Dissipative Hamiltonian Realization and Energy-Based L_2-Disturbance Attenuation Control of Multimachine Power Systems

Yuzhen Wang, Daizhan Cheng, Chunwen Li, and You Ge

***Abstract*—The note considers the L_2-disturbance attenuation of multimachine power systems via dissipative pseudo-Hamiltonian realization of the systems. First, the note expresses multimachine systems as a dissipative Hamiltonian system. Then, the note investigates the energy-based control design of L_2-disturbance attenuation of multimachine power systems and proposes a decentralized simple control strategy. Simulations on a six-machine system show that the achieved L_2-disturbance attenuation control strategy is very effective.**

***Index Terms*—Dissipative Hamiltonian realization, L_2-disturbance attenuation, n-machine power system.**

I. Introduction

Recently, Hamiltonian function method [1]–[5] has drawn a considerable attention in the control of power systems and got a lot of achievements [1], [6]–[11]. The method, in general, can thoroughly use the internal structural properties of power systems during control designs, and the controllers designed by the method are relatively simple in form, easy and effective in operation.

It is well known that a key step in using Hamiltonian function method to investigate control problems is to express the system under consideration as a dissipative Hamiltonian system, i.e., to complete dissipative Hamiltonian realization (DHR). With Hamiltonian function method many significant achievements have been obtained for single-machine infinite-bus systems [6], [8], [10], but for multimachine power systems the situation is quite different. The model structure of multimachine systems is so complicated that the systems' DHR problem has become an open puzzle. Very recently, certain contributions have been made for the DHR of multimachine power systems [9], [11]. However, the dissipative Hamiltonian realization problem of multimachine systems still remains a long-unresolved problem, which turns to be the bottleneck of the energy-based control design of multimachine systems.

This note, based on a widely used model of multimachine power systems [12]–[14], has obtained a DHR form of multimachine power systems by using prefeedback technique. Unlike the traditional way, we consider a feedback Hamiltonian realization directly and then adjust the operating point to the preassigned point. The new approach can be simply described as: prefeedback—DHR—operating point adjustment.

Then as an application of the DHR, the note investigates the energy-based control design of L_2-disturbance attenuation of multimachine power systems and proposes a decentralized simple control strategy. Simulations on a six-machine system show that the achieved L_2-disturbance attenuation control strategy is very effective.

Manuscript received November 1, 2002; revised April 8, 2003. Recommended by Associate Editor M. Reyhanoglu. This work was supported by Project 973 of China under Grant G1998020307 and Grant G1998020308 and by the China Postdoctoral Science Foundation.

Y. Wang is with the Department of Automation, Tsinghua University, Beijing 100084, China, and also with the School of Control Science and Engineering, Shandong University, Jinan 250061, China (e-mail: yzwang@tsinghua.edu.cn).

D. Cheng is with the Institute of Systems Science, Chinese Academy of Sciences, Beijing 100080, China (e-mail: dcheng@iss03.iss.ac.cn).

C. Li and Y. Ge are with the Department of Automation, Tsinghua University, Beijing 100084, China.

Digital Object Identifier 10.1109/TAC.2003.815037

The rest of this note is organized as follows. Section II deals with the DHR of multimachine power systems. Section III investigates the energy-based control design of L_2-disturbance attenuation of multimachine systems. Section IV presents some simulation results and Section V is the conclusion.

II. DHR of Multimachine Power Systems

Consider the following n-machine power system, each generator of which is described by the third-order dynamic model [12]–[14]:

$$\begin{cases} \dot{\delta}_i = \omega_i - \omega_0 \\ \dot{\omega}_i = \frac{\omega_0}{M_i} P_{mi} - \frac{D_i}{M_i}(\omega_i - \omega_0) - \frac{\omega_0}{M_i} P_{ei}, \\ \dot{E}'_{qi} = -\frac{1}{T_{d0i}} E_{qi} + \frac{1}{T_{d0i}} u_{fi}, \end{cases} \tag{2.1}$$

$$P_{ei} = G_{ii} {E'}^2_{qi} + E'_{qi} \sum_{j=1, j\neq i}^{n} B_{ij} E'_{qj} \sin(\delta_i - \delta_j)$$

$$E_{qi} = E'_{qi} + I_{di}(x_{di} - x'_{di})$$

$$I_{di} = B_{ii} E'_{qi} - \sum_{j=1, j\neq i}^{n} B_{ij} E'_{qj} \cos(\delta_i - \delta_j)$$

$$i = 1, 2, \ldots, n$$

where δ_i is the power angle of the ith generator, in radian; ω the rotor speed of the ith generator, in rad/s, $\omega_0 = 2\pi f_0$; E'_{qi} the q-axis internal transient voltage of the ith generator, in per unit; x'_{di} the d-axis transient reactance of the ith generator, in per unit; u_{fi} the voltage of the field circuit of the ith generator, the control input in per unit; M_i the inertia coefficient of the ith generator, in seconds; D_i the damping constant, in per unit; T_{d0i} the field circuit time constant, in seconds; x_{di} the d-axis reactance, in per unit; P_{mi} the mechanical power, assumed to be constant, in per unit; P_{ei} the active electrical power, in per unit; I_{di} the d-axis current, in per unit; E_{qi} the internal voltage, in per unit; $Y_{ij} = G_{ij} + jB_{ij}$ the admittance of line i-j, in per unit; $Y_{ii} = G_{ii} + jB_{ii}$ the self-admittance of bus i, in per unit.

