Speed regulation of permanent magnet synchronous motor via feedback dissipative Hamiltonian realisation

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Abstract: Here, the speed regulation of permanent magnet synchronous motors (PMSM) is investigated through feedback dissipative Hamiltonian realisation. Feedback laws for precise and uncertain cases are constructed to transfer the dynamics of PMSM into dissipative Hamiltonian forms. When the load torque is unknown, to realise the speed regulation, an update law is embedded into the dissipative Hamiltonian structure. Simulations show that the controllers designed in this way are efficient.

1 Introduction

In recent years, permanent magnet synchronous motors (PMSM) have received more attention because of their advantages over many other kinds of motors, such as induction motors and DC motors. Generally speaking, they have high power density, torque-to-interia ratio and efficiency. PMSMs play an important role in motion control applications and are broadly used as electric drives. However, it is not an easy task to design a controller of high performance in order to achieve the speed regulation, not only because of the strong coupling between the motor speed and the electrical quantities, also because of the different kinds of uncertainties, for example parameter and modelling uncertainty.

Various nonlinear analysis tools have been used by many authors to investigate the speed control of PMSM, such as sliding-mode control technique [1], adaptive backstepping method [2, 3], feedback linearisation control [4] and so on. Recently, passivity property has drawn considerable attention in nonlinear control design [5–11], for example, Hamiltonian system method [12–16] and IDA-PBC technique [17]. Particularly, the speed regulation of PMSM was investigated by Petrovic *et al.* [18] using interconnection and damping assignment-passivity-based control (IDA-PBC). In this paper, we propose feedback dissipative Hamiltonian realisation (FDHR) of dynamics of PMSM for both precise and uncertain cases to achieve the speed regulation of PMSM. The adaptive control of Hamiltonian systems was first proposed by Xi [19], where it is successfully applied to power systems. This paper also develops an adaptive control technique of Hamiltonian systems to deal with the speed regulation of PMSM with parametric uncertainty in load torque and stator resistance. We first consider the Hamiltonian realisation and the update law of the estimated load torque simultaneously, thus the dynamics of the estimated load torque is naturally embedded into the closed-loop dissipative Hamiltonian system. Then, the update law for the stator resistance is constructed by the certainty-equivalence method [20].

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2 Mathematical model of PMSM

When described in $d{-}q$ frame, a typical PMSM can be represented as the following dynamic model [21, 22]

$$\begin{aligned} L_d \frac{\mathrm{d}i_d}{\mathrm{d}t} &= -R_s i_d + n_p \omega L_q i_q + u_d \\ L_q \frac{\mathrm{d}i_q}{\mathrm{d}t} &= -R_s i_q - n_p \omega L_d i_d - n_p \omega \Phi + u_q \\ J \frac{\mathrm{d}\omega}{\mathrm{d}t} &= \frac{3}{2} n_p [(L_d - L_q) i_d i_q + \Phi i_q] - \tau_L \end{aligned} \tag{1}$$

where i_d and i_q are the $d{-}q$ axis currents, u_d and u_q the $d{-}q$ axis voltages, R_s the stator resistance, L_d and L_q the $d{-}q$ axis stator inductors, n_p the number of pole pairs, Φ the flux linkage of the permanent magnet, J the rotor moment of inertia, and τ_L the load torque.

Define

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} L_d i_d \\ L_q i_q \\ J\omega \end{pmatrix}, \quad \boldsymbol{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_d \\ u_q \end{pmatrix}$$

and denote $a = R_s/L_d$, $b = n_p/J$, $c = R_s/L_q$, $d = \Phi b$, $e = (3(L_d - L_q)/2L_d L_q) n_p$ and $h = (3n_p\Phi)/2L_q$, then system (1) can be represented as

$$\dot{x} = \boldsymbol{f}_0(x) + \boldsymbol{G}_0 \boldsymbol{u} \tag{2}$$

where

$$\boldsymbol{f}_0(x) = \begin{pmatrix} -ax_1 + bx_2x_3 \\ -cx_2 - bx_1x_3 - dx_3 \\ ex_1x_2 + hx_2 - \tau_L \end{pmatrix}$$

$$\boldsymbol{G}_0 = (g_1\, g_2) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

The control objective is to regulate the rotor speed to any pre-specified value $\bar{\omega}$. In real physical systems, some parameters are unknown because of various reasons. Both precise and uncertain cases are discussed in the sequel.

3 Feedback dissipative Hamiltonian realisation

A generalised Hamiltonian system is defined as [8]

$$\dot{x} = \boldsymbol{F}(x)\nabla H(x) \tag{3}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ and $\boldsymbol{F}(x) \in \mathbb{R}^{n\times n}$ are called the structure matrices and $H(x)$ the Hamiltonian function. If the structure matrix $\boldsymbol{F}(x)$ satisfies

$$\boldsymbol{F}(x) + \boldsymbol{F}^{\mathrm{T}}(x) \le 0 \tag{4}$$

then we call system (3) a dissipative Hamiltonian system. In this case, $\boldsymbol{F}(x)$ can be decomposed as

$$\boldsymbol{F}(x) = J(x) - R(x)$$

where $J(x)$ is skew symmetric and $R(x)$ the symmetric, positive semi-definite.

Consider an affine nonlinear system

$$\dot{x} = \boldsymbol{f}(x) + \boldsymbol{G}(x)\boldsymbol{u} \tag{5}$$

where $x \in \mathbb{R}^n$, $\boldsymbol{u} \in \mathbb{R}^m$ and $\boldsymbol{G}(x)$ are of full rank.

Definition 1 [23]: System (5) is said to have a feedback Hamiltonian realisation if there exists a control law $\boldsymbol{u} = \phi(x)$ such that the closed-loop system is of form (3).

System (5) is said to have a feedback dissipative Hamiltonian realisation (FDHR) if the closed loop is a dissipative Hamiltonian system, that is, its structure matrix satisfies (4).

With uncertain parameters, system (5) becomes

$$\dot{x} = \boldsymbol{f}(x, \theta) + \boldsymbol{G}(x, \theta)\boldsymbol{u} \tag{6}$$

where $\theta \in \mathbb{R}^p$ are uncertain parameters.

Definition 2: System (6) is said to have an adaptive feedback Hamiltonian realisation if there exists a control law

$$\begin{aligned} \boldsymbol{u} &= \phi(x, \hat{\theta}) \\ \dot{\hat{\theta}} &= \eta(x, \hat{\theta}) \end{aligned} \tag{7}$$

such that the closed loop is of the form

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{\theta}} \end{pmatrix} = \boldsymbol{F}(x, \hat{\theta}, \theta) \begin{pmatrix} \nabla_x H(x, \hat{\theta}, \theta) \\ \nabla_{\hat{\theta}} H(x, \hat{\theta}, \theta) \end{pmatrix}$$

with $\hat{\theta} \in \mathbb{R}^p$ and $\boldsymbol{F}$ an $(n+p) \times (n+p)$ matrix.

Moreover, if $\boldsymbol{F}$ is dissipative, then system (6) is said to have an adaptive feedback dissipative Hamiltonian realisation (AFDHR).

Remark 1: If system (6) has an AFDHR with the Hamiltonian function positive-definite with respect to $(0, \theta) \in \mathbb{R}^{n+p}$, then (7) is its adaptive stabiliser, embedded into the closed-loop Hamiltonian structure.

According to Definition 1, a feedback dissipative Hamiltonian realisation of system (5) means finding a feedback law $\boldsymbol{u} = \phi(x)$, a Hamiltonian function $H(x)$ and a dissipative structure matrix $\boldsymbol{F}(x)$ such that the matching equation [24]

$$\boldsymbol{f}(x) + \boldsymbol{G}(x)\phi(x) = \boldsymbol{F}(x)\nabla H(x) \tag{8}$$

holds. In general, this leads to a set of partial differential equations. But for a real physical system, according to its physical meaning and the control objectives, we may find a natural candidate Hamiltonian function, then (8) becomes a set of algebraic equations. A necessary and sufficient condition for the existence of feedback dissipative Hamiltonian realisation for fixed $\boldsymbol{F}(x)$ and Hamiltonian function $H(x)$ is as follows.

