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Finite time convergent control using terminal sliding mode

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Abstract: A method for terminal sliding mode control design is discussed. As we know, one of the strong points of terminal sliding mode control is its finite-time convergence to a given equilibrium of the system under consideration, which may be useful in specific applications. The proposed method, different from many existing terminal sliding model control design methods, is studied, and then feedback laws are designed for a class of nonlinear systems, along with illustrative examples.

Keywords: Terminal sliding mode control; Finite-time convergence; Nonlinear systems

1 Introduction

Variable structure system control, as one of the most active research areas of control theory and one of the powerful practical tools, has been studied for many decades. Sliding mode controllers are constructed in order to keep controlled systems to given constraint surfaces and also make the systems insensitive to some certain external and internal disturbances. Many results on sliding mode control and its extensions can be found in the literature including [1 ~ 4].

Recent years have also witnessed an increasing interest in terminal sliding mode control. This may be due to the fact that this non-smooth feedback may possess faster convergent rates (related to finite-time convergence) and superior robustness properties in practice. Terminal sliding mode control approach is to render the closed-loop system finite-time to converge in finite time to the desired position of the system under consideration, rather than only to a sliding surface. In this way, the dynamic response of the closed-loop system may be improved. Theoretical results and their application to robotic systems can be found in the references like [5 ~ 8], though the control methods may cause singularity problems. [5] proposed a two-phase control scheme to avoid the singularity of their original control law. Also, [7] proposed another idea to construct the discontinuous sliding modes with finite time convergence. In addition, it is worthwhile to point out that, besides discontinuous terminal sliding mode control, continuous finite-time control draws much research attention as well [9 ~ 12].

The aim of the paper is to give a new approach to terminal sliding mode controller, which may remove singularities outside of sliding surfaces. The rest of the paper is organized as follows. In Section 2, the problem formulation is given, while in Section 3 theoretical results to construct sliding mode controllers and discussion on terminal sliding mode control are shown. Then, in Section 4, terminal sliding mode control laws are built for a class of nonlinear systems. Finally, concluding remarks are given in Section 5.

2 Problem formulation

Consider the nonlinear control system

$$\dot{x} = f(x) + g(x)u, \ u \in \mathbb{R}, \ x \in \mathbb{R}^n,$$

Let $s:\mathbb{R}^n \to \mathbb{R}$ be a continuous function and the sliding surface is defined as $S = \{x \in \mathbb{R}^n : s(x) = 0\}$. The corresponding motion, satisfying $s(x) \equiv 0$, is called a sliding dynamics with respect to the constraint function s. Conventionally, an ideal sliding motion on the sliding mode is described as

$$\begin{cases} s(x) = 0, \\ \dot{s}(x) = L_{f+gu}s(x) = 0. \end{cases}$$

In other words, S should be invariant if possible feedback, called an equivalent control, can be constructed.

The basic idea of sliding mode control is as follows. Choose a sliding manifold; then use the sliding mode control to drive the state outside of the manifold into the manifold; finally, using u_{eq} to render the state in the sliding mode along the plane to the desired equilibrium. Therefore, the stability problem basically changes to a problem how to find a suitable sliding mode and a sliding mode controller.

In this paper, we focus on a single-input control system of the form:

$$\begin{cases} \dot{x}_1 = f_1(x), \\ \quad \vdots \\ \dot{x}_{n-1} = f_{n-1}(x), \\ \dot{x}_n = f_n(x) + u, \end{cases} \tag{1}$$

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with $x = (x_1,\cdots,x_n)^T \in \mathbb{R}^n$, $f_i(x)$ smooth, and $f_i(0) = 0$ for $i = 1,\cdots,n$.

The sliding mode design procedure can be divided roughly into two main steps:

1) Find a sliding surface satisfying transverse condition, that is, there is an i such that

$$\frac{\partial s}{\partial x_i} \neq 0, \tag{2}$$

(without loss of generality, we take $i = n$ in the sequel) and that $s(x) \equiv 0$ implies $x \to 0$;

2) Construct a stabilizing feedback law in the form of

$$u(x) = \begin{cases} u_{eq}(x), & \text{if } s = 0, \\ u^+(x), & \text{if } s > 0, \\ u^-(x), & \text{if } s < 0. \end{cases}$$

Most sliding mode control laws usually make the controlled system (1) converge to their sliding surfaces in finite time, and then, along the sliding surfaces, the systems converges to the equilibrium $x = 0$ of system (1) as time goes to infinity. However, terminal sliding mode control laws achieve more by steering the states to reach the equilibrium in finite time.

