Stabilization of Time-Varying Hamiltonian Systems

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***Abstract*—This paper investigates the stabilization problem of time-varying port-controlled Hamiltonian (PCH) systems through energy-shaping. First, the closed-loop form of a time-varying PCH system (with certain feedback) is embedded into an extended system. Then by restricting the extended system to its invariant Casimir manifold, the energy function (Hamiltonian) of the original PCH system could be shaped as a candidate of Lyapunov function. Then the stabilization problem is considered by using the shaped Hamiltonian function. When the system has unknown parameters, the adaptive stabilization is considered, and the above stabilization result is used to construct an adaptive stabilizer. Finally, the method developed is used to power systems with periodic disturbances.**

***Index Terms*—Adaptive stabilizer, Casimir function, energy-shaping, Hamiltonian system stabilization.**

I. Introduction

In recent years, the port-controlled Hamiltonian systems have attracted more and more attention [7], [11], [12]. One of the advantages of this approach is as follows. When the (energy-shaped) Hamiltonian is used as the Lyapunov function, since it is part of the system, it can represent some essential system properties. For instance, when multiple equilibriums of a system are considered, one Hamiltonian (as Lyapunov function) can be used for all of them.

Port-controlled form plays a fundamental role in applying Hamiltonian function approach [14]. The transfer of a general nonliner system into a port-controlled Hamiltonian system is the first key issue in its applications [6].

The Hamiltonian function approach has been used for the control and stabilization of power systems [4], [5], [16], [19]. Particularly, in a recent work, the method is applied to multimachine case [15]. In practice, some disturbances of an excitation system could be periodic. Particularly, when the disturbance is coupled with rotors it is very likely to be periodic. This is a motivation for us to consider time-varying Hamiltonian systems.

A port-controlled Hamiltonian (PCH) system (with dissipation) is defined as follows[17]:

$$\begin{cases} \dot{x} = (J(x) - R(x))\dfrac{\partial H(x)}{\partial x} + G(x)u \\ y = G^T(x)\dfrac{\partial H(x)}{\partial x} \end{cases} \tag{1}$$

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where

$$M(x) := J(x) - R(x)$$

is called the structure matrix of the system, which determines a second-order tensor field on state space; $J(x)$ is a skew-symmetric matrix corresponding to a power continuous interconnection in the system; $R(x)$ is a symmetric positive semidefinite matrix corresponding to the energy dissipation of the system.

We give some concepts related to time-varying Hamiltonian systems, which will be used in the sequel.

Definition 1:

1) A function $c(x)$ is called a Casimir function of system (1) if its differential $dc(x)$, satisfies
$$dc(x)M(x) = 0. \tag{2}$$
2) In [10], a time-varying function $V(x,t)$ is said to be positive definite, denoted by $V(x,t) > 0$, if there exists a (time independent) positive definite function $W(x) > 0$, such that
$$V(x,t) \geq W(x) \qquad \forall t \geq 0.$$

Remark 1:

1) In fact, the classical definition of a Casimir function is
$$M(x) \bigtriangledown c(x) = 0. \tag{3}$$
But for classical Hamiltonian system, since the structure matrix $M(x) = J(x)$ is skew symmetric, it is the same as (2). But, in general, as the structure matrix $M(x)$ is neither symmetric nor skew symmetric, they are not the same. Therefore, we call $c(x)$ satisfying (2) [(3)], the left (right) Casimir function. A right Casimir function can be used to modify the Hamiltonian function (adding it to the Hamiltonian function will not change the system at all). A left Casimir function is constant along a trajectory of the system (with zero inputs). In this paper, we consider only the left Casimir functions.
2) In time-varying case, the structure matrix becomes $M(x,t)$. But the Casimir function we considered is still a time independent function $c(x)$ satisfying $dc(x)M(x,t) = 0$.

Many real physic systems can be depicted by this form. A method to shape the energy function via interconnection was first proposed and developed by Ortega, van der Schaft, Maschke, and Escobar [12], [17]. The main idea of this method is to interconnect the plant system (1) with a source system

$$\begin{cases} \dot{\xi} = (J_c(\xi) - R_c(\xi))\frac{\partial H_c(\xi)}{\partial \xi} + G_c(\xi)u_c \\ y_c = G_c^T(\xi)\frac{\partial H_c(\xi)}{\partial \xi} \end{cases} \tag{4}$$

where $H_c(\xi)$ is a designed Hamiltonian function for dynamic control extension. A standard feedback interconnection is $u = -y_c + e, u_c = y + e_c$, where e, e_c are reference signals. When $e = 0, e_c = 0$, the resulting system becomes

$$\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} J(x) - R(x) & -G(x)G_c^T(\xi) \\ G_c(\xi)G^T(x) & J_c(\xi) - R_c(\xi) \end{pmatrix} \begin{pmatrix} \frac{\partial H(x)}{\partial x} \\ \frac{\partial H_c(\xi)}{\partial \xi} \end{pmatrix}. \tag{5}$$

For system (5), a set of Casimir functions in the form $\xi_i - c_i(x), i = 1, 2, \ldots, n_c$, might be found to construct an invariant Casimir manifold. A necessary and sufficient condition was revealed in [12], which states that $\xi_i - c_i(x), i = 1, 2, \ldots, n_c$ are Casimir functions of system (5) if and only if

$$\begin{cases} \frac{\partial^T C(x)}{\partial x} J(x) \frac{\partial C(x)}{\partial x} = J_c(\xi) \\ R(x)\frac{\partial C(x)}{\partial x} = R_c(\xi) = 0 \\ \frac{\partial^T C(x)}{\partial x} J(x) = G_c(\xi)G^T(x) \end{cases} \tag{6}$$

where $C(x) = (c_1(x), c_2(x), \ldots, c_{n_c}(x))$. If these kind of Casimir functions exist, an invariant manifold can be defined as $\mathcal{M} = \{(x, \xi) \,|\, \xi_i = c_i(x) + d_i, i = 1, 2, \ldots, n_c\}$ where d_i are constants. Thus, the restriction of system (5) on $\mathcal{M}$ is

$$\dot{x} = (J(x) - R(x)) \frac{\partial H_a(x)}{\partial x} \tag{7}$$

where $H_a(x) = H(x) + H_c(c_1(x) + d_1, c_2(x) + d_2, \ldots, c_{n_c}(x) + d_{n_c})$ is called the shaped energy function. If $H_a(x)$ satisfies positive definite condition, then it can be regarded as a Lyapunov function and system (1) can be stabilized by the source system (4).

The Hamiltonian function approach has been used for the control and stabilization of power systems with possible constant disturbance [4], [5], [19]. In practice, some disturbances of an excitation system could be periodic. This is one motivation for us to consider time-varying Hamiltonian systems.

Our goal is to generalize the method of Casimir function approach to time-varying Hamiltonian systems. As mentioned in the above, in time-varying case, we require the Casimir functions being independent of time t. Therefore, if we formally shape the energy function by using the aforementioned method, condition (6) must be satisfied. But it is not reasonable to find a function $C(x) = (c_1(x), c_2(x), \ldots, c_{n_c}(x))$, such that the corresponding second condition of (6) holds, i.e., the following (8), called "obstacle dissipation" [13], is satisfied:

$$R(x,t) \frac{\partial C(x)}{\partial x} = 0, \qquad \text{for any} \quad t \in \mathbb{R}. \tag{8}$$

To overcome this obstacle, we have to change the way of interconnection so that under this new interconnection the Casimir functions of new composed system need not satisfy the second equation of (6). Later on, you will see that the new interconnection is a generalization of the previous interconnection.

