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Stabilizer design of planar switched linear systems

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ABSTRACT

This paper considers the stabilization of planar switched linear control systems. First, a structure property of not completely controllable pair (A, b) is revealed. Based on it, a simply verifiable, necessary and sufficient condition for the planar switched linear control system to be feedback stabilizable, is presented under the assumption that the switching law is designable. The proof provides a design technique for stabilizer and switching law.

1. Introduction

Currently, research on switched systems has attracted more and more attention from researchers. Switched systems can be considered as the simplest hybrid systems. Among switched systems, switched linear systems are the simplest. For a survey on hybrid systems and switched system, we refer to [2,3,7,14] and a recent book [20].

Although switched linear systems are considered as the simplest hybrid systems, the simplicity in form does not decrease their practical values. They appear in many engineering problems, such as in automobile areas, in rocket control and biological engineering etc,.

Stabilization of a switched linear system still remains a very challenging problem. Concerning this problem, there are two aspects. One is to stabilize a switched linear system without control channel, namely, to stabilize it by designing a suitable switching sequence, or switching law. The other is to stabilize a switched linear control system. For systems of this type, we have more choice. That is, this type of system can be stabilized by designing a switching law and constructing a controller.

Concerning stabilization of switched linear systems, there are a lot of sources, here we name some as [4,6,8,12,13,17,18,21] etc. Topics on other aspects of the switched systems can be found in [5,16,19,22] etc.

Consider a switched linear control system

$$\dot{x} = A_{\sigma(t)}x + b_{\sigma(t)}u, \quad x \in \mathbb{R}^2 \tag{1.1}$$

where the function $\sigma : [0, +\infty) \to \{1, 2, \ldots, N\}$ is of piece-wise constant.

Using linear feedback $u(t) = F_\sigma x(t)$, a necessary and sufficient condition for the stabilization of planar switched linear systems was proposed in [4]. The problem considered there is the quadratic stabilization for systems with arbitrary switching. As for switched linear control systems with designable switching law, the following result is well known [10]:

Proposition 1. *A sufficient condition for quadratic stabilizability is there exist gain matrices F_i, $i \in \{1, \ldots, N\}$, such that the matrix pencil*

$$\left\{ \sum_{i=1}^{N} w_i(A_i + B_iF_i) \,\middle|\, w_i \geq 0, \sum_{i=1}^{N} w_i = 1 \right\} \tag{1.2}$$

contains a Hurwitz matrix. For the case of $N = 2$, the condition is also necessary.

This paper provides a straightforward verifiable necessary and sufficient condition for linear feedback stabilizability. In addition, a detailed design technique for the controller and switching law is provided.

Since the switching law is designable, throughout this paper we assume the following:

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A1 Each subsystem (A_k, b_k) contains at least one unstable uncontrollable eigenvalue.

If one of the subsystems is stabilizable then the problem becomes trivial.

The paper is organized as follows: Section 2 reveals an interesting structure property of a planar uncontrollable linear system, which plays a key role in the following discussion. Section 3 presents the necessary and sufficient condition. Section 4 is an illustrative example. Section 5 is the conclusion.

2. A system structure property

Under the assumption **A1**, it is obvious that all subsystems are not controllable. In this section we prove a property of uncontrollable pair (A, b). Suppose $A \in \mathbb{R}^{2\times 2}$ and $0 \neq b \in \mathbb{R}^2$, then the pair (A, b) is completely controllable if, and only if

$$\mathrm{Rank}\{b, Ab\} = 2. \tag{2.1}$$

Now, we assume (A, b) is not completely controllable, which implies that $\mathrm{Rank}\{b, Ab\} = 1$ since $b \neq 0$. Therefore, b is an eigenvector associated with some eigenvalue of the matrix A. Suppose $Ab = \lambda b$, then λ must be a real number since both A and b are real.

Linear control theory tells us that there is a nonsingular matrix $T \in \mathbb{R}^{2\times 2}$, such that the following equalities hold [9,11]:

$$T^{-1}AT = \begin{pmatrix} \lambda & a \\ 0 & \mu \end{pmatrix} \triangleq \tilde{A}, \qquad T^{-1}b = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \triangleq \tilde{b}. \tag{2.2}$$

Instead of discussing (A, b) directly, we examine $(\tilde{A}, \tilde{b})$. From (2.2), it is obvious that μ is an uncontrollable real eigenvalue of $\tilde{A}$. Moreover, $\lambda = \mu$ is possible.

