Output Regulation for Nonlinear Systems: Some Recent Theoretical and Experimental Results

Tzyh-Jong Tarn, Peerayot Sanposh, Daizhan Cheng, and Mingjun Zhang

***Abstract*—This brief summarizes recent theoretical and presents experimental results of output regulation for nonlinear systems. The main contribution of this brief consists of the first experimental study of output regulation using a two-link underactuated mechanical system—the Pendubot.**

***Index Terms*—Nonlinear systems, output regulation, Pendubot, underactuated mechanical systems.**

I. Introduction

OUTPUT regulation is the problem of finding a control law by which the output of the system that we are interested in can asymptotically track another output generated by an exosystem, and simultaneously reject undesired disturbances. The control law must also asymptotically stabilize the system whenever the exosignal is absent. Without loss of generality, both the exo-output and the disturbances can be viewed as an output of an augmented exosystem.

The output regulation problem originated from trajectory tracking for linear systems. By viewing the trajectory as an output of an exosystem, trajectory tracking can be regarded as a subclass of global output regulation. In general, if the disturbances do not affect the response of the system, the problem of trajectory tracking can be easily solved. However, this is seldom the case practically. The disturbances always result from unmodeled dynamics, parameter uncertainties, and measurement noises. For the case of measurement noises, some tradeoffs between steady-state tracking errors and responses due to the disturbances must be considered. The output regulation for linear systems has been solved by Francis and Wonham [2].

For output regulation of nonlinear systems, Isidori and Byrnes [7] gave necessary and sufficient conditions for solvability of the problem. In general, the solvability is based on existence of a center manifold, which is controlled invariant, zero-error and exponentially attractive. Their work also includes a geometric theory describing the relationships between the output-zeroing manifold and the vector field after applying the control law. The controller can be viewed as an interconnection of two controllers, which exponentially stabilizes the plant when the exosignal is absent, and imposes a signal which converges to a steady response. On the other hand, Isidori [8] states another viewpoint of the interconnection of the control laws: one control signal renders the zero-error manifold invariant and the other signal asymptotically stabilizes the augmented systems. Later on, Huang and Rugh [3] showed that it is sufficient to have a zeroth-order and controlled invariant manifold that is locally defined and attractive. Approximations of the manifold were presented in [4] and [9], where Taylor's series expansion and Kronecker product were used to solve the regulator equations. Huang [5] presented an example of how to construct a control law to enable the system of interest to track with a trajectory without concern for stabilization. Marconi *et al.* [10] applied existing theory to a vertical taking off and landing vehicle. All of previous work addressed local properties. Until recently, the work of Serrani and Isidori [11] was related to global robust output regulation of single-input–single-output (SISO) nonlinear systems in a special form. Their work and the aforementioned literature is restricted to a special class of exosystems.

This brief surveys recent theoretical results on solvability and feedback regulator construction for output regulation of nonlinear systems. These theories do not restrict the number of inputs and outputs. An experimental study for output regulation of the Pendubot is then given. The main contribution of this brief consists of the first experimental study of nonlinear output regulation using a special under-actuated mechanical systems—the Pendubot. The brief is organized as follows. The problem statement is given in Section II. The output regulation by state feedback is presented in Section III. In Section IV, the experimental study is presented. The conclusion is given in Section V.

II. Problem Statement

Consider a nonlinear system in the following form:

$$\dot{x} = f(x) + g(x)u + p(x)w, \qquad y = h(x) \tag{1}$$

where state $x \in R^n$, input $u \in R^m$, output $y \in R^p$, and w is a *disturbance* signal generated by an *exosystem* as follows:

$$\dot{w} = s(w),\ y_d = q(w) \tag{2}$$

where exosignal $w \in R^d$ and a tracking signal $y_d \in R^p$.

For the nonlinear plant (1) and exosystem (2), the output y tracks the signal y_d. The tracking error can be defined

$$e := y - y_d. \tag{3}$$

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T.-J. Tarn is with the Department of Electrical and Systems Engineering, Washington University, St. Louis, MO 63130 USA (e-mail: tarn@wuauto.wustl.edu).

P. Sanposh is with the Department of Electrical Engineering, Kasetsart University, Bangkok 10900, Thailand (e-mail: fengpyp@ku.ac.th).

D. Cheng is with the Institute of Systems Science, Chinese Academy of Sciences, Beijing 100080, P. R. China (e-mail: dcheng@iss03.iss.ac.cn).