This paper considers the stabilization of time-varying Hamiltonian systems. Casimir function approach of time-invariant Hamiltonian systems has been extended to time-varying case. When some disturbances exist, an adaptive control law is designed to stabilize the system. As a case study, the results are applied to the stabilization of excitation systems with periodic disturbances.

The paper is arranged as follows. In Section II, we propose a new way to handle the energy shaping problem for time-varying Hamiltonian systems. In Section III, we investigate the stabilization of time-varying Hamiltonian systems. Based on these results, the adaptive stabilization problem is studied in Section IV. Finally, in Section V, the previously obtained results are used to investigate the stabilization of a class of power systems with periodic disturbances. Some concluding remarks are given in Section VI.

II. Energy-Shaping Via Casimir Method

Consider a general time-varying Hamiltonian system

$$\dot{x} = M(x,t) \frac{\partial H(x,t)}{\partial x} + G(x,t)u \tag{9}$$

and its extended system

$$\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} M(x,t) & M(x,t)B(x) \\ B^T(x)M(x,t) & B^T(x)M(x,t)B(x) \end{pmatrix} \times \begin{pmatrix} \frac{\partial H_a}{\partial x} \\ \frac{\partial H_a}{\partial \xi} \end{pmatrix} \tag{10}$$

where $x \in R^n, \xi \in R^{n_c}, M(x,t)$ is an $n \times n$ matrix, which is called the structure matrix of system (9), $G(x,t)$ is an $n \times m$ matrix, $B(x)$ is an $n \times n_c$ matrix, and $H_a(x, \xi) \triangleq H(x,t) + H_c(\xi, t)$ with $H_c(\xi, t)$ a smooth function. In fact, (10) is a time-varying version of its original time-invariant form proposed in [11].

Our aim is to shape the energy function of system (9) by embedding its closed-loop form with a proper control $u = \alpha(x,t)$ into its extended system (10). First, we look for n_c Casimir functions of system (10), define an invariant manifold (multilevel set) of it, and then restrict system (10) on this invariant manifold. As you will see, for properly chosen $B(x)$ and $M(x,t)$, the restricted system is exactly the same as system (9) with proper control. First, let's prove a simple lemma, which is basically from [11], as a time-varying generalization.

Lemma 1: If there exists functions $c_1(x), \ldots, c_{n_c}(x)$, such that

$$B(x) = \frac{\partial C(x)}{\partial x}$$

where $C(x) = (c_1(x), \ldots, c_{n_c}(x))$, then $\xi_i - c_i(x), i = 1, 2, \ldots, n_c$, are Casimir functions of system (10).

Proof: A straightforward computation shows that

$$(-B^T(x) \quad I) \times \begin{pmatrix} M(x,t) & M(x,t)B(x) \\ B^T(x)M(x,t) & B^T(x)M(x,t)B(x) \end{pmatrix} \equiv 0.$$

Since $B^T(x) = (\partial^T C(x))/(\partial x)$, we have

$$\begin{pmatrix} -\frac{\partial^T C(x)}{\partial x} & I \end{pmatrix} \times \begin{pmatrix} M(x,t) & M(x,t)B(x) \\ B^T(x)M(x,t) & B^T(x)M(x,t)B(x) \end{pmatrix} = 0 \tag{11}$$

which means that for any $j \in \{1, 2, \ldots, n_c\}$

$$\begin{pmatrix} -\frac{\partial^T c_j(x)}{\partial x} & e_j \end{pmatrix} \times \begin{pmatrix} M(x,t) & M(x,t)B(x) \\ B^T(x)M(x,t) & B^T(x)M(x,t)B(x) \end{pmatrix} = 0.$$

That is, $\xi_i - c_i(x), i = 1, 2, \ldots, n_c$, are Casimir functions of system (10). □

Note that (11) shows that the Casimir functions $\xi_i - c_i(x)$ are constant along the trajectories of system (10).

Then we can define a multilevel set

$$\mathcal{M} = \{(x, \xi) \,|\, \xi_i = c_i(x) + d_i\} \subset \mathbb{R}^{n+n_c}$$

where $d_i, i = 1, 2, \ldots, n_c$, are constants depending on the initial values of system (10). Obviously, $\mathcal{M}$ is an invariant manifold of system (10). When restricting on $\mathcal{M}$, system (10) becomes

$$\begin{aligned} \dot{x} &= M(x,t)\frac{\partial}{\partial x}(H(x,t)+H_c(c_1(x)+d_1, \ldots, c_m(x)+d_m, t)) \\ &= M(x,t)\frac{\partial H_a(x,t)}{\partial x}. \end{aligned} \tag{12}$$

We need the following assumption.

A1: There exists an $n_c \times m$ matrix $G_c(x,t)$, such that

$$M(x,t)B(x) = -G(x,t)G_c^T(x,t). \tag{13}$$

Then we have the following.

Proposition 1: Under the assumption A1, the restricted system of (10) on $\mathcal{M}$ is the same as the closed-loop system of (9) with controls

$$u(x,t) = -G_c^T(x,t)\left.\frac{\partial H_c(\xi,t)}{\partial \xi}\right|_{\xi_i = c_i(x)+d_i}. \tag{14}$$

Proof: Using the control (14) to system (9), we have

$$\begin{aligned} \dot{x} &= M(x,t)\frac{\partial H(x,t)}{\partial x} - G(x,t)G_c^T(x,t)\left.\frac{\partial H_c(\xi,t)}{\partial \xi}\right|_{\xi_i=c_i(x)+d_i} \\ &= M(x,t)\frac{\partial H(x,t)}{\partial x} + M(x,t)B(x)\left.\frac{\partial H_c(\xi,t)}{\partial \xi}\right|_{\xi_i=c_i(x)+d_i} \\ &= M(x,t)\frac{\partial H(x,t)}{\partial x} + M(x,t)\frac{\partial C(x)}{\partial x}\left.\frac{\partial H_c(\xi,t)}{\partial \xi}\right|_{\xi_i=c_i(x)+d_i} \\ &= M(x,t)\frac{\partial(H(x,t)+H_c(x,t))}{\partial x} \\ &= M(x,t)\frac{\partial H_a(x,t)}{\partial x} \end{aligned}$$

where $H_c(x,t) = H_c(c_1(x)+d_1, \ldots, c_m(x)+d_m, t)$. Obviously, it coincides with the one in (12). □

Remark 2: Equation (13) holds if and only if

$$M(x,t)B(x) \in \mathrm{Span}\{G(x,t)\}. \tag{15}$$

Therefore, the key is to choose $c_1(x), \ldots, c_{n_c}(x)$, such that (15) holds.

In the following discussion, we focus on the special case that the structure matrix $M(x,t)$ takes in the form

$$M(x,t) = J(x,t) - R(x,t) \tag{16}$$

where $J(x,t)$ is skew-symmetric and $R(x,t)$ is positive semi-definite. That is, we consider only the forced Hamiltonian system with dissipation as

$$\begin{cases} \dot{x} = (J(x,t) - R(x,t))\frac{\partial H(x,t)}{\partial x} + G(x,t)u \\ y = G^T(x,t)\frac{\partial H(x,t)}{\partial x} \end{cases} \tag{17}$$

where $x \in R^n, u, y \in R^m, G(x,t) = (g_1(x,t), \ldots, g_m(x,t))$.

