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# Simultaneous stabilization of a set of nonlinear port-controlled Hamiltonian systems☆

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**Abstract**

This paper investigates simultaneous stabilization of a set of nonlinear port-controlled Hamiltonian (PCH) systems and proposes a number of results on the design of simultaneous stabilization controllers for the PCH systems. Firstly, the case of two PCH systems is studied. Using the dissipative Hamiltonian structural properties, the two systems are combined to generate an augmented PCH system, with which some results on the control design are then obtained. For the case that there exist parametric uncertainties in the two systems' Hamiltonian structures, an adaptive simultaneous stabilization controller is proposed. When there are external disturbances and parametric uncertainties in the two systems, two simultaneous stabilization controllers are designed for the systems: one is a robust controller and the other is a robust adaptive one. Secondly, the case of more than two PCH systems is investigated, and a new result is proposed for the simultaneous stabilization of the systems. Finally, two illustrative examples are studied by using the results proposed in this paper. Simulations show that the simultaneous stabilization controllers obtained in this paper work very well.


*Keywords:* PCH System; Simultaneous stabilization; Augmented PCH structure; Zero-state detectability; $L_2$ disturbance attenuation; Adaptive/robust simultaneous stabilization controller

## 1. Introduction

In recent years, port-controlled Hamiltonian (PCH) systems, proposed by Maschke and van der Schaft (1992) and van der Schaft and Maschke (1995), have been well investigated in a series of works, see, e.g., Dalsmo and van der Schaft (1999), Fujimoto, Sakurama, and Sugie (2003), Fujimoto and Sugie (2001), Maschke, Ortega, and van der Schaft (2000), Nijmeijer and van der Schaft (1990), Ortega, vanderSchaft, Maschke, and Escobar (2002) and van der Schaft (1999). A constructive procedure was proposed in Maschke et al. (2000) to modify the Hamiltonian function of forced Hamiltonian systems with dissipation to generate a Lyapunov function for non-zero equilibria. In Dalsmo and van der Schaft (1999), it was shown that a power-conserving interconnection of port-controlled generalized Hamiltonian systems leads to an implicit generalized Hamiltonian system, and a power-conserving partial interconnection to an implicit PCH system, respectively. Several nice results were obtained in Ortega et al. (2002) for passivity-based control, after thorough investigation of interconnection and damping assignment passivity-based control of PCH systems. Via generalized canonical transformations, a very important method was provided in Fujimoto et al. (2003) for trajectory tracking control of time-varying PCH systems. The Hamiltonian function, the sum of potential energy and kinetic energy in physical systems, is a good candidate of Lyapunov functions for many physical systems. Due to this, the PCH system has drawn a good deal of attention in practical control designs.

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Up to now, the energy-based approach has been used in various control problems (Brambilla & D'Amore, 2001; Escobar, van der Schaft, & Ortega, 1999; Galaz, Ortega, Bazanella, & Stankovic, 2003; Macchelli & Melchiorri, 2004; Ortega, Galaz, Astolfi, Sun, & Shen, 2005; Shen, Ortega, Lu, Mei, & Tamura, 2000; Sun, Shen, Ortega, & Liu, 2001; Wang, Cheng, Li, & Ge, 2003; Xi, Cheng, Lu, & Mei, 2002). In these applications, a key to success is to design effective feedback controllers.

In practical control designs, due to system's uncertainty, failure modes or systems with various modes of operation, the simultaneous stabilization problem has often to be taken into account. The problem is concerned with designing a single controller which can simultaneously stabilize a set of systems. Since it is one of the important research topics in the area of robust control (Blondel, 1994; Ho-Mock-Qai & Dayawansa, 1999; Wu, 2005), the simultaneous stabilization problem has drawn a considerable attention, and up to now a lot of important results have been obtained for the problem, see, e.g., Blondel (1994), Cao, Sun, and Lam (1999), Ho-Mock-Qai and Dayawansa (1999), Howitt and Luus (1991), Miller and Chen (2002), Miller and Rossi (2001), Schmitendorf and Hollot (1989) and Wu (2005). In Blondel (1994), based on the frequency domain approach, it was shown that the simultaneous stabilizability of more than two linear systems is rationally undecidable. A useful sufficient condition was presented in Schmitendorf and Hollot (1989) for the existence of a linear state feedback simultaneous stabilization controller for a set of single-input linear systems. Linear periodically time-varying controllers were successfully employed in Miller and Chen (2002) and Miller and Rossi (2001) for the simultaneous stabilization and disturbance attenuation of a collection of linear systems. In some recent works (Ho-Mock-Qai & Dayawansa, 1999; Wu, 2005), the simultaneous stabilization problem was investigated for nonlinear systems. In Ho-Mock-Qai and Dayawansa (1999), some nice results were proposed for the existence of simultaneous stabilization controllers of a set of nonlinear systems. The control Lyapunov function approach was successfully used in Wu (2005) to derive necessary and sufficient conditions for the existence of feedback time-invariant simultaneous stabilization controllers of single-input nonlinear systems. It should be pointed out that except the references (Ho-Mock-Qai & Dayawansa, 1999; Wu, 2005), almost all of the results mentioned above were derived for linear systems. For a set of nonlinear systems, designing a simultaneous stabilization controller is not an easy task, and accordingly there are relatively fewer results for nonlinear systems. Particularly, there are, to the authors' best knowledge, fewer works on the simultaneous stabilization of Hamiltonian systems.

In this paper, we investigate the simultaneous stabilization problem for a set of nonlinear PCH systems, and propose a number of new results on the design of simultaneous stabilization controllers for the PCH systems. Firstly, we study the case of two PCH systems, which is an important case in the field of robust control of PCH systems. Using the dissipative Hamiltonian structural properties, the two Hamiltonian systems are combined to obtain an augmented PCH system, with which several results on the control design are then obtained. For the case that the two systems have parametric uncertainties in their Hamiltonian structures, an adaptive simultaneous stabilization controller is proposed. When there are external disturbances and parametric uncertainties in the two systems, two simultaneous stabilization controllers are obtained for the systems: one is a robust controller and the other is a robust adaptive one. Secondly, the case of more than two PCH systems is studied, and a new control design method is presented for the systems. Finally, two illustrative examples with simulations are given to support the results proposed in the paper.

The control design methods and results proposed in this paper differ fundamentally from those mentioned earlier. The major technique used in this paper is a novel one: the system-augmentation technique, which is developed by fully exploiting the Hamiltonian structural properties. In addition, the simultaneous stabilization controllers obtained in this paper can globally simultaneously stabilize the PCH systems under consideration if the Hamiltonian functions respectively have a unique minimum. Moreover, the conditions of the main results proposed in this paper are easy to check, and the control design methods can be applied in practice to design (robust and/or adaptive) simultaneous stabilization controllers for a set of PCH systems.

It is well worth pointing out that, together with Hamiltonian realization (Wang, Cheng, & Hu, 2005; Wang, Li, & Cheng, 2003), the results proposed in this paper have made it possible to set up a new way to the simultaneous stabilization of ordinary nonlinear systems. The new way can be called energy-based control design (ECD) approach, which should contain the following two steps: (i) Hamiltonian realization, i.e., express nonlinear systems as the form of Hamiltonian systems; (ii) design a simultaneous stabilization controller for the achieved Hamiltonian systems. In some recent works (Wang et al., 2005; Wang, Li et al., 2003), the Hamiltonian realization problem was well investigated and several realization methods were provided for nonlinear systems. With these realization methods and the results proposed in this paper, we can deal with the simultaneous stabilization problem for some classes of nonlinear systems.

The paper is organized as follows. The case of two PCH systems is investigated in Sections 2–4. Section 2 studies the ordinary simultaneous stabilization of two PCH systems. In Section 3, we consider the case that the two systems have parametric uncertainties in their Hamiltonian structures. Section 4 deals with the case that there exist both external disturbances and parametric uncertainties in the two systems. In Section 5, we study the case of more than two PCH systems. Section 6 gives two illustrative examples, which is followed by the conclusion in Section 7.

## 2. Simultaneous stabilization of two PCH systems

This section investigates simultaneous stabilization of two nonlinear PCH systems, and proposes some results on the control design for the two systems.

Consider the following two PCH systems (Maschke & van der Schaft, 1992; van der Schaft & Maschke, 1995):

$$\Sigma_1: \begin{cases} \dot{x} = [J_1(x) - R_1(x)]\dfrac{\partial H_1(x)}{\partial x} + g_1(x)u, \\ y = g_1^{\mathrm{T}}(x)\dfrac{\partial H_1(x)}{\partial x}, \end{cases} \tag{2.1}$$

$$\Sigma_2: \begin{cases} \dot{\xi} = [J_2(\xi) - R_2(\xi)]\dfrac{\partial H_2(\xi)}{\partial \xi} + g_2(\xi)u, \\ \eta = g_2^{\mathrm{T}}(\xi)\dfrac{\partial H_2(\xi)}{\partial \xi}, \end{cases} \tag{2.2}$$

where $x, \xi \in \mathbb{R}^n$ and $y, \eta \in \mathbb{R}^m$ are the states and outputs of the two systems, respectively; $u \in \mathbb{R}^m$ is the control input; $J_i(x) = -J_i^{\mathrm{T}}(x) \in \mathbb{R}^{n\times n}$, $0 \leqslant R_i(x) \in \mathbb{R}^{n\times n}$, $g_i(x) \in \mathbb{R}^{n\times m}$, $H_i(x)$ is the Hamiltonian function with a local strict minimum at $x_e^{(i)}$, $i=1,2$, and $x_e^{(1)} = x_0$, $x_e^{(2)} = \xi_0$.

Obviously, when $u=0$, the systems (2.1) and (2.2) are stable, but not asymptotically stable at their equilibria. The objective of this section is as follows.