Lemma 1 [24]: For fixed $H(x)$ and $\boldsymbol{F}(x)$, which satisfy (4), there exists a feedback such that (8) holds if and only if the projected matching equation

$$\boldsymbol{G}^{\perp}(x)(\boldsymbol{f}(x) - \boldsymbol{F}(x)\nabla H(x)) = 0 \tag{9}$$

holds for an arbitrary full-rank left annihilator $\boldsymbol{G}^{\perp}(x)$ of $\boldsymbol{G}(x)$.

Full-rank left annihilator of $\boldsymbol{G}(x)$ is an $(n-m) \times n$ matrix $\boldsymbol{G}^{\perp}(x)$, which satisfies $\boldsymbol{G}^{\perp}(x)\boldsymbol{G}(x) = 0$ and $\mathrm{rank}(\boldsymbol{G}^{\perp}(x)) = n - \mathrm{rank}(\boldsymbol{G}(x))$ [24].

For system (6), without loss of generality, we consider the AFDHR for the case

$$\boldsymbol{G} = \begin{pmatrix} \boldsymbol{G}_m \\ O_{n-m} \end{pmatrix}$$

with $\boldsymbol{G}_m$ being an $m \times m$ matrix of full rank. We denote

$$\boldsymbol{f}(x, \theta) = \begin{pmatrix} \boldsymbol{f}_m(x, \theta) \\ \boldsymbol{f}_{n-m}(x, \theta) \end{pmatrix}$$

with $\boldsymbol{f}_m$ and $\boldsymbol{f}_{n-m}$ represent vectors containing the first m and the last $n-m$ components of $\boldsymbol{f}$, respectively. Similarly, for an $(n+p) \times (n+p)$ matrix $\boldsymbol{F}$, we denote

$$\boldsymbol{F} = \begin{pmatrix} \boldsymbol{F}_m \\ \boldsymbol{F}_{n-m} \\ \boldsymbol{F}_{\mathrm{p}} \end{pmatrix}$$

We have the following lemma.

Lemma 2: Assume that a pair of fixed $H(x, \hat{\theta}, \theta)$ and $\boldsymbol{F}(x, \hat{\theta}, \theta)$, satisfying (4), is given. Moreover, $\boldsymbol{F}_{\mathrm{p}}\nabla H$ and $\boldsymbol{G}_m^{-1}(\boldsymbol{F}_m\nabla H - \boldsymbol{f}_m)$ are assumed to be independent of the uncertain parameter θ. Then, system (6) has an AFDHR if and only if

$$\boldsymbol{f}_{n-m} - \boldsymbol{F}_{n-m}\nabla H = 0 \tag{10}$$

where ∇H stands for $\nabla_{(x,\hat{\theta})}H$.

Proof: The necessity is obvious. In the following, we assume that (10) holds. Denote

$$\tilde{\boldsymbol{G}} = \begin{pmatrix} \boldsymbol{G}_m \\ O_{n-m} \\ O_p \end{pmatrix}$$

and choose

$$\tilde{\boldsymbol{G}}^{\perp} = \begin{pmatrix} O_{(n-m)\times m} & I_{n-m} & O_{(n-m)\times p} \\ O_{p\times m} & O_{p\times(n-m)} & I_p \end{pmatrix}$$

Then

$$\begin{aligned} \tilde{\boldsymbol{G}}^{\perp}\left[\begin{pmatrix} f \\ \eta \end{pmatrix} - \boldsymbol{F}\nabla H\right] &= \begin{pmatrix} \boldsymbol{f}_{n-m} - \boldsymbol{F}_{n-m}\nabla H \\ \eta - \boldsymbol{F}_p\nabla H \end{pmatrix} \\ &= \begin{pmatrix} O \\ \eta - \boldsymbol{F}_p\nabla H \end{pmatrix} \end{aligned}$$

As $\boldsymbol{F}_p\nabla H$ is independent of θ, we can choose $\eta = \boldsymbol{F}_p\nabla H$, thus the matching equation (9) holds. According to Lemma 1, system (6) has an AFDHR by adaptive controller

$$\begin{aligned} \boldsymbol{u} &= \boldsymbol{G}_m^{-1}(\boldsymbol{F}_m\nabla H - \boldsymbol{f}_m) \\ \dot{\hat{\theta}} &= \boldsymbol{F}_p\nabla H \end{aligned} \tag{11}$$

because $\boldsymbol{G}_m^{-1}(\boldsymbol{F}_m\nabla H - \boldsymbol{f}_m)$ is independent of θ. □

4 Control design

In this section, we investigate the design technique to achieve the control goal through transferring the original system into a dissipative Hamiltonian form. According to the control objective, we choose a candidate Hamiltonian function that is minimised at the desired equilibrium point. Then, find a suitable control that transfers system (2) into a dissipative Hamiltonian system. For convenience, we first impose a pre-feedback to the original system to simplify the controller design.

4.1 Pre-feedback

For simplicity, we first use a pre-feedback

$$\begin{aligned} u_1 &= ax_1 - bx_2x_3 + v_1 \\ u_2 &= cx_2 + bx_1x_3 + dx_3 + v_2 \end{aligned} \tag{12}$$

to convert system (2) to

$$\dot{x} = \boldsymbol{f}(x) + \boldsymbol{G}v \tag{13}$$

where

$$\begin{aligned} \boldsymbol{f}(x) &= \begin{pmatrix} 0 \\ 0 \\ ex_1x_2 + hx_2 - \tau_L \end{pmatrix} \\ \boldsymbol{G} = G_0 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \end{aligned} \tag{14}$$

Note that if all the parameters in the pre-feedback are exactly known, then the design of controllers for system (13) is rather obvious. In the next, we also discuss the case when the stotor resistance R_s is unknown (precisely, a and $c = (L_d/L_q)a$ are unknown), so the parameters a and c in the pre-feedback should be replaced by their estimated values $\hat{a}$ and $(L_d/L_q)\hat{a}$ respectively.

In this paper, the speed regulation problem of PMSM means to design a control law such that the rotor velocity ω (or x_3) is regulated to any pre-specified value $\bar{\omega}$ (or $\bar{x}_3 = J\bar{\omega}$). From system (13), for any feedback law, equilibrium points of the closed-loop system must satisfy

$$e\bar{x}_1\bar{x}_2 + h\bar{x}_2 - \tau_L = 0$$

As we will see, for any $\bar{x}_1$ satisfying $e\bar{x}_1 + h \neq 0$ and any given $\bar{x}_3$, there exists a feedback law to asymptotically stabilise the point $\bar{x} = (\bar{x}_1, (\tau_L/(e\bar{x}_1 + h)), \bar{x}_3)^T$. In the following sections, we will investigate how to design feedback law to transfer system (2) into a dissipative Hamiltonian system.

Remark 2: From the third equation of system (1), we know the driving torque τ_D is

$$\tau_D = n_p[(L_d - L_q)i_d + \Phi]i_q$$

and the composition of torques is the difference between the driving torque and the load torque, that is $\tau = \tau_D - \tau_L$. So, if the load torque $\tau_L \neq 0$, the assumption $e\bar{x}_1 + h \neq 0$ (or equivalently $(L_d - L_q)\bar{i}_d + \Phi \neq 0$) means that the driving torque τ_D does not vanish when the rotor speed approaches its equilibrium value. (Of course, in this case, $\bar{i}_q \neq 0$.) In the case $\tau_L = 0$, as we will see, our results are still valid, this is because the driving torque and the load torque can still be balanced by regulating i_q to zero, that is, $\bar{i}_q = 0$. So, in this paper, we ignore the trivial case $e\bar{x}_1 + h = 0$ for simplicity.

4.2 For precise model

This section considers the case that all the parameters in the system are precisely known. In this case, the following result is obtained.