M. Zhang is with the Life Sciences and Chemical Analysis Division, Agilent Technologies, Palo Alto, CA 94304 USA (e-mail: mingjunzhang@ieee.org).

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Assume $f(0) = 0$, $g(0) = 0$, $p(0) = 0$, $h(0) = 0$, $s(0) = 0$ and $q(0) = 0$, all vector fields related to the plant (1) and exosystem (2) are smooth with respect to their arguments, and all states can be measured, then the state feedback regulation problem can be defined as finding a control law $u = \alpha(x, w)$, such that:

1) the equilibrium point $x = 0$ of $\dot{x} = f(x) + g(x)\alpha(x, 0)$ is locally asymptotically stable;
2) there exists a neighborhood $U \subset R^n \times R^d$ of (0,0) such that, for each initial condition $(x(0), w(0)) \in U$, the solutions of (1) and (2) satisfy $\lim_{t\to\infty} e(t) = 0$. ◃

If only the output-tracking error e is measurable, error feedback regulation problems can be formulated similarly [1], [7]. As stated in [7], the error feedback regulation problem is solvable if and only if the state feedback regulation problem is solvable. This brief will focus on state feedback regulation problem, whose solving techniques may be extended to solve error feedback regulation problems.

III. Output Regulation by State Feedback

Isidori and Byrnes [7] gave the necessary and sufficient conditions for the solvability of local output regulation for nonlinear systems, which assumes the following conditions.

Assumption 3.1: The point $w = 0$ is stable and there exists a neighborhood $\hat{W} \subset R^d$ of the origin with the property that each initial condition $w(0) \in \hat{W}$ is Poisson stable. ◃

Assumption 3.2: The pair $f(x), g(x)$ has a stabilizable linear approximation at $x = 0$. ◃

Then the following theorem can be obtained.

Theorem 1 [7]: Under Assumptions 3.1–3.2, the output regulation by state feedback is solvable if and only if there exist C^k $(k \geq 2)$ mappings $x = \pi(w)$ with $\pi(0) = 0$, and $u = \gamma(w)$ with $\gamma(0) = 0$, both defined in a neighborhood $\hat{W} \subset R^d$, satisfying the conditions

$$\begin{aligned} \frac{\partial \pi}{\partial w} s(w) &= f(\pi(w)) + g(\pi(w))\gamma(w) + p(\pi(w))w \\ 0 &= h(\pi(w)) - q(w). \end{aligned} \tag{4}$$

◃

This theorem is based on the existence of a center manifold which is locally attractive and controlled-invariant under $u = \gamma(w)$. Moreover, the output-tracking error is identically zero on this manifold. The control law

$$u = \gamma(w) + K(x - \pi(w)) \tag{5}$$

where $\mathrm{eig}(\partial f(0)/\partial x + (\partial g(0)/\partial x)K) := \mathrm{eig}(A + BK) \in C^-$, solves the state feedback regulator problem. In general, it is difficult to solve (4). However, without considering convergence of the solution, the equation can be solved using Taylor series expansion [9].

Instead of choosing $u = \gamma(w)$, Cheng *et al.* [1] proposed a control in the form of $u = \gamma(x, w)$, which can render the manifold invariant. Moreover, the number of inputs and outputs for the proposed control may not be equal to each other and the conditions can be easily verified. The following condition is assumed in addition to Assumptions 3.1–3.2.

Assumption 3.3: The plant decoupling matrix D has full rank at $x = 0$ ◃

$$D = \begin{bmatrix} L_{g_1} L_f^{r_1 - 1} h_1(x) & \cdots & L_{g_m} L_f^{r_1 - 1} h_1(x) \\ L_{g_1} L_f^{r_2 - 1} h_2(x) & \cdots & L_{g_m} L_f^{r_2 - 1} h_2(x) \\ \vdots & \vdots & \vdots \\ L_{g_1} L_f^{r_p - 1} h_p(x) & \cdots & L_{g_m} L_f^{r_p - 1} h_p(x) \end{bmatrix}.$$

The following theorem can be obtained.