*Simultaneous stabilization*: Design an output feedback controller $u = u(y,\eta)$ such that under the feedback control, the two systems (2.1) and (2.2) are simultaneously asymptotically stable.

For the simultaneous stabilization of the above two systems, we have the following result.

**Theorem 2.1.** *Assume that there exists a symmetric matrix $K \in \mathbb{R}^{m\times m}$ such that*

$$\begin{cases} R_1(x) + K_{11}(x,x) > 0, \\ R_2(\xi) - K_{22}(\xi,\xi) > 0, \end{cases} \tag{2.3}$$

*where*

$$K_{ij}(x,\xi) = g_i(x)Kg_j^{\mathrm{T}}(\xi), \quad i,j=1,2. \tag{2.4}$$

*Then, the output feedback*

$$u = -K(y-\eta) \tag{2.5}$$

*can simultaneously stabilize the systems* (2.1) *and* (2.2).

**Proof.** Substituting (2.5) into the systems (2.1) and (2.2), we obtain

$$\dot{X} = [J(X) - R(X)]\frac{\partial H(X)}{\partial X}, \tag{2.6}$$

where

$$X = [x^{\mathrm{T}}, \xi^{\mathrm{T}}]^{\mathrm{T}}, \quad H(X) = H_1(x) + H_2(\xi),$$

$$J(X) = \begin{bmatrix} J_1(x) & K_{12}(x,\xi) \\ -K_{12}^{\mathrm{T}}(x,\xi) & J_2(\xi) \end{bmatrix}, \tag{2.7}$$

and

$$R(X) = \mathrm{Diag}\{R_1(x) + K_{11}(x,x),\ R_2(\xi) - K_{22}(\xi,\xi)\}. \tag{2.8}$$

Obviously, $J(X)$ is skew-symmetric, and from (2.3) $R(X)$ is positive definite. Thus, the system (2.6) is an augmented strictly dissipative Hamiltonian system.

Let $X_0 := [x_0^{\mathrm{T}}, \xi_0^{\mathrm{T}}]^{\mathrm{T}}$. Since $\nabla H_1(x_0) = 0$ and $\nabla H_2(\xi_0) = 0$, we know that $X_0$ is the equilibrium of the system (2.6), where $\nabla H_i(x) = \partial H_i/\partial x$, $i=1,2$. On the other hand, it can be seen that $H(X)$ has a local strict minimum at $X_0$. From the properties of dissipative PCH systems (Ortega et al., 2002; Wang, Li et al., 2003), the system (2.6) is asymptotically stable at $X_0$, which means that $x \to x_0$ and $\xi \to \xi_0$. Therefore, under the feedback (2.5), the systems (2.1) and (2.2) are simultaneously stabilized. □

**Remark 2.2.** Combining the two PCH systems to obtain an augmented Hamiltonian system is called the system-augmentation (SA) technique in this paper. This technique plays an important role in the control design of this paper.

In the following, we use the SA technique to give a relatively more general result for the simultaneous stabilization of the systems (2.1) and (2.2).

**Theorem 2.3.** *Assume that there exist a matrix $K_0(x,\xi) \in \mathbb{R}^{n\times n}$ and a symmetric matrix $K \in \mathbb{R}^{m\times m}$ such that*

(i) *the following inequalities hold true*:

$$\bar{R}_1(x) := R_1(x) + K_{11}(x,x) \geqslant 0,$$
$$\bar{R}_2(\xi) := R_2(\xi) - K_{22}(\xi,\xi) \geqslant 0, \tag{2.9}$$

*where $K_{ij}$ are defined in* (2.4), $i,j=1,2$;

(ii) *the systems*

$$\dot{x} = [J_1(x) - \bar{R}_1(x)]\frac{\partial H_1}{\partial x},$$
$$\dot{\xi} = [J_2(\xi) - \bar{R}_2(\xi)]\frac{\partial H_2}{\partial \xi} \tag{2.10}$$

*are zero-state detectable with respect to $\bar{y} := \bar{R}_1^{1/2}(x)\nabla H_1(x)$ and $\bar{\eta} := \bar{R}_2^{1/2}(\xi)\nabla H_2(\xi)$* (van der Schaft, 1999), *respectively, where $\bar{R}_i^{1/2}(x)$ is defined as $\bar{R}_i(x) = [\bar{R}_i^{1/2}(x)]^2$, $i=1,2$;*

(iii) *either*

$$K_{12}(x,\xi) = K_0(x,\xi)\bar{R}_2(\xi)$$
$$\text{or} \quad K_{21}(\xi,x) = K_0(x,\xi)\bar{R}_1(x) \tag{2.11}$$

*holds true.*

*Then,*

$$u = -K(y-\eta)$$

*can simultaneously stabilize the two systems* (2.1) *and* (2.2).

**Proof.** Substituting $u = -K(y-\eta)$ into the systems (2.1) and (2.2), similar to the proof of Theorem 2.1, we obtain the system (2.6). Now, consider the energy flow of the system (2.6).

Using Condition (i), we have

$$\begin{aligned}\dot{H}(X) &= -\frac{\partial H^{\mathrm{T}}}{\partial X}R(X)\frac{\partial H}{\partial X}\\ &= -\nabla^{\mathrm{T}}H_1(x)\bar{R}_1(x)\nabla H_1(x)\\ &\quad -\nabla^{\mathrm{T}}H_2(\xi)\bar{R}_2(\xi)\nabla H_2(\xi)\leqslant 0,\end{aligned}$$

from which we know that the system (2.6) is stable. Furthermore, it is easy to see from the dynamic system theory (Khalil, 1996) that the system (2.6) converges to the largest invariant set contained in

$$\begin{aligned}S &= \{X|\dot{H}=0\}\\ &= \{(x,\xi)|\bar{R}_1^{1/2}(x)\nabla H_1(x)=0,\ \bar{R}_2^{1/2}(\xi)\nabla H_2(\xi)=0\}.\end{aligned}$$

Without loss of generality, we assume that $K_{12}(x,\xi)=K_0(x,\xi)\bar{R}_2(\xi)$ holds in Condition (iii) (as for the other equation, we can follow the similar argument). With $K_{12}(x,\xi)=K_0(x,\xi)\bar{R}_2(\xi)$, we have $\bar{R}_2^{1/2}(\xi)\nabla H_2(\xi)=0\Rightarrow K_{12}(x,\xi)\nabla H_2(\xi)=0$. Thus, when $\bar{R}_2^{1/2}(\xi)\nabla H(\xi)=0$, the system (2.6) can be decomposed as

$$\begin{cases}\dot{x}=[J_1(x)-\bar{R}_1(x)]\nabla H_1(x),\\ \dot{\xi}=[J_2(\xi)-\bar{R}_2(\xi)]\nabla H_2(\xi)-K_{21}(\xi,x)\nabla H_1(x).\end{cases}\tag{2.12}$$

Because the first system of (2.10) being zero-state detectable with respect to $\bar{y}$, $\bar{R}_1^{1/2}(x)\nabla H_1(x)\equiv 0\Rightarrow x\to x_0$ $(t\to\infty)$, from which it follows that $\nabla H_1(x)\to 0$. Then, as $t\to\infty$, the second part of system (2.12) becomes

$$\dot{\xi}=[J_2(\xi)-\bar{R}_2(\xi)]\nabla H_2(\xi).$$

Since the second system of (2.10) is zero-state detectable with respect to $\bar{\eta}$, $\bar{R}_2^{1/2}(\xi)\nabla H_2(\xi)\equiv 0\Rightarrow\xi\to\xi_0$. Thus, the above largest invariant set only contains one point, i.e., $X_0:=[x_0^{\mathrm{T}},\xi_0^{\mathrm{T}}]^{\mathrm{T}}$. From LaSalle's invariance principle (Khalil, 1996), the system (2.6) is asymptotically stable. Therefore, $x\to x_0$ and $\xi\to\xi_0$. That is, under the feedback law $u=-K(y-\eta)$, the systems (2.1) and (2.2) can be simultaneously stabilized. □

**Remark 2.4.** Notice that $\bar{R}_1(x)$ and $\bar{R}_2(\xi)$ denote the dissipative parts of the two systems, and that $\bar{y}^2(t)=-\dot{H}_1(x(t))$ and $\bar{\eta}^2(t)=-\dot{H}_2(\xi(t))$. The outputs $\bar{y}$ and $\bar{\eta}$ can be interpreted as a kind of energy dissipation output, which is the physical interpretations of $\bar{y}$ and $\bar{\eta}$.

**Remark 2.5.** In Theorem 2.3, $K$ and $K_0(x,\xi)$ can be determined in the following steps:

(1) find $K$ according to Conditions (i) and (ii);
(2) compute $K_{ij}(x,\xi)$ and $\bar{R}_i(x)$;
(3) find $K_0(x,\xi)$ from $K_{12}(x,\xi)=K_0(x,\xi)\bar{R}_2(\xi)$ or $K_{21}(\xi,x)=K_0(x,\xi)\bar{R}_1(x)$.

**Remark 2.6.** (1) The form of the control $u=-K(y-\eta)$ is a suitable choice that can provide an augmented dissipative Hamiltonian structure for the systems (2.1) and (2.2). Other forms such as $u=-K(y+\eta)$ and $u=-K(y-2\eta)$ cannot play the role of providing an augmented dissipative Hamiltonian structure for the two systems.

(2) Noticing that $K$ is only demanded to be symmetric, not positive (semi-)definite, the condition $R_2(\xi)-K_{22}(\xi,\xi)=R_2(\xi)-g_2(\xi)Kg_2^{\mathrm{T}}(\xi)\geqslant 0$ can be satisfied for many PCH systems. In fact, it is as demanding as the condition $R_1(x)+K_{11}(x,x)\geqslant 0$ (see Example 6.1).

