization criterion for delayed CDNs was derived. It is shown that the new criteria can provide larger sampling period than some existing criteria using the integral inequality method and reciprocally convex approach. Two numerical examples are presented to show the validity of the proposed techniques. In future work, the new sampled-data approach will be extended to networked fuzzy systems [38]–[40], Markovian-jump systems [41], and others.

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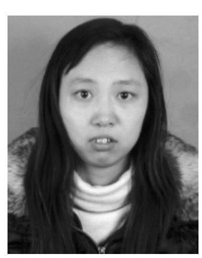

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