Remark 3: Equation (16) is equivalent to

$$M(x,t) + M^T(x,t) \le 0 \tag{18}$$

which is assumed hereafter. In fact, for any matrix $M(x,t)$ which satisfies (18), we have

$$\begin{aligned} M(x,t) &= \frac{1}{2}(M(x,t) - M^T(x,t)) + \frac{1}{2}(M(x,t) + M^T(x,t)) \\ &\triangleq J(x,t) - R(x,t) \end{aligned}$$

where

$$\begin{aligned} J(x,t) &\triangleq \frac{1}{2}(M(x,t) - M^T(x,t)) \\ R(x,t) &\triangleq -\frac{1}{2}(M(x,t) + M^T(x,t)). \end{aligned}$$

It is obvious that $J(x,t)$ is skew-symmetric and $R(x,t)$ is positive semi-definite.

If $R(x,t)B(x) = 0$, from the modeling perspective, system (10) can be regarded as the interconnection of system (17) with a source system

$$\begin{cases} \dot{\xi} = (J_c(\xi,t) - R_c(\xi,t))\frac{\partial H_c(\xi)}{\partial \xi} + \bar{G}_c(\xi,t)u_c \\ y_c = \bar{G}_c^T(\xi,t)\frac{\partial H_c(\xi)}{\partial \xi} \end{cases} \tag{19}$$

via the standard power-conserving interconnection $u = -y_c, u_c = y$, where

$$\begin{cases} \frac{\partial^T C(x)}{\partial x} J(x,t) \frac{\partial C(x)}{\partial x} = J_c(\xi,t) \\ R(x,t)\frac{\partial C(x)}{\partial x} = R_c(\xi,t) = 0 \\ \frac{\partial^T C(x)}{\partial x} J(x,t) = \bar{G}_c(\xi,t)G^T(x,t). \end{cases} \tag{20}$$

Degenerated to time-invariant case, the condition (20) coincides with the conditions in [12].

But in general, just as discussed in Section I, since the structure matrix is time-varying, while $C(x)$ is required to be independent of t, the second equation of (20) is very difficult to satisfy. In the present paper, we removed the second condition of (20), thus, the choice of Camisir functions has relatively more freedom than that in [12]. In this case, if the structure matrix $J(x,t) - R(x,t)$ is invertible, then we can introduce a new output function of system (17) as

$$\tilde{y} = G^T(x,t)(J(x,t)+R(x,t))^{-1}(J(x,t)-R(x,t))\frac{\partial H(x,t)}{\partial x} \tag{21}$$

then system (17) with control (14) can still be regarded as its interconnection with (19), where

$$\begin{cases} J_c(\xi,t) = B^T(x)J(x,t)B(x) \\ R_c(\xi,t) = B^T(x)R(x,t)B(x) \\ \bar{G}_c(\xi,t) = G_c(x,t) \end{cases}$$

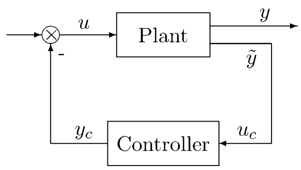

Fig. 1. Interconnection.

via interconnection constraints $u = -y_c, u_c = \tilde{y}$ (shown in Fig. 1).

Remark 4: The output defined in (21) is similar to the one defined in [9] and [3]. But they are also different. In [9] or [3], the output is proven to be a passive one, but ours is not. Our purpose here is to realize similar interconnection constraints as $u = -y_c$ and $u_c = \tilde{y}$. If the exact output in [9] and [3] is used, this interconnection will be destroyed.

III. Stabilization

In the previous section, we studied how the energy function of a time-varying Hamiltonian system could be shaped through the Casimir method. In this section, the shaped Hamiltonian function is used as a candidate Lyapunov function in order to investigate the stabilization of system (17).

Consider system (17), if we choose an output as

$$\bar{y} = G^T(x,t)\frac{\partial H_a(x,t)}{\partial x} \tag{22}$$

then we have the following result.

Theorem 1: Assume the following:

1) there exists functions $c_1(x) + d_1, \ldots, c_{n_c}(x) + d_n$ and $H_c(\xi, t) : \mathbb{R}^{n_c+1} \to \mathbb{R}$, such that

$$H_a(x,t) \triangleq H(x,t) + H_c(c_1(x) + d_1, \ldots, c_{n_c}(x) + d_n, t) \geq H_a(0,t) = 0;$$

$$\frac{\partial H}{\partial t} + \frac{\partial H_c}{\partial t}(c_1(x) + d_1, \ldots, c_{n_c}(x) + d_n, t) \leq 0;$$

2) there exists an $n_c \times m$ matrix $G_c(x,t)$, such that

$$(J(x,t) - R(x,t))\frac{\partial C(x)}{\partial x} = -G(x,t)G_c^T(x,t)$$

where $C(x) = (c_1(x) + d_1, \ldots, c_{n_c}(x) + d_n)$.

Then the control rule

$$u(x,t) = -G_c^T(x,t)\left.\frac{\partial H_c}{\partial \xi}\right|_{\xi_i = c_i(x) + d_i} + v \tag{23}$$

renders the input-output mapping $v \to \bar{y}$ passive with storage function $H_a(x,t)$.

Furthermore, if $H_a(x,t)$ is positive definite, then the closed-loop system is stable for $v = 0$.

Proof: According to the discussions in Section II, the closed-loop system of (17) with control (23) is

$$\dot{x} = (J(x,t) - R(x,t))\frac{\partial H_a(x,t)}{\partial x} + G(x,t)v. \tag{24}$$

Moreover

$$\begin{aligned}\dot{H}_a &= \frac{\partial H_a}{\partial t} - \left(\frac{\partial H_a}{\partial x}\right)^T R(x,t)\frac{\partial H_a}{\partial x} + v^T\bar{y} \\ &\leq -\left(\frac{\partial H_a}{\partial x}\right)^T R(x,t)\frac{\partial H_a}{\partial x} + v^T\bar{y}.\end{aligned} \tag{25}$$

This implies the passivity of the input-output mapping $v \to \bar{y}$.

The second statement is obviously true. □

Since the system is time-dependent, LaSalle's principle (or the Barbashin and Krasovskii's Theorem) fails in general. Therefore, the conditions of Theorem 1 are not enough to assure asymptotic stability. Some other conditions must be found to assure asymptotical stabilizing the system. First, we introduce a lemma whose proof can be found in [8].

Lemma 2: [8] Consider system (17). Assume: 1) the Hamiltonian $H(x,t)$ is positive definite and $(\partial H(x,t))/(\partial t) \leq 0$ holds for all x and t and 2) the system is zero-state detectable, then the feedback

$$u = -y \tag{26}$$

renders the system asymptotically stable.

Suppose, moreover, that $H(x,t)$ is decreasing and that the system is periodic, then feedback (26) renders the system uniformly asymptotically stable.

Theorem 2: Under the conditions of Theorem 1, assume: 1) $H_a(x,t)$ is positive definite and 2) system

$$\begin{cases}\dot{x} = (J(x,t) - R(x,t))\frac{\partial H_a(x,t)}{\partial x} + G(x,t)v \\ \bar{y} = G^T(x,t)\frac{\partial H_a(x,t)}{\partial x}\end{cases} \tag{27}$$

is zero-state detectable, then the control law

$$u(x,t) = -\left[G_c^T(x,t)\left.\frac{\partial H_c}{\partial \xi}\right|_{\xi_i = c_i(x)} + G^T(x,t)\frac{\partial H_a}{\partial x}\right] \tag{28}$$

renders system (17) asymptotically stable.