(3) Both $K_{ij}$ and $\bar{R}_i$ can be obtained with $K$ (see (2.4) and Condition (i)). Thus, when $K$ is found, $K_{12}(x,\xi)=K_0(x,\xi)\bar{R}_2(\xi)$ or $K_{21}(\xi,x)=K_0(x,\xi)\bar{R}_1(x)$ is only a matrix equation with respect to $K_0$, which can generally be solved by MatLab or Maple.

Next, as an application, we apply Theorem 2.1 to investigate the simultaneous stabilization for a class of nonlinear affine systems.

Consider the following two affine systems:

$$\dot{x}=f_1(x)+g_1(x)u,\quad x\in\mathbb{R}^n,\quad f_1(x_0)=0,\tag{2.13}$$

$$\dot{\xi}=f_2(\xi)+g_2(\xi)u,\quad \xi\in\mathbb{R}^n,\quad f_2(\xi_0)=0,\tag{2.14}$$

where $u\in\mathbb{R}^m$ is the control input.

**Corollary 2.7.** *Assume that*

(1) *there exist Lyapunov functions $V_i$ such that*

$$L_{f_i}V_i(x)\leqslant 0,\quad i=1,2;$$

(2) *there exists a symmetric matrix $K\in\mathbb{R}^{m\times m}$ such that*

$$\begin{cases}\dfrac{L_{f_1}V_1(x)}{\|\nabla V_1\|^2}I_n-g_1(x)Kg_1^{\mathrm{T}}(x)<0\quad(x\neq x_0),\\[2mm] \dfrac{L_{f_2}V_2(x)}{\|\nabla V_2\|^2}I_n+g_2(x)Kg_2^{\mathrm{T}}(x)<0\quad(x\neq\xi_0),\end{cases}\tag{2.15}$$

*where $I_n$ is the $n\times n$ identity matrix.*

*Then, under the feedback control*

$$u=-K[g_1^{\mathrm{T}}(x)\nabla V_1(x)-g_2^{\mathrm{T}}(\xi)\nabla V_2(\xi)],$$

*the systems* (2.13) *and* (2.14) *can be simultaneously stabilized.*

**Proof.** With Condition (1), the systems (2.13) and (2.14) can be expressed as (Wang, Li et al., 2003)

$$\begin{cases}\dot{x}=[J_1(x)-R_1(x)]\nabla V_1(x)+g_1(x)u,\\ \dot{\xi}=[J_2(\xi)-R_2(\xi)]\nabla V_2(\xi)+g_2(\xi)u,\end{cases}\tag{2.16}$$

where $J_i^{\mathrm{T}}(x)=-J_i(x)$ and $R_i(x)\geqslant 0$.

On the other hand, it can be seen from Wang, Li et al. (2003) that Condition (2) implies that (2.3) holds for the two systems when $x\neq x_0$ and/or $\xi\neq\xi_0$. Thus, all the conditions of Theorem 2.1 are satisfied. From Theorem 2.1, the corollary follows directly. □

## 3. Adaptive simultaneous stabilization of two PCH systems

In this section, we consider the case that the systems (2.1) and (2.2) have parametric uncertainties in their Hamiltonian structures, and *design an adaptive output feedback law to simultaneously stabilize the two systems*.

When the systems (2.1) and (2.2) involve parametric uncertainties in their Hamiltonian structures, their dynamics become (Shen et al., 2000)

$$\begin{cases} \dot{x} = [J_1(x, p_1) - R_1(x, p_1)]\dfrac{\partial H_1(x, p_1)}{\partial x} + g_1(x)u, \\ y = g_1^{\mathrm{T}}(x)\dfrac{\partial H_1(x)}{\partial x}, \end{cases} \tag{3.1}$$

$$\begin{cases} \dot{\xi} = [J_2(\xi, p_2) - R_2(\xi, p_2)]\dfrac{\partial H_2(\xi, p_2)}{\partial \xi} + g_2(\xi)u, \\ \eta = g_2^{\mathrm{T}}(\xi)\dfrac{\partial H_2(\xi)}{\partial \xi}, \end{cases} \tag{3.2}$$

where $p_1,\ p_2 \in \mathbb{R}^s$ are unknown vectors denoting the parametric uncertainties of the two Hamiltonian structures, respectively; for simplicity, we still denote the structural matrices by $J_i$ and $R_i$, and when $p_i = 0$, $J_i(x, 0) = J_i(x)$, $R_i(x, 0) = R_i(x)$ and $H_i(x, 0) = H_i(x)$, $i = 1, 2$; $p_1$ and $p_2$ are assumed to be small enough to keep the two dissipative Hamiltonian structures unchanged, i.e., $J_i^{\mathrm{T}}(x, p_i) = -J_i(x, p_i)$ and $R_i(x, p_i) \geqslant 0$, $i = 1, 2$.

For the adaptive simultaneous stabilization of the systems (3.1) and (3.2), we have the following result.

**Theorem 3.1.** *Assume that*

(i) *there exists a symmetric matrix* $K \in \mathbb{R}^{m\times m}$ *such that*

$$\begin{cases} \bar{R}_1(x, p_1) := R_1(x, p_1) + K_{11}(x, x) > 0, \\ \bar{R}_2(x, p_2) := R_2(x, p_2) - K_{22}(x, x) > 0, \end{cases} \tag{3.3}$$

*where* $K_{ij}(x, \xi) = g_i(x)Kg_j^{\mathrm{T}}(\xi)$, $i, j = 1, 2$;

(ii) *there exists* $\Phi \in \mathbb{R}^{m\times l}$ *such that*

$$[J_i(x, p_i) - R_i(x, p_i)]\Delta_{H_i}(x, p_i) = g_i(x)\Phi\theta, \tag{3.4}$$

*where* $i = 1, 2$,

$$\Delta_{H_i}(x, p_i) := \frac{\partial H_i(x, p_i)}{\partial x} - \frac{\partial H_i(x)}{\partial x}, \tag{3.5}$$

*and* $\theta \in \mathbb{R}^l$ *is an unknown constant vector related to* $p_1$ *and* $p_2$.

*Then, under the following adaptive feedback law:*

$$\begin{cases} u = -K(y - \eta) - \Phi\hat{\theta}, \\ \dot{\hat{\theta}} = Q\Phi^{\mathrm{T}}(y + \eta), \end{cases} \tag{3.6}$$

*the systems* (3.1) *and* (3.2) *can be simultaneously stabilized, where* $\hat{\theta}$ *is the estimate of* $\theta$ *and* $Q \in \mathbb{R}^{l\times l}$ *is a positive definite constant matrix called the adaptation gain.*

**Proof.** Substituting (3.6) into the systems (3.1) and (3.2) and using Condition (ii), we obtain

$$\dot{X} = [J(X, p) - R(X, p)]\frac{\partial H(X)}{\partial X}, \tag{3.7}$$

where $X := [x^{\mathrm{T}}, \xi^{\mathrm{T}}, \hat{\theta}^{\mathrm{T}}]^{\mathrm{T}}$, $p := [p_1^{\mathrm{T}}, p_2^{\mathrm{T}}]^{\mathrm{T}}$,

$$J(X, p) := \begin{bmatrix} J_1(x, p_1) & K_{12}(x, \xi) & -g_1(x)\Phi Q \\ -K_{21}(\xi, x) & J_2(\xi, p_2) & -g_2(\xi)\Phi Q \\ (g_1(x)\Phi Q)^{\mathrm{T}} & (g_2(\xi)\Phi Q)^{\mathrm{T}} & 0 \end{bmatrix},$$

$$R(X, p) := \mathrm{Diag}\{\bar{R}_1(x, p_1), \bar{R}_2(\xi, p_2), 0\} \quad \text{and}$$

$$H(X) := H_1(x) + H_2(\xi) + \tfrac{1}{2}(\theta - \hat{\theta})^{\mathrm{T}}Q^{-1}(\theta - \hat{\theta}). \tag{3.8}$$

Noticing that $K_{12}^{\mathrm{T}}(x, \xi) = K_{21}(\xi, x)$, $J(X, p)$ is skew-symmetric. And from Condition (i), $R(X, p)$ is positive semi-definite. Thus, the system (3.7) is a dissipative Hamiltonian system. From Ortega et al. (2002) and Wang, Li et al. (2003), we know that the system (3.7) is stable.

In the following, we consider the energy flow of the system (3.7). From (3.7), we obtain

$$\begin{aligned} \dot{H}(X) =& -\nabla^{\mathrm{T}}H(X)R(X, p)\nabla H(X) \\ =& -\nabla^{\mathrm{T}}H_1(x)\bar{R}_1(x, p_1)\nabla H_1(x) \\ & -\nabla^{\mathrm{T}}H_2(\xi)\bar{R}_2(\xi, p_2)\nabla H_2(\xi) \leqslant 0. \end{aligned}$$

Thus, the system (3.7) converges to the largest invariant set contained in

$$S = \{X \mid \bar{R}_1^{1/2}(x, p_1)\nabla H_1(x) = 0,\ \bar{R}_2^{1/2}(\xi, p_2)\nabla H_2(\xi) = 0\}.$$

From Condition (i), we know that both $\bar{R}_1^{1/2}(x, p_1)$ and $\bar{R}_2^{1/2}(\xi, p_2)$ are nonsingular, which implies that $\bar{R}_1^{1/2}(x, p_1)\nabla H_1(x) = 0 \Rightarrow x = x_0$, and $\bar{R}_2^{1/2}(\xi, p_2)\nabla H_2(\xi) = 0 \Rightarrow \xi = \xi_0$. Therefore, $S = \{[x_0^{\mathrm{T}}, \xi_0^{\mathrm{T}}, \hat{\theta}^{\mathrm{T}}]^{\mathrm{T}}\}$, with which it is easy to see that $x \to x_0$ and $\xi \to \xi_0$, as $t \to \infty$. That is, under the adaptive feedback (3.6), the systems (3.1) and (3.2) can be simultaneously stabilized. □

Now, we apply Theorem 3.1 to investigate adaptive simultaneous stabilization of the following two uncertain nonlinear systems:

$$\dot{x} = f_1(x) + \Delta f_1(x) + g_1(x)u, \quad x \in \mathbb{R}^n, \tag{3.9}$$

$$\dot{\xi} = f_2(\xi) + \Delta f_2(\xi) + g_2(\xi)u, \quad \xi \in \mathbb{R}^n, \tag{3.10}$$

where $f_1(x_0) = 0$, $f_2(\xi_0) = 0$, $u \in \mathbb{R}^m$ is the control input, and $\Delta f_i(x)$, $i = 1, 2$, are the uncertain parts of the two systems, respectively.