Denote $x_{i1} = \delta_i, x_{i2} = \omega_i - \omega_0, x_{i3} = E'_{qi}, (\omega_0/M_i)P_{mi} = a_i, (D_i/M_i) = b_i, (\omega_0/M_i)G_{ii} = c_i, (\omega_0/M_i) = d_i, (1/T_{\mathrm{doi}}) = e_i, (x_{di} - x'_{di}/T_{\mathrm{doi}}) = h_i, (1/T_{\mathrm{doi}})u_{fi} = u_i$, then (2.1) can be written as

$$\begin{cases} \dot{x}_{i1} = x_{i2}, \\ \dot{x}_{i2} = a_i - b_i x_{i2} - c_i x_{i3}^2 \\ \qquad -d_i x_{i3} \sum_{j=1, j\neq i}^{n} B_{ij} x_{j3} \sin(x_{i1} - x_{j1}), \\ \dot{x}_{i3} = -(e_i + h_i B_{ii}) x_{i3} + u_i \\ \qquad +h_i \sum_{j=1, j\neq i}^{n} B_{ij} x_{j3} \cos(x_{i1} - x_{j1}) \end{cases}$$

$$i = 1, 2, \ldots, n. \tag{2.2}$$

Tying every means, we find it almost impossible to express system (2.2) into a Hamiltonian system directly. Prefeedback seems necessary. Then the problem becomes how to design a suitable prefeedback law to provide (2.2) a dissipative Hamiltonian structure. After analyzing the form of (2.2), we find that the term $-c_i x_{i3}^2$ in the second equation is a key factor in the DHR, because this term does not have its (skew-) symmetric counterpart and destroys the system's dissipative structure. Based on the term $-c_i x_{i3}^2$, the forms of the three equations of (2.2) and Poincare lemma, a rough calculation shows that the prefeedback law should be a nonlinear one related to $x_{i1} x_{i3}$. Finally, it turns out that the following prefeedback control law (2.3) works:

$$u_i = -\frac{2c_i h_i}{d_i} x_{i1} x_{i3} + v_i, \qquad i = 1, 2, \ldots, n. \tag{2.3}$$

Theorem 2.1: Under the prefeedback control (2.3), the multimachine system (2.1) has an overall dissipative Hamiltonian realization.

Proof: Substituting (2.3) into (2.2), we have (2.4), as shown at the bottom of the page, where $i = 1, 2, \ldots, n, g = (0, 0, 1)^T$. Set formal Hamiltonian-like functions as

$$H_i = -\frac{a_i}{d_i}x_{i1} + \frac{c_i}{d_i}x_{i1}x_{i3}^2 + \frac{1}{2d_i}x_{i2}^2 - x_{i3}\sum_{j=1,j\neq i}^{n} B_{ij}x_{j3}\cos(x_{i1} - x_{j1}) + \frac{e_i + h_i B_{ii}}{2h_i}x_{i3}^2$$

$i = 1, 2, \ldots, n$, then (2.4) can be described as

$$\dot{x}_i = (J_i - R_i)\frac{\partial H_i}{\partial x_i} + g v_i, \qquad i = 1, 2, \ldots, n \tag{2.5}$$

where

$$J_i = \begin{pmatrix} 0 & d_i & 0 \\ -d_i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad R_i = \begin{pmatrix} 0 & 0 & 0 \\ 0 & b_i d_i & 0 \\ 0 & 0 & h_i \end{pmatrix}$$

$x_i = (x_{i1}, x_{i2}, x_{i3})^T$. Here, (2.5) is a formal dissipative Hamiltonian-like system.

Note that this formal structure does not provide a Hamiltonian structure to the overall system, because in each individual subsystem the cross-variables are frozen as constants. In the following, we look for a real Hamiltonian function of the n generators, which is considered as the total energy of the whole system. Set

$$\begin{aligned} H(x) &= \sum_{i=1}^{n} H_i + \frac{1}{2}\sum_{i=1}^{n} x_{i3}\sum_{j=1,j\neq i}^{n} B_{ij}x_{j3}\cos(x_{i1} - x_{j1}) \\ &= \sum_{i=1}^{n}\left(-\frac{a_i}{d_i}x_{i1} + \frac{c_i}{d_i}x_{i1}x_{i3}^2 + \frac{1}{2d_i}x_{i2}^2 + \frac{e_i + h_i B_{ii}}{2h_i}x_{i3}^2\right) \\ &\quad - \frac{1}{2}\sum_{i=1}^{n} x_{i3}\sum_{j=1,j\neq i}^{n} B_{ij}x_{j3}\cos(x_{i1} - x_{j1}) \end{aligned} \tag{2.6}$$

where $x = (x_1^T, x_2^T, \ldots, x_n^T)^T$. By using relation $B_{ij} = B_{ji}$, it is not difficult to check that

$$\frac{\partial H(x)}{\partial x_{ij}} = \frac{\partial H_i}{\partial x_{ij}}, \qquad i = 1, 2, \ldots, n \quad j = 1, 2, 3. \tag{2.7}$$

In fact, from (2.6), we have

$$\begin{aligned} \frac{\partial H}{\partial x_{i1}} &= -\frac{a_i}{d_i} + \frac{c_i}{d_i}x_{i3}^2 - \frac{1}{2}\frac{\partial}{\partial x_{i1}} \times \left(\sum_{s=1}^{n} x_{s3}\sum_{j=1,j\neq s}^{n} B_{sj}x_{j3}\cos(x_{s1} - x_{j1})\right) \\ &= -\frac{a_i}{d_i} + \frac{c_i}{d_i}x_{i3}^2 + \frac{1}{2}x_{i3}\sum_{j=1,j\neq i}^{n} B_{ij}x_{j3}\sin(x_{i1} - x_{j1}) - \frac{1}{2}\sum_{s=1,s\neq i}^{n} x_{s3}B_{si}x_{i3}\sin(x_{s1} - x_{i1}) \\ &= -\frac{a_i}{d_i} + \frac{c_i}{d_i}x_{i3}^2 + x_{i3}\sum_{j=1,j\neq i}^{n} B_{ij}x_{j3}\sin(x_{i1} - x_{j1}) = \frac{\partial H_i}{\partial x_{i1}}. \end{aligned}$$

Similarly, we get $(\partial H)/(\partial x_{i3}) = (\partial H_i)/(\partial x_{i3})$. On the other hand, it is apparent that $(\partial H)/(\partial x_{i2}) = (\partial H_i)/(\partial x_{i2})$. So, (2.7) holds.