Furthermore, if $H_a(x,t)$ is decreasing and the system (27) is periodic, then feedback (28) renders the system (17) uniformly asymptotically stable.

Proof: According to Theorem 1, the control rule (23) assures system (27) as being passive with storage function $H_a(x,t)$. Since $H_a(x,t)$ is positive definite and system (27) is zero-state detectable, according to Lemma 2, the control

$$v = -\bar{y} = -G^T(x,t)\frac{\partial H_a(x,t)}{\partial x} \tag{29}$$

asymptotically stabilizes system (27). That is, the control (28) asymptotically stabilizes system (17). □

Theorem 1 can be used to time-invariant case. Consider time-invariant forced Hamiltonian system with dissipation:

$$\begin{cases}\dot{x} = (J(x) - R(x))\frac{\partial H(x)}{\partial x} + G(x)u \\ y = G^T(x)\frac{\partial H(x)}{\partial x}.\end{cases} \tag{30}$$

Using Theorem 1 and LaSalle's invariance principle, it is easy to prove the following.

Theorem 3: For system (30), assume:

1) there exists an $n_c \times m$ matrix $G_c(x)$ and functions $C(x) = (c_1(x), \ldots, c_{n_c}(x))$, such that

$$(J(x) - R(x))\frac{\partial C(x)}{\partial x} = -G(x)G_c^T(x);$$

2) there exists a function $H_c : R^{n_c} \to R$, such that $H_a(x) \triangleq H(x) + H_c(c_1(x), \ldots, c_{n_c}(x))$ has strict minimum at x_0; then the control

$$u(x) = -G_c^T(x) \left. \frac{\partial H_c(\xi)}{\partial \xi} \right|_{\xi_i = c_i(x)} \tag{31}$$

stabilizes system (30).
Moreover, if there is no non-zero trajectory of the closed-loop system contained in the set

$$\{x \,|\, R(x) \frac{\partial H_a(x)}{\partial x} = 0\}$$

then the control (31) asymptotically stabilizes system (30).

Remark 5: Theorem 3 is a known result (an equivalent statement can be found in [17]). In fact, Theorem 3 is the classical control by interconnection methodology. It can be seen as the interconnection of system (30) with an integrator $\dot{\xi} = G_c(\xi)u_c, y_c = G_c^T(\xi)(\partial H_c)/(\partial \xi)$ via the standard feedback interconnection $u = -y_c, u_c = y$, which yields the embedded system

$$\dot{x} = (J(x) - R(x)) \frac{\partial H_a(x)}{\partial x}.$$

Also, in the control by interconnection method, the energy function is free, chosen by the designer to assign the desired equilibrium point.[1]

IV. Adaptive Control

In this section, we consider a time-varying Hamiltonian system with uncertain parameters

$$\begin{cases} \dot{x} = (J(x,t) - R(x,t)) \frac{\partial H(x,\theta,t)}{\partial x} + G(x,t)u \\ y = G^T(x,t) \frac{\partial H(x,\theta,t)}{\partial x} \end{cases} \tag{32}$$

where $\theta = (\theta_1, \ldots, \theta_p)^T$ are uncertain parameters. When J, R, and G are constant matrices and the Hamiltonian $H(x,\theta)$ is positive and linearly depends on the uncertain parameters θ, it is investigated in [19]. In the present paper, we still suppose that the unknown parameters in the Hamiltonian function are linear, i.e.

A2: $H(x,\theta,t) = L_0(x,t) + \sum_{i=1}^{p} L_i(x,t)\theta_i = L_0(x,t) + L(x,t)\theta$, where $L_0(x,t)$, $L_i(x,t)$, $i = 1,2,\ldots,p$, are smooth functions and $L(x,t) = (L_1(x,t), \ldots, L_p(x,t))$. But we do not require that $H(x,\theta,t)$ is positive definite. We construct an adaptive control law for system (32) through the following two steps.

Step 1) Suppose all the parameters θ_i in the system are known, apply the method proposed in the previous section to shape the energy function and design a feedback control law

$$u = \alpha(x,\theta,t)$$

which can stabilize system (32).

Step 2) Construct an update law and substitute the unknown parameters θ in the feedback by its estimate $\hat{\theta}$, i.e., design a control law

$$\begin{cases} \dot{\hat{\theta}} = \beta(x,\hat{\theta},t) \\ u = \alpha(x,\hat{\theta},t). \end{cases}$$

Step 1: Suppose all the parameters in the system are known and assume the following.

A3: There exists a vector function $C(x) = (c_1(x), \ldots, c_{n_c}(x))$, a matrix $G_c(x,t)$, and functions $L_0^c(\xi,t), L_i^c(\xi,t), i = 1,2,\ldots,p$, such that

$$(J(x,t) - R(x,t)) \frac{\partial C(x)}{\partial x} = -G(x,t)G_c^T(x,t) \tag{33}$$

and

$$\begin{aligned} \bar{H}(x,\theta,t) &\triangleq H(x,\theta,t) + H_c(C(x),\theta,t) \\ &= (L_0(x,t) + L_0^c(C(x),t)) + (L(x,t) + L_c(C(x),t)) \end{aligned}$$

is positive definite with respect to x and $(\partial \bar{H})/(\partial t) \leq 0$, where

$$H_c(\xi,\theta,t) = L_0^c(\xi,t) + L_c(\xi,t)\theta$$

and $L_c(\xi,t) = (L_1^c(\xi,t), \ldots, L_p^c(\xi,t)), \xi \in \mathbb{R}^m$.

According to Theorem 3, the control law

$$u = -G_c^T(x,t) \left. \frac{\partial H_c(\xi,\theta,t)}{\partial \xi} \right|_{\xi = C(x)} \tag{34}$$

stabilizes system (32).

Step 2: Replace the unknown parameter θ in (34) by its estimate $\hat{\theta}$, i.e., consider adaptive controller

$$\begin{cases} u = -G_c^T(x,t) \left. \frac{\partial H_c(\xi,\hat{\theta},t)}{\partial \xi} \right|_{\xi = C(x)} \\ \dot{\hat{\theta}} = \beta(x,\hat{\theta},t) \end{cases} \tag{35}$$

where $\beta(x,\hat{\theta},t)$ is to be designed. Substituting (35) into system (32), we obtain

$$\begin{aligned} \dot{x} &= (J(x,t) - R(x,t)) \frac{\partial H(x,\theta,t)}{\partial x} \\ &\quad - G(x,t)G_c^T(x,t) \left. \frac{\partial H_c(\xi,\hat{\theta},t)}{\partial \xi} \right|_{\xi = C(x)} \\ &= (J(x,t) - R(x,t)) \frac{\partial \bar{H}(x,\theta)}{\partial x} \\ &\quad - G(x,t)G_c^T(x,t) \left[\left. \frac{\partial H_c(\xi,\hat{\theta},t)}{\partial \xi} \right|_{\xi = C(x)} - \left. \frac{\partial H_c(\xi,\theta,t)}{\partial \xi} \right|_{\xi = C(x)} \right] \\ &= (J(x,t) - R(x,t)) \frac{\partial \bar{H}(x,\theta)}{\partial x} \\ &\quad - G(x,t)G_c^T(x,t) \left. \left(\frac{\partial L_c(\xi,t)}{\partial \xi} \right) \right|_{\xi = C(x)} (\hat{\theta} - \theta). \end{aligned} \tag{36}$$

[1]This novel remark is pointed by an anonymous referee.