**Corollary 3.2.** *Assume that*

(1) $x_0$ *and* $\xi_0$ *are, respectively, the unique zero point of* $f_1(x)$ *and* $f_2(\xi)$;

(2) *the Jacobian matrix of* $f_i(x)$, *denoted by* $J_{f_i}(x)$, *is strictly dissipative*,[1] $i=1,2$;
(3) *there exists an* $m \times n$ *symmetric matrix* $K$ *such that*

$$\begin{cases} J_{f_1}^{-1}(x) + J_{f_1}^{-\mathrm{T}}(x) - 2g_1(x)Kg_1^{\mathrm{T}}(x) < 0, \\ J_{f_2}^{-1}(x) + J_{f_2}^{-\mathrm{T}}(x) + 2g_2(x)Kg_2^{\mathrm{T}}(x) < 0; \end{cases}$$

(4) *there exists* $\Phi \in \mathbb{R}^{m\times l}$ *such that*

$$\Delta f_i(x) = g_i(x)\Phi\theta, \quad i=1,2, \tag{3.11}$$

*where* $\theta \in \mathbb{R}^l$ *is an unknown constant vector related to* $\Delta f_i(x)$, $i=1,2$.

*Then, under the following adaptive feedback law*:

$$\begin{cases} u=-K[g_1^{\mathrm{T}}(x)J_{f_1}^{\mathrm{T}}(x)f_1(x)-g_2^{\mathrm{T}}(\xi)J_{f_2}^{\mathrm{T}}(\xi)f_2(\xi)]-\Phi\hat{\theta}, \\ \dot{\hat{\theta}} = Q\Phi^{\mathrm{T}}[g_1^{\mathrm{T}}(x)J_{f_1}^{\mathrm{T}}(x)f_1(x) + g_2^{\mathrm{T}}(\xi)J_{f_2}^{\mathrm{T}}(\xi)f_2(\xi)], \end{cases} \tag{3.12}$$

*the systems* (3.9) *and* (3.10) *can be simultaneously stabilized, where* $\hat{\theta}$ *is the estimate of* $\theta$ *and* $0 < Q \in \mathbb{R}^{l\times l}$ *is the adaptation gain.*

**Proof.** Since $J_{f_i}(x)$ is strictly dissipative, from Wang, Li et al. (2003) $J_{f_i}$ is nonsingular, $i=1,2$. From Wang, Li et al. (2003), the systems (3.9) and (3.10) can be, respectively, expressed as

$$\dot{x} = J_{f_1}^{-\mathrm{T}}(x)\nabla H_1(x) + \Delta f_1(x) + g_1(x)u, \tag{3.13}$$

$$\dot{\xi} = J_{f_2}^{-\mathrm{T}}(\xi)\nabla H_2(\xi) + \Delta f_2(\xi) + g_2(\xi)u, \tag{3.14}$$

where $H_i(x) = \frac{1}{2}f_i^{\mathrm{T}}(x)f_i(x)$. From Condition (1), we know that $H_1(x)$ and $H_2(\xi)$ have a unique minimum at $x_0$ and $\xi_0$, respectively. On the other hand, $J_{f_i}^{\mathrm{T}}(x)$ is strictly dissipative, too. According to Wang, Li et al. (2003), $J_{f_i}^{-\mathrm{T}}(x)$ is strictly dissipative and can be expressed as $J_{f_i}^{-\mathrm{T}}(x) = J_i(x) - R_i(x)$, where $J_i(x)$ is skew-symmetric and $R_i(x) > 0$, $i=1,2$. Thus, the systems (3.13) and (3.14) can be rewritten as

$$\begin{aligned} &\dot{x} = [J_1(x) - R_1(x)]\nabla H_1(x) + \Delta f_1(x) + g_1(x)u, \\ &\dot{\xi} = [J_2(\xi) - R_2(\xi)]\nabla H_2(\xi) + \Delta f_2(\xi) + g_2(\xi)u. \end{aligned} \tag{3.15}$$

Moreover, it can be seen from Wang, Li et al. (2003) that Condition (3) implies that

$$R_1(x) + K_{11}(x,x) > 0, \quad R_2(\xi) - K_{22}(\xi,\xi) > 0, \tag{3.16}$$

where $K_{ij}(x,\xi) = g_i(x)Kg_j^{\mathrm{T}}(\xi)$, $i,j=1,2$.

With (3.16) and Condition (4), similar to the proof of Theorem 3.1, we can show that under the adaptive feedback law

$$\begin{cases} u = -K[g_1^{\mathrm{T}}(x)\nabla H_1(x) - g_2^{\mathrm{T}}(\xi)\nabla H_2(\xi)] - \Phi\hat{\theta}, \\ \dot{\hat{\theta}} = Q\Phi^{\mathrm{T}}[g_1^{\mathrm{T}}(x)\nabla H_1(x) + g_2^{\mathrm{T}}(\xi)\nabla H_2(\xi)], \end{cases} \tag{3.17}$$

the two systems given in (3.15) can be simultaneously stabilized. Noticing that $\nabla H_i(x) = J_{f_i}^{\mathrm{T}} f_i(x)$ $(i=1,2)$, (3.17) is exactly (3.12). Thus, the proof is completed. □

[1] An $n\times n$ matrix $M(x)$ is called dissipative if it can be expressed as $M(x) = J(x) - R(x)$, with $J(x)$ skew-symmetric and $R(x) \geqslant 0$. Furthermore, if $R(x) > 0$, $M(x)$ is called strictly dissipative.

**Corollary 3.3.** *Consider the systems* (3.9) *and* (3.10). *Assume that*

(1) *there exist Lyapunov functions* $V_i$ *such that*

$$L_{f_i}V_i(x) \leqslant 0, \quad i=1,2;$$

(2) *there exists a symmetric matrix* $K \in \mathbb{R}^{m\times m}$ *such that*

$$\begin{cases} \dfrac{L_{f_1}V_1(x)}{\|\nabla V_1\|^2}I_n - g_1(x)Kg_1^{\mathrm{T}}(x) < 0 \quad (x \neq x_0), \\ \dfrac{L_{f_2}V_2(x)}{\|\nabla V_2\|^2}I_n + g_2(x)Kg_2^{\mathrm{T}}(x) < 0 \quad (x \neq \xi_0); \end{cases} \tag{3.18}$$

(3) *there exists* $\Phi \in \mathbb{R}^{m\times l}$ *such that*

$$\Delta f_i(x) = g_i(x)\Phi\theta, \quad i=1,2, \tag{3.19}$$

*where* $\theta \in \mathbb{R}^l$ *is an unknown constant vector related to* $\Delta f_i(x)$, $i=1,2$.

*Then, under the following adaptive feedback*:

$$\begin{cases} u = -K[g_1^{\mathrm{T}}(x)\nabla V_1(x) - g_2^{\mathrm{T}}(\xi)\nabla V_2(\xi)] - \Phi\hat{\theta}, \\ \dot{\hat{\theta}} = Q\Phi^{\mathrm{T}}[g_1^{\mathrm{T}}(x)\nabla V_1(x) + g_2^{\mathrm{T}}(\xi)\nabla V_2(\xi)], \end{cases} \tag{3.20}$$

*the systems* (3.9) *and* (3.10) *can be simultaneously stabilized, where* $0 < Q \in \mathbb{R}^{l\times l}$.

**Proof.** From the proof of Corollary 2.7, we know that under Condition (1), the two systems can be expressed as the forms of (3.15), where $H_i(x) = V_i(x)$ and $R_i(x) \geqslant 0$, $i=1,2$. On the other hand, from Wang, Li et al. (2003) we can show that Condition (2) implies that

$$R_1(x) + K_{11}(x,x) > 0, \quad R_2(\xi) - K_{22}(\xi,\xi) > 0, \tag{3.21}$$

where $K_{ij}(x,\xi) = g_i(x)Kg_j^{\mathrm{T}}(\xi)$, $i,j=1,2$.

With (3.21) and Condition (3), from the proof of Corollary 3.2, we know that under the adaptive feedback law (3.20) the systems (3.9) and (3.10) can be simultaneously stabilized. □

## 4. Robust simultaneous stabilization of two PCH systems

In this section, we consider the case that there are external disturbances and parametric uncertainties in the systems (2.1) and (2.2). We design two controllers to simultaneously stabilize the systems: one is a robust controller and the other is a robust adaptive one.

### 4.1. Robust simultaneous stabilization

Consider the systems (2.1) and (2.2) with external disturbances as follows:

$$\begin{cases} \dot{x} = [J_1(x) - R_1(x)]\dfrac{\partial H_1}{\partial x} + g_1(x)u + \bar{g}_1(x)w, \\ y = g_1^{\mathrm{T}}(x)\dfrac{\partial H_1}{\partial x}, \end{cases} \tag{4.1}$$

$$\begin{cases} \dot{\xi} = [J_2(\xi) - R_2(\xi)]\dfrac{\partial H_2}{\partial \xi} + g_2(\xi)u + \bar{g}_2(\xi)w, \\ \eta = g_2^{\mathrm{T}}(\xi)\dfrac{\partial H_2}{\partial \xi}, \end{cases} \tag{4.2}$$

where $w \in \mathbb{R}^q$ is the disturbance, $\bar{g}_i(x) \in \mathbb{R}^{n\times q}$, $i = 1, 2$, and other variables are the same as in Section 2.