Equation (2.7) indicates that $H(x)$ is the real Hamiltonian function for the n generators. From (2.5) and (2.7), it turns out that the overall system is expressed as

$$\dot{x} = (J - R)\frac{\partial H}{\partial x} + Gv \tag{2.8}$$

where $J = \mathrm{Diag}\{J_1, J_2, \ldots, J_n\}, R = \mathrm{Diag}\{R_1, R_2, \ldots, R_n\}, G = \mathrm{Diag}\{g, g, \ldots, g\}_{3n\times n}$, and $v = (v_1, v_2, \ldots, v_n)^T$ □.

Remark 2.2: Since J_i is skew-symmetric and $R_i \geq 0, J$ is skew-symmetric and $R \geq 0$. Therefore, (2.8) is our desired DHR of multimachine power systems.

Remark 2.3: The model (2.1) does not take into account the transfer conductances $G_{ij}\ (i \neq j)$. In power systems, since $G_{ij} \ll B_{ij}, i \neq j$ [9], [12], $G_{ij}\ (i \neq j)$ can be neglected in the modeling compared with B_{ij} [12]–[14]. In model (2.1), $G_{ii} \neq 0$, which exactly presents a part of the network load.

Before ending this section, we consider the problem of working point adjustment. For the following controlled system:

$$\dot{x} = f(x) + g(x)u \tag{2.9}$$

when $u = 0$ the equilibrium point is called the working point. Assume x_0 is the working point of (2.9) with zero input, i.e., $f(x_0) = 0$; moreover, using a control $u = \varphi(x) + v$ the system is converted to

$$\dot{x} = M\nabla H + gv$$

where $\nabla H = (\partial H)/(\partial x)$. In general, the equilibrium may be shifted, i.e., $M\nabla H(x_0) \neq 0$. Let $\psi(x)$ be such that

$$\begin{cases} \varphi(x_0) + \psi(x_0) = 0, \\ M^{-1}g\psi(x) = \nabla H' \end{cases} \tag{2.10}$$

for some smooth function H' (where M is assumed invertible). We can prove that Proposition 2.4 holds.

Proposition 2.4: Control law $u = \varphi(x) + \psi(x) + v$ provides (2.9) a Hamiltonian realization as

$$\dot{x} = M\nabla\tilde{H} + gv \tag{2.11}$$

where $\tilde{H} = H + H'$. Moreover, for (2.11), the working point x_0 remains.

III. L_2-Disturbance Attenuation of Multimachine Power Systems

This section deals with the L_2-disturbance attenuation of multimachine power systems. First, we investigate the L_2-disturbance attenuation of port-controlled Hamiltonian (PCH) systems. As for the concept and properties of L_2-disturbance attenuation, please refer to [8] and [15].

A. L_2-Disturbance Attenuation of PCH Systems

Consider the following PCH system [3], [5]:

$$\begin{cases} \dot{x} = (J(x) - R(x))\nabla H + g_1(x)u + g_2(x)w \\ z = h(x)g_1^T(x)\nabla H, \end{cases} \tag{3.1}$$

where $x \in R^n, u \in R^m, R(x) \geq 0, H(x)$ is positive definite near the equilibrium concerned and $h(x)$ is a weighting matrix.

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = \begin{pmatrix} 0 & d_i & 0 \\ -d_i & -b_i d_i & 0 \\ 0 & 0 & -h_i \end{pmatrix} \times \begin{pmatrix} -\frac{a_i}{d_i} + \frac{c_i}{d_i}x_{i3}^2 + x_{i3}\sum_{j=1,j\neq i}^{n} B_{ij}x_{j3}\sin(x_{i1} - x_{j1}) \\ \frac{1}{d_i}x_{i2} \\ \frac{e_i + h_i B_{ii}}{h_i}x_{i3} + \frac{2c_i}{d_i}x_{i1}x_{i3} - \sum_{j=1,j\neq i}^{n} B_{ij}x_{j3}\cos(x_{i1} - x_{j1}) \end{pmatrix} + g v_i \tag{2.4}$$

Given a disturbance attenuation level $\gamma > 0$ and take $z = h(x)g_1^T(x)\nabla H$ as the penalty signal, then we have the following result.

Theorem 3.1: For the given disturbance attenuation level $\gamma > 0$, if

$$R(x) - \frac{1}{2\gamma^2}\left[g_2(x)g_2^T(x) - g_1(x)g_1^T(x)\right] \geq 0 \tag{3.2}$$

then the L_2-disturbance attenuation problem of (3.1) can be solved by feedback control law

$$u = -\left[\frac{1}{2}h^T(x)h(x) + \frac{1}{2\gamma^2}I_m\right]g_1^T(x)\nabla H \tag{3.3}$$

and γ-dissipation inequality

$$\begin{aligned}\dot{H} + (\nabla H)^T &\left[R(x) - \frac{1}{2\gamma^2}\left(g_2(x)g_2^T(x)\right.\right.\\ &\left.\left.-g_1(x)g_1^T(x)\right)\right]\nabla H\\ &\leq \frac{1}{2}\{\gamma^2\|w\|^2 - \|z\|^2\}\end{aligned} \tag{3.4}$$

holds along trajectories of the closed loop system consisted of (3.1) and (3.3), where I_m is the $m \times m$ identity matrix.

Proof: It is easy to know from (3.1) and (3.3) that

$$\begin{aligned}\frac{dH}{dt} &= -(\nabla H)^T R(x)\nabla H + (\nabla H)^T g_1 u + (\nabla H)^T g_2 w\\ &= -(\nabla H)^T R(x)\nabla H - \frac{1}{2}\left\|\gamma w - \frac{1}{\gamma}g_2^T\nabla H\right\|^2\\ &\quad + \frac{1}{2}(\gamma^2\|w\|^2 - \|z\|^2) + \frac{1}{2}(\nabla H)^T g_1 h^T h g_1^T \nabla H\\ &\quad - \nabla H^T g_1\left(\frac{1}{2}h^T h + \frac{1}{2\gamma^2}I_m\right)g_1^T\nabla H\\ &\quad + \frac{1}{2\gamma^2}\nabla H^T g_2 g_2^T \nabla H\\ &= -\nabla H^T\left[R - \frac{1}{2\gamma^2}g_2 g_2^T + \frac{1}{2\gamma^2}g_1 g_1^T\right]\nabla H\\ &\quad + \frac{1}{2}(\gamma^2\|w\|^2 - \|z\|^2) - \frac{1}{2}\left\|\gamma w - \frac{1}{\gamma}g_2^T\nabla H\right\|^2.\end{aligned}$$