Definition 1 Consider a system

$$\dot{x} = f(x) + g(x)u,\ x \in \mathbb{R}^n, \tag{3}$$

where $f(0) = 0$ and $g(0) \neq 0$. $u = \mu(x)$ is called a finite-time convergent controller if the equilibrium $x = 0$ of the closed-loop system (3) under this feedback law is finite-time convergent, namely, for any initial condition $x(0) = x^0 \in \mathbb{R}^n$, there is a settling time $T \geq 0$, which satisfies:

$$\lim_{t \to T} x(t;0,x^0) = 0,$$

and

$$x(t;0,x^0) = 0,\ \text{if } t > T$$

for every solution $x(t;0,x^0)$ to the closed-loop system (3). Moreover, if the controller is also a sliding mode controller, it is called a terminal sliding mode controller.

In what follows, for simplicity, we assume that s is selected in the following form to satisfy the transverse condition (2):

$$s(x) = x_n - h(x_1,\cdots,x_{n-1}),\ h(0,\cdots,0) = 0,$$

where $h(x_1,\cdots,x_{n-1})$ is a continuous function. Thus, $s(x)$ is obtained once $h(x)$ is fixed. Here we give an assumption for the selection of $h(x)$, that is,

$$|h(x_1,\cdots,x_{n-1})| < K_0 + L_0 \sum_{i=1}^{n-1} |x_i| \tag{4}$$

for suitable positive numbers K_0 and L_0.

Many choices of sliding modes satisfy condition (4). For example, $s(x) = \sum_{i=1}^{n-1} a_i x_i$ is a widely used form, and taking $K_0 = 1$ and $L_0 = \max\{a_i, i = 1,\cdots,n-1\}$ will make (4) hold.

3 Main results

At first, two lemmas are introduced, whose proofs are quite obvious and therefore omitted here.

Lemma 1 For any $0 < \alpha < 1$ and $M_0 > 0$, there is a $M_1 > 0$ such that $|z|^\alpha \leq M_0 + M_1|z|$ holds for all $z \in \mathbb{R}$.

Lemma 2 Suppose that a, b, and $m > 1$ are all positive numbers. Then $(a^m + b^m)^{1/m} \leq a + b$.

Denote sgn $(\cdot)$ as the sign function. Then we have a result related to the convergence to sliding surfaces for system (1):

Theorem 1 If the sliding surface S is taken with $h(x)$ satisfying (4), then the control law

$$u = -f_n(x) + v, \tag{5}$$

where

$$v = -K(1 + \sum_{i=1}^{n-1} |f_i(x)|)\,\text{sgn}\,(s),$$
$$K > \max\{L_0, 1\},\ s \neq 0, \tag{6}$$

will ensure that the system (1) reaches S in finite time.

Proof The task, in fact, is to prove that, for any initial condition $x(0) \neq 0$ with $s(x(0)) \neq 0$, feedback (5) can make $s(x(t)) = 0$ in finite time.

Let $x(0) = x^0 = (x_1^0,\cdots,x_{n-1}^0,x_n^0)$ be the initial condition. The trajectory with the initial condition is denoted by $x(t;0,x(0))$, or $x(t)$ for simplicity.

We first study the case when $s(x(0)) > 0$, namely, $x_n^0 > h(x_1^0,\cdots,x_{n-1}^0)$. We will prove that there is $T > 0$ such that $s(x(T)) = 0$ by contradiction.

Suppose that there is not any $T > 0$ such that $s(x(T)) = 0$. Because $s(x(0)) > 0$ and $s(x(t))$ is continuous, $s(x(t)) > 0$ for any $t > 0$, namely, $x_n(t) > h(x_1(t),\cdots,x_{n-1}(t))$ for any $t > 0$. Therefore,

$$\begin{aligned}\dot{x}_n &= -K(1 + \sum_{i=1}^{n-1} |f_i(x)|)\,\text{sgn}\,(s) \\ &= -K(1 + \sum_{i=1}^{n-1} |f_i(x)|).\end{aligned} \tag{7}$$

Integrating both sides of (7) gives

$$\begin{aligned} x_n(t) - x_n^0 &= -Kt - K\sum_{i=1}^{n-1}\int_0^t |f_i(x)|\,dt \\ &\leq -Kt - K\sum_{i=1}^{n-1} \left|\int_0^t \dot{x}_i dt\right| \\ &\leq -Kt - K\sum_{i=1}^{n-1} |x_i(t)| + K\sum_{i=1}^{n-1} |x_i^0|, \end{aligned}$$

which implies

$$x_n(t) \leq -(Kt - x_n^0 - K\sum_{i=1}^{n-1} |x_i^0|) - K\sum_{i=1}^{n-1} |x_i(t)|.$$