Given a disturbance attenuation level $\gamma > 0$, choose

$$z = \Lambda(y + \eta) \tag{4.3}$$

as the penalty function, where $\Lambda \in \mathbb{R}^{s\times m}$ is a weighting matrix. Our objective of this subsection is described as follows:

*Robust simultaneous stabilization*: Design an $L_2$ disturbance attenuation control law such that under the law

**R1:** The two systems are simultaneously asymptotically stable when $w$ vanishes;
**R2:** The $L_2$ gain (from $w$ to $z$) of the closed-loop system is less than $\gamma$.

For the above control problem, we have the following results.

**Theorem 4.1.** *Consider the systems* (4.1) *and* (4.2), *with the penalty function* (4.3) *and the given level* $\gamma > 0$. *If*

(i) *there exists a symmetric matrix* $K \in \mathbb{R}^{m\times m}$ *such that*

$$\begin{cases} R_1(x) + K_{11}(x, x) > 0, \\ R_2(\xi) - K_{22}(\xi, \xi) > 0, \end{cases} \tag{4.4}$$

*where* $K_{ij}(x, \xi) = g_i(x)Kg_j^{\mathrm{T}}(\xi)$, $i, j = 1, 2$;
(ii) $\bar{g}_1 = g_1$ *and* $\bar{g}_2 = g_2$,

*then*

$$u = -K(y - \eta) - \left[\frac{1}{2}\Lambda^{\mathrm{T}}\Lambda + \frac{1}{2\gamma^2}I_m\right](y + \eta) \tag{4.5}$$

*is an* $L_2$ *disturbance attenuation controller such that both* R1 *and* R2 *hold true for the systems* (4.1) *and* (4.2).

**Proof.** Rewrite (4.5) as follows:

$$\begin{cases} u = -K(y - \eta) + v, \\ v = -\left[\dfrac{1}{2}\Lambda^{\mathrm{T}}\Lambda + \dfrac{1}{2\gamma^2}I_m\right](y + \eta). \end{cases} \tag{4.6}$$

Substituting the first part of (4.6) into the systems (4.1) and (4.2), it can be seen from the proof of Theorem 2.1 that the systems (4.1) and (4.2) can be expressed as

$$\dot{X} = [J(X) - R(X)]\frac{\partial H}{\partial X} + G(X)v + \bar{G}(X)w, \tag{4.7}$$

where $X$, $J(X)$, $R(X)$ and $H(X)$ are given in (2.7) and (2.8), $G(X) := [g_1^{\mathrm{T}}(x), g_2^{\mathrm{T}}(\xi)]^{\mathrm{T}}$ and $\bar{G}(X) := [\bar{g}_1^{\mathrm{T}}(x), \bar{g}_2^{\mathrm{T}}(\xi)]^{\mathrm{T}}$.

On the other hand, the penalty function (4.3) can be rewritten as

$$z = \Lambda G^{\mathrm{T}}(X)\nabla H(X). \tag{4.8}$$

Moreover, from Conditions (i) and (ii), we have

$$R(X) + \frac{1}{2\gamma^2}[G(X)G^{\mathrm{T}}(X) - \bar{G}(X)\bar{G}^{\mathrm{T}}(X)] = R(X) > 0. \tag{4.9}$$

Consider the system (4.7) with (4.8) and (4.9). From Wang, Cheng et al. (2003), an $L_2$ disturbance attenuation controller of the system (4.7) can be designed as

$$v = -\left[\frac{1}{2}\Lambda^{\mathrm{T}}\Lambda + \frac{1}{2\gamma^2}I_m\right]G^{\mathrm{T}}(X)\frac{\partial H}{\partial X} \tag{4.10}$$

and furthermore, the $\gamma$-dissipation inequality

$$\dot{H} + \frac{\partial^{\mathrm{T}} H}{\partial X}R(X)\frac{\partial H}{\partial X} \leqslant \frac{1}{2}\{\gamma^2\|w\|^2 - \|z\|^2\} \tag{4.11}$$

holds along the trajectories of the closed-loop system consisting of (4.7) and (4.10).

Noticing that (4.10) is exactly the second part of (4.6), the feedback law (4.5) is an $L_2$ disturbance attenuation controller for the systems (4.1) and (4.2). According to Wang, Cheng et al. (2003), the $L_2$ gain from $w$ to $z$ is less than $\gamma$. Moreover, since $\nabla^{\mathrm{T}}H(X)R(X)\nabla H(X) > 0$, from (4.11) we know that the system (4.7) is asymptotically stable when $w = 0$, that is, $x \to x_0$ and $\xi \to \xi_0$ (as $t \to \infty$). Thus, R1 and R2 hold under the feedback law (4.5). □

**Theorem 4.2.** *Consider the systems* (4.1) *and* (4.2), *with the penalty function* (4.3) *and the given level* $\gamma > 0$. *If*

(i) *there exists a symmetric matrix* $K \in \mathbb{R}^{m\times m}$ *such that*

$$\begin{cases} \widetilde{R}_1(x) := R_1(x) + K_{11}(x, x) \\ \qquad\qquad + \dfrac{1}{2\gamma^2}[g_1(x)g_1^{\mathrm{T}}(x) - \bar{g}_1(x)\bar{g}_1^{\mathrm{T}}(x)] > 0, \\ \widetilde{R}_2(\xi) := R_2(\xi) - K_{22}(\xi, \xi) \\ \qquad\qquad + \dfrac{1}{2\gamma^2}[g_2(\xi)g_2^{\mathrm{T}}(\xi) - \bar{g}_2(\xi)\bar{g}_2^{\mathrm{T}}(\xi)] > 0, \end{cases} \tag{4.12}$$

*where* $K_{ij}(x, \xi) = g_i(x)Kg_j^{\mathrm{T}}(\xi)$, $i, j = 1, 2$;
(ii) $g_1g_2^{\mathrm{T}} = 0$ *and* $\bar{g}_1\bar{g}_2^{\mathrm{T}} = 0$,

*then*

$$u = -K(y - \eta) - \left[\frac{1}{2}\Lambda^{\mathrm{T}}\Lambda + \frac{1}{2\gamma^2}I_m\right](y + \eta) \tag{4.13}$$

*is an* $L_2$ *disturbance attenuation controller such that both* R1 *and* R2 *hold true for the systems* (4.1) *and* (4.2).

**Proof.** The proof is similar to that of Theorem 4.1, and thus omitted. □

### *4.2. Robust adaptive simultaneous stabilization*

We now consider the case that there are both disturbances and parametric uncertainties in the systems (2.1) and (2.2).

In this case, the two systems can be expressed as

$$\begin{cases} \dot{x} = [J_1(x, p_1) - R_1(x, p_1)]\dfrac{\partial H_1(x, p_1)}{\partial x} \\ \qquad + g_1(x)u + \bar{g}_1(x)w, \\ y = g_1^{\mathrm{T}}(x)\dfrac{\partial H_1(x)}{\partial x}, \end{cases} \tag{4.14}$$

$$\begin{cases} \dot{\xi} = [J_2(\xi, p_2) - R_2(\xi, p_2)]\dfrac{\partial H_2(\xi, p_2)}{\partial \xi} \\ \qquad + g_2(\xi)u + \bar{g}_2(\xi)w, \\ \eta = g_2^{\mathrm{T}}(\xi)\dfrac{\partial H_2(\xi)}{\partial \xi}, \end{cases} \tag{4.15}$$

where $w \in \mathbb{R}^q$ is the systems' disturbance; and $p_1$, $p_2$ and other variables are the same as in Section 3.

Given a disturbance attenuation level $\gamma > 0$, choose

$$z = \Lambda(y + \eta) \tag{4.16}$$

as the penalty function, where $\Lambda \in \mathbb{R}^{r\times m}$ is a weighting matrix. The aim of this subsection is described as follows:

*Robust adaptive simultaneous stabilization*: Design an adaptive $L_2$ disturbance attenuation control law such that under the law both $R1$ and $R2$ hold true for the systems (4.14) and (4.15).

For the robust adaptive simultaneous stabilization problem, we have the following results.

**Theorem 4.3.** *Assume that*

(i) *there exists a symmetric matrix $K \in \mathbb{R}^{m\times m}$ such that*

$$\begin{cases} \bar{R}_1(x, p_1) := R_1(x, p_1) + K_{11}(x, x) > 0, \\ \bar{R}_2(\xi, p_2) := R_2(\xi, p_2) - K_{22}(\xi, \xi) > 0, \end{cases} \tag{4.17}$$

*where $K_{ij}(x, \xi) = g_i(x)Kg_j^{\mathrm{T}}(\xi)$, $i, j = 1, 2$;*

(ii) *there exists $\Phi \in \mathbb{R}^{m\times l}$ such that*

$$[J_i(x, p_i) - R_i(x, p_i)]\Delta_{H_i}(x, p_i) = g_i(x)\Phi\theta, \tag{4.18}$$

*where $i = 1, 2$, $\Delta_{H_i}(x, p_i)$ is given in* (3.5) *and $\theta \in \mathbb{R}^l$ is an unknown vector related to $p_1$ and $p_2$;*

(iii) *$\bar{g}_1 = g_1$ and $\bar{g}_2 = g_2$.*

*Then*,

$$\begin{cases} u = -K(y-\eta) - \left[\dfrac{1}{2}\Lambda^{\mathrm{T}}\Lambda + \dfrac{1}{2\gamma^2}I_m\right](y+\eta) - \Phi\hat{\theta}, \\ \dot{\hat{\theta}} = Q\Phi^{\mathrm{T}}(y + \eta) \end{cases} \tag{4.19}$$

*is an adaptive $L_2$ disturbance attenuation controller such that both* R1 *and* R2 *hold true for the systems* (4.14) *and* (4.15), *where $\hat{\theta}$ is the estimate of $\theta$ and $Q \in \mathbb{R}^{l\times l}$ is a positive definite constant matrix* (*the adaptation gain*).