So

$$\begin{aligned}\frac{dH}{dt} &+ \nabla H^T\left[R - \frac{1}{2\gamma^2}\left(g_2 g_2^T - g_1 g_1^T\right)\right]\nabla H\\ &= \frac{1}{2}(\gamma^2\|w\|^2 - \|z\|^2) - \frac{1}{2}\left\|\gamma w - \frac{1}{\gamma}g_2^T\nabla H\right\|^2\\ &\leq \frac{1}{2}(\gamma^2\|w\|^2 - \|z\|^2)\end{aligned}$$

which is (3.4). Because $R - (1/(2\gamma^2))[g_2 g_2^T - g_1 g_1^T] \geq 0$, (3.3) with $H(x)$ is a solution to the L_2-disturbance attenuation of (3.1). □

Remark 3.2: Theorem 3.1 is motivated by [8, Th. 1]. When $g_1(x) \equiv g_2(x)$, Theorem 3.1 degenerates to [8, Th. 1].

B. Energy-Based L_2-Disturbance Attenuation of Multimachine Systems

Consider n-machine power system (2.1) affected by external disturbances. Then, the system can be rewritten as

$$\begin{cases}\dot{\delta}_i = \omega_i - \omega_0\\ \dot{\omega}_i = \frac{\omega_0}{M_i}P_{mi} - \frac{D_i}{M_i}(\omega_i - \omega_0) - \frac{\omega_0}{M_i}P_{ei} + \varepsilon_{i1}\\ \dot{E}'_{qi} = -\frac{1}{T_{d0i}}E_{qi} + \frac{1}{T_{d0i}}u_{fi} + \varepsilon_{i2}\end{cases}$$
$$i = 1, 2, \ldots, n \tag{3.5}$$

where $\varepsilon_{i1}, \varepsilon_{i2}$ are bounded disturbances, and the other variables and parameters are the same as in Section II.

The L_2-disturbance attenuation problem of (3.5) can be described as follows: Given penalty signals

$$\begin{aligned}z_i = r_i(\delta_i, \omega_i, E'_{qi})&\left\{\frac{(1 + k_{i0}T_{\mathrm{doi}})E'_{qi} - T_{\mathrm{doi}}\bar{u}_i}{x_{di} - x'_{di}}\right.\\ &\left.+\ 2G_{ii}\delta_i E'_{qi} + I_{di}\right\}, \qquad i = 1, 2, \ldots, n\end{aligned} \tag{3.6}$$

a disturbance attenuation level $\gamma > 0$ and a desired equilibrium $(\delta_i^{(0)}, 0, E_{qi}^{\prime(0)}), i = 1, 2, \ldots, n$. Find a feedback control strategy $u_{fi} = \alpha_i(x), i = 1, 2, \ldots, n$, and a storage function $V(x)$ which is positive definite near the desired equilibrium such that γ-dissipation inequality

$$\dot{V} + Q(x) \leq \frac{1}{2}\{\gamma^2\|\varepsilon\|^2 - \|z\|^2\} \qquad \forall\varepsilon \tag{3.7}$$

holds along all trajectories of the closed-loop system consisted of (3.5) and the feedback control strategy, where $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)^T, \varepsilon_i = (\varepsilon_{i1}, \varepsilon_{i2}), z = (z_1, z_2, \ldots, z_n)^T, x = (x_1^T, x_2^T, \ldots, x_n^T)^T, x_i = (\delta_i, \omega_i - \omega_0, E'_{qi})^T$, $\|\cdot\|$ is the euclidean norm, $Q(x) \geq 0$ is a given nonnegative function, r_i weighting functions, and k_{i0} and $\bar{u}_i$ are suitably large numbers (may be adjusted).

Remark 3.3: Penalty signals (3.6) can be rewritten as

$$z_i = \frac{r_i(\delta_i, \omega_i, E'_{qi})}{x_{di} - x'_{di}}\left(E_{qi} - u_{fi}^{\alpha}\right), \qquad i = 1, 2, \ldots, n$$

where E_{qi} are the internal voltage signals and $u_{fi}^{\alpha} := T_{\mathrm{doi}}\bar{u}_i - 2G_{ii}(x_{di} - x'_{di})\delta_i E'_{qi} - k_{i0}T_{\mathrm{doi}}E'_{qi}$, which are the excitation signals to be designed [see (3.8) and (3.13)]. Thus, the penalty signals z_i $(i = 1, 2, \ldots, n)$ have clear physical meaning.

Now, we give the energy-based control design for the above L_2-disturbance attenuation problem. From the DHR in Section II, (3.5) can be expressed as

$$\dot{x}_i = (J_i - R_i)\frac{\partial H(x)}{\partial x_i} + g v_i + g_1\varepsilon_i^T \qquad i = 1, 2, \ldots, n \tag{3.8}$$

where

$$g_1 = \begin{pmatrix}0 & 0\\ 1 & 0\\ 0 & 1\end{pmatrix} \qquad \varepsilon_i = (\varepsilon_{i1}, \varepsilon_{i2}).$$

For the convenience of design, we let $\varepsilon_i = 0$ in (3.8) tentatively. Since the feedback law in the dissipative Hamiltonian realization of (3.5) can cause the equilibrium point to be shifted, we first, based on Proposition 2.4, design a feedback control law which stabilizes system (3.8) to the desired operating point $(\delta_i^{(0)}, 0, E_{qi}^{\prime(0)}), i = 1, 2, \ldots, n$.