Take

$$V(x,\hat{\theta},t)=\bar{H}(x,\theta,t)+\frac{1}{2}(\hat{\theta}-\theta)^T\Gamma(\hat{\theta}-\theta) \tag{37}$$

as a candidate Lyapunov function, where Γ is a positive definite matrix, then the derivative of $V(x,\hat{\theta},t)$ along system (32)–(35) is

$$\begin{aligned}\dot{V}=&\frac{\partial\bar{H}(x,\theta,t)}{\partial t}-\left(\frac{\partial\bar{H}(x,\theta,t)}{\partial x}\right)^T R(x,t)\frac{\partial\bar{H}(x,\theta,t)}{\partial x}\\&-\left(\frac{\partial\bar{H}(x,\theta,t)}{\partial x}\right)^T G(x,t)G_c^T(x,t)\\&\times\left(\frac{\partial L_c(\xi,t)}{\partial\xi}\right)\bigg|_{\xi=C(x)}(\hat{\theta}-\theta)+(\hat{\theta}-\theta)^T\Gamma\beta(x,\hat{\theta},t)\\=&\frac{\partial\bar{H}(x,\theta,t)}{\partial t}-\left(\frac{\partial\bar{H}(x,\theta,t)}{\partial x}\right)^T R(x,t)\frac{\partial\bar{H}(x,\theta,t)}{\partial x}\\&-(\hat{\theta}-\theta)^T\left[\left(\frac{\partial L_c(\xi,t)}{\partial\xi}\right)^T\bigg|_{\xi=C(x)}G_c(x,t)G^T(x,t)\right.\\&\left.\times\frac{\partial\bar{H}(x,\theta,t)}{\partial x}-\Gamma\beta(x,\hat{\theta},t)\right].\end{aligned} \tag{38}$$

If we choose

$$\beta(x,\hat{\theta},t)=\Gamma^{-1}\left(\frac{\partial L_c(\xi,t)}{\partial\xi}\right)^T\bigg|_{\xi=C(x)}\times G_c(x,t)G^T(x,t)\frac{\partial\bar{H}(x,\hat{\theta},t)}{\partial x}$$

then

$$\begin{aligned}\dot{V}=&\frac{\partial\bar{H}(x,\theta,t)}{\partial t}-\left(\frac{\partial\bar{H}(x,\theta,t)}{\partial x}\right)^T R(x,t)\frac{\partial\bar{H}(x,\theta,t)}{\partial x}\\&+(\hat{\theta}-\theta)^T\left[\left(\frac{\partial L_c(\xi,t)}{\partial\xi}\right)^T\bigg|_{\xi=C(x)}G_c(x,t)\right.\\&\left.\times G^T(x,t)\left(\frac{\partial\bar{L}(x,t)}{\partial x}\right)\right](\hat{\theta}-\theta)\\=&\frac{\partial\bar{H}(x,\theta,t)}{\partial t}-\left(\frac{\partial\bar{H}(x,\theta,t)}{\partial x}\right)^T R(x,t)\frac{\partial\bar{H}(x,\theta,t)}{\partial x}\\&+(\hat{\theta}-\theta)^T Q(x,t)(\hat{\theta}-\theta)\end{aligned} \tag{39}$$

where $\bar{L}(x,t)=L(x,t)+L_c(C(x),t)$ and

$$Q(x,t)=\left(\frac{\partial L_c(\xi,t)}{\partial\xi}\right)^T\bigg|_{\xi=C(x)}\times G_c(x,t)G^T(x,t)\left(\frac{\partial\bar{L}(x,t)}{\partial x}\right).$$

If we assume the following.

A4: $Q(x,t)+Q^T(x,t)\leq 0$, then $\dot{V}(x,\hat{\theta},t)\leq 0$, which implies that system (32)–(35) is stable.

According to the above analysis, we conclude the following.

Theorem 4: Under assumptions A2–A4, system (32) with uncertain parameters can be stabilized by adaptive controller

$$\begin{cases}u=-G_c^T(x,t)\left.\frac{\partial H_c(\xi,\hat{\theta},t)}{\partial\xi}\right|_{\xi=C(x)}\\ \dot{\hat{\theta}}=\Gamma^{-1}\left(\frac{\partial L_c(\xi,t)}{\partial\xi}\right)^T\bigg|_{\xi=C(x)}G_c(x,t)G^T(x,t)\frac{\partial\bar{H}(x,\hat{\theta})}{\partial x}\end{cases} \tag{40}$$

where Γ is any positive-definite matrix.

Remark 6: In fact, the closed-loop system (32)–(39) can be expressed as the following Hamiltonian system: (we take $\Gamma=I_p$ for simplicity)

$$\begin{pmatrix}\dot{x}\\ \dot{\hat{\theta}}\end{pmatrix}=\begin{pmatrix}J(x,t)-R(x,t) & -P(x,t)\\ P^T(x,t) & Q(x,t)\end{pmatrix}\times\begin{pmatrix}\frac{\partial V(x,\hat{\theta},t)}{\partial x}\\ \frac{\partial V(x,\hat{\theta},t)}{\partial\hat{\theta}}\end{pmatrix} \tag{41}$$

where $V(x,\hat{\theta},t)$ is as in (37) and

$$P(x,t)=G(x,t)G_c^T(x,t)\left(\frac{\partial L_c(\xi,t)}{\partial\xi}\right)\bigg|_{\xi=C(x)}.$$

Therefore, condition *A4* can be replaced by the assumption that system (41) is dissipative, i.e.,

$$\begin{pmatrix}R(x,t) & 0\\ 0 & -\frac{1}{2}[Q(x,t)+Q^T(x,t)]\end{pmatrix}\geq 0.$$

Remark 7: We need some other conditions to assume the asymptotical stability of the closed-loop system, such as generalizations of Barbashin–Krasovski–LaSalle Theorem [2]. In the following, we give a result of asymptotical stability for periodical systems. The result will be applied in Section V.

Consider a continuous time-varying nonlinear periodic system

$$\dot{x}=f(x,t)$$

where

$$f(x,t+\tau)=f(x,t)\qquad\forall x,t. \tag{42}$$

Assume $f(x,t)$ is a complete vector field, so for each (x_0,t_0), the unique solution exists for all $t>t_0$. Moreover, there exists a positive definite function $V(x)$ with $\dot{V}(x,t)=\nabla V(x)\cdot f(x,t)\leq 0$, then the origin is asymptotically stable if $\dot{V}(x,t)$ is not identically zero for all nontrivial solutions of system (42) [1]. Using this result, we have the following.

Theorem 5: Suppose conditions *A2, A3,* and *A4* hold, and it follows:

1) $J(x,t), R(x,t), G(x,t)$, and $H_c(\xi,t)$ are all periodic in t with the same period T;
2) $\bar{H}(x,\theta,t)=\bar{H}(x,\theta)$, which is independent of t;
3) $\dot{V}(x,\hat{\theta},t)$ is not identically zero for any solution of system (32)–(35) other than $(0,\theta)$, where $V(x,\hat{\theta})$ is defined in (37).