Theorem 2: Under Assumptions 3.1–3.3 and by Theorem 2.7 [6], if the output regulation problem is solvable, then the control can be expressed by

$$u = \gamma(x, w) + \tilde{K}(x - \pi(w))$$

where

$$\gamma(x, w) = -D^\dagger \begin{bmatrix} L_f^{r_1} h_1(x) - L_s^{r_1} q_1(w) + L_p L_f^{r_1 - 1} h_1(x) \\ L_f^{r_2} h_2(x) - L_s^{r_2} q_2(w) + L_p L_f^{r_2 - 1} h_2(x) \\ \vdots \\ L_f^{r_p} h_p(x) - L_s^{p_1} q_p(w) + L_p L_f^{r_p - 1} h_p(x) \end{bmatrix}$$

and $\tilde{K} = K - \partial\gamma(0, 0)/\partial x$. The matrix K is selected such that $\mathrm{eig}(A + BK) \in C^-$. The symbol $D^\dagger$ denotes the pseudo-inverse of a matrix. ◃

Consider the nonlinear systems (1), and further assume:

Assumption 3.4: The decoupling matrix D has full rank over $x \in U \subset R^n$, a neighborhood of the origin. ◃

Then, the following necessary conditions are held for the existence of an output regulator.

Theorem 3 (Necessary Conditions): Under Assumption 3.4 and by Theorem 2.7 [6], if the output regulation problem by state feedback is solvable, and $x = 0$ of the system $\dot{x} = f(x) + g(x)\bar{u}$ is globally asymptotically stabilizable by a smooth control law $\|\bar{u}\| = \beta(x)$, then the following conditions are held.

1) There exists a positive–definite and proper smooth function $V(x)$ such that:
   a) if $L_g V(x) = 0$ then $L_f V(x) < 0$ for all $x \neq 0$;
   b) for each $\epsilon > 0$, there exists $\delta > 0$ such that, if $x \neq 0, \|x\| < \delta$, then there exists $\bar{u}$ with $\|\bar{u}\| < \epsilon$ satisfying $L_f V(x) + L_g V(x)\bar{u} < 0$.
2) Let $M := \{(x, w) \in R^n \times R^d : h(x) = q(w)\}$, the controller which forces the trajectory $(x(t), w(t))$ to stay on the manifold M is given by

$$\begin{aligned} u &= \gamma(x, w) \\ &= -D^\dagger \begin{bmatrix} L_f^{r_1} h_1 - L_s^{r_1} q_1 + L_p L_f^{r_1 - 1} h_1 \\ L_f^{r_2} h_2 - L_s^{r_2} q_2 + L_p L_f^{r_2 - 1} h_2 \\ \vdots \\ L_f^{r_p} h_p - L_s^{p_1} q_p + L_p L_f^{r_p - 1} h_p \end{bmatrix}. \end{aligned} \tag{6}$$

3) There exists $M_0 \subseteq M$ such that M_0 is globally attractive. ◃

The condition (1) is a necessary and sufficient condition for $\dot{x} = f(x) + g(x)\bar{u}$ to be globally asymptotically stabilized by the control $\bar{u} = \beta x$ known as the Artstein–Sontag Theorem [8]. Proof of Theorem 2 can be done by continuously differentiating $h(x(t)) = q(w(t)), \forall t \geq 0$, and solving for the input u.

Note that in the case of $m < P$, there must be some redundancies in the system outputs. If $m \geq p$, then the previous control $u = \gamma(x, w)$ is such that $\gamma(x, w) = \min_{u \in \mathcal{U}} \|u\|$ among all other feasible control laws.

Theorem 4 (Sufficient Conditions): Under Assumption 3.4 and by Theorem 2.7 [6], the output regulation by state feedback is solvable if all of the following conditions are satisfied.

1) There exists a positive–definite and proper smooth function $V(x)$ such that:
   a) if $L_g V(x) = 0$ then $L_f V(x) < 0$ for all $x \neq 0$;
   b) for each $\epsilon > 0$, there exists $\delta > 0$ such that, if $x \neq 0, \|x\| < \delta$, then there exits u with $\|u\| < \epsilon$ such that $L_f V(x) + L_g V(x) u < 0$.
2) There exists a globally defined, smooth mapping $\pi : R^d \to R^n$ such that

$$\frac{\partial \pi}{\partial w} s(w) = f(\pi(w)) + g(\pi(w))\gamma(\pi(w), w) + p(\pi(w)) w \quad (7)$$

   subject to the constraint $h(\pi(w)) - q(w) = 0$.
3) There exists a smooth and proper mapping $W : R^n \to R$ such that:
   a) $W(e_\pi) \geq c\|e_\pi\|$ for all $e_\pi \in R^n$ and for some constant $c > 0$ with $W(0) = 0$;
   b) $\dot{W}(e_\pi, w) \leq -\lambda W(e_\pi)$ for all $(e_\pi, w) \in R^n \times R^d$ and for some constant $\lambda > 0$;
   c) $\nabla W(e_\pi) g(x)$ has full column rank over $R^n \times R^n / \{0\}$.