Recall condition (4), i.e.,

$$|h(x_1,\cdots,x_{n-1}) < K_0 + L_0 \sum_{i=1}^{n-1} |x_i|.$$

When

$$t_1 > \frac{K_0 + |x_n^0|}{K} + \sum_{i=1}^{n-1} |x_i^0|,$$

we have

$$x_n(t_1) < -K_0 - L_0\sum_{i=1}^{n-1}|x_i(t_1)| \leqslant h(x_1(t_1),\cdots,x_{n-1}(t_1)),$$

that is, $s(x(t_1)) < 0$. This contradiction shows that there exists $T_1 > 0$ such that $s(x(T_1)) = 0$.

Similarly, when $s(x(0)) < 0$, that is, $x_n^0 < h(x_1^0,\cdots,x_{n-1}^0)$, there is a $T_2 > 0$ such that $s(x(T_2)) = 0$. Let $T = \max\{T_1, T_2\}$, then the assertion follows. □

An important merit of sliding mode control is that it provides robustness to uncertainties. Consider system (1) with $f_i, i = 1,\cdots,n$ uncertain. Suppose

$$|f_\tau(x)| \leqslant \phi_i(x),\ i = 1,\cdots,n,$$

where functions $\phi_i(x), i = 1,\cdots,n$, are known and $\phi_i(0)$ may not be zero. Then we have

Theorem 2 If $h(x_1,\cdots,x_{n-1})$ satisfies (4), we can take control law in the form of

$$u(x) = -K(1 + \sum_{i=1}^{n}\phi_i(x))\mathrm{sgn}\ (s),\ s \neq 0 \tag{8}$$

with $K > \max\ \{K_0, L_0, 1\}$, such that the uncertain system (1) reaches S in finite time.

Proof Consider the case when $s(x(0)) > 0$ at first. Then similar to (7), we have

$$\dot{x}_n = f_n(x) - K(1 + \sum_{i=1}^{n}\phi_i(x))\mathrm{sgn}\ (s) \leqslant -K(1 + \sum_{i=1}^{n-1}\phi_i(x)), \tag{9}$$

because $f_n(x) - K\phi_n(x) \leqslant 0$ (noting that $K \geqslant 1$). Then, integrating both sides of (7) gives

$$x_n(t) = x_n^0 - Kt - K\sum_{i=1}^{n-1}\int_0^t\phi_i(x)\mathrm{d}t,$$

and again,

$$x_n(t) \leqslant -(Kt - x_n^0 - K\sum_{i=1}^{n-1}|x_i^0|) - K\sum_{i=1}^{n-1}|x_i(t)|.$$

Then, following the same procedure of the proof of Theorem 1, we can reach the conclusion. □

Terminal sliding mode control is to find a suitable sliding surface such that the state outside of the selected sliding surface is finite-time convergent to it and the system state in the sliding surface tends to the equilibrium in finite time. Here we sketch our design idea as follows. At first, we select $s(x) = x_n - h(x_1,\cdots,x_{n-1})$ such that it satisfies condition (4) and

$$\begin{cases}\dot{x}_1 = f_1(x_1,\cdots,x_{n-1},h),\\ \quad\vdots\\ \dot{x}_{n-2} = f_{n-2}(x_1,\cdots,x_{n-1},h),\\ \dot{x}_{n-1} = f_{n-1}(x_1,\cdots,x_{n-1},h),\end{cases} \tag{10}$$

is finite time convergent on $S = \{x \in \mathbb{R}^n : s(x) = 0\}$ (the selection of such $h(x)$ will be discussed in next section). Then we use the control given in (5) or (8) to guarantee finite-time convergence to S and force the state to move along S.

Remark 1 Conventionally, we need to construct $u = u_{\mathrm{eq}}$ on S based on the knowledge of $f_i, i = 1,\cdots,n$, which is taken to guarantee $s(x(t)) \equiv 0$ for any $t \geqslant 0$ with the initial condition $s(x(0)) = 0$. However, in some uncertain situations, u_{eq} may not always keep an uncertain system on the surface S, which may lead to some theoretical problems. Moreover, the finite time convergence may result in the singularity of u_{eq} (or equivalently in some sense, the existence problem of the sliding surface S, which should be kept invariant by u_{eq}), mostly for third or higher order systems. Therefore, u_{eq} may not be implemented in practice and the singularity problem of terminal sliding mode control may be essential and may not be theoretically solved with conventional ideas (Levant constructed sliding modes $\phi_{i-1,r}$ and also commented "none of these sliding modes really exists" in [7]), and the concepts from differential inclusions may have to be employed for understanding.