Next, we point out that the eigenvectors, associated with the uncontrollable eigenvalue μ of the matrix $\tilde{A} + \tilde{b}\tilde{c}^T$, can be made orthogonal to any prescribed vector, by choosing suitable control matrix $\tilde{c}^T$.

Proposition 2. *Suppose $\tilde{A}$ and $\tilde{b}$ are in form (2.2) and $\tilde{v} = (\tilde{v}_1, \tilde{v}_2)$ is an arbitrary vector in the space $\mathbb{R}^2$. Then, a row vector $\tilde{c} = (\tilde{c}_1, \tilde{c}_2)$ can be found such that the eigenvectors associated with the uncontrollable eigenvalue of the matrix $\tilde{A} + \tilde{b}\tilde{c}^T$ is orthogonal to $\tilde{v}$.*

Proof. It is not difficult to write out the expression of the matrix $\tilde{A} + \tilde{b}\tilde{c}^T$ as

$$\begin{pmatrix} \lambda + \tilde{c}_1 & a + \tilde{c}_2 \\ 0 & \mu \end{pmatrix}, \tag{2.3}$$

whose eigenvalues are $\lambda + \tilde{c}_1$ and μ. And one eigenvector associated with the eigenvalue μ has the form:

$$(\tilde{x}_1, \tilde{x}_2)^T = (-(a + \tilde{c}_2), \lambda + \tilde{c}_1 - \mu)^T. \tag{2.4}$$

Now we check if the following equality

$$-(a + \tilde{c}_2)\tilde{v}_1 + (\lambda + \tilde{c}_1 - \mu)\tilde{v}_2 = 0 \tag{2.5}$$

can be hold by suitably choosing $\tilde{c}_1$ and $\tilde{c}_2$. Obviously, if $(\tilde{v}_1, \tilde{v}_2) = (0, 0)$, then any $(\tilde{c}_2, \tilde{c}_2)$ satisfies (2.5). For $(\tilde{v}_1, \tilde{v}_2) \neq (0, 0)$, the formula of $(\tilde{c}_1, \tilde{c}_2)$ can be obtained explicitly. Suppose $\tilde{v}_1 \neq 0$, then the solution for Eq. (2.5) is

$$(\tilde{c}_1, \tilde{c}_2) = \left(\tilde{c}_1, \frac{\tilde{v}_2}{\tilde{v}_1}\tilde{c}_1 - \frac{(\mu - \lambda)\tilde{v}_2 + a\tilde{v}_1}{\tilde{v}_1}\right) \tag{2.6}$$

where $\tilde{c}_1$ is free. While for $\tilde{v}_2 \neq 0$, the solution is

$$(\tilde{c}_1, \tilde{c}_2) = \left(\frac{\tilde{v}_1}{\tilde{v}_2}\tilde{c}_2 + \frac{\tilde{v}_1}{\tilde{v}_2}a + (\mu - \lambda), \tilde{c}_2\right) \tag{2.7}$$

where $\tilde{c}_2$ is free. □

Remark 2.1. The formula (2.7) shows that when $\tilde{v}_1 = 0$, the solution becomes $(\tilde{c}_1, \tilde{c}_2) = (\mu - \lambda, \tilde{c}_2)$, which makes all the diagonal elements of the matrix $\tilde{A}$ equal to μ. Since $\tilde{c}_2$ is free, we can make the matrix $\tilde{A}$ a multiple of the identity matrix I.

Remark 2.2. In the case of the first component of $\tilde{v}$, $\tilde{v}_1 = 0$, there holds $\tilde{b} \perp \tilde{v}$, which means that the eigenvector associated with μ is a multiple of $\tilde{b}$. This implies that the matrix $\tilde{A} + \tilde{b}\tilde{c}^T$ assumes μ as its eigenvalue of multiplicity 2. Thus, if we want the eigenvector associated with μ to be orthogonal to $\tilde{v}$, the controllable eigenvalue of $\tilde{A}$ can only be controlled to be μ, which is not a stable eigenvalue as we designed. For this reason, we need to suppose $\tilde{b}$ not perpendicular to $\tilde{v}$.

Remark 2.3. When $\tilde{v}_1 \neq 0$, which means that $\tilde{b}$ is not perpendicular to $\tilde{v}$. Then (2.6) shows that $\tilde{c}^T$ can perform two functions, one is to make the eigenvectors associated with μ perpendicular to $\tilde{v}$, the other is to make the another eigenvalue of $\tilde{A} + \tilde{b}\tilde{c}^T$ arbitrarily negatively large.