   where, the error e_π and the time derivative $\dot{W}(e_\pi, w)$ are defined as

$$e_\pi := x - \pi(w) \quad (8)$$

$$\dot{W}(e_\pi, w) := \frac{\partial W}{\partial e}[\dot{x}|_{x = e_\pi + \pi(w)} - \dot{\pi}(w)]. \quad (9)$$

◁

Again, condition (1) is always true since it is necessary. Proof of the results can be done by showing that the controlled-invariant manifold of the extended system, which consists of the plant and the dynamic control, is globally attractive.

If Assumptions 3.1–3.4 are true and by Theorem 2.7 [6], Theorem 3 and Theorem 4 allows us to write the control law in the form of

$$u = \gamma(x, w) + \tilde{K}(x - \pi(w)) \quad (10)$$

where $\tilde{K} = K - \partial\gamma(0, 0)/\partial x$. The advantage is that this allows us to express the control law which renders the zero-error manifold invariant explicitly. A nice thing about the result is that, different from [11], the proposed output regulation theorem does not require a special structure of the system.

IV. Output Regulation of the Pendubot

The Pendubot is a two-link underactuated mechanical system, whose first link (shoulder) is actuated and the second link (elbow) is unactuated. Many approaches have been proposed to control the Pendubot [12]. However, no output regulator for the Pendubot has been reported in the open literature. It would be interesting to see how the proposed output regulator works for the Pendubot. Unfortunately, global output regulation cannot be applied to control of the Pendubot, since one of the necessary conditions, condition 3, in Theorem 3 is not satisfied. Here, local output regulator is applied for tracking control of the end position of the Pendubot's second link. In the meantime, keep the second link upright. The dynamic model of the Pendubot is given in as follows:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + e(q) = \tau$$

where $q = (q_1, q_2)^T$, $v \in R$ and

$$\begin{aligned} M(q) &= \begin{pmatrix} \theta_1 + \theta_2 + 2\theta_3 \cos q_2 & \theta_2 + \theta_3 \cos q_2 \\ \theta_2 + \theta_3 \cos q_2 & \theta_2 \end{pmatrix} \\ C(q, \dot{q}) &= \begin{pmatrix} -\theta_3 (\sin q2) \dot{q_2} & -\theta_3 (\sin q2)(\dot{q_1} + \dot{q_2}) \\ \theta_3 (\sin q_2) \dot{q_1} & 0 \end{pmatrix} \\ e(q) &= \begin{pmatrix} \theta_4 g \cos(q_1) + \theta_5 g \cos(q_1 + q_2) \\ \theta_5 g \cos(q_1 + q_2) \end{pmatrix} \\ \tau &= \begin{pmatrix} v \\ 0 \end{pmatrix}. \end{aligned} \quad (11)$$

In the previous model, parameters $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$ are obtained through reparameterization, whose values are (0.0308, 0.0106, 0.0095, 0.2086, 0.0630). The end position of the second link of the Pendubot can be expressed as follows:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} l_{c1} \cos(q_1) + l_{c2} \cos(q_1 + q_2) \\ l_{c1} \sin(q_1) + l_{c2} \sin(q_1 + q_2) \end{pmatrix}. \quad (12)$$

where l_{c1} and l_{c2} are the length of the first and second links, respectively. A picture and the dynamic model of the Pendubot may be seen in [12].

A. Experiment Design

The desired output $y_{d1} = w_1 = A \sin(\omega t)$ is used for one-dimensional (1-D) regulation. The signal y_{d1} can be generated by an appropriate linear system of the form

$$\dot{w} = Sw, \; y_{d1} = Qw. \quad (13)$$

Consequently, the tracking error in this case is

$$e_1 = y_1 - y_{d1}. \quad (14)$$

For two-dimensional (2-D) output regulation, the variation of y_2 can be approximated as a sinusoidal signal that can also be generated by a linear system of the form (13). The tracking error in this case is

$$e = (e_1, e_2)^T = (y_1 - y_{d1}, y_2 - y_{d2})^T. \quad (15)$$

In this experiment, linear quadratic regulator are first used for swing up control to bring the two links of the Pendubot to the upright position, i.e., $q_1 = \pi/2$ and $q_2 = 0$. Then, the controller will be switched to the proposed output regulator to track the end of the second link along a desired trajectory. In the meantime, keep the second link up straight. Detailed control algorithm is as follows:

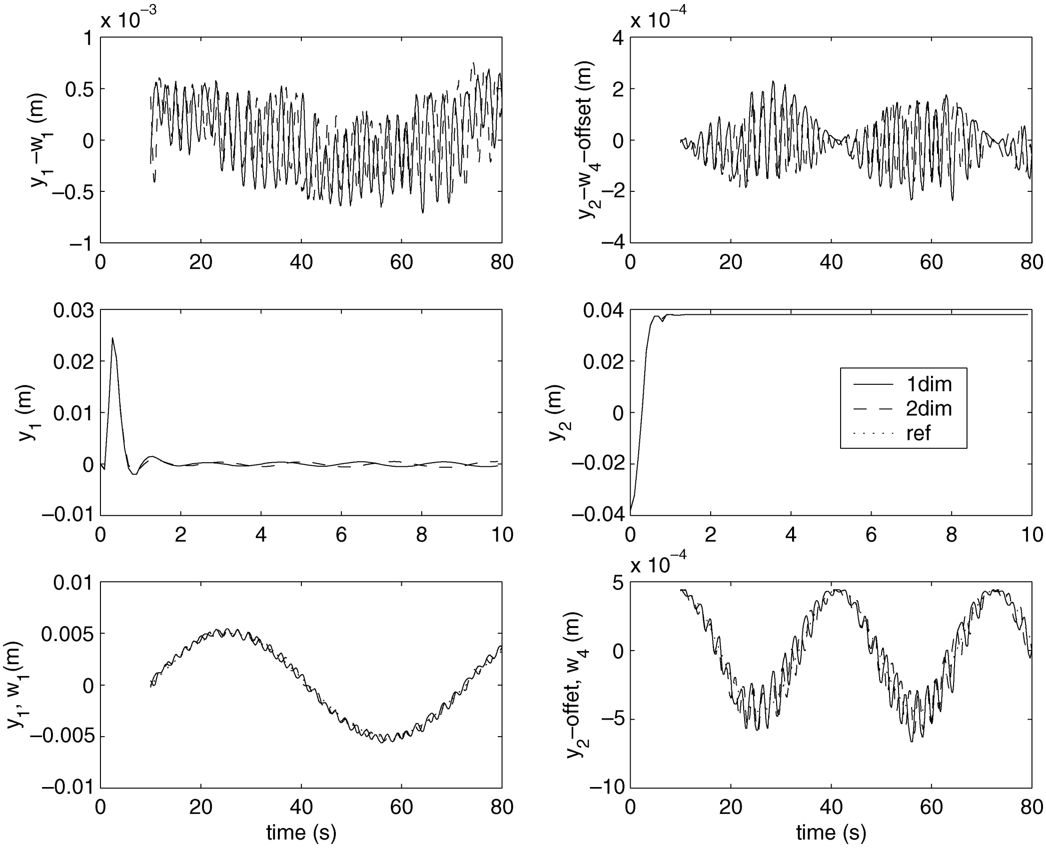

Fig. 1. Experimental results for the up position via output feedback.

- First, use swing up control to bring both links close to the neighborhood of the equilibrium state. This can be done through partial feedback linearization and PD control. The gain for PD control is chosen as $(k_p, k_d) = (58.0, 8.9)$. Then, apply linear quadratic regulator for feedback stabilization. The gain matrix is chosen as $K = [16.4615, 16.2422, 3.1287, 2.0658]$.
- After stabilizing both links for about 10 s, the control for 1-D regulation is given by

$$\begin{aligned} u(x,w) &= (L_g L_f h_1(x))^{-1} \left(L_f^2 h_1(x) - L_S^2 q_1(w) \right) \\ &\quad + \tilde{K}(x - \pi(w)) \\ &= \gamma(x,w) + \tilde{K}(x - \pi(w)) \end{aligned} \tag{16}$$

where the matrix $\tilde{K} = K - \partial\gamma(0,0)/\partial(x)$, and K is chosen so that $\text{eig}(\partial f(0)/\partial x + (\partial g(0)/\partial x)K)$ is in the open left-half plane.

