Shear Force Feedback Control of Flexible Robot Arms

Zheng-Hua Luo, Nobuyuki Kitamura, and Bao-Zhu Guo

Abstract—**For flexible robots with rotational joints it has been shown previously, by the first author, that direct strain feedback can damp out vibrations very satisfactorily. In this paper, a simple sensor-based output feedback control law, called shear force feedback, is newly proposed to control vibrations arising from structural flexibility of robots of Cartesian or SCARA types. Closed-loop exponential stability of such shear force feedback system is proved. Experimental results on set point control and trajectory tracking control are reported. It is found that the simple PI + shear force feedback can yield good performance for both robot motion and vibration suppression.**

I. Introduction

Structural flexibility in robotic systems is becoming an issue of increasing concern. The demands for high speed, low cost, and low energy consumption are main motivations for control of lightweight flexible robots. Most of early researches on control of flexible robots concentrated on model-based controller design [1], [2], [11]. However, these model-based controllers, originally designed for the demands of high performance, may not be easy to implement due to uncertainties in design models, large variations of loads on the robot's end-effector, ignored high frequency dynamics (related to control and observation spillovers), and the high order of the designed controllers. Thus, it is highly desirable to seek simple and robust controllers for the control of flexible robot arms. Indeed, there have been many studies related to this problem. Various methods such as PD feedback of robot joint variables, PD with gravity compensation [5], and PD with feedforward [8] have been proposed for control of robot flexibility.

In [7], [9], and [12], it is shown that the simple strain feedback can damp out vibrations in robots with rotational joints. However, it should be pointed out that many industrial robots, especially those widely used in automatic manufacturing assembly line, are either of SCARA or Cartesian types (see, respectively, Figs. 1 and 2). Although there may exist many causes for the vibrations of these robots, we restrict ourselves to the suppression of vibration arising from structural flexibility of the robot tip arm. In this case, would the strain feedback laws proposed in [9] still work? Unfortunately, the answer is in the negative. This assertion is verified experimentally. The physical background of this phenomenon lies in the fact that vibrations of robot arms with rotational joints are caused by angular acceleration at the arm's root end, while the vibrations in Cartesian robots are caused by linear acceleration at the arm's root end. Theoretically, it can be shown, by using the operator theory in [9], that the strain feedback control action does not introduce a damping operator for Cartesian robots, as it does for robots with rotational joints.

The purpose of this paper is to present a new kind of simple control method called *shear force feedback control* for vibration suppression in flexible robots of Cartesian and SCARA types. In Section II, we explain what is meant by shear force feedback control. We discuss how to implement this simple control law. We also show the closed-loop exponential stability under the shear force feedback. In Section III, we present experimental results for shear force feedback with both set point control and trajectory tracking control, for an X–Y Cartesian robot. Conclusions are drawn in Section IV.

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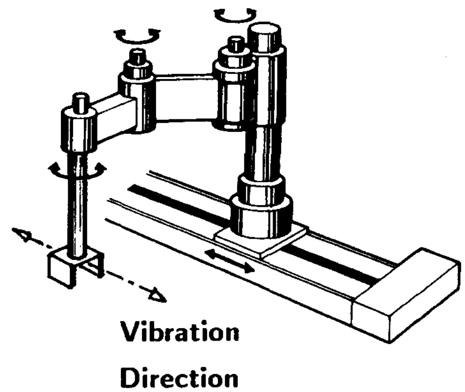

Fig. 1. A SCARA robot with a long tip arm.

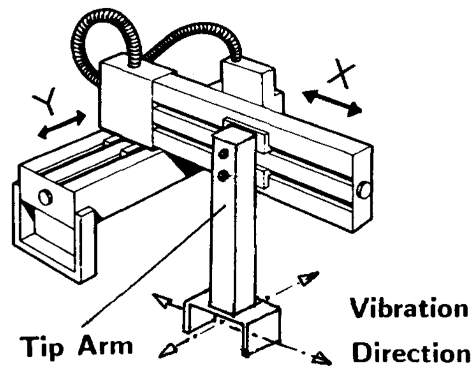

Fig. 2. A Cartesian robot with a long tip arm.

II. Shear Force Feedback Control

Consider a Cartesian or SCARA robot with a long tip arm as illustrated in Figs. 1 and 2. In industry, these robots are widely used in handling objects, assembling parts, packing and detecting detects etc. A primary demand for these tasks is high speed movement of robots in order to increase productivity. When a robot with a long tip arm moves at a high speed, vibration is unavoidable. In many industrial applications, this vibration can be a main problem in task implementation. In many circumstances, no effective method is available to solve this problem except to slow down the robot, or to wait till after the movement for vibrations to die down. This obviously is not efficient and motivates us to develop active vibration control to overcome this difficulty.

