with the smooth function $\nu_{k-1}(\cdot)$ determined later renders

$$\begin{aligned}\dot{V}_k \leq\ & z_k z_{k+1} - \frac{2n-k+1}{2}\sum_{i=1}^{k} z_i^2 \\ & - (n-k)q\sum_{i=1}^{k} z_i^2\hat{\rho} + \frac{k}{2}\|z\|^2 \\ & + \sum_{i=2}^{k} z_i \left\{\nu_{i-1} - \frac{\partial\alpha_{i-1}}{\partial\hat{\Theta}}\dot{\hat{\Theta}}\right\} \\ & + (\Theta - \hat{\Theta})^T\Gamma^{-1}(\beta_k - \dot{\hat{\Theta}}) \end{aligned} \tag{30}$$

whenever the Razumikhin condition holds, and

$$\beta_k(\tilde{x}_k, \hat{\Theta}) = \beta_{k-1} + \Gamma^{-1}\Psi_{k1}^T(\tilde{x}_k, \hat{\Theta})z_k + \Gamma^{-1}\Psi_{k2}^T(\tilde{x}_k) \tag{31}$$

where

$$\Psi_{k1} = \left[\left(\varphi_k - \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_i}\varphi_i\right) \quad qz_k(1+E_k)\right]$$
$$\Psi_{k2} = \left[0 \quad q\sum_{i=1}^{k-1} z_i^2\right].$$

The aforementioned inductive argument shows that (28) holds for $k = n$. Hence, at the last step, the Lyapunov–Razumikhin function is constructed as

$$V(z, \hat{\Theta}) = V_{n-1}(\tilde{z}_{n-1}, \hat{\Theta}) + \frac{1}{2}z_n^2 \tag{32}$$

then, choosing the control law u and $\dot{\hat{\Theta}}$ as (19) renders the time derivative of V along the trajectories of (18) to satisfy

$$V(z, \hat{\Theta}) \leq -\frac{1}{2}\|z\|^2 + \sum_{i=2}^{n} z_i \left\{\nu_{i-1}(\tilde{x}_i, \hat{\Theta}) - \frac{\partial\alpha_{i-1}}{\partial\hat{\Theta}}\dot{\hat{\Theta}}\right\} \tag{33}$$

whenever the Razumikhin condition $\|\tilde{z}_t\| < q\|\tilde{z}\|$ holds.

Now, we determine the smooth functions $\nu_i(\cdot)$ $(i = 1, 2, \ldots, n-1)$. By using the determination of $\dot{\hat{\Theta}}$ in (19) and the recursion of β_n, we choose $\nu_i(\cdot)$ as

$$\begin{aligned}\nu_i =\ & \frac{\partial\alpha_i}{\partial\hat{\Theta}}\beta_{i+1} + (n-i-1)\frac{\partial\alpha_i}{\partial\hat{\Theta}}\Gamma^{-1}\Psi_{(i+2)2}^T \\ & + \sum_{j=1}^{i-1} z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\Theta}}\Gamma^{-1} \\ & \left\{\Psi_{(i+1)1}^T + (n-i-1)[0 \ \ qz_{i+1}]\right\}. \end{aligned} \tag{34}$$

Substituting (34) into (33), we obtain

$$\dot{V}(z, \hat{\Theta}) \leq -\frac{1}{2}\|z\|^2, \quad \text{if } \|\tilde{z}_t\| < q\|\tilde{z}\|. \tag{35}$$

Therefore, from (32) and (35), it follows obviously that the conditions (4) and (5) are satisfied. In view of Theorem 1, we conclude that the closed-loop system is stable in the sense of Lyapunov, and $z(t) \to 0$ as $t \to \infty$. In consequence, from (20) with $\alpha_i(0, \cdot) = 0$, it follows that $x(t) \to 0$ as $t \to \infty$ for any given $\phi(\cdot)$ and $\hat{\Theta}(0)$. ■

IV. Conclusion

In this note, the adaptive stabilization problem for nonlinear time-delay systems is investigated. The main contributions are to establish the LaSalle–Yoshizawa-like condition that ensures the convergence of partial states with the stability of the solution for a class of functional differential equations, and to show that with the proposed condition a design method to adaptive control of nonlinear time-delay systems can be developed. It should be noted that the proposed controller is of delay-independent.