Proposition 1: Suppose all of the parameters in system (1) are precisely known, then the speed regulation problem of PMSM can be solved by the feedback

$$\begin{aligned} \boldsymbol{u}_d &= -\Gamma_1 L_d(i_d - \bar{i}_d) - \frac{3Jn_p(L_d - L_q)}{2L_dk_1} i_q(\omega - \bar{\omega}) \\ &\quad + R_s i_d - n_p L_q i_q \omega \\ \boldsymbol{u}_q &= -\Gamma_2 L_q\left(i_q - \frac{2\tau_L}{3n_p((L_d - L_q)\bar{i}_d + \Phi)}\right) \\ &\quad - \frac{3Jn_p((L_d - L_q)\bar{i}_d + \Phi)}{2L_qk_2}(\omega - \bar{\omega}) \\ &\quad + R_s i_q + n_p L_d i_d \omega + \Phi n_p \omega \end{aligned} \tag{15}$$

where Γ_1, Γ_2, k_1 and k_2 are positive numbers.

Proof: Obviously, achieving the regulating objective is equivalent to asymptotically stabilising the equilibrium $\bar{x}$. In order to stabilise the desired equilibrium point $\bar{x} = (\bar{x}_1, (\tau_L/(e\bar{x}_1 + h)), \bar{x}_3)^T$, we first choose $H(x) = 1/2(x - \bar{x})^T \boldsymbol{K}(x - \bar{x})$ as a candidate Hamiltonian function, where

$$\boldsymbol{K} = \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{pmatrix}$$

is a positive definite matrix. Suppose there is a feedback $v = \phi(x) = (\phi_1(x) \quad \phi_2(x))^T$ such that

$$\boldsymbol{f}(x) + \boldsymbol{G}\phi = \boldsymbol{F}(x)\nabla H(x) \tag{16}$$

where $\boldsymbol{F}(x)$ is an $n \times n$ dissipative matrix. According to Lemma 1 [24], such a feedback exists if and only if

$$\boldsymbol{G}^{\perp}(\boldsymbol{f}(x) - \boldsymbol{F}(x)\nabla H(x)) = 0 \tag{17}$$

holds for a full-rank left annihilator $\boldsymbol{G}^{\perp}$ of $\boldsymbol{G}$. It is easy to verify that one of such left annihilators of $\boldsymbol{G}$ is

$$\boldsymbol{G}^{\perp} = (0 \quad 0 \quad 1)$$

Suppose the last row of $\boldsymbol{F}(x)$ is $(\alpha \quad \beta \quad \gamma)$, that is

$$\boldsymbol{F}(x) = \begin{pmatrix} * & * & * \\ * & * & * \\ \alpha & \beta & \gamma \end{pmatrix}$$

then (17) becomes

$$(\alpha \quad \beta \quad \gamma)\nabla H(x) = ex_1x_2 + hx_2 - \tau_L$$

It is easy to see that one of its solutions is

$$\alpha = \frac{ex_2}{k_1}, \quad \beta = \frac{e\bar{x}_1 + h}{k_2}, \quad \gamma = 0$$

In order to render the resulting Hamiltonian system dissipative, that is

$$\boldsymbol{F}(x) + \boldsymbol{F}^{\mathrm{T}}(x) \leq 0$$

we choose

$$\boldsymbol{F}(x) = \begin{pmatrix} -\Gamma_1 & 0 & -\dfrac{ex_2}{k_1} \\ 0 & -\Gamma_2 & -\dfrac{e\bar{x}_1 + h}{k_2} \\ \dfrac{ex_2}{k_1} & \dfrac{e\bar{x}_1 + h}{k_2} & 0 \end{pmatrix}$$

where Γ_1 and Γ_2 are arbitrary positive constants. We can easily solve for the feedback $v = \phi(x)$ from (16) as

$$\begin{aligned} \phi_1(x) &= -\Gamma_1 \frac{\partial H(x)}{\partial x_1} - \frac{ex_2}{k_1}\frac{\partial H(x)}{\partial x_3} \\ \phi_2(x) &= -\Gamma_2 \frac{\partial H(x)}{\partial x_2} - \frac{(e\bar{x}_1 + h)}{k_2}\frac{\partial H(x)}{\partial x_3} \end{aligned} \tag{18}$$

Thus, the closed-loop system is

$$\dot{x} = (J(x) - R(x))\nabla H(x) \tag{19}$$

where

$$J(x) = \frac{1}{2}(\boldsymbol{F}(x) - \boldsymbol{F}^{\mathrm{T}}(x)) = \begin{pmatrix} 0 & 0 & -\dfrac{ex_2}{k_1} \\ 0 & 0 & -\dfrac{(e\bar{x}_1 + h)}{k_2} \\ \dfrac{ex_2}{k_1} & \dfrac{(e\bar{x}_1 + h)}{k_2} & 0 \end{pmatrix}$$

$$R(x) = -\frac{1}{2}(\boldsymbol{F}(x) + \boldsymbol{F}^{\mathrm{T}}(x)) = \begin{pmatrix} \Gamma_1 & 0 & 0 \\ 0 & \Gamma_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \geq 0$$

As the Hamiltonian function is positive definite and the closed loop is dissipative, it is stable. In order to prove that $\bar{x}$ is asymptotically stable, we calculate the derivative of $H(x)$ along the trajectories of the closed loop as follows

$$\begin{aligned} \dot{H}(x) &= -\mathrm{d}H(x)R(x)\nabla H(x) \\ &= -\Gamma_1(x_1 - \bar{x}_1)^2 - \Gamma_2\left(x_2 - \frac{\tau_{\mathrm{L}}}{e\bar{x}_1 + h}\right)^2 \leq 0 \end{aligned}$$

So, $\dot{H}(x)$ is positive semi-definite and

$$M \triangleq \{\boldsymbol{x} | \dot{H}(x) = 0\} = \left\{ x | x_1 = \bar{x}_1 \text{ and } x_2 = \frac{\tau_{\mathrm{L}}}{e\bar{x}_1 + h} \right\}$$

In the following, we will show that the only solution of the closed-loop system contained in M is $\bar{x}$. Thus, according to LaSelle's invariance principle, the closed-loop system is asymptotically stable. In fact, the closed-loop system is

$$\begin{aligned} \dot{x}_1 &= -\frac{k_3 e}{k_1} x_2(x_3 - \bar{x}_3) - \Gamma_1 k_1(x_1 - \bar{x}_1) \\ \dot{x}_2 &= -\frac{k_3(e\bar{x}_1 + h)}{k_2}(x_3 - \bar{x}_3) - \Gamma_2 k_2(x_2 - \bar{x}_2) \\ \dot{x}_3 &= ex_2(x_1 - \bar{x}_1) + (e\bar{x}_1 + h)(x_2 - \bar{x}_2) \end{aligned} \tag{20}$$

Suppose $(\bar{x}_1 \quad \bar{x}_2 \quad x_3(t))^{\mathrm{T}}$ is a solution contained in M, then from the second equation of the this closed-loop system, we have $x_3(t) \equiv \bar{x}_3$. Recall the form of M, the conclusion follows.

Combining the control (18) with (12), we obtain an asymptotical stabiliser for system (2) as

$$\begin{aligned} u_1 &= -\Gamma_1(x_1 - \bar{x}_1) - \frac{ex_2}{k_1}(x_3 - \bar{x}_3) + ax_1 - bx_2x_3 \\ u_2 &= -\Gamma_2\left(x_2 - \frac{\tau_L}{e\bar{x}_1 + h}\right) - \frac{(e\bar{x}_1 + h)}{k_2}(x_3 - \bar{x}_3) \\ &\quad + cx_2 + bx_1x_3 + dx_3 \end{aligned} \tag{21}$$

which is equivalent to (15). □

4.3 Adaptive control

In this section, we consider uncertain cases. First, assume that the load torque is uncertain. Then, consider a more general case when the stator resistance is also unknown. The adaptive controls are constructed, respectively, to solve the speed regulation problem.