For clarity of statement, we only consider vibration control of Cartesian robots. The problems associated with SCARA robots are essentially the same and can thus be treated in a similar way. Since any motion in the X–Y plane can be decomposed into its X and Y components, the vibrations in the X-direction and Y-direction can be considered independently. Fig. 3 shows motion of an X–Y robot in the X-direction. M represents a moving body driven by control motor. One end of the flexible arm is attached to this moving body. Let the vibration magnitude of flexible arm at time t and position r be $w(t, r)$. It is not difficult to show that the dynamic model for

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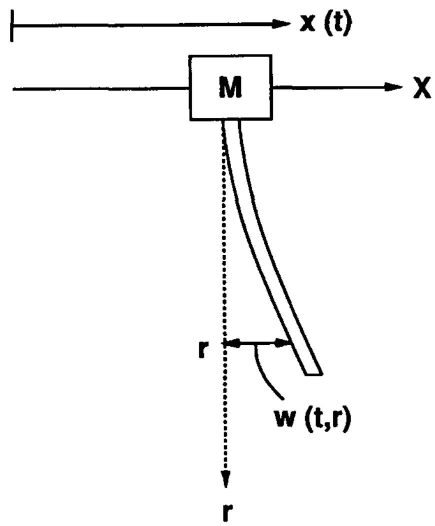


Fig. 3. Vibration of the tip flexible arm of a Cartesian robot along the X-axis.

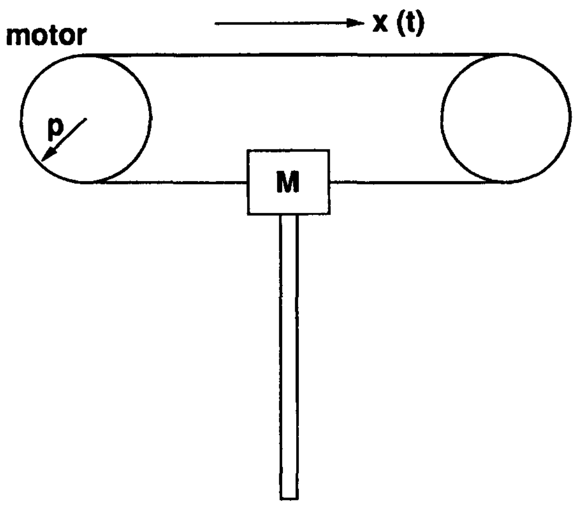


Fig. 4. An illustration of transformation from rotational motion to linear motion via timing belt.

vibration of flexible arm in the X -direction can be written as

$$\begin{cases} \ddot{w}(t, r) + \alpha w''''(t, r) = -\ddot{x}(t), & r \in (0, \ell) \\ w(t, 0) = w'(t, 0) = 0 \\ w''(t, \ell) = w'''(t, \ell) = 0 \\ w(0, r) = w_0(r), \quad \dot{w}(0, r) = w_1(r) \end{cases} \tag{1}$$

where $\ddot{x}(t)$ denotes acceleration of the moving body M, $\dot{w}$ denotes time derivative and w' spatial derivative. $\alpha > 0$ is a constant determined by the arm material. $w_0(r)$ and $w_1(r)$ are respectively the initial displacement and initial velocity. In deriving this dynamic model, we have assumed that the arm material is uniform and the vibration magnitude is small.

A. Shear Force Feedback

In the dynamic model (1), the acceleration $\ddot{x}(t)$ of the moving body acts as control input for the vibration of the flexible arm. We can measure shear force at the root end of the arm which is proportional to $w'''(t, 0)$. We can also control the motion of motor so that

$$\dot{x}(t) = -kw'''(t, 0) \tag{2}$$

where k is an arbitrary constant. Therefore, we have,

$$\ddot{x}(t) = -k\dot{w}'''(t, 0). \tag{3}$$

Substituting (3) into (1) yields the shear force feedback controlled closed-loop system equation:

$$\begin{cases} \ddot{w}(t, r) + \alpha w''''(t, r) - k\dot{w}'''(t, 0) = 0 & r \in (0, \ell) \\ w(t, 0) = w'(t, 0) = 0 \\ w''(t, \ell) = w'''(t, \ell) = 0 \\ w(0, r) = w_0(r), \quad \dot{w}(0, r) = w_1(r). \end{cases} \tag{4}$$

At this point, we have at least three questions which need to be clarified.

1) How to measure the shear force $w'''(t, 0)$?
2) How to control the motor so that (2) is satisfied?
3) Is the partial differential (4) exponentially stable?