For 2-D regulation, the control will be

$$\begin{aligned} u(x,w) &= -\begin{pmatrix} L_g L_f h_1 \\ L_g L_f h_2 \end{pmatrix}^{\dagger} \begin{pmatrix} L_f^2 h_1 - L_s^2 q_1 \\ L_f^2 h_2 - L_s^2 q_2 \end{pmatrix} \\ &\quad + \bar{K}(x - \pi(w)) \\ &= \bar{\gamma}(x,w) + \bar{K}(x - \pi(w)) \end{aligned} \tag{17}$$

where $\bar{K} = K - \partial\bar{\gamma}(0,0)/\partial(x)$ and the function $\bar{\gamma}(x,w)$ is the least square solution.

TABLE I
QUANTITATIVE RESULTS FOR THE UP POSITION VIA OUTPUT FEEDBACK

case	max $\lvert u_{ss} \rvert$	y_2	y_2 offset	max $\lvert e_{1ss} \rvert$	max $\lvert e_{2ss} \rvert$
1D	0.993	4.0329e-3	0.377	0.746e-2	0.237e-2
2D	0.899	5.3525e-3	0.376	0.750e-2	0.207e-2

The parameters for the exosignal are $A = 0.05$ and $\omega = 0.1$. The center manifold $x = \pi(w)$ is given by

$$x = \begin{pmatrix} -65.6319 & 0 \\ 65.6420 & 0 \\ 0 & -6.5632 \\ 0 & 6.5642 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} + 0\ w^{[2]} + O(w^{[3]}) \tag{18}$$

which is approximated by methods proposed in [4] and [9]. The center manifold for 2-D tracking is from least-square solution.

B. Experimental Results

Fig. 1 shows end-position of the link two for both 1-D and 2-D regulations, where y_1 and y_2 are corresponding to the coordinates of end-position for the link two. Two figures in the middle row show responses of the link two with swing-up and stabilization control. Both links are stabilized at about 10 s. Then, the controller switches to output regulation for trajectory tracking. Two figures in the bottom row show coordinates of link two and w_1 and w_4 generated by the computer. Two figures in the first row show differences of the coordinates with respect to exosignals. Based on our observation and the results in Table I, it seems that the performance of 2-D tracking control is better than that

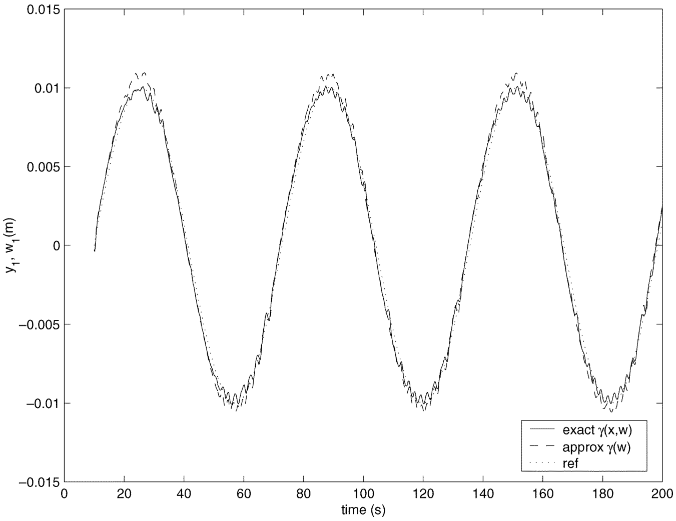

Fig. 2. Output regulation using $u = \gamma(w) + K(x - \pi(w))$.

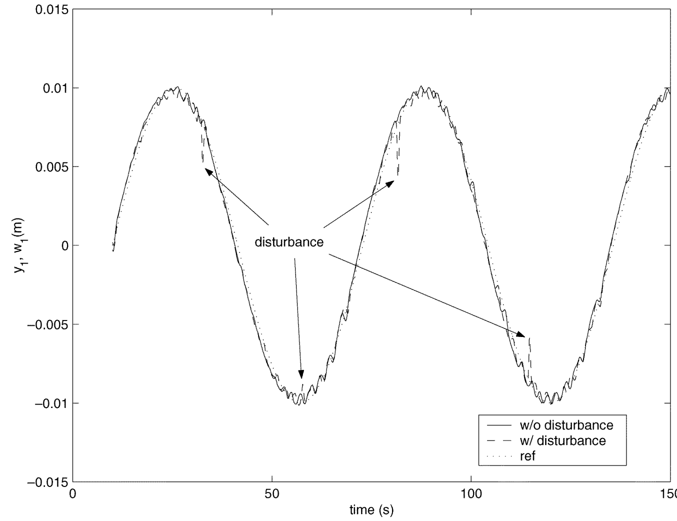

Fig. 4. End-position of link two with disturbances.