**Proof.** Rewrite (4.19) as follows:

$$\begin{cases} \begin{cases} u = -K(y - \eta) - \Phi\hat{\theta} + v, \\ \dot{\hat{\theta}} = Q\Phi^{\mathrm{T}}(y + \eta), \end{cases} \\ v = -\left[\dfrac{1}{2}\Lambda^{\mathrm{T}}\Lambda + \dfrac{1}{2\gamma^2}I_m\right](y + \eta). \end{cases} \tag{4.20}$$

Substituting the first part of (4.20) into the systems (4.14) and (4.15), from the proof of Theorem 3.1 and Condition (ii), we obtain

$$\dot{X} = [J(X, p) - R(X, p)]\frac{\partial H(X)}{\partial X} + G(X)v + \bar{G}(X)w, \tag{4.21}$$

where $X = [x^{\mathrm{T}}, \xi^{\mathrm{T}}, \hat{\theta}^{\mathrm{T}}]^{\mathrm{T}}$, $p = [p_1^{\mathrm{T}}, p_2^{\mathrm{T}}]^{\mathrm{T}}$, $R(X, p) = \mathrm{Diag}\{\bar{R}_1(x, p_1), \bar{R}_2(\xi, p_2), 0\}$,

$$J(X, p) = \begin{bmatrix} J_1(x, p_1) & K_{12}(x, \xi) & -g_1(x)\Phi Q \\ -K_{12}^{\mathrm{T}}(x, \xi) & J_2(\xi, p_2) & -g_2(\xi)\Phi Q \\ (g_1(x)\Phi Q)^{\mathrm{T}} & (g_2(\xi)\Phi Q)^{\mathrm{T}} & 0 \end{bmatrix},$$

$$H(X) = H_1(x) + H_2(\xi) + \tfrac{1}{2}(\theta - \hat{\theta})^{\mathrm{T}}Q^{-1}(\theta - \hat{\theta}), \tag{4.22}$$

$G(X) := [g_1^{\mathrm{T}}(x), g_2^{\mathrm{T}}(\xi), 0]^{\mathrm{T}} \in \mathbb{R}^{2n+l}$ and $\bar{G}(X) := [\bar{g}_1^{\mathrm{T}}(x), \bar{g}_2^{\mathrm{T}}(\xi), 0]^{\mathrm{T}} \in \mathbb{R}^{2n+l}$.

On the other hand, the penalty function (4.16) can be rewritten as $z = \Lambda G^{\mathrm{T}}(X)\partial H/\partial X$. Furthermore, from Conditions (i) and (iii), $R(X, p) + (1/2\gamma^2)[G(X)G^{\mathrm{T}}(X) - \bar{G}(X)\bar{G}^{\mathrm{T}}(X)] = R(X, p) \geqslant 0$.

From Wang, Cheng et al. (2003), an $L_2$ disturbance attenuation controller for the system (4.21) can be designed as

$$\begin{aligned} v &= -\left[\frac{1}{2}\Lambda^{\mathrm{T}}\Lambda + \frac{1}{2\gamma^2}I_m\right]G^{\mathrm{T}}(X)\frac{\partial H}{\partial X} \\ &= -\left[\frac{1}{2}\Lambda^{\mathrm{T}}\Lambda + \frac{1}{2\gamma^2}I_m\right](y + \eta), \end{aligned} \tag{4.23}$$

which is the second part of (4.20), and furthermore, the $\gamma$-dissipation inequality

$$\dot{H} + \frac{\partial^{\mathrm{T}} H}{\partial X}R(X, p)\frac{\partial H}{\partial X} \leqslant \frac{1}{2}\{\gamma^2\|w\|^2 - \|z\|^2\} \tag{4.24}$$

holds along the trajectories of the closed-loop system consisting of (4.21) and (4.23).

From Wang, Cheng et al. (2003), R2 is true under the feedback (4.19). On the other hand, from (4.24) we know that when $w$ vanishes

$$\dot{H} \leqslant -\frac{\partial^{\mathrm{T}} H_1}{\partial x}\bar{R}_1(x, p_1)\frac{\partial H_1}{\partial x} - \frac{\partial^{\mathrm{T}} H_2}{\partial \xi}\bar{R}_2(\xi, p_2)\frac{\partial H_2}{\partial \xi} \leqslant 0,$$

with which it is easy to see that the closed-loop system consisting of (4.21) and (4.23) converges to the largest invariant set contained in

$$S = \left\{X \,|\, \bar{R}_1^{1/2}(x, p_1)\frac{\partial H_1}{\partial x} = 0, \bar{R}_2^{1/2}(\xi, p_2)\frac{\partial H_2}{\partial \xi} = 0\right\}.$$

From Condition (i), $\bar{R}_1^{1/2}(x, p_1)\partial H_1/\partial x = 0 \Rightarrow x = x_0$ and $\bar{R}_2^{1/2}(\xi, p_2)\partial H_2/\partial \xi = 0 \Rightarrow \xi = \xi_0$. Therefore, $x \to x_0$, $\xi \to \xi_0$ as $t \to \infty$, which means that R1 also holds under the feedback (4.19). The proof is thus completed. □

**Theorem 4.4.** *Consider the systems* (4.14) *and* (4.15), *with the penalty function* (4.16) *and the given* $\gamma > 0$. *Assume that*

(i) *there exists a symmetric matrix* $K \in \mathbb{R}^{m\times m}$ *such that*

$$\widetilde{R}_1(x,p_1) := R_1(x,p_1) + K_{11}(x,x) + \frac{1}{2\gamma^2}[g_1(x)g_1^{\mathrm{T}}(x) - \bar{g}_1(x)\bar{g}_1^{\mathrm{T}}(x)] > 0,$$

$$\widetilde{R}_2(\xi,p_2) := R_2(\xi,p_2) - K_{22}(\xi,\xi) + \frac{1}{2\gamma^2}[g_2(\xi)g_2^{\mathrm{T}}(\xi) - \bar{g}_2(\xi)\bar{g}_2^{\mathrm{T}}(\xi)] > 0;$$

(ii) *there exists* $\Phi \in \mathbb{R}^{m\times l}$ *such that* (4.18) *holds true*;
(iii) $g_1 g_2^{\mathrm{T}} = 0$ *and* $\bar{g}_1\bar{g}_2^{\mathrm{T}} = 0$.

*Then*,

$$\begin{cases} u = -K(y-\eta) - \left[\dfrac{1}{2}\Lambda^{\mathrm{T}}\Lambda + \dfrac{1}{2\gamma^2}I_m\right](y+\eta) - \Phi\hat{\theta}, \\ \dot{\hat{\theta}} = Q\Phi^{\mathrm{T}}(y+\eta) \end{cases} \tag{4.25}$$

*is an adaptive* $L_2$ *disturbance attenuation controller such that both* R1 *and* R2 *hold true for the systems* (4.14) *and* (4.15), *where* $\hat{\theta}$ *and* $Q$ *are the same as in Theorem* 4.3.

**Proof.** The proof is similar to that of Theorem 4.3, and thus omitted. □

## 5. Simultaneous stabilization of more than two PCH systems

In this section, we study the case of more than two PCH systems, and propose a new control design method for the simultaneous stabilization of the PCH systems.

Consider the following $N$ PCH systems:

$$\Sigma_i : \begin{cases} \dot{x}_i = [J_i(x_i) - R_i(x_i)]\dfrac{\partial H_i}{\partial x_i} + g_i(x_i)u, \\ y_i = g_i^{\mathrm{T}}(x_i)\dfrac{\partial H_i}{\partial x_i}, \quad i = 1,2,\ldots,N, \end{cases} \tag{5.1}$$

where $x_i \in \mathbb{R}^{n_i}$; $u \in \mathbb{R}^m$ is the control input; $y_i \in \mathbb{R}^m$ are the outputs of the $N$ systems, respectively; $J_i(x_i) = -J_i^{\mathrm{T}}(x_i) \in \mathbb{R}^{n_i\times n_i}$, $0 \leqslant R_i(x_i) \in \mathbb{R}^{n_i\times n_i}$, $g_i(x_i) \in \mathbb{R}^{n_i\times m}$, and $H_i(x_i)$ is the Hamiltonian function with a local strict minimum at $x_e^{(i)}$, $i = 1,2,\ldots,N$.

Assume that $(i_1, i_2, \ldots, i_N)$ is an arbitrary permutation of $\{1,2,\ldots,N\}$ and that $L$ is a positive integral satisfying $1 \leqslant L \leqslant N-1$. Let $M_1 = n_{i_1} + \cdots + n_{i_L}$ and $M_2 = n_{i_{L+1}} + \cdots + n_{i_N}$.