As for the first question, there is no commercial shear force sensor known to us. However, the bending strain which is proportional to $w''(t, 0)$ can be easily measured by cementing strain gauge foils at the arm's root end. Similarly, by cementing strain gauge foils at location $r = \varepsilon$ (ε is a small constant), close to the root end $r = 0$, we can measure $w''(t, \varepsilon)$. Therefore, one method for measuring shear force, $w'''(t, 0)$, is to approximate it by $[w''(t, \varepsilon) - w''(t, 0)]/\varepsilon$. This method is adopted in our experiments. Although it is commonly recognized that derivation may introduce noise, it is found, by experiments, that such treatment is quite acceptable and the noise is not severe enough to cause problems.

For the second question, we can choose a servo motor with a motor driver of high gain speed reference type. For such a case, the input voltage $V_{ref}(t)$ to the motor driver is approximately proportional to angular velocity $\dot{\theta}(t)$ of the motor, i.e.,

$$V_{ref}(t) = k_f \dot{\theta}(t) \tag{5}$$

where $k_f > 0$ is the back emf constant of the motor. The rotational motion of the control motor is transferred into linear motion in the X-direction via timing belt as shown in Fig. 4 where p is the radius of driving wheel. Then, clearly,

$$\dot{x}(t) = p\dot{\theta}(t). \tag{6}$$

Combining (2) and (6), we obtain

$$\dot{\theta}(t) = \frac{1}{p}\,\dot{x}(t) = -\frac{k}{p}\,w'''(t, 0). \tag{7}$$

Substituting (7) into (5) yields

$$V_{ref}(t) = -\frac{kk_f}{p}\,w'''(t, 0). \tag{8}$$

This is to say, that if we measure the shear force $w'''(t, 0)$ and determine the control voltage $V_{ref}(t)$ according to (8), then (2) is satisfied as desired and the control closed-loop equation is given by (4).

The third question is the most important and will be answered in next section.

B. Closed-Loop Exponential Stability of Shear Force Feedback

This section is devoted to studying the stability of system (4) when feedback gain $k > 0$. We are especially interested in the exponential stability. We note that when $k = 0$, i.e., there is no shear force feedback, (4) is a conservative system whose energy never decays theoretically. Our objective is to show that the third term $-k\dot{w}'''(t, 0)$ on the left-hand side of (4) is a damping term and shear force feedback control can exponentially decay the solution of (4).

The partial differential in (4) is a nonstandard one and does not appear in the existing literature. So a unique solution is not obvious. Fortunately, it can be shown, using the concept of A-dependent operators proposed in [9], that there exists a unique solution of (4) for any $k > 0$ and smooth initial conditions w_0, w_1. We claim that this solution is exponentially stable. To this end, we introduce a new variable

$$y(t, r) = w''(t, r). \tag{9}$$

Assume that the initial conditions $w_0(r)$ and $w_1(r)$ are sufficiently smooth that the solution of (4) admits continuous spatial derivatives of up to sixth order. Taking twice the spatial derivative of both sides of the first equation of (4) yields

$$\ddot{y}(t, r) + \alpha y''''(t, r) = 0. \tag{10}$$

From the boundary conditions of (4), we see that $\ddot{w}(t, 0) = \ddot{w}'(t, 0) = 0$. Hence it is easily verified that

$$\begin{cases} y(t, \ell) = w''(t, \ell) = 0 \\ y'(t, \ell) = w'''(t, \ell) = 0 \\ y''(t, 0) = w''''(t, 0) = -\dfrac{1}{\alpha}\left[\ddot{w}(t, 0) - k\dot{w}'''(t, 0)\right] \\ \qquad = \dfrac{k}{\alpha}\dot{w}'''(t, 0) = \dfrac{k}{\alpha}\dot{y}'(t, 0) \\ y'''(t, 0) = w'''''(t, 0) = \ddot{w}'(t, 0) = 0. \end{cases} \tag{11}$$

Also, introducing a new variable x according to $x = \ell - r$ and paying attention to the following equalities

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial r}\frac{\partial r}{\partial x} = -\frac{\partial y}{\partial r}$$
$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial r^2}$$
$$\frac{\partial^3 y}{\partial x^3} = -\frac{\partial^3 y}{\partial r^3}$$
$$\frac{\partial^4 y}{\partial x^4} = \frac{\partial^4 y}{\partial r^4},$$

we obtain the following partial differential equation and the corresponding boundary conditions

$$\begin{cases} \ddot{y}(t, x) + \alpha y''''(t, x) = 0 \\ y(t, 0) = 0,\ y'(t, 0) = 0 \\ y''(t, \ell) = -\dfrac{k}{\alpha}\dot{y}'(t, \ell) \\ y'''(t, \ell) = 0. \end{cases} \tag{12}$$

This is a standard boundary moment controlled flexible system which often appears in existing literature. Energy stored in system (12) is dissipative provided $k > 0$. To see this, define an energy function for system (12) as

$$E(t) = \frac{1}{2}\int_0^\ell [\dot{y}(t, x)]^2\, dx + \frac{\alpha}{2}\int_0^\ell [y''(t, x)]^2\, dx. \tag{13}$$