4 Construction of controllers

In this section, based on the discussion in Section 3, we consider how to construct terminal sliding mode controllers for a class of systems in form of

$$\begin{cases}\dot{x}_1 = x_2^{m_1},\\ \quad\vdots\\ \dot{x}_{n-1} = x_n^{m_{n-1}},\\ \dot{x}_n = f_n(x) + u,\end{cases} \tag{11}$$

where $m_i, i = 1,\cdots,n-1$ are positive odd integer.

Remark 2 The terminal sliding mode control for systems in the form of (4) with $m_1 = \cdots = m_{n-1} = 1$ has been studied ([7]). In fact, the study can be carried out on the nonlinear systems with regular relative degrees (as did in [3,7]), which can be written in the form of:

$$\begin{cases}\dot{x} = Ax + Bu,\\ \dot{z} = q(z,x),\end{cases}$$

with (A, B) controllable.

As shown in [5,6], the existing terminal sliding modes for system (11) with $m_1 = \cdots = m_{n-1} = 1$ were usually constructed in a recursive structure as follows:

$$s_0 = x_1,\cdots,s_{n-1} = \dot{s}_{n-2} + b_{n-2}s_{n-2}^{(q_{n-1}/p_{n-1})},$$

where $(n-i-1)/(n-i) < q_{i+1}/p_{i+1} < 1$ with p_{i+1}, q_{i+1}, $i = 0,\cdots,n-2$ as positive odd integers and $b_i > 0$, $i = 0,\cdots,n-2$. Then the control law is taken as

$$u = u_{\mathrm{eq}} - K\mathrm{sgn}\ (s_{n-1}). \tag{12}$$

$u_{\mathrm{eq}}(x)$ is given to keep $s(x(t)) = 0$ once $s(x(0)) = 0$, but this may not be satisfied because the singularity problems of terminal sliding modes or uncertainties will occur so as to make the feedback law (12) fail. In [7], another idea for terminal sliding mode was given for the system with $m_1 = \cdots = m_{n-1} = 1$. Denote

$$N_{1,n} = |x_1|^{(n-1)/n},\cdots,$$

$$N_{i,n} = (|x_1|^{p/n} + |x_2|^{p/(n-1)} + \cdots + |x_{i-1}|^{p/(n-i+1)})^{(n-i)/p}$$

for $i = 1,\cdots,n-1$ and $p > 0$. Then the sliding mode controller is given as

$$u = -f_n(x) - \beta_0 \mathrm{sgn}\ (\phi_{p-1,p}), \tag{13}$$

with properly chosen positive parameters β_i, $i = 0,1,\cdots,n-1$ and a (recursive) sliding mode:

$$\phi_{0,n} = x_1, \cdots,$$
$$\phi_{i,n} = x_{i+1} + \beta_i N_{i,n} \mathrm{sgn}\ (\phi_{i-1,n}),$$
$$i = 1,\cdots,n-1,$$

Our approach is dlifferent. Take $s = x_n - h(x_1,\cdots,x_{n-1})$ with $h = v_{n-1}$ such that

$$\begin{cases} \dot{x}_1 = x_2^{m_1}, \\ \vdots \\ \dot{x}_{n-2} = x_{n-1}^{m_{n-2}}, \\ \dot{x}_{n-1} = u_{n-1}(t) = v_{n-1}(x_1,\cdots,x_{n-1}) \end{cases} \tag{14}$$

is finite-time convergent to the origin. In fact, system (14) admits a continuous finite-time convergent 'feedback law' $u_{n-1} = v_{n-1}(x_1,\cdots,x_{n-1})$ (e.g., [9,11,12]). Thus, in our design, we select $h = v_{n-1}^{1/m_{n-1}}$ in one of the forms provided in [9,11]. For example, we can construct v_{n-1} based on:

$$v_{i+1} = -l_{i+1}[x_{i+1}^{\beta_i} - v_i(x_1,\cdots,x_i)^{\beta_i}]^{\frac{r_{i+1}+k_0}{r_{i+1}\beta_i}}, \tag{15}$$

where $v_0 = 0$, $l_{i+1} > 0$, for $i = 0,\cdots,n-2$ are suitable constants, $r_1 = 1,\cdots,r_{i+1} = r_i + k_0 > 0$, $i = 2,\cdots,n-2$; $k_0 = \frac{p_0}{q_0} - 1 < 0$, and

$$\beta_0 = 1,\ \beta_i r_{i+1} \geqslant \beta_{i-1} r_i,$$
$$\beta_i = \frac{p_i}{q_i},\ i = 1,\cdots,n-2, \tag{16}$$

with p_i, q_i, $i = 0,\cdots n-2$ are positive odd integers [11].

