EXTENDED CASIMIR APPROACH TO CONTROLLED HAMILTONIAN SYSTEMS*

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Abstract In this paper, we first propose an extended Casimir method for energy-shaping. Then it is used to solve some control problems of Hamiltonian systems. To solve the H_∞ control problem, the energy function of a Hamiltonian system is shaped to such a form that could be a candidate solution of HJI inequality. Next, the energy function is shaped as a candidate of control ISS-Lyapunov function, and then the input-to-state stabilization of port-controlled Hamiltonian systems is achieved. Some easily verifiable sufficient conditions are presented.

Key words Casimir function, Energy-shaping, H_∞ control, Input-to-state stabilization.

1 Introduction

Port-controlled Hamiltonian system is an important class of nonlinear systems, which has been investigated by a number of authors in the last three decades. Its importance lies not only on the fact that many real physical systems, such as mechanical or electrical systems, can be represented in the form of Hamiltonian systems, but also on its many advantages comparing with general controlled nonlinear systems, such as its structural properties which are very convenient for system synthesis and control design. Most of affine nonlinear systems can be transformed into Hamiltonian systems through Hamiltonian realization (HR) or feedback Hamiltonian realization (FHR)[1−4], thus Hamiltonian system theory is applicable to control problems of various nonlinear systems. Many tools have been developed or applied by many researchers to study Hamiltonian systems, among which passivity-based control has been proved to be very powerful to handle many robust control problems of physical systems[5−11]. Hamiltonian function of a Hamiltonian system can be interpreted as the total energy of the system. It plays an important role in the study of Hamiltonian systems, because the total energy can be used as a natural candidate of Lyapunov function. For instance, Hamiltonian approach is particularly suitable for power systems. Some nice applications include the disturbance attenuation of power systems via Hamiltonian approach[2,11−13] and the stabilization of multi-machine systems[3,14−16].

In many cases, the energy function of a given Hamiltonian system doesn't take the desirable form, then we have to shape the energy function into a suitable form to meet certain designing purposes[7,8].

In this paper we first propose a method to shape the energy function of a Hamiltonian system. Using it, two control problems of Hamiltonian systems are investigated. One is the suboptimal H_∞ control problem. The other one is the input-to-state stabilization problem.

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As the first application, we consider the problem of suboptimal H_∞ control which means L_2-gain control with internal stability[6]. A system has finite L_2-gain if the Hamiltonian-Jacobi-Issacs inequality holds for a nonnegative solution $V(x)$. In [11], the H_∞ control of a Hamiltonian system with positive definite Hamiltonian function has been investigated. In general, the Hamiltonian function of a system may not be positive definite, thus the approach introduced in [11] may fail to be applicable. In our approach, this obstacle could be overcome by shaping the energy function.

As the second application, the input-to-state stabilization problem is considered. The concept of input-to-state stability was firstly introduced by E. D. Sontag. A system is input-to-state stable if and only if it has a so-called ISS Lyapunov function. Input-to-state stabilizing a system means finding a control law such that the resulting closed-loop system has an ISS Lyapunov function (i.e., control ISS Lyapunov function). See [17–21].

It is well known that both solving a Hamiltonian-Jacobi-Issacs inequality and finding a control ISS Lyapunov function for a given general nonlinear system are far from easy tasks. But for a Hamiltonian system, it is natural to expect that the Hamiltonian function, if it is or can be shaped to a suitable form, suggests a candidate solution of HJI inequality and/or a candidate control ISS Lyapunov function with suitable controls. It is one basic motivation of this paper.

The rest of this paper is arranged as follows: In Section 2, the Casimir method of energy-shaping is extended; Base on the result in Section 2, H_∞ control and input-to-state stabilization are investigated in Section 3 and Section 4 respectively; Section 5 is the conclusion.

2 Energy-Shaping of Hamiltonian Systems

Consider a controlled Hamiltonian system with dissipation

$$\dot{x} = (J(x) - R(x))\frac{\partial H(x)}{\partial x} + G(x)u, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state variable, $u \in \mathbb{R}^m$ is the input. $J(x)$ and $R(x)$ are respectively skew-symmetric and positive semi-definite matrices. $G(x)$ is an $n \times m$ matrix. $H(x)$ is the Hamiltonian function of the system.