Choose control law

$$v_i = -k_i x_{i3} + \bar{u}_i, \qquad i = 1, 2, \ldots, n \tag{3.9}$$

where k_i and $\bar{u}_i$ are constant numbers to be determined. Substitute (3.9) into (3.8) $(\varepsilon_i = 0)$ and note that $(J_i - R_i)$ is invertible, then we have

$$\dot{x}_i = (J_i - R_i)\frac{\partial H(x)}{\partial x_i} + (J_i - R_i)\frac{\partial \bar{H}_i}{\partial x_i}$$

where $\bar{H}_i = (k_i)/(2h_i)x_{i3}^2 - (\bar{u}_i/h_i)x_{i3}$. Choosing a new Hamiltonian function as

$$H_\alpha = H(x) + \sum_{i=1}^{n}\bar{H}_i \tag{3.10}$$

the closed-loop system (3.8) with control (3.9) can be expressed as

$$\dot{x}_i = (J_i - R_i)\frac{\partial H_\alpha}{\partial x_i}, \qquad 1, 2, \ldots, n \tag{3.11}$$

which is also a dissipative Hamiltonian system.

In the following, we investigate the properties of the Hamiltonian function H_α. It is easy to know that H_α can be expressed as

$$\begin{aligned} H_\alpha = \sum_{i=1}^{n} \Bigg\{ & -\frac{a_i}{d_i}x_{i1} + \left(\frac{k_i}{2h_i} + \frac{c_i}{d_i}x_{i1} - \frac{1}{2}\sum_{j>i}^{n} B_{ij}\right) x_{i3}^2 \\ & + \frac{1}{2d_i}x_{i2}^2 + \frac{e_i + h_i B_{ii}}{2h_i}\left(x_{i3} - \frac{\bar{u}_i}{e_i + h_i B_{ii}}\right)^2 \Bigg\} \\ & + \frac{1}{2}\sum_{i=1}^{n}\sum_{j>i}^{n} B_{ij}x_{i3}^2 - \sum_{i=1}^{n}\frac{\bar{u}_i^2}{2h_i(e_i + h_i B_{ii})} \\ & - \frac{1}{2}\sum_{i=1}^{n} x_{i3}\sum_{j=1, j\neq i}^{n} B_{ij}x_{j3}\cos(x_{i1} - x_{j1}). \end{aligned}$$

Use relation $B_{ij} = B_{ji}$ and set

$$\begin{aligned} H_\beta = \sum_{i=1}^{n} \Bigg\{ & -\frac{a_i}{d_i}x_{i1} + \left(\frac{k_i}{2h_i} + \frac{c_i}{d_i}x_{i1} - \frac{1}{2}\sum_{j>i}^{n} B_{ij}\right) x_{i3}^2 \\ & + \frac{1}{2d_i}x_{i2}^2 + \frac{e_i + h_i B_{ii}}{2h_i}\left(x_{i3} - \frac{\bar{u}_i}{e_i + h_i B_{ii}}\right)^2 \Bigg\} \\ & + \frac{1}{2}\sum_{i=1}^{n}\sum_{j>i}^{n} B_{ij}(|x_{i3}| - |x_{j3}|)^2 \\ & - \sum_{i=1}^{n}\frac{\bar{u}_i^2}{2h_i(e_i + h_i B_{ii})} \\ H_\eta = \sum_{i=1}^{n} \Bigg\{ & -\frac{a_i}{d_i}x_{i1} + \left(\frac{k_i}{2h_i} + \frac{c_i}{d_i}x_{i1} - \frac{1}{2}\sum_{j>i}^{n} B_{ij}\right) x_{i3}^2 \\ & + \frac{1}{2d_i}x_{i2}^2 + \frac{e_i + h_i B_{ii}}{2h_i}\left(x_{i3} - \frac{\bar{u}_i}{e_i + h_i B_{ii}}\right)^2 \Bigg\} \\ & + \frac{1}{2}\sum_{i=1}^{n}\sum_{j>i}^{n} B_{ij}(|x_{i3}| + |x_{j3}|)^2 \\ & - \sum_{i=1}^{n}\frac{\bar{u}_i^2}{2h_i(e_i + h_i B_{ii})} \end{aligned}$$

then we can easily get

$$H_\beta \leq H_\alpha \leq H_\eta. \tag{3.12}$$

Because $x_{i1} \in [-\pi, \pi]$, we can select suitably large numbers k_i such that H_β is bounded from below. Now, let $k_i = k_{i0}$ such that H_β is bounded from below. From (3.12), H_α is also bounded from below and for $\forall l > 0$ the set $\{x | H_\alpha(x) \leq l\}$ is compact. From [11] and properties of the power system itself, $H_\alpha(x)$ has a strict local minimum at the operating point.

From (3.11), we have

$$\frac{dH_\alpha}{dt} = -\sum_{i=1}^{n}\frac{b_i}{d_i}x_{i2}^2 - \sum_{i=1}^{n} h_i f_i^2 \leq 0$$

where

$$\begin{aligned} f_i := \frac{e_i + h_i B_{ii}}{h_i}x_{i3} - \sum_{j=1, j\neq i}^{n} B_{ij}x_{j3}\cos(x_{i1} - x_{j1}) \\ + \frac{2c_i}{d_i}x_{i1}x_{i3} + \frac{k_{i0}}{h_i}x_{i3} - \frac{\bar{u}_i}{h_i}. \end{aligned}$$

Since $H_\alpha(x)$ has a strict local minimum at the operating point, (3.11) is stable at the operating point. Moreover, the system converges to the largest invariant set contained in

$$\begin{aligned} S &= \left\{x : \frac{dH_\alpha}{dt} = 0\right\} \\ &= \{x : x_{i2} = 0, f_i = 0, i = 1, 2, \ldots, n\}. \end{aligned}$$

From $x_{i2} \equiv 0$, we can conclude that

$$a_i - c_i x_{i3}^2 - d_i x_{i3}\sum_{j=1, j\neq i}^{n} B_{ij}x_{j3}\sin(x_{i1} - x_{j1}) = 0$$

$i = 1, 2, \ldots, n$. So, points in the largest invariant set satisfy

$$\begin{cases} a_i - c_i x_{i3}^2 - d_i x_{i3}\sum_{j=1, j\neq i}^{n} B_{ij}x_{j3}\sin(x_{i1} - x_{j1}) = 0 \\ x_{i2} = 0, \qquad f_i = 0 \quad i = 1, 2, \ldots, n \end{cases}$$

which is exactly the condition the equilibrium satisfies. From the LaSalle invariant principle, the closed-loop system (3.11) is asymptotically stable.