Now, we divide the $N$ systems into two sets: $\{i_1,\ldots,i_L\}$ and $\{i_{L+1},\ldots,i_N\}$, which can, respectively, be rewritten as

$$\begin{cases} \dot{X}_1 = [\bar{J}_1(X_1) - \bar{R}_1(X_1)]\dfrac{\partial \bar{H}_1}{\partial X_1} + G_1(X_1)u, \\ Y_1 = G_1^{\mathrm{T}}(X_1)\dfrac{\partial \bar{H}_1}{\partial X_1}, \end{cases} \tag{5.2}$$

$$\begin{cases} \dot{X}_2 = [\bar{J}_2(X_2) - \bar{R}_2(X_2)]\dfrac{\partial \bar{H}_2}{\partial X_2} + G_2(X_2)u, \\ Y_2 = G_2^{\mathrm{T}}(X_2)\dfrac{\partial \bar{H}_2}{\partial X_2}, \end{cases} \tag{5.3}$$

where $X_1 = [x_{i_1}^{\mathrm{T}},\ldots,x_{i_L}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{M_1}$, $X_2 = [x_{i_{L+1}}^{\mathrm{T}},\ldots,x_{i_N}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{M_2}$,

$$\bar{J}_1(X_1) = -\bar{J}_1^{\mathrm{T}}(X_1) = \mathrm{Diag}\{J_{i_1}(x_{i_1}),\ldots,J_{i_L}(x_{i_L})\},$$
$$\bar{R}_1(X_1) = \mathrm{Diag}\{R_{i_1}(x_{i_1}),\ldots,R_{i_L}(x_{i_L})\} \geqslant 0,$$
$$\bar{J}_2(X_2) = -\bar{J}_2^{\mathrm{T}}(X_2) = \mathrm{Diag}\{J_{i_{L+1}}(x_{i_{L+1}}),\ldots,J_{i_N}(x_{i_N})\},$$
$$\bar{R}_2(X_2) = \mathrm{Diag}\{R_{i_{L+1}}(x_{i_{L+1}}),\ldots,R_{i_N}(x_{i_N})\} \geqslant 0,$$
$$G_1(X_1) = [g_{i_1}^{\mathrm{T}}(x_{i_1}),\ldots,g_{i_L}^{\mathrm{T}}(x_{i_L})]^{\mathrm{T}},$$
$$G_2(X_2) = [g_{i_{L+1}}^{\mathrm{T}}(x_{i_{L+1}}),\ldots,g_{i_N}^{\mathrm{T}}(x_{i_N})]^{\mathrm{T}},$$

and

$$\bar{H}_1(X_1) = \sum_{j=1}^{L} H_{i_j}(x_{i_j}),$$

$$\bar{H}_2(X_2) = \sum_{j=L+1}^{N} H_{i_j}(x_{i_j}). \tag{5.4}$$

Obviously, the systems (5.2) and (5.3) are two dissipative PCH systems, with which we can obtain the following result.

**Theorem 5.1.** *Assume that there exist a symmetric matrix* $K \in \mathbb{R}^{m\times m}$, *a permutation* $(i_1, i_2, \ldots, i_N)$ *of* $\{1,2,\ldots,N\}$ *and a positive integral* $L$ $(1 \leqslant L \leqslant N-1)$ *such that*

$$\begin{cases} \bar{R}_1(X_1) + K_{11}(X_1,X_1) > 0, \\ \bar{R}_2(X_2) - K_{22}(X_2,X_2) > 0, \end{cases} \tag{5.5}$$

*where*

$$K_{ij}(X_i,X_j) = G_i(X_i)KG_j^{\mathrm{T}}(X_j), \quad i,j = 1,2. \tag{5.6}$$

*Then, the output feedback*

$$u = -K(y_{i_1} + \cdots + y_{i_L} - y_{i_{L+1}} - \cdots - y_{i_N}) \tag{5.7}$$

*can simultaneously stabilize the* $N$ *systems given in* (5.1).

**Proof.** From the proof of Theorem 2.1, we know that when the dimension of $x$ is not equal to that of $\xi$, Theorem 2.1 still holds, with which Theorem 5.1 can be easily proved. In fact, it is easy to see that under the theorem's conditions, the systems (5.2) and (5.3) satisfy all the conditions of Theorem 2.1. Using Theorem 2.1, we know that the feedback law

$$u = -K(Y_1 - Y_2) \tag{5.8}$$

can simultaneously stabilize the two systems (5.2) and (5.3), which are equivalent to the $N$ systems given in (5.1). Noticing that (5.8) is exactly the feedback (5.7), the proof is completed. □

## 6. Illustrative examples

In this section, we give two examples to show how to apply the results obtained in this paper to investigate the simultaneous stabilization problem.

**Example 6.1.** Consider the following two PCH systems:

$$\begin{cases} \dot{x} = [J_1(x,p) - R_1(x,p)]\dfrac{\partial H_1(x,p)}{\partial x} + g_1 u + \bar{g}_1 w, \\ y = g_1^{\mathrm{T}} \dfrac{\partial H_1(x)}{\partial x}, \end{cases} \tag{6.1}$$

$$\begin{cases} \dot{\xi} = [J_2(\xi,p) - R_2(\xi,p)]\dfrac{\partial H_2(\xi,p)}{\partial \xi} + g_2 u + \bar{g}_2 w, \\ \eta = g_2^{\mathrm{T}} \dfrac{\partial H_2(\xi)}{\partial \xi}, \end{cases} \tag{6.2}$$

where $x = [x_1, x_2, x_3]^{\mathrm{T}} \in \mathbb{R}^3$, $\xi = [\xi_1, \xi_2, \xi_3]^{\mathrm{T}} \in \mathbb{R}^3$, $p$ is an unknown constant satisfying $|p| < 1$, $w \in \mathbb{R}^2$ is the disturbance, $u = [u_1, u_2]^{\mathrm{T}} \in \mathbb{R}^2$ is the control,

$$H_1(x,p) = \tfrac{1}{2}[x_1^2 + 2x_2^2 + (x_3 + p)^2],$$
$$H_2(\xi,p) = \tfrac{1}{2}[(\xi_1 + p)^2 + \xi_2^2 + (\xi_3 + p)^2],$$

$$J_1(x,p) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad J_2(\xi,p) = \begin{bmatrix} 0 & 0 & 2+p \\ 0 & 0 & 0 \\ -2-p & 0 & 0 \end{bmatrix},$$

$R_1(x,p) = \mathrm{Diag}\{1, 0, 2+p\} \geqslant 0$, $R_2(\xi,p) = \mathrm{Diag}\{2+p, 3+p, 0\} \geqslant 0$, and

$$g_1 = \bar{g}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}, \quad g_2 = \bar{g}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}. \tag{6.3}$$

Choosing $K = \mathrm{Diag}\{1, -1\}$, a straightforward computation shows that $R_1(x,p) + g_1 K g_1^{\mathrm{T}} = \mathrm{Diag}\{1,\ 1,\ 1+p\} > 0$, $R_2(\xi,p) - g_2 K g_2^{\mathrm{T}} = \mathrm{Diag}\{1+p,\ 3+p,\ 1\} > 0$.

On the other hand, it is easy to obtain

$$\Delta_{H_1} = \frac{\partial H_1(x,p)}{\partial x} - \frac{\partial H_1(x)}{\partial x} = [0,\ 0,\ p]^{\mathrm{T}},$$

$$\Delta_{H_2} = \frac{\partial H_2(\xi,p)}{\partial \xi} - \frac{\partial H_2(\xi)}{\partial \xi} = [p,\ 0,\ p]^{\mathrm{T}}.$$

Letting $\Phi = [0,\ -1]^{\mathrm{T}}$ and $\theta = p(2+p)$, it is easy to check that $[J_i(x,p) - R_i(x,p)]\Delta_{H_i} = g_i \Phi \theta$ holds for $i = 1, 2$.

Thus, all the conditions of Theorem 4.3 are satisfied. From Theorem 4.3, a robust adaptive simultaneous stabilizer for the systems (6.1) and (6.2) can be designed as

$$\begin{cases} u = -K(y - \eta) - \left[\dfrac{1}{2}\Lambda^{\mathrm{T}}\Lambda + \dfrac{1}{2\gamma^2} I_2\right](y + \eta) - \Phi\hat{\theta}, \\ \dot{\hat{\theta}} = Q\Phi^{\mathrm{T}}(y + \eta), \end{cases}$$

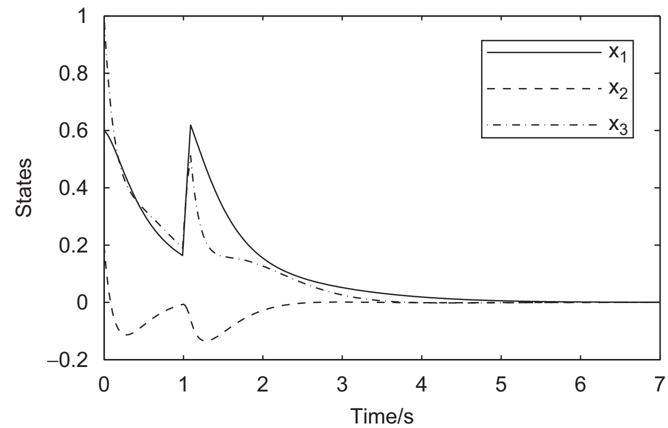


Fig. 1.

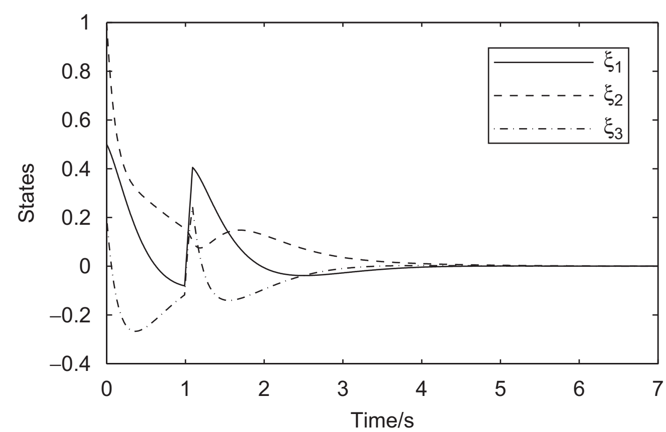


Fig. 2.

that is,

$$\begin{cases} u_1 = -2x_2 + \xi_1 - \left(\dfrac{1}{2}r_1^2 + \dfrac{1}{2\gamma^2}\right)(2x_2 + \xi_1), \\ u_2 = x_3 - \xi_3 - \left(\dfrac{1}{2}r_2^2 + \dfrac{1}{2\gamma^2}\right)(x_3 + \xi_3) + \hat{\theta}, \\ \dot{\hat{\theta}} = -Q(x_3 + \xi_3), \end{cases} \tag{6.4}$$

where $\Lambda = \mathrm{Diag}\{r_1, r_2\} > 0$ is the weighting matrix, $Q > 0$ is a real number used as the adaptation gain, and $\gamma > 0$ is the disturbance attenuation level.