Then system (32) can be asymptotically stabilized by controller (40). □

Next, we consider a general Hamiltonian system

$$\dot{x}=(J(x,\theta,t)-R(x,\theta,t))\frac{\partial H(x,\theta,t)}{\partial x}+G(x,\theta,t)u. \tag{43}$$

In the above system, $\theta \in R^p$ is the unknown constant parameter. We assume that the following.

A3: There exists a function $C(x,\theta) = (c_1(x,\theta),\ldots,c_m(x,\theta))$, a matrix $G_c(x,\theta)$, and a function $H_c(\xi,\theta,t), \xi \in R^m$ such that

$$(J(x,\theta,t) - R(x,\theta,t))\frac{\partial C(x,\theta)}{\partial x} = -G(x,\theta,t)G_c^T(x,\theta,t) \tag{44}$$

and

$$\bar{H}(x,t) = H(x,\theta,t) + H_c(C(x,\theta),\theta,t)$$

is positive definite with respect to x and $(\partial\bar{H})/(\partial t) \le 0$.

A8: There exists $L_0^c(x,t)$, $L_0(x,t) \in R^m, m\times r$ matrices $L_c(x,t)$, $L(x,t)$, and a constant vector $p \in R^r$ such that

$$G_c^T(x,\theta,t)\left.\frac{\partial H_c(\xi,\theta,t)}{\partial \xi}\right|_{\xi=c(x,\theta)} = L_0^c(x,t) + L_c(x,t)p. \tag{45}$$

and

$$G^T(x,\theta,t)\frac{\partial \bar{H}(x,\theta,t)}{\partial x} = L_0(x,t) + L(x,t)p. \tag{46}$$

A4': $L_c^T(x,t)L(x,t) + L^T(x,t)L_c(x,t) \le 0$.

Then we have the following result.

Theorem 6: Under assumptions *A3', A8,* and *A4',* system (43) can be stabilized by the following control law:

$$\begin{cases} u = -L_0^c(x,t) - L_c(x,t)\hat{p} \\ \dot{\hat{p}} = L_c(x,t)L_0(x,t) + L_c(x,t)L(x,t)\hat{p}. \end{cases} \tag{47}$$

Proof: It is easy to verify the closed-loop of system (43) with control (47) as

$$\begin{pmatrix}\dot{x}\\ \dot{\hat{p}}\end{pmatrix} = \left\{\begin{pmatrix} J & -GL_c \\ L_c^TG^T & -\frac{1}{2}\left(L_c^TL - L^TL_c\right)\end{pmatrix} - \begin{pmatrix} R & 0 \\ 0 & -\frac{1}{2}\left(L_c^TL + L^TL_c\right)\end{pmatrix}\right\}\begin{pmatrix}\frac{\partial V}{\partial x}\\ \frac{\partial V}{\partial \hat{p}}\end{pmatrix} \tag{48}$$

where $V(x,\hat{p},t) = \bar{H}(x,\theta,t) + (1/2)(\hat{p}-p)^T(\hat{p}-p)$. According to assumptions A3' and A4', system (48) is a dissipative Hamiltonian system with positive definite Hamiltonian function, so the equilibrium $(0,p)$ is stable. □

As a summary, this section contains three main results: Theorems 4, 5, and 6. Theorem 5 is a result about the asymptotical stability. It will be used in Section V for the stabilization of power systems with periodic disturbances. Theorems 4 and 6 are results of stability. The following numerical example is an application of Theorem 6.

Example 1: Consider time-varying system

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \left\{\begin{pmatrix} 0 & x_2 \\ -x_2 & 0\end{pmatrix} - \begin{pmatrix} 1-\sin t & 0 \\ 0 & \theta\end{pmatrix}\right\}\begin{pmatrix}\frac{\partial H}{\partial x_1}\\ \frac{\partial H}{\partial x_2}\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix}u \tag{49}$$

where $H = (1/2)x_1^2(1-\sin x_2) + (1/2)x_2^2 + (1/2\theta)x_2^2(2x_1 - 1) + x_1$ and $\theta > 0$ is an uncertain parameter. We choose $c(x_1,x_2) = (1/2)x_2^2 - \theta x_1$ and $G_c(x_1,x_2,t) = -\theta(1-\sin t) - x_2^2$, then we have

$$(J(x,t) - R(x,t))\frac{\partial c(x)}{\partial x} = -GG_c^T(x,t).$$

If H_c is selected as $H_c(\xi) = (1/\theta)\xi$, then

$$\bar{H} \triangleq H(x,\theta) + H_c(c(x),\theta) = \frac{1}{2}x_1^2(1-\sin x_2) + \frac{1}{2}x_2^2 + \frac{1}{\theta}x_1x_2^2$$

which is locally positive definite. Furthermore

$$G_c^T(x,t)\left.\frac{\partial H_c(\xi)}{\xi}\right|_{\xi=c(x)} = L_0^c(x,t) + L_c(x,t)p$$

$$G^T\frac{\partial \bar{H}(x)}{\partial x} = L_0(x,t) + L(x,t)p$$

where $L_0^c(x,t) = -(1-\sin t), L_c(x,t) = -x_2^2, L_0(x,t) = x_1(1-\sin x_2), L(x,t) = x_2^2, p = (1/\theta)$. It is obvious that

$$Q(x) = \frac{1}{2}(L_c^TL + L^TL_c) = -x_2^4$$

is negative semi-definite. Thus, we obtain an adaptive stabilizer

$$\begin{cases} u = 1 - \sin t + x_2^2\hat{p} \\ \dot{\hat{p}} = -x_2^2\left[x_1(1-\sin x_2) + x_2^2\hat{p}\right]. \end{cases} \tag{50}$$

According to Theorem 6, the Lyapunov function for the closed-loop system is

$$V = \bar{H}(x_1,x_2) + \frac{1}{2}(\hat{p}-p)^2.$$

Then the derivative of V is

$$\begin{aligned}\dot{V} = &-(1-\sin t)\left[x_1(1-\sin x_2) + \frac{1}{\theta}x_2^2\right]^2 \\ &- \theta\left[-\frac{1}{2}x_1^2\cos x_2 + x_2 + \frac{1}{2\theta}x_1x_2\right]^2 - x_2^4(\hat{p}-p)^2 \\ \le\, & 0\end{aligned} \tag{51}$$

thus, $\lim_{t\to\infty} V(t)$ exists since $V(t)$ is decreasing and bounded from below. The stability of the closed-loop system implies that any trajectory $(x_1(t),x_2(t),\hat{p}(t))$ of the closed-loop system is bounded on R_+, and so is $(\dot{x}_1(t),\dot{x}_2(t),\dot{\hat{p}}(t))$ since the system under consideration is periodic in t. Thus, by (51) it is easy to check $\ddot{V}$ is also bounded on R_+, so $\dot{V}$ is uniformly continuous on R_+. By Barbalat's lemma, we have $\dot{V} \to 0$ as $t \to \infty$. This implies that $x_1 \to 0$ and $x_2 \to 0$ as $t \to +\infty$ by (51). Fig. 2 is a simulation for $x_1(0) = 3, x_2(0) = -4, \hat{p}(0) = 3$, and $\theta = 2$. It shows that the controller is efficient.