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Comment on "Coordination of Groups of Mobile Autonomous Agents Using Nearest Neighbor Rules"

Lixin Gao and Daizhan Cheng

***Abstract*—This note provides some corrections and generalizations to the aforementioned paper.**

***Index Terms*—Cooperative control, graph theory, multiagent systems, switched systems.**

In [1], each agent's heading was updated using a simple neighbor rule, and it was shown that for a large class of switching signals and for any initial set of headings that the headings of all agents will converge to a steady-state value. The approach in [1] is based on bidirectional information exchange, modeled by an undirected graph. In this comment, we point that a part of the Proofs of Theorems 4 and 5, the main results about leader following case in [1], is questionable. The objective of this comment is to correct the Proofs of Theorems 4 and 5 of [1] and then extend the results to directed graph case.

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We first cite some notions from [1] to make this note more readable. For the leader following case in [1], the system consists of n autonomous agents, labeled 1 through n, plus one additional agent, labeled 0, which acts as the group's leader. Agents i's update rule is of the form

$$\theta_i(t+1) = \frac{1}{1+n_i(t)+b_i(t)}\left(\theta_i(t) + \sum_{j\in\mathcal{N}_i(t)} \theta_j(t) + b_i(t)\theta_0\right) \quad (1)$$

where $\mathcal{N}_i(t)$ is the set of agent i's neighbors, $n_i(t)$ is the number of the neighbors within $\mathcal{N}_i(t)$, and $b_i(t)$ is 1 whenever agent 0 is a neighbor of agent i and 0, otherwise.

The neighbor relationship between agents can be conveniently described by a simple directed graph. Each such graph $\mathcal{G}$ has vertex set $\mathcal{V} = \{v_0, v_1, \ldots, v_n\}$ and $\varepsilon \subset \mathcal{V}\times\mathcal{V}$ is the set of edges of the graph $\mathcal{G}$. A graph is call undirected (or bidirectional) if $\forall(v_i, v_j) \in \varepsilon \Rightarrow (v_j, v_i) \in \varepsilon$. If $(v_i, v_j) \in \varepsilon$, then v_j is said to be a neighbor of v_i, which means the information flow is from agent j to agent i. From those definitions we know that the leading agent labeled node 0 does not have any neighbor. In the sequel we use the symbol $\bar{\mathcal{P}}$ to denote a set indexing the class of all simple digraphs $\bar{\mathcal{G}}_{\mathcal{P}}$ defined on vertices $\{v_0, v_1, \ldots, v_n\}$. We define the subgraph $\mathcal{G}_{\mathcal{P}}$ of $\bar{\mathcal{G}}_{\mathcal{P}}$ on vertex set $\{v_1, v_2, \ldots, v_n\}$ which is obtained from $\bar{\mathcal{G}}_{\mathcal{P}}$ by deleting vertex 0 and all edges incident on vertex 0. A path from vertex v_i to vertex v_j is a sequence of distinct vertices starting v_i and ending with v_j such that consecutive pairs of vertices make an edge of digraph. If there is a path from one node v_i to another node v_j, then v_j is said to be reachable from v_i. If a node v_i is reachable from every other node of the digraph, then it is said to be globally reachable. A directed graph $\mathcal{G}$ is called weakly connected if there exits a node which is globally reachable, and a digraph is strongly connected if and only if any two distinct nodes of the graph can be connected via a path. A weakly connected undirected graph must be strongly connected, so it is simply termed as a connected graph. By the union of a collection of simple graphs, $\{\bar{\mathcal{G}}_{p_1}, \bar{\mathcal{G}}_{p_2}, \ldots, \bar{\mathcal{G}}_{p_m}\} \subset \bar{\mathcal{G}}_{\mathcal{P}}$, each with vertex set $\mathcal{V}$, we mean a simple graph $\bar{\mathcal{G}}$ with vertex set $\mathcal{V}$ and edge set equaling the union of the edge sets of all of the graphs in the collection. We say that such a collection is jointly weakly connected if the union of its members is a weakly connected graph. For undirected graph, the jointly weakly connected union must be jointly connected. More information is available in [2].

For matrices $M, N, M > N$ means $M - N$ is a positive matrix, where by a positive matrix is meant a matrix with all positive entries. The norm $\|R\|$ of a nonnegative matrix R is the induced infinity norm of matrix R which is the largest row sums of a nonnegative matrix. We denote the matrix obtained by replacing all of R's nonzero entries with 1 by $\lceil R\rceil$. Note that $R > 0$ if and only if $\lceil R\rceil > 0$. Any nonnegative matrices A, B, C with positive diagonal elements satisfy $\lceil ABC\rceil \geq \lceil AC\rceil$ and $\lceil AB\rceil = \lceil\lceil A\rceil\lceil B\rceil\rceil$.