Next we point out the validity of the proposition when the concerned matrices are not in the canonical forms.

In fact,

$$T^{-1}\left(A - \mu I + bc^T T^{-1}\right) T\tilde{x} = \left(\tilde{A} + \tilde{b}\tilde{c}^T\right)\tilde{x} = 0, \tag{2.8}$$

from which we can see that the eigenvector associated with μ of the matrix $A + bc^T$ is

$$x = T\begin{pmatrix} -(a + \tilde{c}_2) \\ \lambda + \tilde{c}_1 - \mu \end{pmatrix} \tag{2.9}$$

where $c^T = \tilde{c}^T T^{-1}$.

The above arguments establish the correspondence between x, c and $\tilde{x}, \tilde{c}$, respectively.

Proposition 3. *Let $A \in \mathbb{R}^{2\times 2}$ and $b \in \mathbb{R}^2$. Suppose*

$$\mathrm{Rank}\{b, Ab\} = 1, \tag{2.10}$$

then for any vector $v \in \mathbb{R}^2$, there exists at least one row vector c^T such that the eigenvector associated with the uncontrollable eigenvalue of the matrix $A + bc^T$ is orthogonal to v.

Corollary 1. *Under the hypothesis of the above proposition, if v is any vector that is not perpendicular to b, then there exists at least one row vector c^T such that the eigenvector associated with the uncontrollable eigenvalue of the matrix $A + bc^T$ is orthogonal to v, and the controllable eigenvalue of $A + bc^T$ is arbitrarily negatively large.*

3. Stabilization via designed switching law and controls

In this section we present a necessary and sufficient condition for the stabilizability of the planar switched linear control systems (1.1).

In order to avoid the trival case, we assume **A1**. First, we need a definition.

Definition 1. System (1.1) is said to be asymptotically stabilizable if there exist a piecewise constant switching sequence $\sigma(t)$ and a piecewise control law $u_{\sigma(t)}$ such that the trajectories of (1.1) asymptotically approach the origin from any initial state $x_0 \in \mathbb{R}^2$.

We now present the main result of this section, and also the main conclusion of the article.

Theorem 1. *Consider the switched planar linear control system (1.1) with $N = 2$. Assume (A_1, b_1) and (A_2, b_2) are both unstabilizable systems. Then the switched system is asymptotically stabilizable if and only if b_1 and b_2 are linearly independent.*

To prove this result, we need the following lemmas.

Lemma 3.1 ([15]). *If a matrix A has distinct eigenvalues, it is simple.*

Lemma 3.2 ([1,15]). *Let A be a simple matrix with eigenvalues $\lambda_1, \ldots \lambda_n$ and associated eigenvectors $X_1, \ldots, X_n$. Then there are left eigenvectors $Y_1, \ldots, Y_n$, for which $Y_j^T X_i = \delta_{i,j}$, $1 \le i, j \le n$, and*

$$A = \sum_{i=1}^{n} \lambda_i X_i Y_i^T. \tag{3.1}$$

Lemma 3.3 ([1,15]). *If a simple matrix A takes the representation (3.1), then*

$$e^{At} = \sum_{i=1}^{n} e^{\lambda_i t} X_i Y_i^T. \tag{3.2}$$

Now we are ready to prove our main theorem:

Proof. (Necessity) Assure b_1 and b_2 are linearly dependent. Then after a suitable change of coordinates each subsystem can be expressed as

$$\dot{x} = \begin{pmatrix} \lambda_i & a_i \\ 0 & \mu_i \end{pmatrix} + \begin{pmatrix} c_i \\ 0 \end{pmatrix} u, \quad i = 1, 2 \tag{3.3}$$

where $c_i \neq 0$, $i = 1, 2$.

According to **A1**, no matter how to choose controls and/or switching laws, the second component x_2 can never converge to zero.

(Sufficiency) Consider the subsystem

$$\dot{x} = A_1 x + b_1 u. \tag{3.4}$$

Denote the set of the eigenvalues of the matrix A_1 by $\{\lambda_{11}, \lambda_{12}\}$ and assume λ_{11} is controllable, while λ_{12} uncontrollable [11,9]. For any row vector c_1^T, denote the set of eigenvalues of the matrix $A_1 + b_1 c_1^T$ by $\{\lambda'_{11}, \lambda_{12}\}$. So, we can select suitable c_1^T, such that λ'_{11} is negatively sufficiently large. To be more specific, we make λ'_{11} satisfy

$$\lambda'_{11} < -|\lambda_{22}| \tag{3.5}$$

where λ_{22} is the uncontrollable eigenvalue of the matrix A_2.