V. Application to Power Systems

In excitation systems, it happens that the disturbances could be periodic. Consider an excitation system with periodic disturbances w_1 and w_2

$$\begin{cases} \dot{\delta} = \omega - \omega_0 \\ \dot{\omega} = \frac{\omega_0}{M}P_m - \frac{D}{M}(\omega-\omega_0) - \frac{\omega_0 E_q' V_s}{M x_{d\Sigma}'}\sin\delta + w_1 \\ \dot{E}_q' = -\frac{1}{T_d'}E_q' + \frac{1}{T_{d_0}}\frac{x_d - x_d'}{x_{d\Sigma}'}V_s\cos\delta + \frac{1}{T_{d_0}}V_f + w_2 \end{cases} \tag{52}$$

where $w_1 = \eta_1(\omega-\omega_0)\sin t, w_2 = \eta_2\sin t$. The physical meanings of the variables with their units are listed in Table I (where "pu" means "per unit" for normalized parameters).

Denote

$$\begin{cases} x_1 = \delta \\ x_2 = \omega - \omega_0 \\ x_3 = E_q' \end{cases}$$

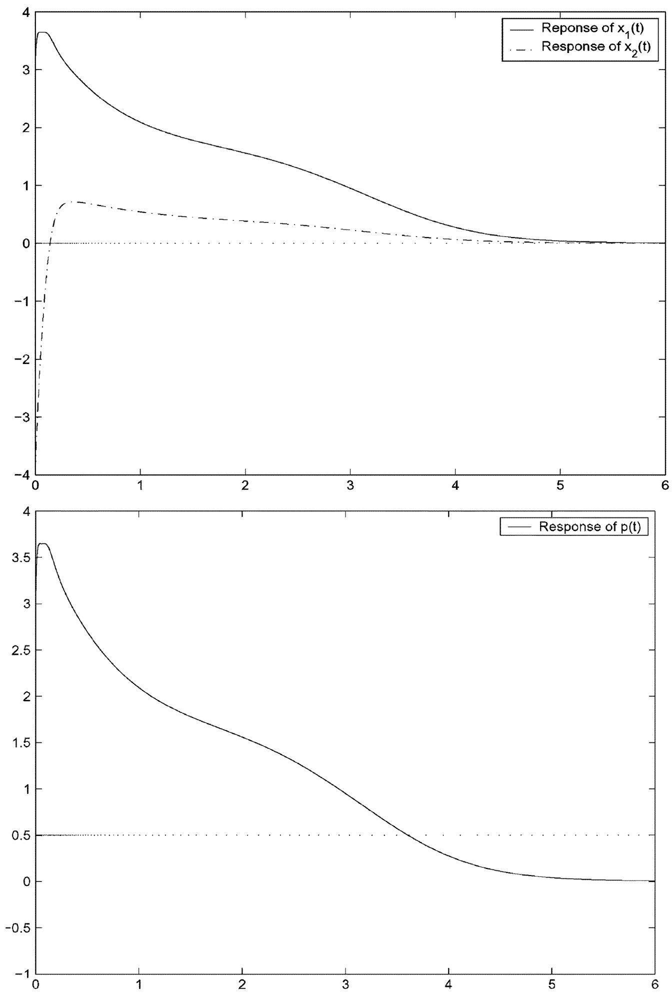

Fig. 2. Simulation for Example 1.

TABLE I
PARAMETER ILLUSTRATION

Parameter	Physical Meaning	Unit
δ	the rotor angle	rad
ω	the rotor speed	rad/s
E'_q	the internal transient voltage	pu
P_m	the mechanical power	pu
M	the inertia coefficient of the generator	s
D	the damping constant	pu
$P_e = E_q^0 V_s / x_{d\Sigma}^0 \sin\delta$	the active electrical power	pu
T'_d	the stator closed loop time constant	s
T_{d0}	the excitation circuit time constant	s
x_d	the d-axis synchronous reactance of the generator	pu
x'_d	the d-axis transient reactance	pu
V_f	the voltage of the field circuit of the generator	pu

and $a = (\omega_0/M)P_m, b = (D/M), c = (\omega_0 V_s)/(M x'_{d\Sigma}), d = (1)/(T'_d), e = (1)/(T_{d_0})((x_d - x'_d)V_s)/(x'_{d\Sigma}), \lambda = (c/e)$, and regard $u = V_f$ as control. We assume d and η_1, η_2 are unknown parameters and $|\eta_1| < b$, then the system can be written as the following Hamiltonian system:

$$\dot{x} = (J - R)\frac{\partial H}{\partial x} + Gu \tag{53}$$

where

$$J = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & b - \eta_1 \sin t & 0 \\ 0 & 0 & \frac{1}{\lambda} \end{pmatrix}, \quad G = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$H(x,t) = L_0(x,t) + L_1(x,t)d + L_2(x,t)\eta_2$$

$$L_0(x,t) = -cx_3 \cos x_1 - ax_1 + \frac{1}{2}x_2^2$$

$$L_1(x,t) = \frac{1}{2}\lambda x_3^2, \quad L_2(x,t) = -\lambda x_3 \sin t.$$

We take $C(x) = x_3$ and $G_c(x,t) = (1/\lambda)$, then condition (33) is satisfied.

Suppose $\bar{x} = (\bar{x}_1, \bar{x}_2, \bar{x}_3)^T$ is the equilibrium to be stabilized and $h(x_3) = L_0^c(x_3,t) + L_1^c(x_3,t)d + L_2^c(x_3,t)\eta_2$ is the function such that $\bar{H}(x,t) = H(x,t) + h(x_3,t)$ has a minimum at $\bar{x}$, then $\nabla \bar{H}\,|_{\bar{x}} = 0$ and $\mathrm{Hess}\{\bar{H}\}\,|_{\bar{x}} > 0$, i.e., the following conditions must be satisfied:

$$\begin{cases} c\bar{x}_3 \sin \bar{x}_1 - a = 0 \\ \bar{x}_2 = 0 \\ \frac{dh}{dx_3}\Big|_{\bar{x}_3} = c\cos \bar{x}_1 - \lambda d \bar{x}_3 + \lambda \eta_2 \sin t \end{cases} \tag{54}$$

$$\begin{cases} c\bar{x}_3 \cos \bar{x}_1 > 0 \\ \frac{d^2 h}{dx_3^2}\Big|_{\bar{x}_3} > \frac{c \sin^2 \bar{x}_1}{\bar{x}_3 \cos \bar{x}_1} - \lambda d. \end{cases} \tag{55}$$

Thus, we choose

$$\begin{cases} L_0^c(x_3,t) = \frac{1}{2}px_3^2 + (c\cos\bar{x}_1 - p\bar{x}_3)x_3 \\ L_1^c(x_3,t) = -\frac{1}{2}\lambda x_3^2 \\ L_2^c(x_3,t) = \lambda x_3 \sin t \end{cases}$$

where p is a parameter satisfying $p > (c\sin^2 \bar{x}_1)/(\bar{x}_3 \cos \bar{x}_1)$. Then, we have

$$\bar{H}(x,\theta) = -cx_3 \cos x_1 - ax_1 + \frac{1}{2}x_2^2 + \frac{1}{2}px_3^2 + (c\cos\bar{x}_1 - p\bar{x}_3)x_3$$

which is independent of t. It is easy to calculate that $Q(x,t) = 0$ and

$$\begin{pmatrix} \dot{\hat{d}} \\ \dot{\hat{\eta}}_2 \end{pmatrix} = \beta = \begin{pmatrix} -x_3(-c\cos x_1 + px_3 + c\cos\bar{x}_1 - p\bar{x}_3) \\ \sin t(-c\cos x_1 + px_3 + c\cos\bar{x}_1 - p\bar{x}_3) \end{pmatrix}$$

$$u = -\frac{1}{\lambda}(px_3 + c\cos\bar{x}_1 - p\bar{x}_3 - \lambda\hat{d}x_3 + \lambda\hat{\eta}_2 \sin t).$$