Let $\theta = (\theta_1, \theta_2, \ldots, \theta_n)^T$ and $\bar{\theta} = \mathrm{col}(\theta, \theta_0)$, then the set of agent heading update rules defined by (1) can be expressed as

$$\bar{\theta}(t+1) = \bar{F}_{\sigma(t)}\bar{\theta}(t) \quad (2)$$

where $\sigma:\{0, 1, \ldots\} \to \bar{\mathcal{P}}$ is a switching signal whose value at time t is the index of the neighbor graph $\bar{\mathcal{G}}_p$, and matrix $\bar{F}_p$ is nonnegative and stochastic matrix associated with neighbor graph $\bar{\mathcal{G}}_p$. Let $B_p = \mathrm{diag}(b_1, b_2, \ldots, b_n)$, and let A_p denote the $n\times n$ adjacency matrix of the n-agent graph and D_p the corresponding diagonal matrix of valences of $\mathcal{G}_p$. The matrix $\bar{F}_p$ is partitioned as

$$\bar{F}_p = \begin{bmatrix} F_p & H_p\mathbf{1} \\ 0 & 1 \end{bmatrix} \quad (3)$$

where $\mathbf{1} = (1, 1, \ldots, 1)^T \in \mathcal{R}^n$, F_p is

$$F_p = (I + D_p + B_p)^{-1}(I + A_p) \quad (4)$$

and H_p is

$$H_p = (I + D_p + B_p)^{-1}B_p. \quad (5)$$

Denote the heading error vector by $\epsilon(t) := \theta(t) - \theta_0\mathbf{1}$, from (1) and (2) we can get

$$\epsilon(t+1) = F_{\sigma(t)}\epsilon(t). \quad (6)$$

Now, we give a simple counterexample to Lemma 5 in [1].

Assume the adjacency matrix $\bar{A}_{p_i}$ of neighbor graph $\bar{\mathcal{G}}_{p_i}, i = 1, 2$, are

$$\bar{A}_{p1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \bar{A}_{p2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

The last row and column label the neighbor relationship with the leader. Then, we have

$$\bar{F}_{p1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \bar{F}_{p2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}.$$

It is obvious that the union of $\{\bar{\mathcal{G}}_{p1}$ and $\bar{\mathcal{G}}_{p2}\}$ is jointly connected, which means all conditions of [1, Lemma 5] are satisfied. However, it is easy to calculate that for any $i > 2$ we have

$$\sum_{k=1}^{2} F_{p_k}^{i-1} H_{p_k}\mathbf{1} = \begin{bmatrix} 0 \\ \left(\frac{1}{2}\right)^i \end{bmatrix}$$

which is not greater than 0. It violates the conclusion of [1, Lemma 5]. Therefore, [1, Lemma 5] is incorrect. As a result, the main results for the leader following case, Theorems 4 and 5 of [1] are questionable.

For switching sequence $\overbrace{p_1, \ldots, p_1}^{i}, \overbrace{p_2, \ldots, p_2}^{i}$, the reader can verify directly that

$$\overbrace{\bar{F}_{p_2}\ldots\bar{F}_{p_2}}^{i}\overbrace{\bar{F}_{p_1}\ldots\bar{F}_{p_1}}^{i} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \left(\frac{1}{2}\right)^{i+1} & \left(\frac{1}{2}\right)^{i+1} & 1-\left(\frac{1}{2}\right)^i \\ 0 & 0 & 1 \end{bmatrix}$$

from which we conclude that $\|\overbrace{F_{p_2}\ldots F_{p_2}}^{i}\overbrace{F_{p_1}\ldots F_{p_1}}^{i}\| = 1$, which shows that [1, Prop. 2] is incorrect too.