4.3.1 Load torque is uncertain: For PMSMs, it is very likely that the load torque is unknown. The following control is constructed to solve the problem.

Proposition 2: Suppose that the load torque τ_{L} is uncertain, then the speed regulation problem of PMSM can be solved by the following adaptive controller

$$\begin{aligned} u_1 &= -\bar{\Gamma}_1(i_d - \bar{i}_d) - \frac{3}{2}\bar{\Gamma}_2(L_d - L_q)i_q(\omega - \bar{\omega}) \\ &\quad + R_{\mathrm{s}}i_d - n_{\mathrm{p}}L_q i_q \omega \\ u_2 &= -\bar{\Gamma}_3\left(i_q - \frac{2\hat{\tau}_{\mathrm{L}}}{3n_{\mathrm{p}}[(L_d - L_q)\bar{i}_d + \Phi]}\right) \\ &\quad - \left[\frac{3}{2}\bar{\Gamma}_4((L_d - L_q)\bar{i}_d + \Phi) \right. \\ &\quad \left. + \frac{2\bar{\Gamma}_5}{3[(L_d - L_q)\bar{i}_d + \Phi]}\right](\omega - \bar{\omega}) \\ &\quad + R_{\mathrm{s}}i_q + n_{\mathrm{p}}L_d i_d \omega + n_{\mathrm{p}}\Phi\omega \\ \dot{\hat{\tau}}_{\mathrm{L}} &= -\bar{\Gamma}_6(\omega - \bar{\omega}) \end{aligned} \tag{22}$$

where $\bar{\Gamma}_i$ $(i = 1, 2, \ldots, 6)$ are positive numbers.

Proof: Note that τ_{L} does not appear in control law (12), thus we can directly investigate system (13). Our objective is to construct an adaptive law

$$\dot{\hat{\tau}}_{\mathrm{L}} = \eta(x, \hat{\tau}_{\mathrm{L}}) \tag{23}$$

to estimate the uncertain load torque and a feedback law

$$v = \varphi(x, \hat{\tau}_{\mathrm{L}}) = \begin{pmatrix} \varphi_1(x, \hat{\tau}_{\mathrm{L}}) \\ \varphi_2(x, \hat{\tau}_{\mathrm{L}}) \end{pmatrix}$$

to make $(\bar{x},\tau_L)$ asymptotically stable. Combining (13) and (23), we have

$$\dot{z} = \tilde{f}(x, \hat{\tau}_L) + \tilde{\boldsymbol{G}} v \tag{24}$$

where $z = \begin{pmatrix} x \\ \hat{\tau}_L \end{pmatrix}$ and

$$\tilde{\boldsymbol{f}}(x, \hat{\tau}_L) = \begin{pmatrix} 0 \\ 0 \\ e x_1 x_2 + h x_2 - \tau_L \\ \eta(x, \hat{\tau}_L) \end{pmatrix}, \quad \tilde{\boldsymbol{G}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Similar to the discussion in Section 4.2, according to the control objective, we first choose a candidate Hamiltonian function. Then, using this Hamiltonian function, we can find a suitable control and an adaptive law to transfer system (24) into a dissipative Hamiltonian system.

Choosing

$$H(z) = \frac{\lambda_1}{2}(x_1 - \bar{x}_1)^2 + \frac{\lambda_2}{2}\left(x_2 - \frac{\hat{\tau}_L}{e\bar{x}_1 + h}\right)^2 + \frac{\lambda_3}{2}(x_3 - \bar{x}_3)^2 + \frac{\lambda_4}{2}(\hat{\tau}_L - \tau_L)^2$$

where $\lambda_i > 0$ $(i = 1, 2, 3, 4)$ are adjustable parameters, we have

$$\nabla H(z) = \begin{pmatrix} \lambda_1(x_1 - \bar{x}_1) \\ \lambda_2\left(x_2 - \frac{\hat{\tau}_L}{e\bar{x}_1 + h}\right) \\ \lambda_3(x_3 - \bar{x}_3) \\ \lambda_4(\hat{\tau}_L - \tau_L) - \frac{\lambda_2}{e\bar{x}_1 + h}\left(x_2 - \frac{\hat{\tau}_L}{e\bar{x}_1 + h}\right) \end{pmatrix}$$

Suppose that

$$\tilde{\boldsymbol{F}}(z) = \begin{pmatrix} * & * & * & 0 \\ * & * & * & 0 \\ \alpha & \beta & \gamma & \xi \\ \lambda & \mu & \nu & 0 \end{pmatrix}$$

According to Lemma 2, it is easy to check that there is an adaptive controller such that the closed-loop system becomes

$$\dot{z} = \tilde{\boldsymbol{F}}(z)\nabla H(z)$$

if and only if

$$(\alpha \quad \beta \quad \gamma \quad \xi)\nabla H(z) = e x_1 x_2 + h x_2 - \tau_L \tag{25}$$

A particular solution of (25) is

$$\alpha = \frac{e x_2}{\lambda_1}, \quad \beta = \frac{e\bar{x}_1 + h}{\lambda_2} + \frac{1}{\lambda_4(e\bar{x}_1 + h)}, \quad \xi = \frac{1}{\lambda_4}, \quad \gamma = 0$$

In order to assure the dissipation of $\tilde{\boldsymbol{F}}(z)$, we choose

$$\tilde{\boldsymbol{F}}(z) = \begin{pmatrix} -\Gamma_1 & 0 & -\alpha & 0 \\ 0 & -\Gamma_2 & -\beta & 0 \\ \alpha & \beta & 0 & \xi \\ 0 & 0 & -\xi & 0 \end{pmatrix}$$

Thus, according to (11), the corresponding control and adaptive laws are

$$\begin{aligned} v_1 &= -\Gamma_1 \frac{\partial H(z)}{\partial x_1} - \frac{e x_2}{\lambda_1}\frac{\partial H(z)}{\partial x_3} \\ v_2 &= -\Gamma_2 \frac{\partial H(z)}{\partial x_2} - \left[\frac{e\bar{x}_1 + h}{\lambda_2} + \frac{1}{\lambda_4(e\bar{x}_1 + h)}\right]\frac{\partial H(z)}{\partial x_3} \\ \dot{\hat{\tau}}_L &= -\frac{1}{\lambda_4}\frac{\partial H(z)}{\partial x_3} \end{aligned} \tag{26}$$

Note that in the form of $\nabla H(z)$, the only term that contains τ_L is $\partial H(z)/\partial \hat{\tau}_L$, so τ_L does not appear in the above controller. The resulting closed-loop system is

$$\dot{z} = (\tilde{J}(z) - \tilde{\boldsymbol{R}}(z))\nabla H(z) \tag{27}$$

where

$$\tilde{J}(z) = \frac{1}{2}(\tilde{\boldsymbol{F}}(z) - \tilde{\boldsymbol{F}}^{\mathrm{T}}(z)) = \begin{pmatrix} 0 & 0 & -\frac{e x_2}{\lambda_1} & 0 \\ 0 & 0 & -\left[\frac{e\bar{x}_1 + h}{\lambda_2} + \frac{\xi}{e\bar{x}_1 + h}\right] & 0 \\ \frac{e x_2}{\lambda_1} & \frac{e\bar{x}_1 + h}{\lambda_2} + \frac{\xi}{e\bar{x}_1 + h} & 0 & \xi \\ 0 & 0 & -\xi & 0 \end{pmatrix}$$

$$\tilde{\boldsymbol{R}}(z) = -\frac{1}{2}(\tilde{\boldsymbol{F}}(z) + \tilde{\boldsymbol{F}}^{\mathrm{T}}(z)) = \begin{pmatrix} \Gamma_1 & 0 & 0 & 0 \\ 0 & \Gamma_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \geq 0$$

A straightforward calculation shows that

$$\dot{H}(z) = -\lambda_1\Gamma_1(x_1 - \bar{x}_1)^2 - \lambda_2\Gamma_2\left(x_2 - \frac{\hat{\tau}_L}{e\bar{x}_1 + h}\right)^2 \leq 0$$