We can show that $(\bar{x}, d, \eta_2)$ is asymptotically stable by using Theorem 5. In fact, we only need to check condition 3). Suppose there is a solution $(\tilde{x}, \tilde{d}, \tilde{\eta}_2)$, such that $\dot{V}(\tilde{x}, \tilde{d}, \tilde{\eta}_2) = 0$, then we have

$$\begin{cases} \tilde{x}_2 = 0 \\ -c\cos\tilde{x}_1 + p\tilde{x}_3 + c\cos\bar{x}_1 - p\bar{x}_3 = 0 \\ c\tilde{x}_3 \sin\tilde{x}_1 - a = 0. \end{cases}$$

It is easy to check that in a neighborhood of $\bar{x}$, the only solution to the above equations is $\bar{x}$. So $\dot{\tilde{x}}_3 = 0$, this implies that

$$(\tilde{d} - d)\bar{x}_3 - (\tilde{\eta}_2 - \eta_2)\sin(t) = 0$$

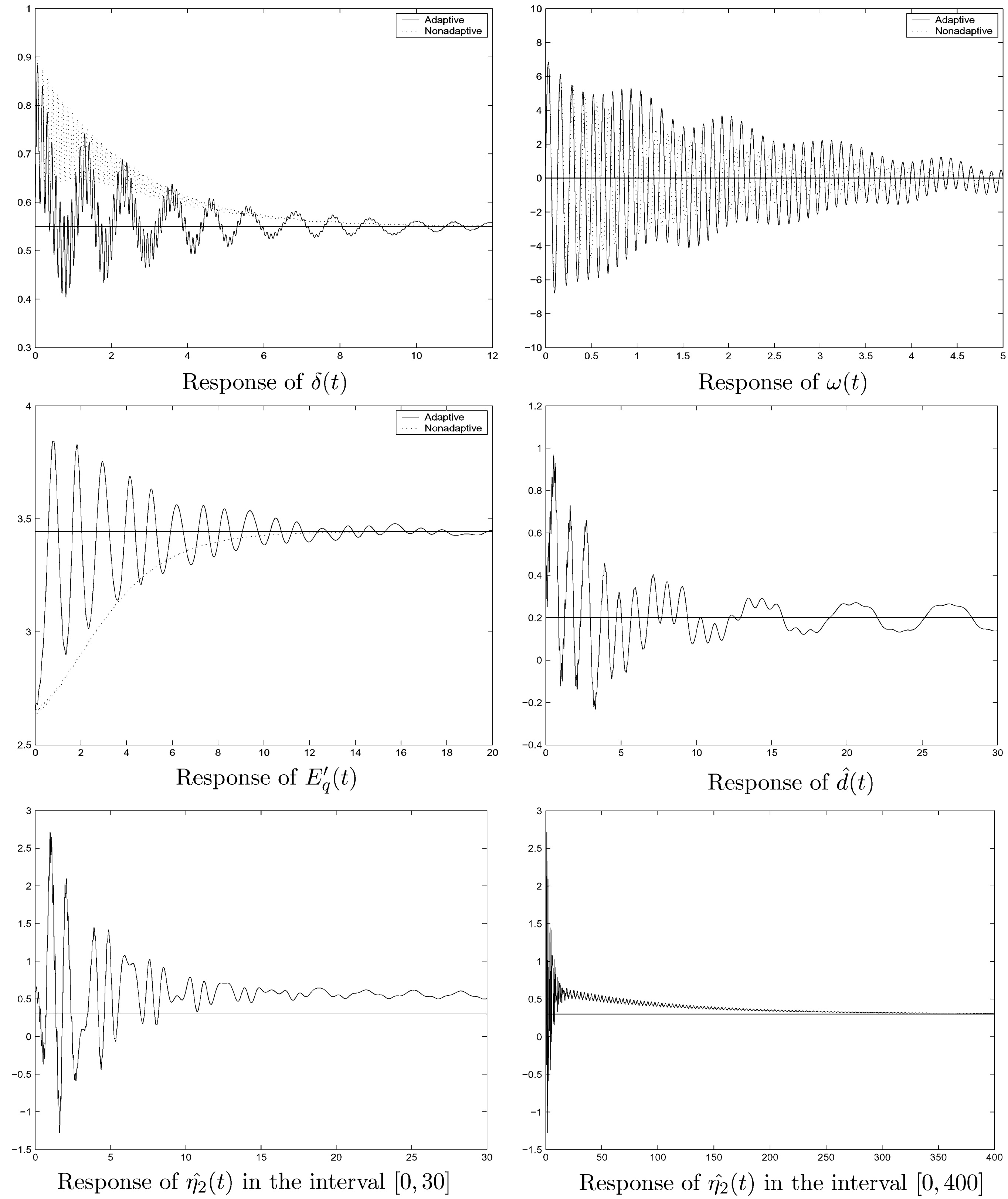

Fig. 3. Simulation for power system.

for any t. Thus, we have $\tilde{d} = d$ and $\tilde{\eta}_2 = \eta_2$.

Fig. 3 shows the simulation results for $D = 3, M = 7.6, \omega_0 = 50\pi, V_s = 20, x'_{d\Sigma} = 0.36, P_m = 100, T'_d = 5, T_d = 5, x_d = 0.9, x'_d = 0.36$ (Z. Xi, D. Cheng, 2000 [18]). We choose $\bar{\delta} = \bar{x}_1 = 0.55, \bar{\omega} - \omega_0 = \bar{x}_2 = 0, \bar{E}'_q = \bar{x}_3 = 3.4437, p = 200, \eta_1 = (1/3), \eta_2 = 0.3$, and initial values $x_1(0) = 0.6, x_2(0) = 0.5, x_3(0) = 2.6437, \hat{d}(0) = 0.3, \hat{\eta}_2(0) = 0.6$.

The above simulation results show the asymptotical stability of the overall system. From the simulation one can also see an unhappy phenomenon: chattering. We found that it is mainly caused by the system's structure. Particularly, as the damping coefficient D increases, the chattering phenomenon is decreased.

Remark 8: From the case study in this section, we may summarize the following.

The method proposed in this paper can be used to treat time-varying Hamintonian systems. When certain uncertainties exist, the proposed adaptive controller can be designed to deal with the uncertainties and reach (asymptotical) stability of the closed-loop systems. The design method is simple and efficient.

There are some drawbacks of this approach. First, stability is relatively easier to be obtained. But for asymptotical stability, so far we can only treat periodic disturbances. Second, it seems that using this design technique, the chattering phenomenon can hardly be eliminated.

VI. Conclusion

In this paper, the Casimir method of Hamiltonian systems has been generalized to time-varying case. In using it, the stabilization and adaptive stabilization problems of time-varying Hamiltonian systems were investigated. The results were applied to single machine power systems with periodic disturbances, and the simulations showed the efficiency of the designed adaptive controller.

Certain problems remain for further study. One is the asymptotical stability with nonperiodic disturbances. The other one is to design a control to reduce the chattering phenomenon in stabilization of power systems.

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