On the other hand, for switching sequence $p_2, \overbrace{p_1, \ldots, p_2}^{i}, p_1$, one can get

$$\bar{F}_{p_1}\overbrace{\bar{F}_{p_2}\ldots\bar{F}_{p_1}}^{i}\bar{F}_{p_2} = \begin{bmatrix} \frac{3^{i-1}}{2^{2i-1}} & \frac{3^{i-1}}{2^{2i}} & 1-\frac{3^i}{2^{2i}} \\ \frac{3^{i-1}}{2^{2i-1}} & \frac{3^{i-1}}{2^{2i}} & 1-\frac{3^i}{2^{2i}} \\ 0 & 0 & 1 \end{bmatrix}$$

for which we can get $\|F_{p_1}\overbrace{F_{p_2}\ldots F_{p_1}}^{i}F_{p_2}\| = (3^i)/(2^{2i}) \to 0, i \to \infty$. In the first case the leader's information can not reach directly to agent 1, but in the second case the leader's information can reach directly to all other agents, which implies that the order of switching graph plays an important role in convergence of systems.

Motivated by this counterexample, we give a new definition about joint connectness of an ordered set of neighbor graph. For an ordered set of $\{\mathcal{G}_{p_1}, \mathcal{G}_{p_2}, \ldots, \mathcal{G}_{p_m}\} \subset \mathcal{G}_{\mathcal{P}}$, where every $\mathcal{G}_{p_k}$ is a simple graph with vertices given by $\{v_1, v_2, \ldots, v_n\}$ and edge set given by $\varepsilon_{p_k}, k = 1, 2, \ldots, m$, a joint path of length l from v_{j_0} to v_{j_l} is an ordered set of distinct nodes $\{v_{j_0}, v_{j_1}, \ldots, v_{j_l}\}$ such that $(v_{j_{i-1}}, v_{j_i}) \in \varepsilon_{p_{k_i}}, i = 1, 2, \ldots, l$, $k_i \in \{1, 2, \ldots, m\}$ and $k_i \geq k_{i+1}$. If there is a joint path from one node v_i to another v_j, then v_j is said to be jointly reachable from v_i. If a node v_j is jointly reachable from every other node in the digraph, then it is said to be jointly globally reachable. Note that if such an ordered set contains at least one weakly connected graph, then the ordered set must be jointly globally reachable.

Lemma 1: Let $\{p_1, p_2, \ldots, p_m\}$ be a set of indices in $\bar{\mathcal{P}}$ for which v_0 is a jointly globally reachable node of the ordered set $\{\bar{\mathcal{G}}_{p_1}, \bar{\mathcal{G}}_{p_2}, \ldots, \bar{\mathcal{G}}_{p_m}\}$. Then, the matrix $(\bar{F}_{p_m}\bar{F}_{p_{m-1}}\ldots\bar{F}_{p_1})^n$ has the form

$$(\bar{F}_{p_m}\bar{F}_{p_{m-1}}\ldots\bar{F}_{p_1})^n = \begin{bmatrix} \left(F_{p_m}F_{p_{m-1}}\ldots F_{p_1}\right)^n & B \\ 0 & 1 \end{bmatrix} \quad (7)$$

where B satisfies $B > 0$.

Proof: The form (7) can be verified by a straightforward computation, which shows

$$B = \sum_{k=1}^{n}\sum_{j=1}^{m}\left(F_{p_m}F_{p_{m-1}}\ldots F_{p_1}\right)^{n-k} \left(F_{p_m}F_{p_{m-1}}\ldots F_{p_{j+1}}\right)H_{p_j}\mathbf{1}. \quad (8)$$

Since v_0 is a jointly globally reachable node of the ordered set $\{\bar{\mathcal{G}}_{p_1}, \bar{\mathcal{G}}_{p_2}, \ldots, \bar{\mathcal{G}}_{p_m}\}$, for any $v_j, j \neq 0$ there exits a joint path from v_j to v_0 and the path length l satisfies $l \leq n$. Assume the joint path is an ordered set of distinct nodes $\{v_j, v_{j_1}, \ldots, v_{j_{l-1}}, v_0\}$ such that $(v_{j_{i-1}}, v_{j_i}) \in \varepsilon_{p_{k_i}}$, $i = 1, 2, \ldots, l, p_{k_i} \in \{p_1, p_2, \ldots, p_m\}$, and $k_i \geq k_{i+1}$, which means the jth element of vector $A_{p_{k_1}}A_{p_{k_2}}\ldots A_{p_{k_{l-1}}}H_{p_{k_l}}\mathbf{1}$ is greater than 0. We can conclude that the jth component of the vector $F_{p_{k_1}}\ldots F_{p_{k_{l-1}}}H_{p_{k_l}}\mathbf{1}$ is greater than 0 by noting that all matrices are nonnegative and using (4). We also have

$$\left[\left(F_{p_m}F_{p_{m-1}}\ldots F_{p_1}\right)^{n-1}F_{p_m}F_{p_{m-1}}\ldots F_{p_{k_l+1}}Hp_{k_l}\mathbf{1}\right] \geq \left[F_{p_{k_1}}F_{p_{k_2}}\ldots F_{p_{k_{l-1}}}H_{p_{k_l}}\mathbf{1}\right]$$

which implies $B > 0$.