Denote the eigenvectors of the matrix $A_1 + b_1 c_1^T$ as v'_{11} and v_{12}. According to Lemma 3.2, there are row vectors u_{11}^T and u_{12}^T such that

$$\tilde{A}_1 \triangleq A_1 + b_1 c_1^T = \lambda'_{11} v'_{11} u_{11}^T + \lambda_{12} v_{12} u_{12}^T \tag{3.6}$$

and

$$u_{11}^T v_{12} = 0, \qquad u_{12}^T v'_{11} = 0. \tag{3.7}$$

Next we consider another subsystem

$$\dot{x} = A_2 x + b_2 u. \tag{3.8}$$

Denote the spectrum of the matrix A_2 by $\{\lambda_{21}, \lambda_{22}\}$, and suppose λ_{21} is controllable, λ_{22} uncontrollable.

By the aforementioned propositions and remarks, there exists a row vector c_2^T such that v_{22}, an eigenvector associated with the eigenvalue λ_{22} of the matrix $A_2 + b_2 c_2^T$, is perpendicular to u_{12}^T, i.e. $u_{12}^T v_{22} = 0$.

The assumption **A1** and the hypotheses of the theorem imply that the rank condition $\text{Rank}\{b_1, A_1 b_1\} = 1$ holds, which means b_1 is an eigenvector associated with the controllable eigenvalue of the matrix A_1. Therefore, $u_{12}^T v'_{11} = 0 \Leftrightarrow u_{12}^T b_1 = 0$. Consequently, the linear independence of b_1 and b_2 implies $u_{12}^T b_2 \neq 0$, that is, u_{12} is not perpendicular to b_2. So, according to the Proposition 3 and Corollary 1, the row vector c_2^T can also be chosen in such a way that the controllable eigenvalue λ'_{21} of the matrix $A_2 + b_2 c_2^T$ is negatively sufficiently large. In particular, it can satisfy $\lambda'_{21} < -|\lambda_{12}|$.

Similar to (3.6) and (3.7), we can find u_{21}^T and u_{22}^T such that

$$\tilde{A}_2 \triangleq A_2 + b_2 c_2^T = \lambda'_{21} v'_{21} u_{21}^T + \lambda_{22} v_{22} u_{22}^T \tag{3.9}$$

and

$$u_{21}^T v_{22} = 0, \qquad u_{22}^T v'_{21} = 0 \tag{3.10}$$

Now we examine the product $e^{(A_1+b_1c_1^T)t_2} e^{(A_2+b_2c_2^T)t_1}$. Obviously, matrices $A_1 + b_1 c_1^T$ and $A_2 + b_2 c_2^T$ satisfy Lemma 3.3. So

$$\begin{cases} e^{(A_1+b_1c_1^T)t_2} = e^{\lambda'_{11}t_2} v'_{11} u_{11}^T + e^{\lambda_{12}t_2} v_{12} u_{12}^T \\ e^{(A_2+b_2c_2^T)t_1} = e^{\lambda'_{21}t_1} v'_{21} u_{21}^T + e^{\lambda_{22}t_1} v_{22} u_{22}^T. \end{cases} \tag{3.11}$$

Therefore,

$$\begin{aligned} e^{(A_1+b_1c_1^T)t_2} e^{(A_2+b_2c_2^T)t_1} &= e^{\lambda'_{11}t_2 + \lambda'_{21}t_1} v'_{11} u_{11}^T v'_{21} u_{21}^T \\ &+ e^{\lambda'_{21}t_1 + \lambda_{12}t_2} v_{12} u_{12}^T v'_{21} u_{21}^T + e^{\lambda'_{11}t_2 + \lambda_{22}t_1} v'_{11} u_{11}^T v_{22} u_{22}^T. \end{aligned} \tag{3.12}$$

From the above equality we can see that the numbers lying on the exponential positions are all negative, while the matrices appearing in the above equality are all fixed. Therefore, for any positive number $\alpha < 1$, we can find a number $M > 0$ such that

$$\left\| e^{(A_1+b_1c_1^T)M} e^{(A_2+b_2c_2^T)M} \right\| \le \alpha, \tag{3.13}$$

which allows us to construct a switching law $\sigma(t)$ in the following way

$$\sigma(t) = \begin{cases} 2, & \text{when } 2kM \le t < (2k+1)M; \\ 1, & \text{when } (2k+1)M \le t < 2(k+1)M. \end{cases}$$
$$k = 0, 1, 2, \ldots. \tag{3.14}$$