In this way, we first make system (14) on sliding surface S finite-time convergent. Moreover, the condition (4) for this h can also be verified using Lemmas 1 and 2 (see the following examples for some details). Then, with the control law in the form of (5) or (8), the state outside of the sliding surface is finite-time convergent to the surface S. Hence, system (11) with $s(x) = x_n - v_{n-1}(x_1,\cdots,x_{n-1})^{1/m_{n-1}}$ and its corresponding control law is finite-time convergent to the origin.

Remark 3 Note that the terminal sliding mode feedback of form (5) is different from (12) and has no singularities when $s \neq 0$. As the results given in related reference including [6] and [7], we have not solved theoretically the singularity problem of u_{eq} on S (for the existence of S), either. In other words, our approach only removes the singularities outside of the sliding surface using Theorem 1 or 2 as discussed in Remark 1. However, for practical design, we may try some simple structures. For instance, for third or higher order systems, on the surface S, we will take $u = \bar{f}_n(x)$, if $f_n(x) = \bar{f}_n(x) + \tilde{f}_n(x)$ with $\tilde{f}_n(x)$ unknown.

For illustration, we give two examples.

Example 1 Consider a second order nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2^{m_1}, \\ \dot{x}_2 = f_2(x) + u, \end{cases} \tag{17}$$

where $m_1 > 0$ is an odd integer.

Take a continuous function $h(x_1) = -x_1^{3/(3m_1+2)}$ and then

$$s = x_2 + x_1^{3/(3m_1+2)}.$$

Thus, for the given h, (4) can be satisfied with $K_0 = 3$ and $L_0 = 2$, due to Lemma 1. Then we can take

$$u(x) = \begin{cases} -f_2(x) - 3(1 + |x_2|)\mathrm{sgn}\ (s), & \text{if } s \neq 0, \\ u_{eq}(x) = -f_2(x) + \dfrac{3}{3m_1+2} x_1^{1/(3m_1+2)}, & \text{if } s = 0. \end{cases} \tag{18}$$

Then the system is convergent to the sliding surface in finite time. Once on the sliding surface ($s = 0$), we have $\dot{x}_1 = -x_1^{3/(m_1+2)}$, which is finite-time convergent to $x_1 = 0$, that is, the origin $x = 0$, because of $s = 0$. This directly leads to the finite-time convergence of the closed-loop system. □

Example 2 Consider

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = f_3(x) + u. \end{cases} \tag{19}$$

First, we employ the method in [6], and we have

$$s_0 = x_1,\ s_1 = \dot{s}_0 + s_0^{7/9},\ s_2 = \dot{s}_1 + s_1^{5/7}.$$

Then the control law is given as

$$u(x) = u_{eq}(x) - K\mathrm{sgn}\ (s_2),\ K > 0, \tag{20}$$

with

$$u_{eq}(x) = -f_3(x) - (s_0^{7/9}) - (s_1^{5/7})$$
$$= -f_3(x) - \frac{7x_3}{9x_1^{2/9}} - \frac{5(x_3 + \frac{7x_2}{9x_1^{2/9}})}{7(x_2 + x_1^{7/9})},$$

which may contain some singularities.