Then the time derivative of $E(t)$ along the solution of (12) is given by

$$\begin{aligned} \dot{E}(t) &= \int_0^\ell \dot{y}(t, x)\ddot{y}(t, x)\, dx + \alpha\int_0^\ell y''(t, x)\dot{y}''(t, x)\, dx \\ &= -\frac{k}{\alpha}[\dot{y}'(t, \ell)]^2 \le 0. \end{aligned}$$

Consequently the energy stored in system (12) is dissipative. Indeed the energy dissipates exponentially, which had been long unproved by using the energy multiplier method, but is finally proved in [3] by making use of a result in Huang [6]. The interested reader is referred to [3] for details.

With $y(t, x)$ shown to be exponentially stable, it is claimed that the solution $w(t, r)$ of (4) is also exponentially stable since $w(t, r)$ can be expressed as

$$w(t, r) = \int_0^r \int_0^s y(t, x)\, dx\, ds$$

and the integration operator is bounded.

It should be pointed out that the exponential stability of a boundary force controlled system such as

$$\begin{cases} \ddot{y}(t, x) + \alpha y''''(t, x) = 0 \\ y(t, 0) = 0,\ y'(t, 0) = 0 \\ y''(t, \ell) = 0 \\ y'''(t, \ell) = k\dot{y}(t, \ell), \quad k > 0. \end{cases} \tag{14}$$

has been proved earlier by Chen *et al.* [4] using the energy multiplier method.

III. Control Experiments

A. Experimental Device

In order to test the shear force feedback control method proposed in this paper, an X–Y Cartesian robot as shown in Fig. 5 was used. The X and Y axes are respectively driven by two dc motors (68 W) with commercial motor drivers of speed reference type. The motor shafts are directly (without gear reduction) coupled to the ball-screw which converts rotational motion of motors into linear motion in the X and Y axes. A moving body can move in the X-axis. The moving body and the X-axis can move together in the Y-axis. The movable ranges of the X and Y axes are 700 mm. Since there is no gear reduction, the moving body can move at a very high speed.

A thin square aluminum rod with length $\ell = 600$ mm, which will be referred as the X–Y robot's tip arm, is attached to the moving body along the Z-axis. The value of the arm material constant α in (1) is identified to be $\alpha = 78.91$ [N· m^3/kg]. When the moving body moves at a high speed in the X–Y plane, significant vibrations are observed in the end-effector of the tip arm. To detect these vibrations, shear force sensors were developed independently for the X and Y axes. Each sensor consists of four precision strain gauges, two of which are cemented on front surface and the other two on the opposite surface of the tip arm. The distance between the two gauge foils on each surface is taken to be 10 mm ($\varepsilon = 10$ mm). Using two (one on front surface and one on the opposite surface) of these, we construct a 1/2 Wheatstone bridge. The remaining two is used in another 1/2 Wheatstone bridge. The amplified difference of outputs of these two Wheatstone bridges is taken as the shear force signal $w'''(t, 0)$ which we need for feedback. Angular positions and velocities were measured with encoders and tachometers attached to the motor shafts. These signals together with the shear force signals were sent to the controller (NEC9801 personal computer with 80386 CPU). The control program was written in C language, and the sampling period was set at 8 ms. Experimental results on set point control and trajectory tracking control are presented in next subsection.

B. Experimental Results on Set Point Control

In practical applications, it is necessary to control not only the vibrations but also positions of the moving body. Note that vibrations should be suppressed without sacrificing performance of position servo (transient responses of position variables etc.). Some of the conventional vibration control methods do not guarantee good responses of robot joint positions and rapid decay of vibrations in robot arms at the same time. When vibrations are well suppressed, usually the responses of robot joint positions become worse (it takes a longer time for the robot to reach a given position, or overshoot can be seen). An important feature of the shear force feedback control method proposed here is that we can suppress vibrations without much affecting good transient responses of robot joint positions. This will be illustrated by experimental results.

Since the X-axis and Y-axis are controlled independently with the same control law, we describe theoretical formulations only for the X-axis in the following. Let x_d (constant) be the desired position of the moving body in the X -axis. To control both the position of the moving body and vibration of the tip arm, we determine motor input reference voltage $V_{ref}(t)$ by the following PI + shear force feedback law

$$\begin{aligned} V_{ref}(t) = -k_1[x(t) - x_d] - k_2 \int_0^t \\ \cdot [x(\tau) - x_d]\, d\tau - kw'''(t, 0) \end{aligned} \tag{15}$$