And, the controller is

$$c_{\sigma(t)} = \begin{cases} c_2^T, & \text{when } 2kM \le t < (2k+1)M; \\ c_1^T, & \text{when } (2k+1)M \le t < 2(k+1)M. \end{cases}$$
$$k = 0, 1, 2, \ldots. \quad \square \tag{3.15}$$

Remark 3.1. From the above theorem we can see that the case for switching systems with $N > 2$ follows immediately. In fact, all that matters is the existence of two subsystems (A_j, b_j) and (A_k, b_k), with b_j, b_k linearly independent.

Remark 3.2. The conclusion derived here is somewhat different than quadratic stabilizability, since the most accepted definition of **quadratically stabilizable** requires the switching sequence depending on the state, i.e. the switching law is of the form $\sigma(x, t)$, while the switching law constructed in this paper depends only on time t.

4. An illustrative example

Since the proof of the main result is constructive, it provides a method to design the switching law and the control. In this section we use an example to illustrate the method presented in this paper.

Example 1. Let

$$A_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \qquad b_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \qquad A_2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$
$$b_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \tag{4.1}$$

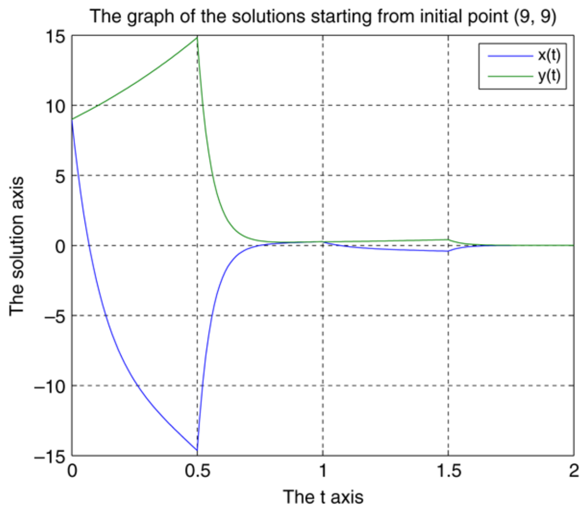

Fig. 1. Figure of Example 1.

Choose $c_2^T = (-9, 9)$. Then a direct computation shows that

$$e^{(A_2+b_2c_2^T)t} = e^{2t}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\end{pmatrix} + e^{-9t}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}\frac{1}{2} & -\frac{1}{2}\end{pmatrix}. \qquad (4.2)$$

Choose $c_1^T = (-10, 0)$ and $c_0^T = (0, -11)$. Then the eigenvectors associated with the uncontrollable eigenvalue 1 of the matrix $A_1 + b_1c_1^T + b_1c_0^T$ is orthogonal to the vector $\left(\frac{1}{2}, \frac{1}{2}\right)$. Also

$$e^{(A_1+b_1c_1^T+b_1c_0^T)t} = e^{-9t}\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1 & 1\end{pmatrix} + e^{t}\begin{pmatrix}-1\\1\end{pmatrix}\begin{pmatrix}0 & 1\end{pmatrix}. \qquad (4.3)$$

A direct computation shows that the following switching sequence and the controller

$$\begin{aligned} \sigma(t) &= \begin{cases} 1, & \text{for } k \le t < k+0.5; \\ 2, & \text{for } k+0.5 \le t < k+1 \end{cases} \quad k = 0, 1, \ldots. \\ c_{\sigma(t)}^T &= \begin{cases} c_1^T + c_0^T, & \text{for } k \le t < k+0.5; \\ c_2^T, & \text{for } k+0.5 \le t < k+1 \end{cases} \quad k = 0, 1, \ldots. \end{aligned} \qquad (4.4)$$

can stabilize the switched control system.

Fig. 1 is the portrait of the solution starting from the initial point $(9, 9)$ and the corresponding phase graph during time interval $[0, 2]$.

5. Conclusions

The stabilization of planar switched linear control systems via switching strategy and control was considered. Excluding the trivial case that a mode is stabilizable, a necessary and sufficient condition was obtained. The constructive proof provides a design technique for both the switching law and the stabilizer.

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