To illustrate the effectiveness of the control law (6.4), we carry out some numerical simulations with the following choices. Initial Conditions: $x(0) = [0.6, 0.2, 1]^{\mathrm{T}}$, $\xi(0) = [0.5, 1, 0.2]^{\mathrm{T}}$; parameters: $r_1 = 0.2$, $r_2 = 0.3$, $Q = 1$, $\gamma = 0.4$. To test the robustness of the controller with respect to external disturbances, a square disturbance of amplitude $[5, 5]^{\mathrm{T}}$ is added to the two systems in the time duration (1s–1.1 s. The simulation results are shown in Figs. 1 and 2, which are the responses of the two systems, respectively.

From Figs. 1 and 2, we know that the robust adaptive control law (6.4) is very effective in simultaneously stabilizing the systems (6.1) and (6.2), and has strong robustness against external disturbances.

**Example 6.2.** Consider the following two affine nonlinear systems:

$$\dot{x} = f_1(x) + \triangle f_1(x) + g_1(x)u,$$
$$\dot{\xi} = f_2(\xi) + \triangle f_2(\xi) + g_2(\xi)u, \tag{6.5}$$

where $x = [x_1,\ x_2,\ x_3]^{\mathrm{T}} \in \mathbb{R}^3$, $\xi = [\xi_1, \xi_2, \xi_3]^{\mathrm{T}} \in \mathbb{R}^3$, $u = [u_1, u_2]^{\mathrm{T}} \in \mathbb{R}^2$,

$$f_1(x) = \begin{bmatrix} -x_1 + x_1x_2 - x_1^3 \\ -x_1^2 - x_2 - x_1^2x_2 \\ -x_3 - x_1^2x_3 \end{bmatrix}, \quad \Delta f_1 = \begin{bmatrix} p_1x_3 + 2p_2x_3 \\ -\dfrac{1}{2}p_2x_1 \\ p_1x_2 + 2p_2x_2 \end{bmatrix},$$

$$f_2(\xi) = \begin{bmatrix} -6\xi_1 - 6\xi_1\xi_3^2 \\ -4\xi_2 - 4\xi_2\xi_3^2 + \xi_3^2 \\ -2\xi_3 - \xi_2\xi_3 - 2\xi_3^3 \end{bmatrix}, \quad \Delta f_2 = \begin{bmatrix} p_2\xi_1 \\ -2p_1\xi_3 - 4p_2\xi_3 \\ p_2\xi_2 \end{bmatrix},$$

$$g_1(x) = \begin{bmatrix} x_3 & 0 \\ 0 & -\dfrac{1}{2}x_1 \\ x_2 & 0 \end{bmatrix}, \quad g_2(\xi) = \begin{bmatrix} 0 & \xi_1 \\ -2\xi_3 & 0 \\ 0 & \xi_2 \end{bmatrix},$$

and $p_1,\ p_2 \in \mathbb{R}^1$ are two unknown parameters.

It can be verified that the two systems given in (6.5) are not simultaneously asymptotically stable when $u=0$. In the following, we use Corollary 3.3 to design an adaptive simultaneous stabilizer for the two systems. Choose Lyapunov functions as

$$V_1(x) = \tfrac{1}{2}(x_1^2 + x_2^2 + x_3^2), \quad V_2(\xi) = \tfrac{3}{4}\xi_1^2 + \tfrac{1}{2}\xi_2^2 + \tfrac{1}{4}\xi_3^2,$$

then, it is easy to know that

$$L_{f_1}V_1(x) = -(1 + x_1^2)(x_1^2 + x_2^2 + x_3^2) < 0,$$
$$L_{f_2}V_2(\xi) = -(1 + \xi_3^2)(9\xi_1^2 + 4\xi_2^2 + \xi_3^2) < 0.$$

Setting $K = \mathrm{Diag}\{1, -2\}$, a straightforward computation shows that

$$\frac{L_{f_1}V_1(x)}{\|\nabla V_1\|^2} I_3 - g_1(x)Kg_1^{\mathrm{T}}(x) < 0,$$
$$\frac{L_{f_2}V_2(\xi)}{\|\nabla V_2\|^2} I_3 + g_2(\xi)Kg_2^{\mathrm{T}}(\xi) < 0.$$

Let

$$\Phi = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix},$$

then it is easy to check that

$$\Delta f_1(x) = g_1(x)\Phi\theta, \quad \Delta f_2(\xi) = g_2(\xi)\Phi\theta,$$

where $\theta = [\theta_1,\ \theta_2]^{\mathrm{T}} := [p_1,\ p_2]^{\mathrm{T}}$.

Therefore, all the conditions of Corollary 3.3 are satisfied. From Corollary 3.3, an adaptive simultaneous stabilization controller for the two systems given in (6.5) can be designed as

$$\begin{cases} u = -K[g_1^{\mathrm{T}}(x)\triangledown V_1(x) - g_2^{\mathrm{T}}(\xi)\triangledown V_2(\xi)] - \Phi\hat{\theta}, \\ \dot{\hat{\theta}} = Q\Phi^{\mathrm{T}}[g_1^{\mathrm{T}}(x)\triangledown V_1(x) + g_2^{\mathrm{T}}(\xi)\triangledown V_2(\xi)], \end{cases}$$

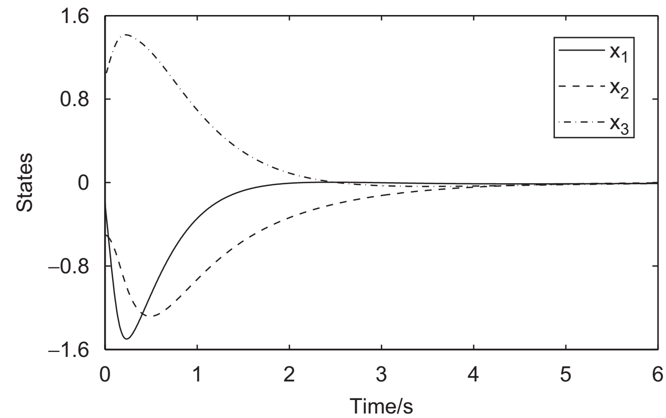


Fig. 3.

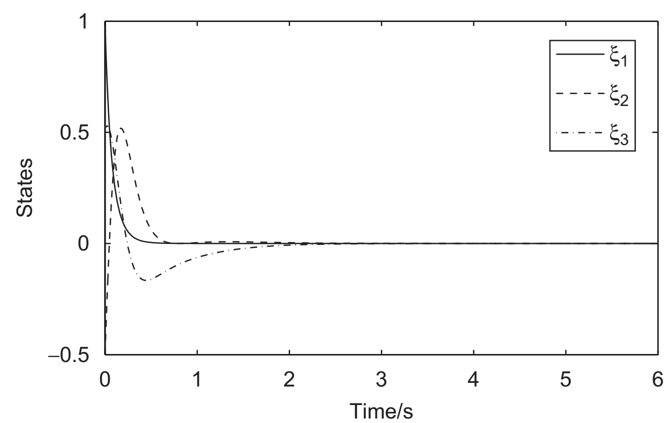


Fig. 4.

that is,

$$\begin{cases} u_1 = -x_1x_3 - x_2x_3 - 2\xi_2\xi_3 - \hat{\theta}_1 - 2\hat{\theta}_2, \\ u_2 = -x_1x_2 - 3\xi_1^2 - \xi_2\xi_3 - \hat{\theta}_2, \\ \dot{\hat{\theta}}_1 = r_1(x_1x_3 + x_2x_3 - 2\xi_2\xi_3), \\ \dot{\hat{\theta}}_2 = 2r_2(x_1x_3 + x_2x_3 - \frac{1}{4}x_1x_2) \\ \qquad -r_2(4\xi_2\xi_3 - \frac{3}{2}\xi_1^2 - \frac{1}{2}\xi_2\xi_3), \end{cases} \tag{6.6}$$

where $Q = \mathrm{Diag}\{r_1, r_2\} > 0$ is the adaptation gain matrix, and $\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2]^{\mathrm{T}}$ is the estimate of $\theta$.

To illustrate the effectiveness of the control law (6.6), some numerical simulations are carried out with the following choices. Initial Condition: $x(0) = [-0.2,\ -0.5,\ 1]^{\mathrm{T}}$, $\xi(0) = [1,\ -0.5,\ 0.5]^{\mathrm{T}}$; parameters: $r_1 = 0.5$, $r_2 = 0.8$. The simulation results are shown in Figs. 3 and 4, which are the responses of the two systems, respectively.

From Figs. 3 and 4, we know that the adaptive control law (6.6) is very effective in simultaneously stabilizing the two uncertain systems given in (6.5).

## 7. Conclusion

This paper has investigated the simultaneous stabilization of a set of nonlinear PCH systems and proposed a number of new results on the design of the simultaneous stabilization controllers. Using the dissipative structural properties, we have combined the two PCH systems to obtain an augmented Hamiltonian system, with which some simultaneous stabilization results are proposed. When the two systems have parametric uncertainties in their Hamiltonian structures, we have presented an adaptive simultaneous stabilization controller. For the case that there are external disturbances and parametric uncertainties in the two PCH systems, two simultaneous stabilization controllers are obtained: one is a robust controller and the other is a robust adaptive one. The case of more than two PCH systems is also studied in this paper, and a new control design method has been proposed for the case. Study on examples and simulations shows that the simultaneous stabilization controllers obtained in this paper work very well.

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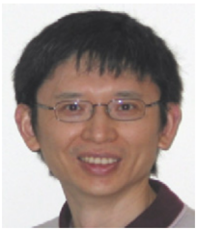

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