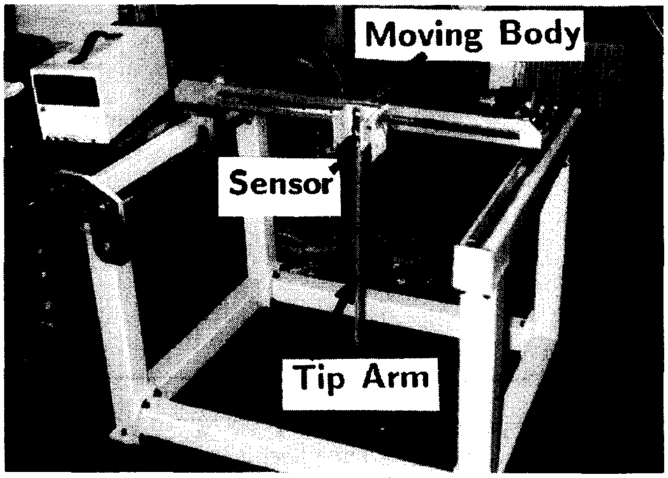


Fig. 5. An X-Y Cartesian robot used for experiments.

where $k_1 \geq 0$ and $k_2 \geq 0$ are PI feedback gains and $k \geq 0$ is the shear force feedback gain. The control closed-loop exponential stability with $k_1 = k_2 = 0$ is given in the last section. When k_1 and k_2 are not zeros, upon substituting (5), (6), and (15) into (1), we obtain the closed-loop system equation as follows

$$\begin{cases} \ddot{w}(t, r) - \frac{p}{k_f} k\dot{w}'''(t, 0) + \alpha w''''(t, r) \\ \quad = \frac{p}{k_f} k_1 \dot{x}(t) + \frac{p}{k_f} k_2 [x(t) - x_d] \\ w(t, 0) = w'(t, 0) = 0 \\ w''(t, \ell) = w'''(t, \ell) = 0 \\ \ddot{x}(t) + \frac{p}{k_f} k_1 \dot{x}(t) + \frac{p}{k_f} k_2 [x(t) - x_d] \\ \quad = -\frac{p}{k_f} k\dot{w}'''(t, 0) \end{cases} \quad (16)$$

This is a coupled hybrid equation consisting of a *stable* partial differential equation and a *stable* ordinary differential equation. Conditions for the closed-loop exponential stability are not easily obtained by the Lyapunov function method. It was not until recently that we succeeded in showing that if the feedback gains k, k_1, and k_2 satisfy

$$k_2 < \frac{\lambda_1}{2} \frac{k_f}{p} \quad (17)$$

and

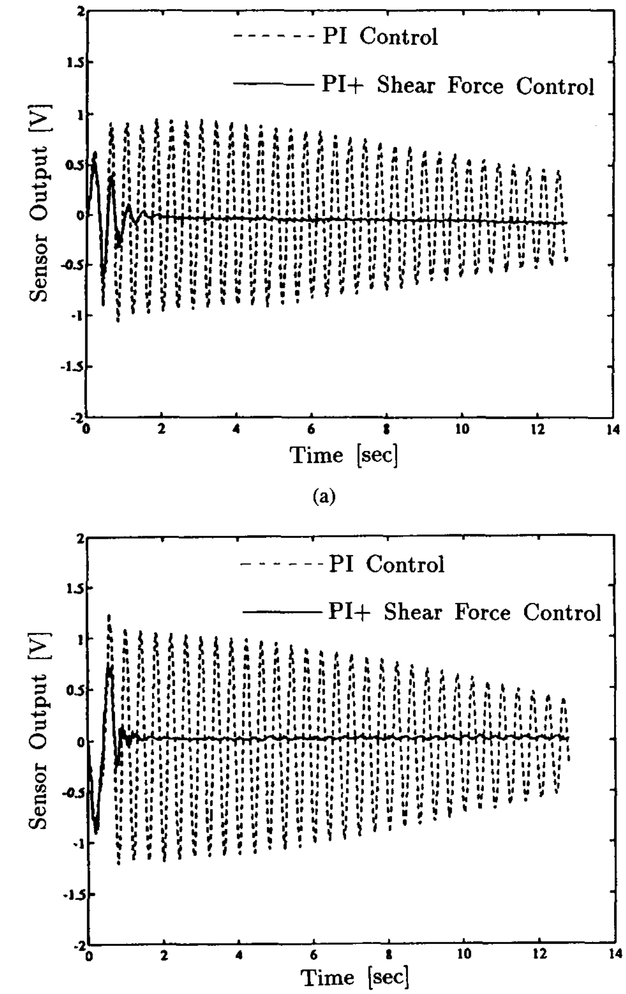
$$\frac{4\ell}{\alpha} \frac{p}{k_f} k_2 k < k_1 < \frac{\alpha}{k\ell} \left(\frac{k_f}{p} \right)^2 \quad (18)$$

then the closed-loop system (16) is exponentially stable. In (17) λ_1 represents the first (smallest) eigenvalue of the positive definite operator $A = \alpha \partial^4 / \partial x^4$ [9]. The derivation of (17) and (18) is too long to be included here and can be found in a separate paper [10]. Usually p/k_f is small and α is large. For instance, in our experiments, $p/k_f = 9.25 \times 10^{-2}$ [m/s · V], $\alpha = 78.91$ [N · m^3/kg] and $\lambda_1 = 246.74$ [rad^2/s^2]. Thus, it can be seen that (17) and (18) are satisfied for a very wide range of the three feedback gains.