Besides, from the aforementioned condition satisfied by the equilibrium point, we know that $\bar{u}_i$ is given as

$$\begin{aligned} \bar{u}_i = (e_i + h_i B_{ii})E_{qi}^{\prime(0)} + \frac{2c_i h_i}{d_i}\delta_i^{(0)}E_{qi}^{\prime(0)} + k_{i0}E_{qi}^{\prime(0)} \\ - h_i\sum_{j=1, j\neq i}^{n} B_{ij}E_{qj}^{\prime(0)}\cos\left(\delta_i^{(0)} - \delta_j^{(0)}\right). \end{aligned}$$

Now, consider (3.8) with $\varepsilon_i \neq 0$ and choose control laws

$$v_i = -k_{i0}x_{i3} + \bar{u}_i + \bar{v}_i, \qquad i = 1, 2, \ldots, n \tag{3.13}$$

where $\bar{v}_i$ are new control inputs. Substituting (3.13) into (3.8) yields

$$\dot{x}_i = (J_i - R_i)\frac{\partial H_\alpha}{\partial x_i} + g\bar{v}_i + g_1\varepsilon_i^T, \qquad i = 1, 2, \ldots, n. \tag{3.14}$$

Then, (3.5) can be expressed as

$$\dot{x} = (J - R)\frac{\partial H_\alpha}{\partial x} + G\bar{v} + G_1\varepsilon \tag{3.15}$$

where $\bar{v} = (\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_n)^T, G = \mathrm{Diag}\{g, g, \ldots, g\}, G_1 = \mathrm{Diag}\{g_1, g_1, \ldots, g_1\}$ and others are the same as before.

Now, we consider the penalty signals z_i and express them into virtual forms. A straightforward computation shows that z_i can be expressed as $z_i = r_i(x_i)g^T(\partial H_\alpha)/(\partial x_i), i = 1, 2, \ldots, n$. Then, we have

$$z = r(x)G^T\frac{\partial H_\alpha}{\partial x} \tag{3.16}$$

where $r(x) = \mathrm{Diag}\{r_1(x_1), r_2(x_2), \ldots, r_n(x_n)\}$, called the weighting matrix.

Theorem 3.4: For the given penalty signals (3.6) and the disturbance attenuation level $\gamma > 0$, if

$$\gamma \geq \gamma^* = \max_{1\leq i\leq n}\left\{\frac{M_i}{\sqrt{2\omega_0 D_i}}\right\} \tag{3.17}$$

then the L_2-disturbance attenuation problem of (3.5) can be solved by feedback laws

$$\begin{aligned} u_{fi} = -2G_{ii}(x_{di} - x'_{di})\delta_i E'_{qi} - k_{i0}T_{\mathrm{doi}}E'_{qi} \\ - \frac{T_{\mathrm{doi}}}{2r_i}\left(r_i^2 + \frac{1}{\gamma^2}\right)z_i + T_{\mathrm{doi}}\bar{u}_i \end{aligned} \tag{3.18}$$

$i = 1, 2, \ldots, n$, and γ dissipation inequality

$$\dot{V}(x) + Q(x) \leq \frac{1}{2}\{\gamma^2\|\varepsilon\|^2 - \|z\|^2\} \tag{3.19}$$

holds along all trajectories of the closed-loop system (3.5) with (3.18), where $V(x) = H_\alpha + c, c = -H_\alpha(x_0), Q(x) = (\nabla H_\alpha)^T P\nabla H_\alpha$ and

$$P = R - \frac{1}{2\gamma^2}\left(G_1G_1^T - GG^T\right) \geq 0. \tag{3.20}$$

Proof: From (3.20), we know

$$\begin{aligned} P = \mathrm{Diag}\{R_1, R_2, \ldots, R_n\} \\ - \frac{1}{2\gamma^2}\mathrm{Diag}\{g_1g_1^T - gg^T, \ldots, g_1g_1^T - gg^T\}. \end{aligned}$$

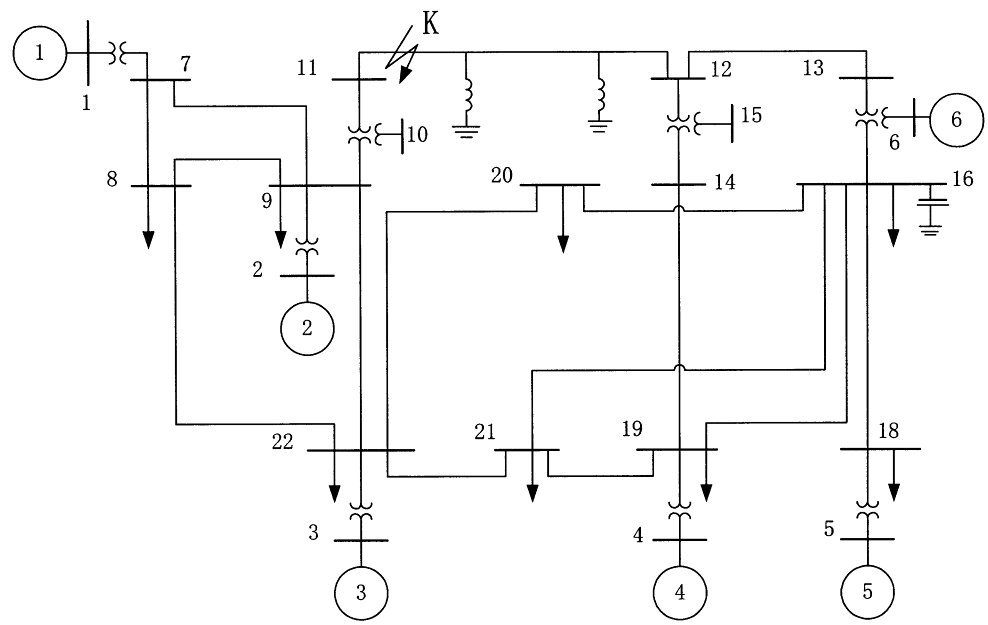

Fig. 1. Six-machine system.