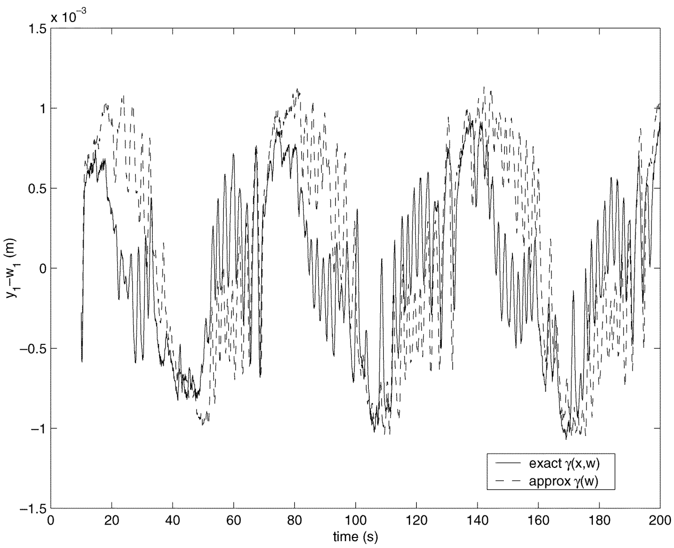

Fig. 3. Output regulation using $u = \gamma(x, w) + \tilde{K}(x - \pi(w))$.

of 1-D tracking control. The better performance may be due to having more information on the end-position of the Pendubot. However, theoretical proof is necessary to make a formal claim.

To compare the two controllers $u = \gamma(w) + K(x - \pi(w))$ and $u = \gamma(x,w) + \tilde{K}(x - \pi(w))$, experiments were conducted for 1-D regulation using both controllers. Figs. 2 and 3 show responses of end-position for the link two using different controllers, respectively. Fig. 3 shows the difference. As shown in the figures, the regulation by the control $u = \gamma(x,w) + \tilde{K}(x - \pi(w))$ is qualitatively better than that by the control $u = \gamma(w) + K(x - \pi(w))$, because in the latter case the term $\gamma(w)$ is approximated. On the contrary, the term $\gamma(x,w)$ is the *exact* control that renders the error manifold invariant. However, the integral of the square of errors are $\int_{10}^{200} (y - y_d)^2_{\gamma(x,w)} dt = 6.9245 \times 10^{-6}$ and $\int_{10}^{200} (y - y_d)^2_{\gamma(w)} dt = 7.2925 \times 10^{-6}$. The result shows that the error using control $\gamma(x,w)$ is more concentrated around zero. The maximum voltage for $\gamma(x,w)$ is 1.4768, which is 0.2097 lower than that for $\gamma(w)$. Thus, the cost using the control $u = \gamma(x,w) + \tilde{K}(x - \pi(w))$ is approximately 14% lower.

To test robustness of the output regulator $u = \gamma(x,w) + \tilde{K}(x - \pi(w))$, disturbances are introduced into the Pendubot through the second link. The disturbances are generated by tapping the link with a rod at $t \approx 40$, 55, 80, and 110 s. As shown in Fig. 4, the second link can still maintain the upright position. In the figure, the solid line represents responses without disturbances, the dashed line means responses with disturbances and the dotted line represents reference trajectory. Moreover, the Pendubot can recover quickly from the disturbances.

V. Conclusion

This brief surveys necessary and sufficient conditions for solvability of global output regulation for general nonlinear systems. The solvability depends on the properties of the system's decoupling matrix and the existence of a zeroth-order, globally-attractive, controlled-invariant manifold. Output regulators are also proposed for nonlinear systems based on the discussions. The output regulator was applied to an under-actuated mechanical systems: the Pendubot. From the experimental study, the output regulator $u = \gamma(x,w) + \tilde{K}(x - \pi(w))$ yields satisfactory results. Moreover, the regulator seems robust enough despite parameter uncertainties, unmodeled dynamics, and noises.

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