For the set point control experiment, the end-effector of the arm is commanded to move from the X–Y coordinate origin to a point $x_d = 100$ mm and $y_d = 100$ mm by simultaneously driving the X and Y axes. Notice that we use the same control law (15) for control of the X and Y axes independently. The PI gains k_1 and k_2 in (15) are determined to be

$$k_1 = 79.20 \, [\text{V/m}], \quad k_2 = 0.50 \, [\text{V/m} \cdot \text{s}]$$

in order to complete the set point control motion in less than 2 s without overshoot in the time response of position of the moving body. After k_1 and k_2 are determined, the shear force feedback gain k can be chosen by trial and error. We begin by choosing a small k. If the vibration suppression is not so satisfactory, we increase k. For $k = 0.85$, the outputs of the X and Y shear force sensors are displayed with real lines in Fig. 6(a) and (b), respectively. The dashed lines in these two figures are the sensor outputs when only PI control is implemented (without shear force feedback). It can be seen that there is not much noise in these sensor outputs, and that the vibrations decay very rapidly when shear force information is used for feedback. What is more attractive is that the time responses of the moving distances of moving body in the X and Y directions do not change much as can be seen from Fig. 7(a) and (b), respectively. The real lines denote responses with shear force control, while the dashed lines denote responses without shear force control. This is because two damping terms $-p/k_f \, k\dot{w}'''(t, 0)$ and $p/k_f \, k_1 \dot{x}(t)$ have been independently introduced, one for vibration and another for motion [see the first and the last equations in (16)]. The corresponding time responses of outputs of tachometers [which is proportional to the input voltages $V_{ref}(t)$] for motors of the X and Y axes are shown in Fig. 8(a) and (b). It can be seen that the responses do not change much before and after shear force feedback. Notice that in implementing the above control law, we do not need to know the precise values of physical parameters of the motor driving systems and the tip arm.



(b)

Fig. 6. Time responses of the outputs of shear force sensor: (a) sensor output along the X-direction; (b) sensor output along the Y-direction.

When the end-effector of the arm is fitted with a payload, the stability of shear force controlled closed-loop system is still guaran-

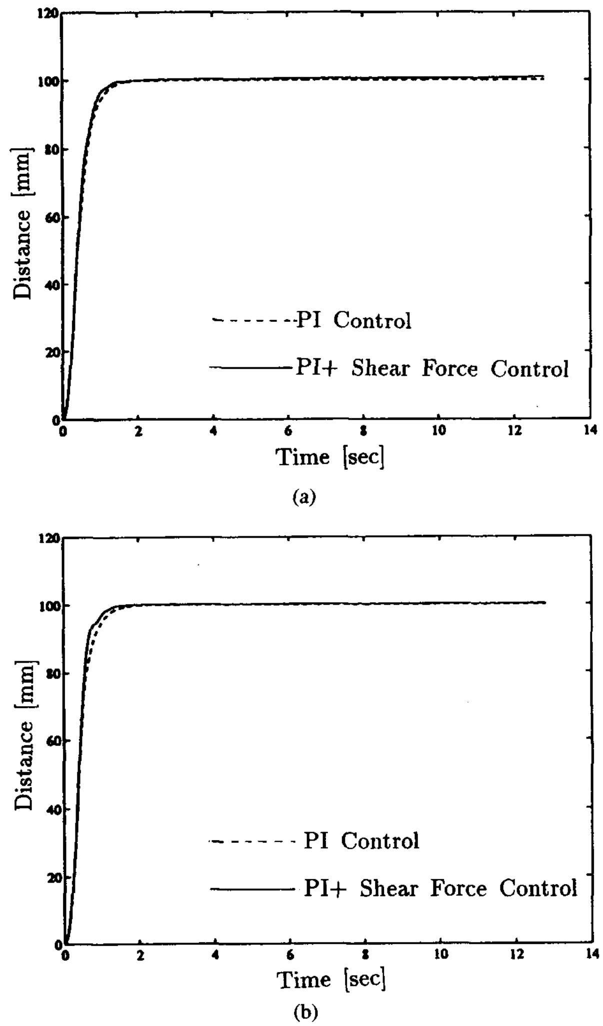


Fig. 7. Time responses of the moving distance of the moving body: (a) the X-axis component; (b) the Y-axis component.