Now, investigate the main diagonal blocks of P:

$$R_i - \frac{1}{2\gamma^2}\left(g_1 g_1^T - gg^T\right) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & b_i d_i & 0 \\ 0 & 0 & h_i \end{pmatrix} - \frac{1}{2\gamma^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Because $\gamma \geq \gamma^* \Rightarrow \gamma \geq (M_i/(\sqrt{2\omega_0 D_i})) \Rightarrow (1/(2\gamma^2)) \leq (\omega_0 D_i)/(M_i^2) = b_i d_i$, thus $R_i - (1/(2\gamma^2))(g_1 g_1^T - gg^T) \geq 0, i = 1, 2, \ldots, n, \Rightarrow P \geq 0$. It is easy to know from Theorem 3.1 that for the given penalty signals (3.16) and the disturbance attenuation level $\gamma \geq \gamma^* > 0$, the L_2-disturbance attenuation problem of (3.15) can be solved by the following feedback law:

$$\bar{v} = -\left[\frac{1}{2}r^T(x)r(x) + \frac{1}{2\gamma^2}I_n\right]G^T\frac{\partial H_\alpha}{\partial x} \tag{3.21}$$

and

$$\dot{H}_\alpha + (\nabla H_\alpha)^T P \nabla H_\alpha \leq \frac{1}{2}\{\gamma^2\|\varepsilon\|^2 - \|z\|^2\}$$

holds along the trajectories of the closed-loop system. This γ dissipation inequality is just (3.19). From (3.21), we have

$$\bar{v}_i = -\frac{1}{2}\left(r_i^2(x_i) + \frac{1}{\gamma^2}\right)g^T\frac{\partial H_\alpha}{\partial x_i}, \qquad i = 1, 2, \ldots, n$$

from which, along with (3.13) and (2.3), we get

$$u_{fi} = T_{\mathrm{doi}}u_i = -\frac{2c_i h_i}{d_i}T_{\mathrm{doi}}x_{i1}x_{i3} - k_{i0}T_{\mathrm{doi}}x_{i3} + T_{\mathrm{doi}}\bar{u}_i - \frac{1}{2}\left(r_i^2 + \frac{1}{\gamma^2}\right)T_{\mathrm{doi}}g^T\frac{\partial H_\alpha}{\partial x_i} \tag{3.22}$$

$i = 1, 2, \ldots, n$. Rewrite (3.22) with the original forms of the variables and parameters, then we have

$$u_{fi} = -2G_{ii}(x_{di} - x'_{di})\delta_i E'_{qi} - k_{i0}T_{\mathrm{doi}}E'_{qi} + T_{\mathrm{doi}}\bar{u}_i - \frac{T_{\mathrm{doi}}}{2r_i}\left(r_i^2 + \frac{1}{\gamma^2}\right)z_i, \qquad i = 1, 2, \ldots, n$$

which is (3.18). □

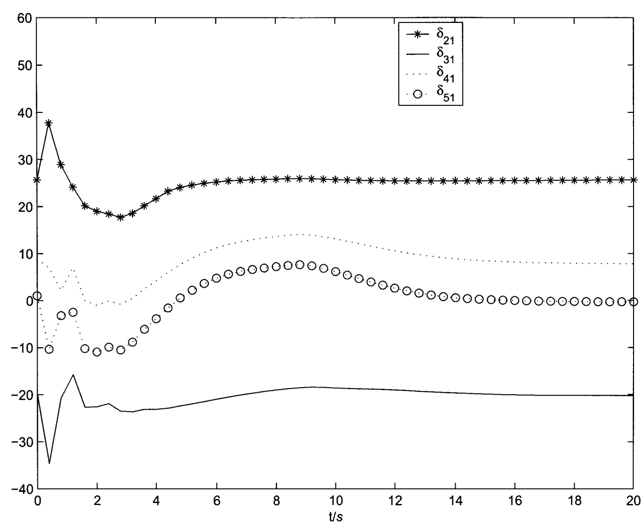

Fig. 2. Responses of $\boldsymbol{\delta}_{i1}$ when $\boldsymbol{\gamma = 2}$.

Remark 3.5:

1) Equation (3.18) is a decentralized control strategy.
2) In practice, k_{i0} and $\bar{u}_i$ can be determined as follows:

$$\begin{cases} k_{i0} = -\frac{2}{T_{\mathrm{doi}}}G_{ii}(x_{di} - x'_{di})\Delta\delta_i^{(0)}, \\ \bar{u}_i = \frac{1}{T_{\mathrm{doi}}}\left(E'^{(0)}_{qi} + I_{di0}(x_{di} - x'_{di})\right), \qquad i = 1, 2, \ldots, n \end{cases} \tag{3.23}$$

where $I_{dio} = B_{ii}E'^{(0)}_{qi} - \sum_{j=1, j\neq i}^{n} B_{ij}E'^{(0)}_{qj}\cos(\delta_i^{(0)} - \delta_j^{(0)}), \Delta\delta_i^{(0)} = \delta_i^{(0)} - \delta_0$ and δ_0 is the power angle of the equivalence infinite-bus system (Note: In practice, we can simply set $\delta_0 = 0$).

Remark 3.6: The desired equilibrium (the preassigned operating point) $(\delta_i^{(0)}, 0, E_i'^{(0)}), i = 1, 2, \ldots, n$ can be given by flow computation of power systems in advance.