Assume that $c_1(x)$, $c_2(x)$, $\cdots$, $c_{n_c}(x)$, $i = 1, 2, \cdots, n_c$, are n_c smooth functions and denote

$$B(x) = \frac{\partial C(x)}{\partial x} := (\nabla c_1(x), \nabla c_2(x), \cdots, \nabla c_{n_c}(x)). \tag{2}$$

Then it is easy to check that $\xi_i - c_i(x)$ are Casimir functions of the system

$$\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} (J(x) - R(x)) & (J(x) - R(x))B(x) \\ B^{\mathrm{T}}(x)(J(x) - R(x)) & B^{\mathrm{T}}(x)(J(x) - R(x))B(x) \end{pmatrix} \begin{pmatrix} \dfrac{\partial \overline{H}}{\partial x} \\ \dfrac{\partial \overline{H}}{\partial \xi} \end{pmatrix}, \tag{3}$$

where $\xi = (\xi_1, \xi_2, \cdots, \xi_{n_c})^{\mathrm{T}} \in \mathbb{R}^{n_c}$, $\overline{H}(x, \xi) \triangleq H(x) + H_c(\xi)$. Define a multi-level set

$$\mathcal{M} = \left\{(x, \xi) \;\middle|\; \xi_i = c_i(x) + c_i\right\} \subset \mathbb{R}^{n+n_c},$$

where c_i, $i = 1, 2, \cdots, n_c$, are constants. Obviously, $\mathcal{M}$ is an invariant manifold of system (3).

When restricted on $\mathcal{M}$, system (3) becomes

$$\begin{aligned}\dot{x} &= (J(x) - R(x))\frac{\partial}{\partial x}(H(x) + H_c(c_1(x) + c_1, \cdots, c_m(x) + c_m)) \\ &= (J(x) - R(x))\frac{\partial \overline{H}(x)}{\partial x}. \qquad (4)\end{aligned}$$

Remark 1 The constants c_i $(i = 1, 2, \cdots, n_c)$ can be decided by certain requirement on the shaped Hamiltonian function $\overline{H}(x)$, e.g., in general, $\overline{H}(x)$ is required to reach local minimum at work point.

Proposition 1 *If there exists an $n_c \times m$ matrix $G_c(x)$ such that*

$$(J(x) - R(x))B(x) = -G(x)G_c^{\mathrm{T}}(x), \qquad (5)$$

where $B(x)$ is defined by equation (2), *then the restricted system of* (3) *on $\mathcal{M}$ is the same as the closed-loop system of* (1) *with control*

$$u(x) = -G_c^{\mathrm{T}}(x)\left.\frac{\partial H_c(\xi)}{\partial \xi}\right|_{\xi_i = c_i(x) + c_i}. \qquad (6)$$

Proof Using control (6) to system (1), we have

$$\begin{aligned}\dot{x} &= (J(x) - R(x))\frac{\partial H(x)}{\partial x} - G(x)G_c^{\mathrm{T}}(x)\left.\frac{\partial H_c(\xi)}{\partial \xi}\right|_{\xi_i = c_i(x) + c_i} \\ &= (J(x) - R(x))\frac{\partial H(x)}{\partial x} + (J(x) - R(x))\frac{\partial C(x)}{\partial x}\left.\frac{\partial H_c(\xi)}{\partial \xi}\right|_{\xi_i = c_i(x) + c_i} \\ &= (J(x) - R(x))\frac{\partial \overline{H}(x)}{\partial x},\end{aligned}$$

where $H_c(x) = H_c(c_1(x) + c_1, c_2(x) + c_2, \cdots, c_m(x) + c_m)$. It obviously coincides with (4) and thus the proof is completed. ∎

According to Proposition 1, system (4) is exactly the closed-loop form of system (1) with control law (6), where $\overline{H}(x)$ is the shaped energy function. If the shaped Hamiltonian function is positive definite and in addition some detectable conditions hold, asymptotical stability can be achieved. In the following sections, we first use the above result to shape the Hamiltonian function, then find conditions to assure that there exist control laws to achieve our specific purposes, including solving H_∞ control and input-to-state stabilization.