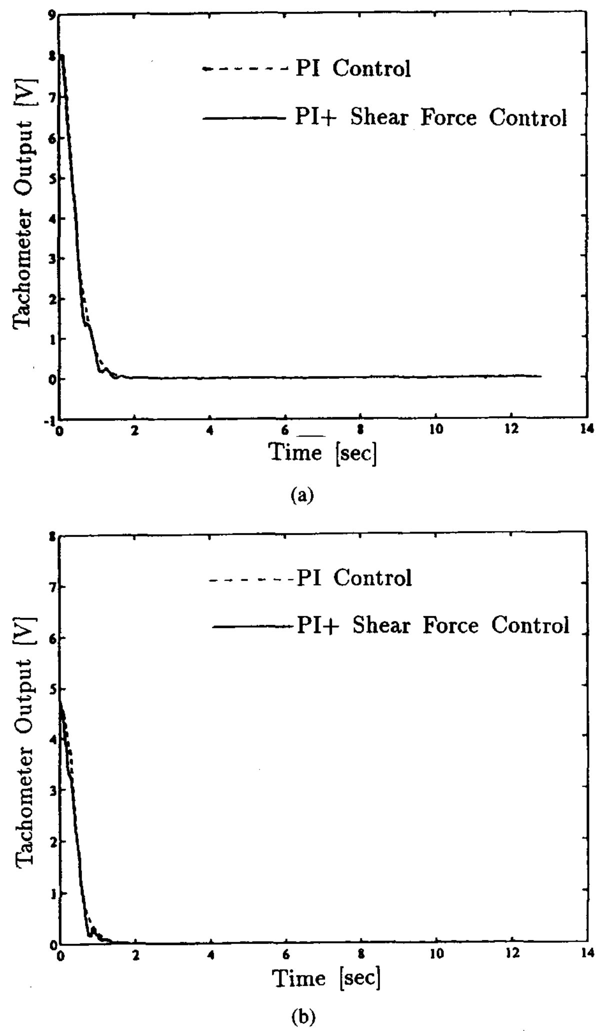


Fig. 8. Time responses of the outputs of tachometers [proportional to the motor reference input voltage $V_{ref}(t)$]: (a) for the X-axis motor; (b) for the Y-axis motor.

teed although the performance may degrade. This point is verified experimentally and can be made clear theoretically.

C. Experimental Results on Trajectory Tracking Control

In set point control stated above, the desired positions x_d and y_d in the X and Y directions are constants. In trajectory tracking control, these desired positions are time varying. We propose the following control law for the X-axis control motor

$$V_{ref}(t) = \frac{k_f}{p}\,\dot{x}_d(t) - k_1[x(t) - x_d(t)] - k_2\int_0^t [x(\tau) - x_d(\tau)]\,d\tau - kw_x'''(t,\,0) \tag{19}$$

and for the Y-axis control motor

$$V_{ref}(t) = \frac{k_f}{p}\,\dot{y}_d(t) - k_1[y(t) - y_d(t)] - k_2\int_0^t [y(\tau) - y_d(\tau)]\,d\tau - kw_y'''(t,\,0) \tag{20}$$

where $w_x'''(t,\,0)$ and $w_y'''(t,\,0)$ are used to distinguish shear force sensor output $w'''(t,\,0)$ in the X and Y directions, respectively. The difference between (15) and (19) is that in trajectory tracking a feedforward term $\dot{x}_d(t)$ [or $\dot{y}_d(t)$] is added. A series of desired trajectories were given to test the performance of the trajectory tracking control law.

For the end-effector of the arm to follow a circular orbit of radius 70 mm 4 times in 6 s, we chose

$$x_d(t) = 70\cos\tfrac{4}{3}\pi t, \qquad y_d(t) = 70\sin\tfrac{4}{3}\pi t.$$

Fig. 9(a) is the trajectory of end-effector when (19) and (20) are implemented with shear force feedback gain $k = 0$. Fig. 9(b) shows the result for $k = 0.34$. It can be seen that the real trajectory approaches the reference trajectory in a few minutes after the start. These two figures demonstrate that vibration suppression is improved a great deal by shear force feedback.

Experiments were also conducted to make the end-effector follow a trigonometric trajectory in the X–Y plane. The results of these experiments are shown in Fig. 10 where the real line represents the end-effector trajectory without shear force feedback and the line with dot marks represents the end-effector trajectory when shear force feedback is applied. It is seen that shear force feedback can improve the responses of the end-effector, especially at corners where vibrations are more likely to occur.