Define

$$\tilde{M} = \{z | \dot{H}(z) = 0\} = \left\{z | x_1 = \bar{x}_1 \text{ and } x_2 = \frac{\hat{\tau}_L}{e\bar{x}_1 + h}\right\}$$

In order to use LaSelle's invariance principle to obtain the asymptotic stability of the closed-loop system, we suppose that $(x_1\ x_2(t)\ x_3(t)\ \tau_L(t))^{\mathrm{T}}$ is a solution of (27), contained in $\tilde{M}$, that is,

$$x_2(t) = \frac{1}{e\bar{x}_1 + h}\hat{\tau}_L(t) \tag{28}$$

Substituting it into (27), we have

$$\begin{aligned} x_2(x_3 - \bar{x}_3) &= 0 \\ \dot{x}_2 &= -\left[\frac{e\bar{x}_1 + h}{\lambda_2} + \frac{\xi}{e\bar{x}_1 + h}\right]\lambda_3(x_3 - \bar{x}_3) \\ \dot{x}_3 &= \xi\lambda_4(\hat{\tau}_L - \tau_L) \\ \dot{\hat{\tau}}_L &= -\xi\lambda_3(x_3 - \bar{x}_3) \end{aligned} \tag{29}$$

The second and the fourth equations of (29), combined with relation (28), imply that $(e\bar{x}_1 + h)(x_3 - \bar{x}_3) = 0$ and this implies $x_3 = \bar{x}_3$, as $e\bar{x}_1 + h \neq 0$. Thus, according to the third equation of (29), we have $\hat{\tau}_L = \tau_L$, and (28) implies that $x_2 = \bar{x}_2$, that is, $\bar{x}$ is the only solution contained in $\tilde{M}$.

Combining the two controls (12) and (26), we obtain the overall adaptive control law

$$
\begin{aligned}
u_1 &= -\lambda_1\Gamma_1(x_1-\bar{x}_1) - \frac{\lambda_3 e x_2}{\lambda_1}(x_3-\bar{x}_3) + ax_1 - bx_2x_3 \\
u_2 &= -\lambda_2\Gamma_2\left(x_2 - \frac{\hat{\tau}_L}{e\bar{x}_1+h}\right) - \lambda_3\left[\frac{e\bar{x}_1+h}{\lambda_2} + \frac{1}{\lambda_4(e\bar{x}_1+h)}\right] \\
&\quad \times (x_3-\bar{x}_3) + cx_2 + bx_1x_3 + dx_3 \\
\dot{\hat{\tau}}_L &= -\frac{\lambda_3}{\lambda_4}(x_3-\bar{x}_3)
\end{aligned} \tag{30}
$$

which is equivalent to (22) with $\bar{\Gamma}_1 = \lambda_1 L_d \Gamma_1$, $\bar{\Gamma}_2 = (\lambda_3 n_p J)/\lambda_1$, $\bar{\Gamma}_3 = \lambda_2 L_q \Gamma_2$, $\bar{\Gamma}_4 = (\lambda_3 n_p J)/\lambda_2 L_q$, $\bar{\Gamma}_5 = (\lambda_3 J L_q)/\lambda_4 n_p$ and $\bar{\Gamma}_6 = \lambda_3 J/\lambda_4$. □

4.3.2 Both load torque and stator resistance are unknown: This section considers a more general uncertain case.

Proposition 3: Suppose that both the load torque τ_L and the stotor resistance R_s are uncertain. Then, the speed regulation problem of PMSM can be solved by the following adaptive controller

$$
\begin{aligned}
u_1 &= -\bar{\Gamma}_1(i_d - \bar{i}_d) - \frac{3}{2}\bar{\Gamma}_2(L_d - L_q)i_q(\omega - \bar{\omega}) \\
&\quad + \hat{R}_s i_d - n_p L_q i_q \omega \\
u_2 &= -\bar{\Gamma}_3\left(i_q - \frac{2\hat{\tau}_L}{3n_p[(L_d-L_q)\bar{i}_d + \Phi]}\right) \\
&\quad - \left[\frac{3}{2}\bar{\Gamma}_4((L_d-L_q)\bar{i}_d + \Phi) \right. \\
&\quad \left. + \frac{2\bar{\Gamma}_5}{3(L_d-L_q)\bar{i}_d + \Phi}\right](\omega - \bar{\omega}) \\
&\quad + \hat{R}_s i_q + n_p L_d i_d \omega + n_p \Phi \omega \\
\dot{\hat{\tau}}_L &= -\bar{\Gamma}_6(\omega - \bar{\omega}) \\
\dot{\hat{R}}_s &= -\bar{\Gamma}_7 i_d(i_d - \bar{i}_d) - \bar{\Gamma}_8 i_q\left(i_q - \frac{2\hat{\tau}_L}{3n_p[(L_d-L_q)\bar{i}_d + \Phi]}\right)
\end{aligned} \tag{31}
$$

where $\bar{\Gamma}_i$ $(i = 1, 2, \ldots, 8)$ are suitable positive numbers.

Proof: We first use the following feedback instead of the pre-feedback (12)

$$
\begin{aligned}
u_1 &= \hat{a}x_1 - bx_2x_3 + v_1 \\
u_2 &= \hat{c}x_2 + bx_1x_3 + dx_3 + v_2
\end{aligned} \tag{32}
$$

where $\hat{c} = L_d/L_q\hat{a}$, v_1, v_2 and the dynamics of $\hat{\tau}_L$ are designed in a similar way as in Section 4.3.1. Then, the closed-loop system becomes

$$\dot{z} = (\tilde{J}(z) - \tilde{R}(z))\nabla H(z) + \bar{\boldsymbol{G}}(x)(\hat{a} - a) \tag{33}$$

where

$$\bar{\boldsymbol{G}}(x) = \begin{pmatrix} x_1 \\ \dfrac{L_d}{L_q}x_2 \\ 0 \\ 0 \end{pmatrix}$$

Taking

$$V(z, \hat{a}) = H(z) + \frac{\Gamma_a}{2}(\hat{a} - a)^2$$

where Γ_a is a positive number, then we have

$$
\begin{aligned}
\dot{V} &= -dH(z)\tilde{R}(z)\nabla H(z) + dH(z)\bar{\boldsymbol{G}}(x)(\hat{a}-a) + \Gamma_a(\hat{a}-a)\dot{\hat{a}} \\
&= -dH(z)\tilde{R}(z)\nabla H(z) + (dH(z)\bar{\boldsymbol{G}}(x) + \Gamma_a\dot{\hat{a}})(\hat{a}-a)
\end{aligned}
$$

Note that the last two elements of $\bar{\boldsymbol{G}}(x)$ are zeros, so the unknown parameter τ_L does not appear in $dH(z)\bar{\boldsymbol{G}}(x)$. Thus, we can take

$$\dot{\hat{a}} = -\frac{1}{\Gamma_a}dH(z)\bar{\boldsymbol{G}}(x) \tag{34}$$

It follows that

$$\dot{V} = -dH(z)\tilde{R}(z)\nabla H(z) \leq 0$$

Define

$$\tilde{M} = \left\{(z, \hat{a}) \middle| \dot{V}(z, \hat{a}) = 0\right\} = \{(z, \hat{a}) | x_1 = \bar{x}_1 \text{ and } x_2 = \frac{\hat{\tau}_L}{e\bar{x}_1 + h}\right\}$$