Then, [1, Prop. 2] can be modified as follows. □

Proposition 1: Let $T > 0$ be a positive integer. There exists a positive number $\lambda < 1$, depending only on T, for which

$$\left\|F_{p_{\bar{t}}}F_{p_{\bar{t}-1}}\ldots F_{p_1}\right\| < \lambda$$

for every sequence of $p_1, p_2, \ldots, p_{\bar{t}}$ at length at most T possessing subsequence of the same sequences $\overbrace{q_1, q_2, \ldots, q_m; \ldots; q_1, q_2, \ldots, q_m}^{n}$ whose associated ordered set of $\{\bar{\mathcal{G}}_{q_1}, \bar{\mathcal{G}}_{q_2}, \ldots, \bar{\mathcal{G}}_{q_m}\}$ has a jointly globally reachable node v_0.

Proof: Note that

$$\left\lceil \bar{F}_{p_{\bar{t}}}\bar{F}_{p_{\bar{t}-1}}\ldots\bar{F}_{p_1}\right\rceil \geq \left\lceil\left(\bar{F}_{q_m}\bar{F}_{q_{m1}}\ldots\bar{F}_{q_1}\right)^n\right\rceil. \quad (9)$$

Using (9) and Lemma 1, the Proposition 1 can be proved via the same ideas as those for the proof of [1, Prop. 2]. □

Theorem 2: Let $\theta(0)$ and θ_0 be fixed and let $\sigma : \{0, 1, \ldots\} \to \bar{\mathcal{P}}$ be a switching signal for which there exists an infinite sequence of contiguous, nonempty, bounded, time-intervals $[t_i, t_{i+1})$, $i \geq 0$, starting at $t_0 = 0$, with the property that the ordered set of neighbor graphs across each such interval has a jointly globally reachable node associated with the leader. Then

$$\lim_{t\to\infty}\theta(t) = \theta_0\mathbf{1}. \quad (10)$$

The proof of this theorem is omitted, because it is mimic to the proof of [1, Th. 4].

Now, we characterize the relationship between the joint weak connectivity of the graphs and the joint global reachability. It is obvious that the union of graphs is jointly weakly connected if the ordered set of graphs has a jointly globally reachable node. On the other hand, a collection can be jointly weakly connected but the associated ordered set has no a jointly globally reachable node. Let Γ denote the subset of $\mathcal{G}_{\mathcal{P}}$ consisting of all connected graphs. Since the set Γ is a finite set, let $\bar{n}$ be the number of elements in Γ. For the contiguous, nonempty, bounded, time-intervals $[t_i, t_{i+1})$, $i \geq 0$, starting at $t_0 = 0$ and having a property that across each such interval the union of neighbor graphs is jointly connected, there must be at least one element in Γ which is the union of neighbor graphs across interval at least n times on any sequence of $n\bar{n}$ contiguous time-intervals. Assume these n intervals are $[t_{k_i}, t_{k_i+1})$, $i = 1, 2, \ldots, n$ satisfy that $k_i < k_{i+1}$ and $k_i < n\bar{n}$, and the union of neighbor graphs of interval $[t_{k_i}, t_{k_i+1})$ is $\bar{\mathcal{G}} \in \Gamma$. Since $\bar{\mathcal{G}}$ is connected, there must exist a path of length $l, l \leq n$, from v_j to v_0 for any node $v_j, j = 1, 2, \ldots, n$, which is an ordered set of distinct nodes $\{v_j, v_{j_1}, \ldots, v_{j_{l-1}}, v_0\}$. In time-interval $[t_0, t_{n\bar{n}})$, for every $j, j = 1, 2, \ldots, n$, there must exist a jointly path from v_j to v_0 by noting that at least a neighbor graph of $[t_{k_{n-i+1}}, t_{k_{n-i+1}+1})$ contains edge $(v_{j_{i-1}}, v_{j_i})$. Thus, the ordered set of neighbor graphs across interval $[t_0, t_{n\bar{n}})$ has a jointly globally reachable node associated with leader. We have proved the following proposition.