3 Suboptimal H_∞ Control

In this section we consider the suboptimal H_∞ control problem of the following forced Hamiltonian system with external disturbances

$$\begin{cases}\dot{x} = (J(x) - R(x))\dfrac{\partial H(x)}{\partial x} + G_1(x)u + G_2(x)w, \\ y = G_1^{\mathrm{T}}(x)\dfrac{\partial H(x)}{\partial x}, \\ z = h(x) + D(x)u,\end{cases} \qquad (7)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ is input, $w \in \mathbb{R}^p$ is exterior disturbance, $J(x)$ is an $n \times n$ skew-symmetric matrix, $R(x)$ is an $n \times n$ positive semi-definite matrix, $H(x)$ is the Hamiltonian function of the system, $G_1(x)$ and $G_2(x)$ are $n\times m$ and $n\times p$ matrices respectively, and $D(x)$ is a $p\times m$ matrix. We always assume that $D^{\mathrm T}(x)D(x)$ is invertible, which is a common assumption for H_∞ control problem.

In order to shape the energy function, we need the following two assumptions:

Assumption 1 There exist functions $c_1(x), c_2(x), \cdots, c_{n_c}(x)$ and an $n_c\times m$ matrix $G_c(x)$ such that

$$\big(J(x)-R(x)\big)\frac{\partial C(x)}{\partial x}=-G_1(x)G_c^{\mathrm T}(x),$$

where $C(x)=(c_1(x),c_2(x),\cdots,c_{n_c}(x))$;

Assumption 2 There exists $H_c(\xi):\mathbb{R}^{n_c}\to\mathbb{R}$ such that

$$\overline{H}(x)\triangleq H(x)+H_c(c_1(x),c_2(x),\cdots,c_{n_c}(x))$$

is positive definite.

According to Proposition 1, the feedback control rule

$$u(x)=\phi(x)+v:=-G_c^{\mathrm T}(x)\left.\frac{\partial H_c(\xi)}{\partial \xi}\right|_{\xi_i=c_i(x)+c_i}+v \tag{8}$$

renders system (7) to

$$\begin{cases}\dot x=(J(x)-R(x))\dfrac{\partial \overline{H}(x)}{\partial x}+G_1(x)v+G_2(x)w,\\ z=\bar h(x)+D(x)v,\end{cases} \tag{9}$$

where

$$\bar h(x)=h(x)+D(x)\phi(x).$$

Here we discard the output y for simplicity. Taking

$$V(x)=\alpha\overline{H}(x),\quad \alpha>0,$$

we expect that $V(x)$ is a possible solution of HJI inequality, where α is an adjustable parameter. Thus we plug $V(x)$ into Hamiltonian-Jacobi-Issacs inequality to get

$$-\alpha d\overline{H}R\nabla\overline{H}+\alpha d\overline{H}G_1v+\frac{\alpha^2}{2\gamma^2}d\overline{H}G_2G_2^{\mathrm T}\nabla\overline{H}+\frac12\bar h^{\mathrm T}\bar h+\bar h^{\mathrm T}Dv+\frac12 v^{\mathrm T}D^{\mathrm T}Dv\le 0, \tag{10}$$

where $\gamma>0$ is a given positive number. The next task is to find out under what conditions there exist functions $v(x)$ such that the above inequality holds. Denote

$$\begin{aligned}A(x)&=\frac12 D^{\mathrm T}(x)D(x),\\ B(\alpha,x)&=\alpha G_1^{\mathrm T}(x)\nabla\overline{H}(x)+D^{\mathrm T}(x)\bar h(x),\\ C(\alpha,x)&=d\overline{H}(x)Q(x)\nabla\overline{H}(x)+\frac12\bar h^{\mathrm T}(x)\bar h(x),\\ Q(x)&=-\alpha R(x)+\frac{\alpha^2}{2\gamma^2}G_2(x)G_2^{\mathrm T}(x).\end{aligned}$$

Then inequality (10) becomes

$$v^{\mathrm T}A(x)v+B^{\mathrm T}(\alpha,x)v+C(\alpha,x)\le 0. \tag{11}$$

Since A is positive definite, there exists a function $v = v(x)$ such that (11) holds, if and only if the following condition, which is named as Assumption 3, holds.

Assumption 3 There exists a positive number $\alpha > 0$ such that

$$\Delta(x) = B^{\mathrm{T}}(\alpha, x)A^{-1}(x)B(\alpha, x) - 4C(\alpha, x) \geq 0. \tag{12}$$

The corresponding feedback control for system (9) is

$$\begin{aligned} v(x) &= -\frac{1}{2}A^{-1}(x)B(\alpha, x) \\ &= -\frac{1}{4}(D^{\mathrm{T}}(x)D(x))^{-1}\left(\alpha G_1^{\mathrm{T}}(x)\nabla\overline{H}(x) + D^{\mathrm{T}}(x)\bar{h}(x)\right), \end{aligned} \tag{13}$$

which renders the L_2-gain of system (9) from w to z be less than or equal to γ.