As we know, any trajectory converges to the largest invariant set Ω in $\tilde{M}$. In the following, we only need to prove that if $(\bar{x}_1\ x_2\ x_3\ \hat{\tau}_L\ \hat{a})^T$ is a solution of the closed-loop systems (33) and (34) contained in $\Omega \subset \tilde{M}$, then $x_3(t) = \bar{x}_3$. In fact, if $x(t)$ is such a solution, we have

$$-\frac{e\lambda_3}{\lambda_1}x_2(x_3 - \bar{x}_3) + \bar{x}_1(\hat{a} - a) = 0 \tag{35}$$

and

$$
\begin{aligned}
\dot{x}_2 &= -\left[\frac{e\bar{x}_1+h}{\lambda_2} + \frac{\xi}{e\bar{x}_1+h}\right]\lambda_3(x_3-\bar{x}_3) + \frac{L_d}{L_q}x_2(\hat{a}-a) \\
\dot{x}_3 &= \xi\lambda_4(\hat{\tau}_L - \tau_L) \\
\dot{\hat{\tau}}_L &= -\xi\lambda_3(x_3 - \bar{x}_3) \\
\dot{\hat{a}} &= 0
\end{aligned} \tag{36}
$$

As $x_2 = 1/e\bar{x}_1 = h\hat{\tau}_L$, we have

$$-\frac{\lambda_3(e\bar{x}_1+h)}{\lambda_2}(x_3-\bar{x}_3) + \frac{L_d}{L_q}x_2(\hat{a}-a) = 0 \tag{37}$$

Consider (35) and (37), the determinant of the coefficient matrix is

$$\det \boldsymbol{A}(t) = -\frac{\lambda_3\bar{x}_1}{\lambda_2}(e\bar{x}_1 + h) + \frac{\lambda_3 L_d e}{\lambda_1 L_q}x_2^2(t)$$

At first, we show that $x_2(t) \equiv$ constant. Otherwise, there exists t_0 such that $\det \boldsymbol{A}(t_0) \neq 0$, thus $x_3(t_0) = \bar{x}_3$ and $\hat{a}(t_0) = a$. Therefore $\hat{a}(t) = a$ for $t \in R$ because $\dot{\hat{a}} = 0$. So, according to (37), $x(t) = \bar{x}_3$ for $t \in R$. Thus, the first equation of (36) implies $\dot{x}_2 \equiv 0$, which is a contradiction. We conclude that $x_2(t) \equiv$ constant.

As x_2 is a constant, we have $\dot{x}_2 = 0$ and thus $\dot{\hat{\tau}}_L = 0$, so $x_3(t) = \bar{x}_3$ and $\hat{\tau}_L = \tau_L$, $x_2(t) = \bar{x}_2$. Thus, the speed regulation is achieved, as $\lim_{t\to\infty} x_3(t) = \bar{x}_3$.

Moreover, if $\bar{x}_1 \neq 0$ or $\tau \neq 0$, then (35) and the first equation of (36) imply $\hat{a} = a$, thus the only solution contained in $\bar{M}$ is $(\bar{z}, a)$. According to LaSelle's invariance principle, the closed loop is asymptotically stable.

The adaptive controller therefore can be constructed as

$$\begin{aligned}
u_1 &= -\lambda_1\Gamma_1(x_1 - \bar{x}_1) - \frac{\lambda_3 e x_2}{\lambda_1}(x_3 - \bar{x}_3) + \hat{a}x_1 - bx_2x_3 \\
u_2 &= -\lambda_2\Gamma_2\left(x_2 - \frac{\hat{\tau}_L}{e\bar{x}_1 + h}\right) - \lambda_3\left[\frac{e\bar{x}_1 + h}{\lambda_2}\right. \\
&\quad \left. + \frac{1}{\lambda_4(e\bar{x}_1 + h)}\right](x_3 - \bar{x}_3) + \frac{L_d}{L_q}\hat{a}x_2 + bx_1x_3 + dx_3 \\
\dot{\hat{\tau}}_L &= -\frac{\lambda_3}{\lambda_4}(x_3 - \bar{x}_3) \\
\dot{\hat{a}} &= -\frac{1}{\Gamma_a}\left[\lambda_1 x_1(x_1 - \bar{x}_1) + \frac{L_d}{L_q}x_2\left(x_2 - \frac{\hat{\tau}_L}{e\bar{x}_1 + h}\right)\right]
\end{aligned} \tag{38}$$

which is equivalent to (31) with $\bar{\Gamma}_1 = \lambda_1 L_d \Gamma_1$, $\bar{\Gamma}_2 = (\lambda_3 n_p J)/\lambda_1$, $\bar{\Gamma}_3 = \lambda_2 L_q \Gamma_2$, $\bar{\Gamma}_4 = (\lambda_3 n_p J)/(\lambda_2 L_q)$, $\bar{\Gamma}_5 = (\lambda_3 J L_q)/(\lambda_4 n_p)$, $\bar{\Gamma}_6 = (\lambda_3 J)/\lambda_4$, $\bar{\Gamma}_7 = (\lambda_1 L_d^3)/\Gamma_a$ and $\bar{\Gamma}_8 = (Ld^2 L_q)/\Gamma_a$. □

Remark 3: In fact, the model of PMSM should include the viscous friction term $B\omega$, that is, the third equation of system (1) should be

$$J\frac{d\omega}{dt} = \frac{3}{2}n_p[(L_d - L_q)i_d i_q + \Phi i_q] - B\omega - \tau_L$$

where B is the viscous friction coefficient. Although we ignored this term, our designing process can still be used when viscous friction is considered. In fact, the term $-B\omega$ can be decomposed as

$$-B\omega = -B(\omega - \bar{\omega}) - B\bar{\omega}$$

The second term in the above equation can be viewed as a part of τ_L. The first term itself contributes to the convergence of ω, and it only adds a positive constant to the third diagonal element of matrix $\boldsymbol{R}$ in (19) (or $\tilde{\boldsymbol{R}}$ in (27)) when incorporated into the closed-loop Hamiltonian structure.

Remark 4: In Petrovic *et al.* [18], an almost globally convergent controller of PMSM was constructed on the basis of IDA-PBC technique, the unknown load torque was treated by an estimator. In the present paper, we use a relatively direct way to achieve AFDHR of PMSM, thus the estimator for the unknown load torque is naturally embedded into the Hamiltonian structure. Besides, the uncertain stator resistance is also tackled.

5 Simulation results

In the following simulations, we set the system parameters as: $J = 0.0008\ \mathrm{kg/m^2}$, $n_p = 4$, $\Phi = 0.175$ Wb, $R_s = 2.875\ \Omega$, $L_d = 0.009$ H and $L_q = 0.008$ H. The following simulations are all performed under the existence of viscous friction and we assume that the viscous friction coefficient is $B = 0.02$.

1. Precise case. In this case, we set load torque $\tau_L = 3$ N m and choose controller parameters as: $\Gamma_1 = 100$, $\Gamma_2 = 500$, $k_1 = k_2 = 1$ and $\bar{i}_d = 0$ A. As we consider viscous friction, τ_L in controller (15) is replaced by $\tau_L + B\bar{\omega}$. The desired rotor speed is set to $\bar{\omega} = 100$ rad/s, $\bar{\omega} = 50$ rad/s and $\bar{\omega} = 120$ rad/s in $t \in [0,\ 1)$, $t \in [1,\ 2)$ and $t \in [2,\ 3]$, respectively. Fig. 1 shows the responses of currents i_d and i_q and tracking performance of rotor speed ω.
2. τ_L is unknown. The controller parameters are chosen to be: $\bar{\Gamma}_1 = 100$, $\bar{\Gamma}_2 = 100$, $\bar{\Gamma}_3 = 200$, $\bar{\Gamma}_4 = 30$, $\bar{\Gamma}_5 = 0.5$ and $\bar{\Gamma}_6 = 0.4$.