Proposition 2: For an infinite sequence of contiguous, nonempty, bounded, time-intervals $[t_i, t_{i+1})$, $i \geq 0$, starting at $t_0 = 0$, the property that the union of neighbor graph across each such interval has a globally reachable node associated with leader is equivalent to the property that the ordered set of neighbor graph across each such interval has a jointly globally reachable node associated with leader.

Remark 1: Proposition 2 implies that the result of [1, Th. 5] is correct by using Theorem 2 of this note. All the results of the leader following case in [1] can be easily extended to the directed graph case. For the leader following in continuous-time case, the proof of [1, Th. 5] should also be similarly modified as what we did in this comment for the discrete-time case. The conditions of [1, Th. 5] can also be replaced by an infinite sequence of contiguous, nonempty, bounded, time-intervals $[t_i, t_{i+1}), i \geq 0$, starting at $t_0 = 0$, with the property that the ordered set of neighbor graphs across each such interval has a jointly globally reachable node associated with leader as in Theorem 2.

Remark 2: Considering possibly time-varying relative confidence of each agent's information variable or relative reliabilities of different information exchange links between agents, we propose the following weighting agent i's update rule instead of update rule (1):

$$\theta_i(t+1) = \frac{1}{a_{ii}(t) + \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t) + b_i(t)} \times \left(a_{ii}(t)\theta_i(t) + \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)\theta_j(t) + b_i(t)\theta_0\right) \quad (11)$$

where $a_{ij}(t)$ is time-variant positive weighting factor, and weighting factor $b_i(t)$ is greater than 0 whenever agent 0 is a neighbor of agent i and 0 otherwise. If all weighting factors $a_{ij}(t), b_i(t) \in [e_{\min}, e_{\max}]$, $e_{\min} > 0$, the similar results of Theorem 2 can be obtained for the update rule (11). For continuous-time case, we propose that agent i uses a hybrid control law of the form

$$u_i(t) = -\left(\sum_{j\in\mathcal{N}_i(t)} a_{ij}(t_i)\left(\theta_i(t) - \theta_j(t)\right) + b_i(t_i)(\theta_i(t) - \theta_0)\right) \quad (12)$$

where $a_{ij}(t_i)$ is a time-variant positive weighting factor, and weighting factor $b_i(t_i)$ is greater than 0 whenever agent 0 is a neighbor of agent i and 0, otherwise. If all weighting factors $a_{ij}(t_i), b_i(t_i) \in [e_{\min}, e_{\max}], e_{\min} > 0$, and all the dwell times satisfy $0 < t_{\min} \leq t_{i+1} - t_i \leq t_{\max}$, some similar results of [1, Th. 5] can also be obtained for the update rule (12).

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Correction to "Homogeneous Observers, Iterative Design, and Global Stabilization of High-Order Nonlinear Systems by Smooth Output Feedback"

Bo Yang and Wei Lin

In [1], there is a typo on page 1074, which needs to be corrected.

- The statement under (5.1): "the mappings $\phi_i : \mathbb{R}^n \to \mathbb{R}, i = 1, \ldots, n$, are C^1 with $\phi_i(0,0) = 0$", should read as "the mappings $\phi_i : \mathbb{R}^{n+1} \to \mathbb{R}, i = 1, \ldots, n$, are C^1 with $\phi_i(0,u) = 0$."

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Erratum to "Hierarchical Interface-Based Supervisory Control—Part I: Serial Case"

Ryan J. Leduc, Bertil. A. Brandin, Mark Lawford, and W. M. Wonham

In the above paper [1], a photo for the second author was missing. The corrected biography with photo is as follows.

Bertil A. Brandin received the B.S. degree in mechanical engineering from the University of New South Wales, Australia, in 1984, and the M.A.Sc. and Ph.D. degrees in electrical engineering from the University of Toronto, Toronto, ON, Canada, in 1989 and 1993, respectively.

From 1993 to 1995, he was Project Leader in a collaborative project on the supervisory control of automated manufacturing systems between the Province of Ontario, Canada, and Région Rhône-Alpe, France. In 1994, he was an Invitied Scientist at the Rockwell Science Center, Thousand Oaks, CA. From 1995 to 2003, he managed R&D on formal verification techniques for software and business process applications at Siemens Corporate Technology, Munich, Germany. He is currently General Manager of Siemens Medical Solutions Health Services, Italy. His research interests include discrete-event systems, supervisory control, formal verification techniques, model-based testing, and diagnostics.

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