In the next, in order to make the closed-loop system of (9) with $w = 0$ asymptotically stable, we calculate the derivative of $V(x)$ along the trajectory of system (9) (with $w = 0$) as

$$\begin{aligned} \dot{V}(x) &= -\alpha d\overline{H}R\nabla\overline{H} + \alpha d\overline{H}G_1 v \\ &\leq -\frac{\alpha^2}{2\gamma^2}d\overline{H}(x)G_2(x)G_2^{\mathrm{T}}(x)\nabla\overline{H}(x) - \frac{1}{2}\left(\bar{h}(x) + D(x)v(x)\right)^{\mathrm{T}}\left(\bar{h}(x) + D(x)v(x)\right) \\ &:= -W(x) \leq 0, \end{aligned} \tag{14}$$

and $W(x) = 0$, if and only if

$$\begin{cases} G_2^{\mathrm{T}}(x)\nabla\overline{H}(x) = 0, \\ \bar{h}(x) - \dfrac{1}{2}D(x)A^{-1}(x)B(\alpha, x) = 0. \end{cases}$$

Define a set

$$M = \left\{x \mid G_2^{\mathrm{T}}(x)\nabla\overline{H}(x) = 0\right\} \bigcap \left\{x \middle| \bar{h}(x) - \frac{1}{2}D(x)A^{-1}(x)B(\alpha, x) = 0\right\}, \tag{15}$$

we have

$$\{x \mid \dot{V}(x) = 0\} \subset M.$$

We assume

Assumption 4 There are no non-zero solutions of closed-loop of system (9) (with $w = 0$) contained in M.

Then according to LaSalle's invariance principle, the closed-loop of (9) with $w = 0$ is asymptotically stable. According to (8), the H_∞ control rule for the original system (7) is

$$\begin{aligned} u(x) &= \phi(x) + v \\ &= \phi(x) - \frac{1}{4}(D^{\mathrm{T}}(x)D(x))^{-1}\left(\alpha G_1^{\mathrm{T}}(x)\nabla\overline{H}(x) + D^{\mathrm{T}}(x)\bar{h}(x)\right) \\ &= -\frac{3}{4}G_c^{\mathrm{T}}(x)\nabla_\xi H_c(\xi)\big|_{\xi=C(x)} - \frac{1}{4}(D^{\mathrm{T}}(x)D(x))^{-1}\left(\alpha G_1^{\mathrm{T}}(x)\nabla\overline{H}(x) + D^{\mathrm{T}}(x)h(x)\right). \end{aligned} \tag{16}$$

Summarizing the above discussion, we obtain the following result.

Theorem 1 *Under the Assumptions* $1-4$*, the suboptimal* H_∞ *control problem can be solved by feedback* (16)*, and its* L_2*-gain from* w *to* z *is less than or equals to* γ*.*

4 Input-to-State Stabilization

Consider a general nonlinear system

$$\dot{x} = f(x, u), \tag{17}$$

where $f(0,0) = 0$. As we know, system (17) is input-to-state stable, if and only if there exists an ISS-Lyapunov function, i.e., a C^1 function $V : \mathbb{R}^n \to \mathbb{R}$ which is positive-definite and proper and furthermore satisfying

$$\frac{\partial V}{\partial x} f(x, u) \leq -\alpha(\|x\|) + \sigma(\|u\|), \quad \forall x \in \mathbb{R}^n, \text{ and } \forall u \in \mathbb{R}^m, \tag{18}$$

where $\alpha \in \mathcal{K}_\infty$ and $\sigma \in \mathcal{K}$.

In this section, we consider the problem of input-to-state stabilization problem of Hamiltonian system

$$\dot{x} = (J(x) - R(x))\frac{\partial H(x)}{\partial x} + G_1(x)u + G_2(x)w. \tag{19}$$

In order to use the result in Section 2, we need the following assumption.

Assumption 5 There exist a function $H_c(\xi) : \mathbb{R}^{n_c} \to \mathbb{R}$ and $\mathcal{K}_\infty$ functions $\underline{\alpha}$, $\overline{\alpha}$, α_0, such that

$$\underline{\alpha}(\|x\|) \leq \overline{H}(x) \leq \overline{\alpha}(\|x\|),$$
$$d\overline{H}(x)\nabla\overline{H}(x) \geq \alpha_0(\|x\|),$$

where $\overline{H}(x) \triangleq H(x) + H_c(c_1(x), c_2(x), \cdots, c_{n_c}(x))$.