IV. Concluding Remarks

A simple shear force feedback control law has been proposed for vibration suppression of Cartesian or SCARA robots with long tip arms. The exponential stability of such shear force feedback controlled closed-loop system, in the absence of internal damping of

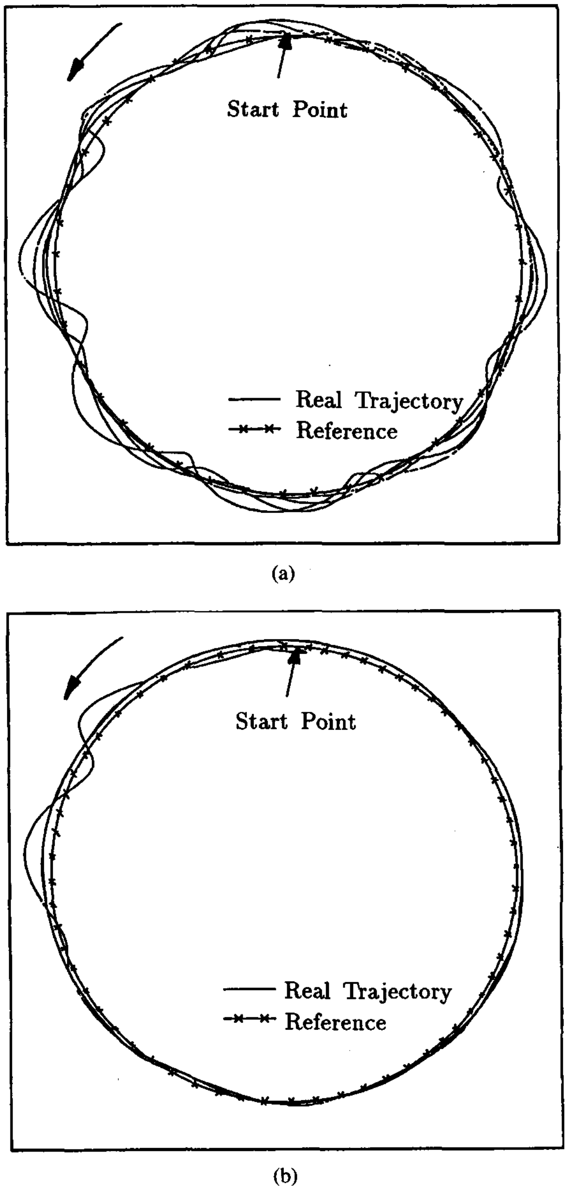


Fig. 9. Traces of end-effector for a circular trajectory: (a) without shear force feedback; (b) with shear force feedback.

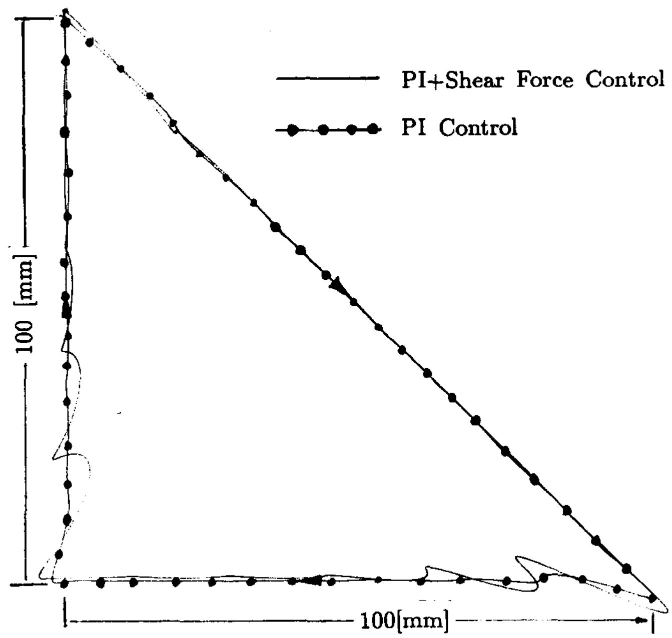


Fig. 10. Traces of end-effector for a trigonometric trajectory. The real line represents trace without shear force feedback; the line with dot marks represents trace with shear force feedback.

the arm material, has been proved by transforming the nonstandard partial differential equation into the standard boundary moment control system. Experiments on both the set point control and the trajectory tracking control were conducted. These experimental results demonstrate that shear force feedback can not only damp out vibrations satisfactorily, but also maintain high performance in the motion of robot. This is very important in practice, since it would make no sense if the vibration suppression were achieved at the price of sacrificing the performance of motion of robot. The initial control objective of flexible robots is simultaneous motion/vibration control and not vibration suppression only.

Several remarks are in order.

- All the above discussions are based on the assumption that the control motor driver is of speed reference type. If the motor is of torque control type, then theoretically it is necessary to feedback the time rate of change of shear force. A sensor that can be used to measure the time rate of change of shear force needs to be developed.
- The control method proposed here can also be used in the handling of flexible materials if the end-effector is equipped with a sensor which can measure shear force at the contact point of flexible materials.

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