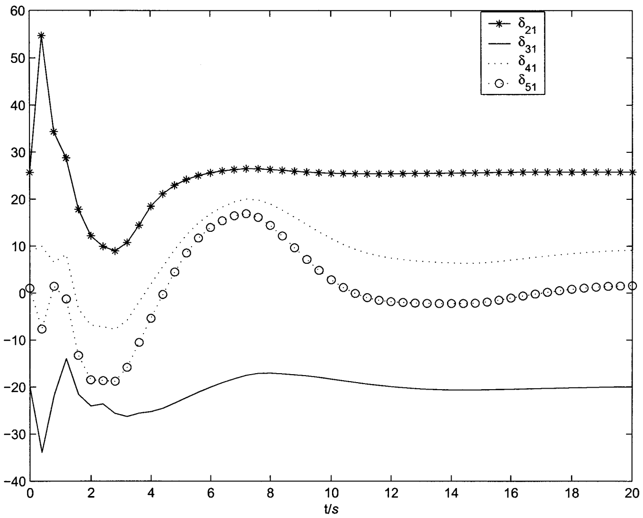

Fig. 3. Responses of $\boldsymbol{\delta}_{i1}$ when $\boldsymbol{\gamma} = \mathbf{10}$.

IV. Simulation

A six-machine system [13], [14] is chosen as an example to demonstrate the effectiveness of the control strategy (3.18) The system is shown in Fig. 1. As for its generator data, we refer to [13] and [14]. The simulation is completed by the PSASP package which is a professional testing system for power systems designed by the China Electrical Power Research Institute, Beijing, China.

In Fig. 1, generator no. 6 is a synchronous condenser and generator no. 1 itself actually represents an equivalent of a large power system, used as the reference here. Equip generators no. 2–no. 5 with controller (3.18). Here, $\gamma^* = (79.5)/(\sqrt{2 \times 314 \times 3}) = 1.8316$. In simulating, we let $r_i = 0.2$ and do with different disturbance attenuation level γ, where k_{i0} and $\bar{u}_i$ are determined by (3.23).

A symmetrical three-phase short-circuit fault is assumed to occur during the time period $0 \sim 0.15$ s at K (see Fig. 1). When $\gamma = 2, 10$, the responses of δ_{i1} (=$\delta_i - \delta_1$, in degree) are given in Figs. 2 and 3 respectively.

Through Figs. 2–3, we can see that the control strategy proposed in the note is very effective and the system's dynamic performance can be improved by reducing the disturbance attenuation level γ.

V. Conclusion

The multimachine power systems have been expressed as a dissipative Hamiltonian system. Based on the dissipative Hamiltonian realization, the L_2-disturbance attenuation of multimachine power systems has been investigated and a decentralized simple control strategy has been proposed. Simulations on a six-machine system show that the achieved L_2-disturbance attenuation control strategy is very effective.

Acknowledgment

The authors would like to thank the anonymous reviewers for many helpful corrections and suggestions for improving the readability of this note.

References

[1] G. Escobar, A. J. van der Schaft, and R. Ortega, "A Hamiltonian viewpoint in the modeling of switching power converters," *Automatica*, vol. 35, no. 3, pp. 445–452, 1999.

[2] R. Ortega, A. J. van der Schaft, B. M. Maschke, and G. Escobar, "Energy-shaping of port-controlled Hamiltonian systems by interconnection," *Proc. 38th IEEE Conf. Decision Control*, pp. 1646–1651, 1999.

[3] R. Ortega, A. J. van der Schaft, B. M. Maschke, and G. Escobar, "Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems," *Automatica*, vol. 38, no. 4, pp. 585–596, 2002.

[4] B. Maschke, R. Ortega, A. J. Van Der Schaft, and G. Escobar, "An energy-based derivation of Lyapunov functions for forced systems with applications to stabilizing control," in *Proc. 14th IFAC World Congr.*, vol. E, Beijing, China, 1999, pp. 409–415.

[5] B. Maschke, R. Ortega, and A. J. van der Schaft, "Energy-based Lyapunov functions for forced Hamiltonian systems with dissipation," *IEEE Trans. Automat. Contr.*, vol. 45, pp. 1498–1502, Aug. 2000.

[6] D. Cheng, Z. Xi, Y. Hong, and H. Qin, "Energy-based stabilization in power systems," in *Proc. 14th IFAC World Congr.*, vol. O, Beijing, China, 1999, pp. 297–303.

[7] D. Cheng, S. Spurgeon, and J. Xiang, "On the development of generalized Hamiltonian realizations," *Proc. 39th IEEE Conf. Decision Control*, vol. 5, pp. 5125–5130, 2000.

[8] T. Shen, R. Ortega, Q. Lu, S. Mei, and K. Tamura, "Adaptive $\boldsymbol{L}_2$ disturbance attenuation of Hamiltonian systems with parameter perturbations and application to power systems," *Proc. 39th IEEE Conf. Decision Control*, vol. 5, pp. 4939–4944, 2000.

[9] Y. Sun, T. Shen, R. Ortega, and Q. Liu, "Decentralized controller design for multimachine power systems on Hamiltonian structure," *Proc. 40th IEEE Conf. Decision Control*, pp. 3045–3050, 2001.

[10] Y. Wang, D. Cheng, and Y. Hong, "Stabilization of synchroous generators with Hamiltonian function approach," *Int. J. Syst. Sci.*, vol. 32, no. 8, pp. 971–978, 2001.

[11] Z. Xi, D. Cheng, Q. Lu, and S. Mei, "Nonlinear decentralized controller design for multimachine power systems using Hamiltonian function method," *Automatica*, vol. 38, pp. 527–534, 2002.

[12] Y. N. Yu, *Electric Power System Dynamics*. New York: Academic, 1983, pp. 95–137.

[13] Q. Lu and Y. Sun, "Nonlinear stabilizing control of multimachine systems," *IEEE Trans. Power Syst.*, vol. 4, pp. 236–241, Feb. 1989.

[14] Q. Lu, Y. Sun, Z. Xu, and T. Mochizuki, "Decentralized nonlinear optimal excitation control," *IEEE Trans. Power Syst.*, vol. 11, pp. 1957–1962, Nov. 1996.

[15] G. Besancon and S. Battiloti, "On output feedback tracking control with disturbance attenuation for Euler-Lagrange systems," *Proc. 37th IEEE Conf. Decision Control*, pp. 3139–3143, 1998.