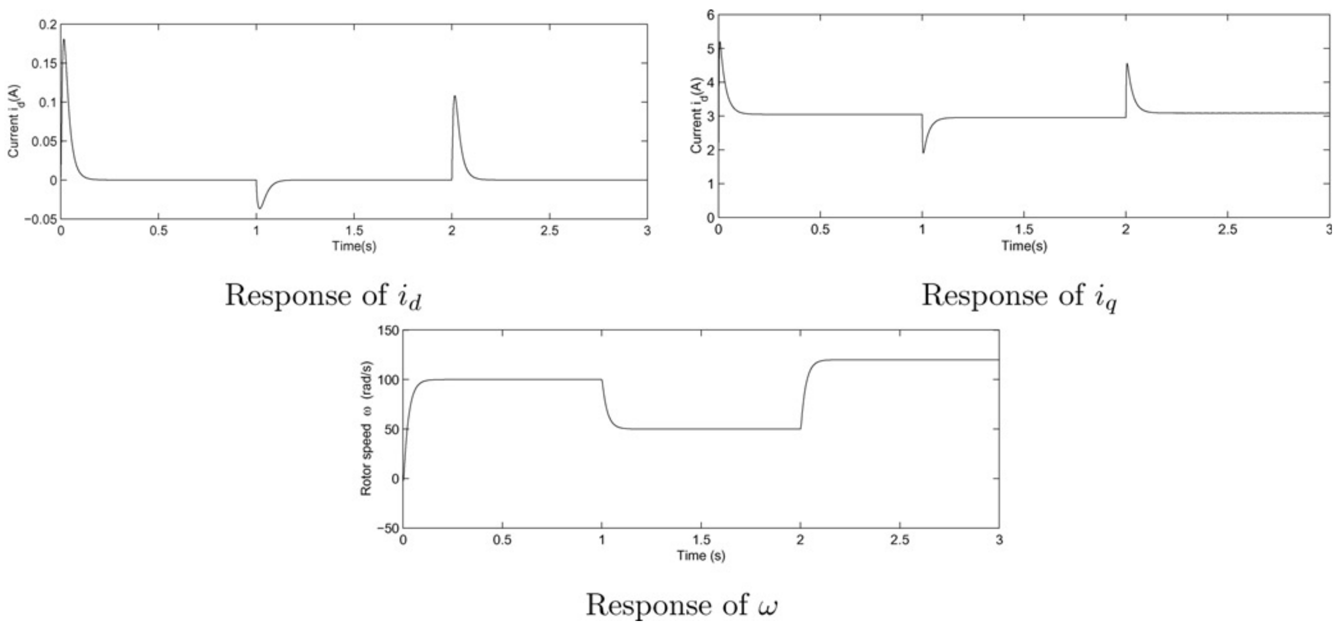


Fig. 1 *Simulations for precise case*

Case 1. In this case, the uncertain load torque $\tau_L = 2$ N m is assumed to be constant. Hereinafter, we assume that the viscous friction coefficient B is uncertain. The desired rotor speed is set to $\bar{\omega} = 100$ rad/s, $\bar{\omega} = 50$ rad/s and $\bar{\omega} = 120$ rad/s in $t \in [0, 4)$, (4, 8] and (8, 12], respectively. Fig. 2*a* shows the responses of i_d, i_q, ω and $\hat{\tau}_L$. As we can see, the rotor speed can rapidly track the reference.

Case 2. In this case, we set the desired rotor speed $\bar{\omega} = 100$ rad/s. The load torque is assumed to be $\tau_L = 0$ N m, 2 N m and 0 N m in $t \in [0, 4)$, [4, 8) and [8, 12], respectively. Fig. 2*b* shows that while the load torque suddenly changes, the rotor speed recovers quickly to the pre-specified value 100 rad/s.

3. τ_L and R_s are unknown. The controller parameters are chosen as $\bar{\Gamma}_1 = 100$, $\bar{\Gamma}_2 = 100$, $\bar{\Gamma}_3 = 200$, $\bar{\Gamma}_4 = 30$, $\bar{\Gamma}_5 = 0.5$, $\bar{\Gamma}_6 = 0.4$; $\bar{\Gamma}_7 = 100$ and $\bar{\Gamma}_8 = 1$.

Case 1. In this case, we assume that $R_s = 2.875\ \Omega$, $\tau_L = 2$ N m. The desired rotor speed is set to $\bar{\omega} = 100$ rad/s, 50 rad/s and 120 rad/s in $t \in [0, 4)$, [4, 8) and [8, 12], respectively. Fig. 3*a* shows the responses.

Case 2. In this case, we assume that $R_s = 2.875\ \Omega$. The desired rotor speed is set to $\bar{\omega} = 100$ rad/s. The load torque is supposed to be $\tau_L = 0$ N m, 2 N m and 0 N m in $t \in [0, 4)$, [4, 8) and [8, 12], respectively. Fig. 3*b* shows the responses.

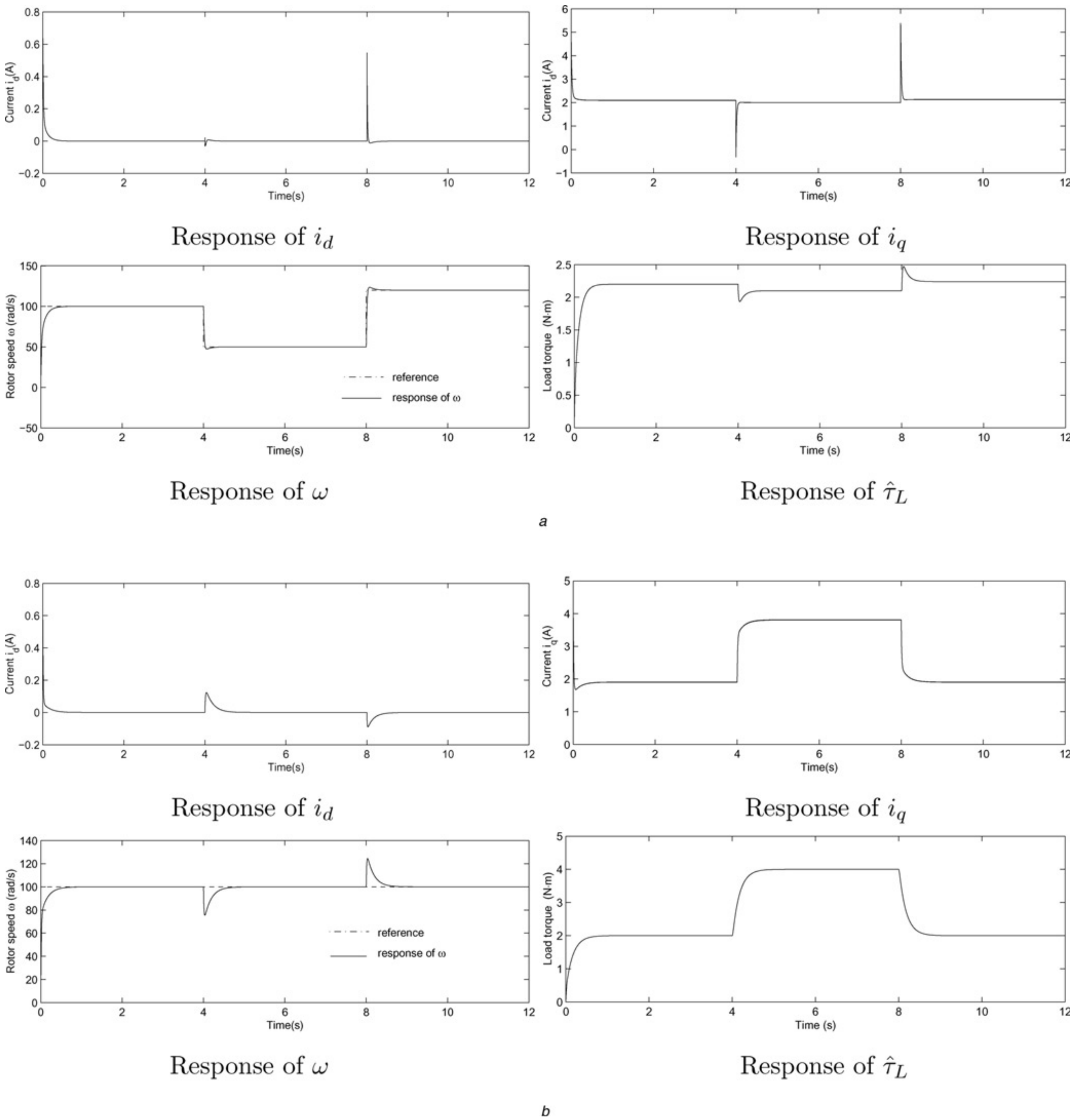
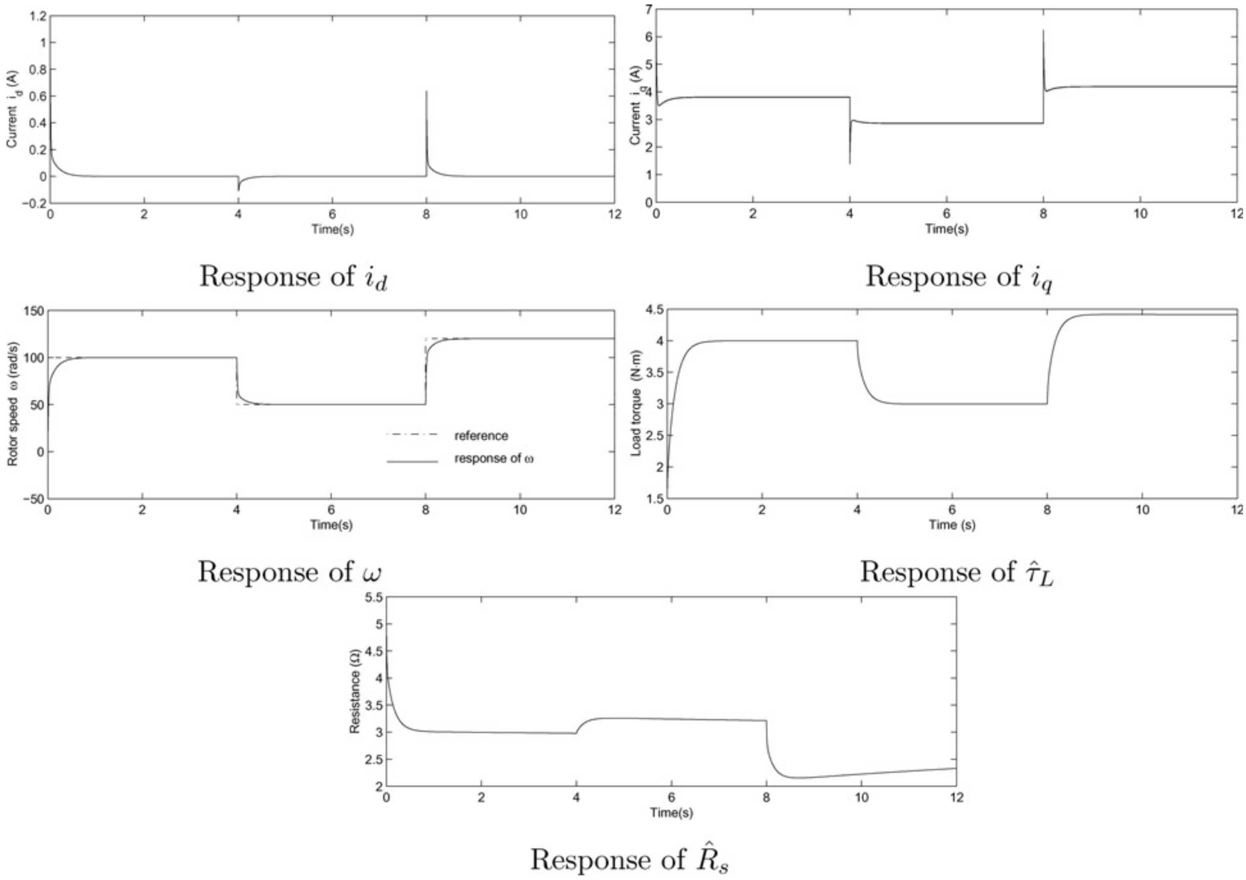