Next, consider our proposed method for terminal sliding mode control. Take a continuous function

$$s(x) = x_3 + x_1^{\alpha_1} + x_2^{\alpha_2},$$
$$0 < \alpha_1 < 1,\ \alpha_2 = \frac{2\alpha_1}{1+\alpha_1}. \tag{21}$$

When $x \in S$ (i.e., $s(x) = 0$), we have

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -x_1^{\alpha_1} - x_2^{\alpha_2}, \end{cases}$$

which is finite-time convergent [9]. Moreover, for $s(x)$

given in (21), (4) can be satisfied with $K_0 = 3$ and $L_0 = 2$, due to Lemma 1. Then, by Theorem 1, we can construct a feedback law of form (5), different from (20),

$$u(x)=\begin{cases}-f_3(x)-3(1+|x_2|+|x_3|)\mathrm{sgn}(s), & \text{if } s\neq 0,\\ u_{\mathrm{eq}}=-f_3(x)-\alpha_1 x_1^{\alpha_1-1}x_2+\alpha_2 x_2^{\alpha_2-1}x_3, & \text{if } s=0.\end{cases} \quad (22)$$

(In the case when $f_3(x)$ is unknown, the feedback can be taken of form (8) based on Theorem 2). Thus, the system is convergent to the sliding surface in finite time, and in the sliding surface, the state gets to the equilibrium infinite time, which implies the finite-time convergence of the closed-loop system. To avoid the singularity (referring to Remarks 1 and 3), we may substitute (22) by a simple form:

$$u(x)=\begin{cases}-f_3(x)-3(1+|x_2|+|x_3|)\mathrm{sgn}(s), \text{if } s\neq 0,\\ -f_3(x), \text{if } s=0.\end{cases} \quad (23)$$

Due to various uncertainties in reality, we cannot expect the exact finite-time convergence, but we can expect the faster convergent rate of this 'finite-time' controller around a given equilibrium than those of conventional 'asymptotic' controllers (with fixed coefficients). Fig. 1 shows two cases with $\alpha_1 = 1/5$ and 1, respectively. Note that when $\alpha_1 = 1$, the sliding mode control become asymptotic with a smooth function $s(x)$ (referring to Remark 4), but trajectory under this control law is still with a little oscillation even around $t = 15$, while feedback (23) makes the trajectory almost become $x_1 = 0$ around $t = 7$. In other words, the convergence in the case of $\alpha_1 = 1/5$ remains faster than that in the case of $\alpha_1 = 1$. □

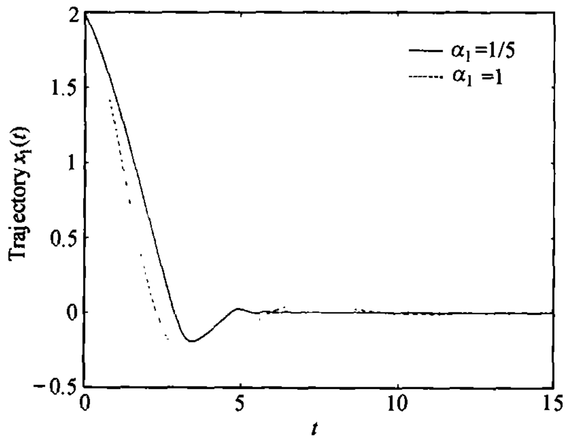

Fig. 1 $x_1(t)$ for initial conditions (2, −0.5, 1) with $\alpha_1 = 1/5$ and 1, respectively.

Remark 4 We would like to make a comparison between conventional methods and our design ideas in the design of asymptotic sliding modes for system (11). It is known that a sliding surface can be obtained with

$$s(x) = \sum_{i=1}^{n} c_i x_i, \quad (24)$$

where $c_i > 0, i = 1, \cdots, n$ (usually taking $c_n = 1$) are Hurwitz coefficients. Then the conventional controller is taken as

$$u_c(x) = u_{eq}(x) - K\mathrm{sgn}(s), \ K > 0,$$

where $u_{eq}(x) = -f_n(x) - \sum_{i=1}^{n-1} c_i x_{i+1}$. This control law will stabilize the system (11) [4]. On the other hand, since, for s given in (24), (4) is also satisfied with $K_0 = 0$ and $L_0 > \max\{c_1, \cdots, c_n\}$, an asymptotic sliding mode controller in the form of (5) can be constructed with our design method:

$$u(x) = -f_n(x) - K(1 + \sum_{i=1}^{n-1} |x_{i+1}|)\mathrm{sgn}(s),$$
$$K > \max\{L_0, 1\}.$$

From Theorem 1, this sliding mode controller can render the system state to converge to the sliding surface in finite time, and then to the origin as $t \to \infty$.

5 Conclusions

Terminal sliding mode control draws attention mostly because of its finite-time convergence to a given equilibrium, in addition to its finite-time convergence to a corresponding sliding surface. In the paper, a new design method for terminal sliding mode control is proposed, and the global control design for some systems is shown. Although some results of terminal sliding mode control have been successfully applied, to set up a complete theory for the analysis and design of these controllers to deal with the problems including singularity removal is still challenging.

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