Theorem 2 *Under Assumptions* 1 *and* 5*, if, in addition, there exist a symmetric matrix* $\Gamma(x)$*, a positive number* $\delta_0 > 0$ *and a function* $\gamma \in \mathcal{K}$ *such that*

$$R(x) + G_1(x)\Gamma(x)G_1^{\mathrm{T}}(x) - \delta_0 G_2(x)G_2^{\mathrm{T}}(x) \geq \gamma(\|x\|)I. \tag{20}$$

Then the control law

$$u(x) = -G_c^{\mathrm{T}}(x) \left.\frac{\partial H_c(\xi)}{\partial \xi}\right|_{\xi_i = c_i(x) + c_i} - (\Gamma(x) + \delta I)G_1^{\mathrm{T}}(x)\frac{\partial \overline{H}(x)}{\partial x} + v, \quad \delta \geq \delta_0 \tag{21}$$

renders the closed-loop system of (19) *input-to-state stable with respect to* v *and* w*.*

Proof According to Proposition 1, by using control law

$$u(x) = -G_c^{\mathrm{T}}(x) \left.\frac{\partial H_c(\xi)}{\partial \xi}\right|_{\xi_i = c_i(x) + c_i} + \bar{v}, \tag{22}$$

system (19) becomes

$$\dot{x} = (J(x) - R(x))\frac{\partial \overline{H}(x)}{\partial x} + G_1(x)\bar{v} + G_2(x)w. \tag{23}$$

It is easy to calculate that

$$\dot{\overline{H}}(x) = -d\overline{H}(x)R(x)\nabla\overline{H}(x) + d\overline{H}(x)G_1(x)\bar{v} + d\overline{H}(x)G_2(x)w. \tag{24}$$

Let

$$\bar{v} = -(\Gamma(x) + \delta I)G_1^{\mathrm{T}}(x)\frac{\partial \overline{H}(x)}{\partial x} + v. \tag{25}$$

Then

$$
\begin{aligned}
\dot{\overline{H}}(x) &= -d\overline{H}(x)R(x)\nabla\overline{H}(x) - d\overline{H}(x)G_1(x)(\Gamma(x)+\delta I)G_1^{\mathrm{T}}(x)\nabla\overline{H}(x) \\
&\quad + d\overline{H}(x)(\,G_1(x)\ G_2(x)\,)\begin{pmatrix} v \\ w \end{pmatrix} \\
&\le -d\overline{H}(x)\left(R(x)+G_1(x)\Gamma(x)G_1^{\mathrm{T}}(x)-\delta_0 G_2(x)G_2^{\mathrm{T}}(x)\right)\nabla\overline{H}(x) + \frac{1}{4\delta_0}(v^{\mathrm{T}}v + w^{\mathrm{T}}w) \\
&\le -\gamma(\|x\|)\alpha_0(\|x\|) + \frac{1}{4\delta_0}(v^{\mathrm{T}}v + w^{\mathrm{T}}w) \\
&:= -\alpha(\|x\|) + \sigma\left(\left\|\begin{pmatrix} v \\ w \end{pmatrix}\right\|\right), \qquad (26)
\end{aligned}
$$

where $\alpha(r) \triangleq \gamma(r)\alpha_0(r)$ and $\sigma(r) \triangleq \frac{1}{4\delta_0}r^2$. Obviously $\sigma \in \mathcal{K}$. According to Assumption 5, $\gamma \in \mathcal{K}$ and $\alpha_0 \in \mathcal{K}_\infty$, so $\alpha \in \mathcal{K}_\infty$. Thus $\overline{H}(x)$ is indeed an ISS-Lyapunov function of the closed-loop system with control (21). ∎

5 Conclusion

An extended Casimir energy method was proposed in this paper. Using this method, two control problems of Hamiltonian systems were investigated. First, the problem of suboptimal H_∞ control of port-controlled Hamiltonian systems was considered. By regarding the shaped Hamiltonian function as a candidate solution to HJI inequality, the suboptimal H_∞ control was designed. Secondly, the input-to-state stabilization problem was considered. Similar to the H_∞ problem, another shaped Hamiltonian function was constructed as a candidate ISS Lyapunov function, and the input-to-state stabilization problem was solved.

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