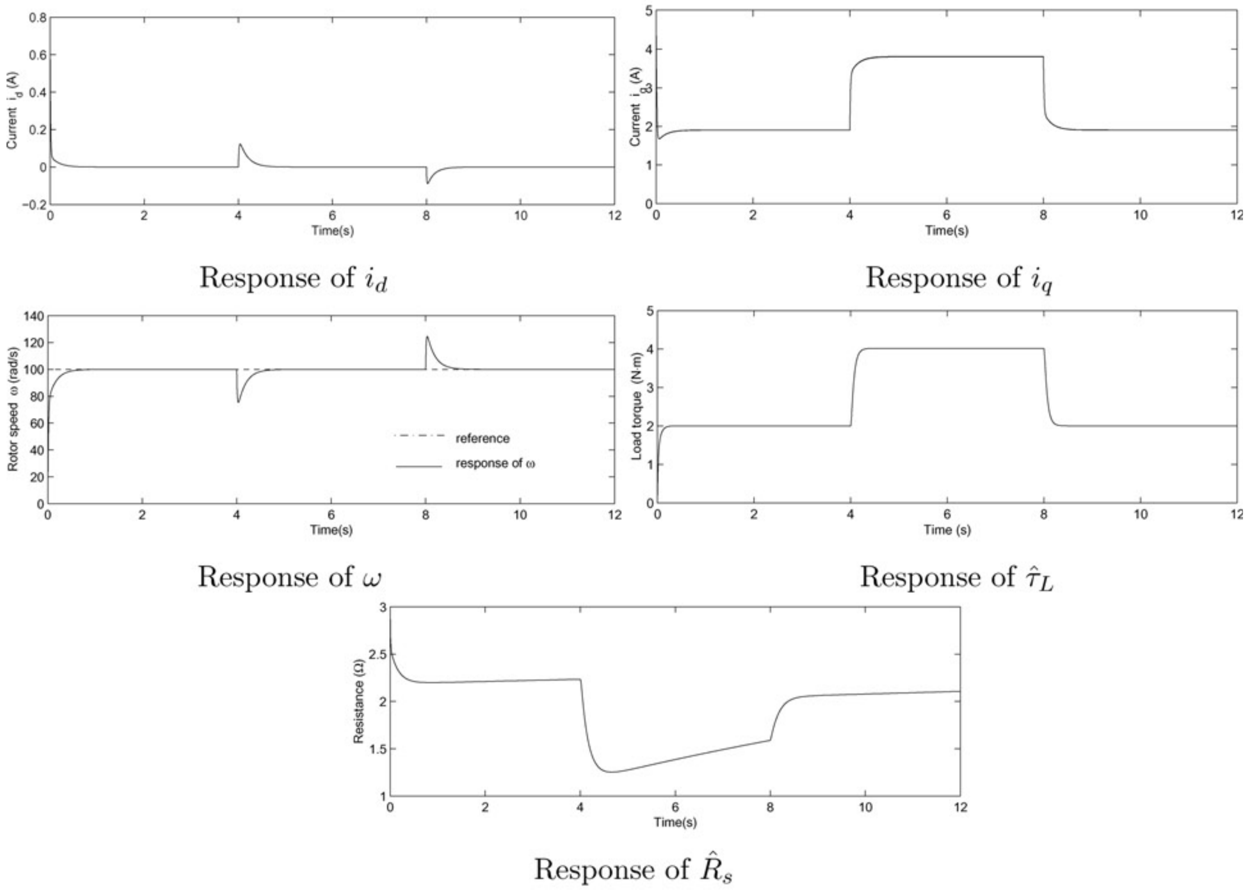
Fig. 2 *Simulations for uncertain load torque*

a Constant load torque

b Piece wise constant load torque



a



b

Fig. 3 *Simulations for uncertain load torque and stator resistance*

a Constant load torque

b Piece wise constant load torque

6 Concluding remarks

In this paper, the adaptive speed regulation of PMSM is investigated through Hamiltonian function approach. State feedback is constructed to transfer the dynamics of PMSM into dissipative Hamiltonian form and then it is used to solve the speed regulation problem. When the load torque (and stator resistance) is (are) unknown, corresponding adaptive controllers are designed to solve the problems. The update laws are embedded into the dissipative Hamiltonian structure. Simulations show that the controllers are efficient.

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